A Field Study of Pixel-Scale Variability of Raindrop Size Distribution in the Mid-Atlantic Region

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The spatial variability of parameters of raindrop size distribution and its derivatives is investigated through a field study where collocated PARSIVEL\textsuperscript{2} and two-dimensional video disdrometers are operated at six sites in Wallops Island, Virginia from December 2013 to March 2014. The three-parameter exponential function is employed to determine the spatial variability across the study domain where the maximum separation distance was 2.3 km. The nugget parameter of exponential function is set to 0.99 and the correlation distance ($d_0$) and shape parameter ($s_0$) are retrieved minimizing root-mean-square error, after fitting it to the correlations of physical parameters. Fits were very good for almost all fifteen physical parameters. The retrieved $d_0$ and $s_0$ were about 4.5 km and 1.1, respectively, for rain rate (RR) when all twelve disdrometers were reporting rainfall with a rain rate threshold of 0.1 mm h\textsuperscript{-1} in one-minute observations. The $d_0$ decreased noticeably when one or more disdrometers were required to report rain. The $d_0$ was considerably different for a number of parameters (e.g. mass weighted diameter) but was about the same for the other parameters (e.g. RR) when rainfall threshold was reset to 12 dB for Ka-band and 18 dB for Ku-band reflectivity following the expected Global Precipitation Measurement mission’s space-borne radar minimum detectable signals. The reduction of the database through elimination of a site did not alter $d_0$ as long as the fit was adequate. The correlations of 5-minute rain accumulations were lower when disdrometer observations were simulated for a rain gauge at different bucket sizes.
1. Introduction

The National Aeronautics and Space Administration (NASA) Global Precipitation Measurement (GPM) mission aims to retrieve the three dimensional hydrometeor size distribution of precipitation through its Dual-frequency Precipitation Radar (DPR) on board the GPM core satellite (Hou et. al. 2014). The retrieval algorithm outputs parameters of gamma model size distribution for each range bin utilizing Ka- and Ku-band radar measurements (Seto et al. 2013). The lowest clutter free range bin near the surface at nadir incidence is approximately 5 km radius and 125 m height. One of the key uncertainties of retrieved size distributions is its spatial variability within a given DPR footprint. The spatial variability is a result of precipitation gradient within the range volume, which may not be completely covered by the precipitation. This variability contributes to Non-Uniform Beam Filling (NUBF) which results in a higher degree of uncertainty in microwave sensor based precipitation estimates where the instantaneous field of view is typically bigger than the DPR footprint (Tokay et al. 2014a).

Scanning radars in range height indicator mode provides the most relevant data source to study the spatial variability of hydrometeor size distribution in the vertical, while gridded radar data at 1x1 km or 2x2 km resolution is a common resource for studying the horizontal spatial variability. The parameters of the size distribution that are derived from radar measurements are based on empirical relationships (Bringi et al. 2004). There is also an uncertainty within the radar pixel
as its size increases away from radar. Disdrometers are an alternative resource for estimating the horizontal spatial variability but are associated with point sampling and it is quite costly to populate a dense network with disdrometers to sample the area of a satellite footprint. Therefore, there have been relatively few field studies conducted to determine the spatial variability of hydrometeor size distributions and all of these dealt with rainfall.

As part of the NASA’s Tropical Rainfall Measurement Mission (TRMM) Ground Validation program, a number of impact type Joss-Waldvogel disdrometers were first deployed during a series of field campaigns and also deployed later at the NASA Wallops Flight Facility on Wallops Island, Virginia. The latter operation allowed for determination of the measurement accuracy of the disdrometers when six units were side-by-side (Tokay et al. 2005) and the spatial variability of raindrop size distribution (DSD) when three units were deployed across the Wallops Island strip, where the minimum and maximum separation distances were 0.65 and 