Efficient Testing Combining Design of Experiment and Learn-to-Fly Strategies

Patrick C. Murphy
Jay M. Brandon
NASA Langley Research Center

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Outline

• Introduction
  – Seeking greater efficiency & performance through experiment design
    • Efficiency gained by collecting the “right amount” of data
    • Performance gained by adding statistical rigor

• System Identification Process in Wind Tunnel
  – Design of Experiment (DOE)
  – Learn-to-Fly (L2F)
  – Blended DOE-L2F
    • First time testing blended concept – strawman approach
    • Work in progress

• Analysis, Results, and Validation Tests
  – DOE Tests
  – L2F Tests
  – Blended DOE-L2F Tests

• Concluding Remarks
Motivation: Seek Efficiency Using Experiment Design

• Wide spectrum of modeling demands
  – Fidelity requirements
  – Aircraft complexity

• Aircraft complexity drive up costs
  – Conventional practice in LaRC 12-foot Wind Tunnel (static test)
    • 100 Hz sample rate, dwell for 10 seconds, average data
    • ~ 2 data pts/min
  – Simple factorial test for L-59
    • 9-Factors: $\alpha$, $\beta$, and 7 control surfaces
    • $2^9 = 512$ test points => 4.26 hours
  – Reasonable data density often requires $5^9$ => 16,276 hours (~8 years)!

• Investigators must tradeoff of cost vs fidelity/complexity
  – Define purpose of model and required fidelity. What is allowable error?
  – Asking for “best possible answer” is not adequate
  – Speeding up the modeling process helps anywhere on spectrum
Aircraft System Identification Process

Model Postulation → Experiment Design → Data Compatibility Analysis → Model Structure Determination & Parameter and State Estimation → Model Validation

Collinearity Diagnostic

Different Data Sets
## Test vehicle for Wind Tunnel Static Test

<table>
<thead>
<tr>
<th>No.</th>
<th>Label</th>
<th>Description</th>
<th>Low Value</th>
<th>High Value</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>aoa</td>
<td>Aircraft alpha</td>
<td>-2</td>
<td>20</td>
<td>deg</td>
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<tr>
<td>2</td>
<td>beta</td>
<td>Aircraft beta</td>
<td>-5</td>
<td>5</td>
<td>deg</td>
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<td>dela_L</td>
<td>Aileron left wing</td>
<td>-25</td>
<td>25</td>
<td>deg</td>
</tr>
<tr>
<td>4</td>
<td>dela_R</td>
<td>Aileron right wing</td>
<td>-25</td>
<td>25</td>
<td>deg</td>
</tr>
<tr>
<td>5</td>
<td>delf_L</td>
<td>Flap left wing</td>
<td>0</td>
<td>40</td>
<td>deg</td>
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<td>6</td>
<td>delf_R</td>
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<td>7</td>
<td>delr</td>
<td>Rudder</td>
<td>-30</td>
<td>30</td>
<td>deg</td>
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<td>8</td>
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<td>Elevator left wing</td>
<td>-30</td>
<td>30</td>
<td>deg</td>
</tr>
<tr>
<td>9</td>
<td>dele_R</td>
<td>Elevator right wing</td>
<td>-30</td>
<td>30</td>
<td>deg</td>
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</table>

- L-59 Albatros
- Czech military trainer
- Low-cost off-the-shelf kit
- 12.5% scale model
- Sport application, RC actuators
Tenets of Design of Experiment (DOE)

Sequential testing proceeds only as model complexity requires

- Orthogonal regressors are uncorrelated
- Replication defines system noise/uncertainty
- Blocking reduces effects of known factors of no direct interest
- Randomization removes effects of unknown systematic errors

Blocks of data, collected as needed, each followed by model validation
Block Designs & Supported Models

- Full factorial design

\[ y = B_0 + \sum_i B_i x_i + \sum_{i \neq j} B_{ij} x_i x_j + \varepsilon \quad i = 1, 2, \ldots, k \]

- Face-centered design (FCD)

\[ y = B_0 + \sum_i B_i x_i + \sum_i B_{ii} x_i^2 + \sum_{i \neq j} B_{ij} x_i x_j + \varepsilon \quad i = 1, 2, \ldots, k \]

- Nested face-centered design

\[ y = B_0 + \sum_i B_i x_i + \sum_i B_{ii} x_i^2 + \sum_{i \neq j} B_{ij} x_i x_j + \sum_i B_{iii} x_i^3 + \varepsilon \quad i = 1, 2, \ldots, k \]
Block 1, DOE Design Metrics (9-factors)

<table>
<thead>
<tr>
<th>Block Type</th>
<th>Blocks</th>
<th>Runs</th>
<th>Design Terms</th>
<th>VIF</th>
<th>% Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>(inclusive)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>¼ Fraction FCD</td>
<td>1</td>
<td>156</td>
<td>Quadratic</td>
<td>9.64</td>
<td>84.4</td>
</tr>
</tbody>
</table>

Maximum Variance Inflation Factor (VIF), reflects lack of orthogonality in design; desire ≤ 10

% Power reflects statistical power of design, manages type-2 error; desire ≥ 80

\[ y = B_0 + \sum_{i} B_i x_i + \sum_{i} B_{ii} x_i^2 + \sum_{i \neq j} B_{ij} x_i x_j + \epsilon \quad i = 1, 2, \ldots, k \]

Validation Test Performed after each block of data
Block 2 added to create Nested FCD

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<td>Quadratic</td>
<td>9.64</td>
<td>84.4</td>
</tr>
<tr>
<td>Nested FCD</td>
<td>1, 2</td>
<td>312</td>
<td>Quadratic</td>
<td>22.41</td>
<td>86.8</td>
</tr>
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</table>

\[
y = B_0 + \sum_{i} B_i x_i + \sum_{i} B_{ii} x_i^2 + \sum_{i \neq j} \sum_{i} B_{ij} x_i x_j + \sum_{i} B_{iii} x_i^3 + \epsilon \quad i = 1, 2, \ldots, k
\]

**Require optimized design points to reduce VIF**
Final DOE Design, 3-blocks

<table>
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<td>1, 2</td>
<td>312</td>
<td>Quadratic</td>
<td>22.41</td>
<td>86.8</td>
</tr>
<tr>
<td>I-optimal</td>
<td>1, 2, 3</td>
<td>366</td>
<td>Quadratic</td>
<td>4.0</td>
<td>99.9</td>
</tr>
</tbody>
</table>

I-optimal block provides test points that minimize prediction error

Validation Test Performed after each block of data
DOE Design for 3 blocks

- FCD (black)
- Nested FCD (red)
- I-optimal (green)
Stepwise Regression Modeling

• Stepwise regression used to select model parameters

\[ y = \beta_0 + \sum_i \beta_i x_i + \sum_i \beta^2_i x_i^2 + \sum_{i \neq j} \beta_{ij} x_i x_j + \sum_i \beta_{iii} x_i^3 + \ldots + \varepsilon \quad i = 1, 2, \ldots, 23 \]

• Primary metrics utilized for model selection:
  – Stepwise Regression significance level: 95% – 99%
  – Standard ANOVA table analysis
  – Lack of Fit (LOF) measure of model error relative to pure error
  – Standard deviation (fit error)
  – PRESS (prediction error sum of squares)
  – Coefficient of Variation (C.V.% = std. dev. / mean)
  – \( e_i / C_{N_{\text{max}}} \% \); \( e_i = C_{N_{\text{measured}}} - C_{N_{\text{predicted}}} \) … desire ≤ 3%
  – \( R^2 \), adjusted \( R^2 \), predicted \( R^2 \), (family of metrics)

\[
R^2 = \frac{\text{variation explained}}{\text{total variation}}; \quad 0 < R^2 < 1
\]
Learn-to-Fly (L2F) Testing

• L2F approach adapted to wind tunnel
  – General L2F approach is real-time global modeling of aerodynamics
  – Applicable to wind tunnel or flight testing
  – Continuous sampling during dynamic test

• This study is a “quasi-static” test
  – Continuous sampling while sweeping target points slowly
  – Batch processing, stepwise regression

• Key to efficiency: Wide-band orthogonal inputs
  – Higher bandwidth (HBW) inputs applied to control surfaces
  – Lower bandwidth (LBW) inputs apply to other factors

• L2F experiment design
  – Test grid is setup for LBW factors
  – LBW trajectories form a nested “FCD-like” design
Learn-to-Fly (L2F) Trajectories

Color points by Standard Order

A: aoa (deg)
B: beta (deg)
Blended DOE-L2F Testing ("quasi-static" test)

- Use key “efficiency features” of both approaches
  - DOE: 4 tenets, sequential testing blocks of data, with validation tests
  - L2F: HBW design for factors that accept wide-band inputs
- Blended design both simplifies and complicates final design
  - Simplifies 9-LBW experiment to a 2-LBW + 7-HBW experiment
  - Complicates evaluation of design metrics
- Strawman blended design
  - Design for 9-LBW experiment ensure all factors are included in design
  - Keep statistical advantages and design metrics of DOE
  - Assume “extra” data between target points enhances modeling
  - Assume blended design is obtained by removing redundant $\alpha-\beta$ targets
  - Blended designs require rig move slow enough to allow full sweep of controls at each $\alpha-\beta$ target point
### Blended DOE-L2F Design Metrics (9-factors)

<table>
<thead>
<tr>
<th>Block Type</th>
<th>Blocks included</th>
<th>Runs</th>
<th>Terms</th>
<th>VIF</th>
<th>% Power</th>
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</thead>
<tbody>
<tr>
<td>Factorial</td>
<td>1</td>
<td>134</td>
<td>Linear + 2FI</td>
<td>1*</td>
<td>99.7</td>
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<tr>
<td>FCD</td>
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<td>156</td>
<td>Quadratic</td>
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<td>84.2</td>
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<tr>
<td>Nested FCD</td>
<td>1, 2, 3</td>
<td>312</td>
<td>Quadratic</td>
<td>22.47</td>
<td>86.7</td>
</tr>
<tr>
<td>I-optimal</td>
<td>1, 2, 3, 4</td>
<td>384</td>
<td>Quadratic</td>
<td>4.85</td>
<td>99.9</td>
</tr>
</tbody>
</table>

*Squared factors are aliased*

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### Some Lessons Learned:

- Fewer blocks required with continuous sampling
- Divide optimal blocks!
- 4th block provided too much data for the blended design.
Blended DOE-L2F Trajectories

![Graph showing Blended DOE-L2F Trajectories](image-url)

- **A**: aoa (deg)
- **B**: beta (deg)

Color points by Standard Order

- 1213
- 1
DOE Model (3 blocks)

Model & Meas data, beta=0, flaps=20, surfaces zero

$C_N$

$\alpha$, deg
DOE Modeling Progression

- Block 1 (FCD)
- 1st in series of sequential tests
- Case #2 – error budget satisfied
- Case #3 – best model is cubic
- Case #4 – minimum PRESS
- Case #6 – minimum Std. Dev

<table>
<thead>
<tr>
<th>case #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>block #</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Design model</td>
<td>FCD</td>
<td>FCD</td>
<td>FCD</td>
<td>Nested FCD</td>
<td>I-Optimal</td>
<td>I-Optimal</td>
<td>I-Optimal</td>
</tr>
<tr>
<td>Model terms (No.)</td>
<td>Linear + 2FI (12)</td>
<td>quadratic + 2FI (20)</td>
<td>cubic + 3FI (32)</td>
<td>cubic + 3FI (38)</td>
<td>cubic + 3FI (68)</td>
<td>cubic + 3FI (81)</td>
<td>cubic + 3FI (128)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9931</td>
<td>0.9988</td>
<td>0.9996</td>
<td>0.9997</td>
<td>0.9995</td>
<td>0.9996</td>
<td>0.9999</td>
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<tr>
<td>Std. Dev.</td>
<td>0.0351</td>
<td>0.0149</td>
<td>0.0095</td>
<td>0.0084</td>
<td>0.0064</td>
<td>0.0060</td>
<td>0.0060</td>
</tr>
<tr>
<td>PRESS</td>
<td>0.1931</td>
<td>0.0411</td>
<td>0.0183</td>
<td>0.0149</td>
<td>0.0208</td>
<td>0.0362</td>
<td>0.0538</td>
</tr>
<tr>
<td>$e_i/C_{M_{max}}$ %</td>
<td>5.68%</td>
<td>0.22%</td>
<td>0.15%</td>
<td>0.11%</td>
<td>0.12%</td>
<td>0.10%</td>
<td>0.10%</td>
</tr>
</tbody>
</table>

*residual $e_i = C_{N_{measured}} - C_{N_{predicted}}$, $**C_{M_{max}} = 1.22$
Validation Test, DOE Block 1

- Residuals vs Run
- Block 1, ¼ FCD
- $C_N$ low $\alpha$ range
- Case #3 model
- 8 fail 3% error

± 3% error test
- 8 points failed
- 9 of 81 allowed

Validation tests reveal true prediction & bias errors
Validation Test, DOE Blocks 1-3

- Residuals vs Run
- Blocks 1-3, opt.
- \( C_N \) low \( \alpha \) range
- Case #3 model
- Similar final stats

\( \pm 3 \% \) error

Model confirmed by validation test; 6 points fail 3% error test
Source of Cubic Terms (DOE blocks 1-3)

Model Error ($\frac{e_i}{C_{N,\text{max}}}$)
($\alpha = 4^\circ$, dela$_L$ = 12.5$^\circ$)
- Case #1, Linear – 4.78%
- Case #2, Quadratic – 1.52%
- Case #3, Cubic – 0.13%
Validation Test, L2F

- Residuals vs Run
- Block L2F
- $C_N$ low $\alpha$ range
- Case #3 model
- Similar final stats

$\pm 3\%$ error

Model confirmed by validation test; 7 points fail 3% error test
Validation Test, Blended DOE-L2F

- Residuals vs Run
- Blocks 1-4
- $C_N$ low $\alpha$ range
- Case #3 model
- Similar final stats

$\pm 3\%$ error

Model confirmed by validation test; 6 points fail 3% error test
Concluding Remarks

• Sequential testing & validation recommended
  – Obtain data sequentially as required
  – Apply validation test after each block of data

• Efficient test methods demonstrated
  – DOE & L2F approaches provide methods to increase efficiency
  – Blending DOE-L2F
    • Currently a “work in progress” but shows promise
    • Presents a challenge in design phase to combine LBW+HBW factors

• Future Test Refinements
  – Fewer blocks required with continuous sampling
  – Smaller optimal blocks
  – Lower sample rates for “quasi-static” tests
  – For “quasi-static” case, lower bandwidth of HBW inputs
  – Design must reflect significant data added by HBW factors
Questions?

• Contact Information
  – patrick.c.murphy@nasa.gov
  – 757-864-4071
  – jay.m.brandon@nasa.gov
  – 757-864-1142

“All models are wrong, but some are useful” – George E. P. Box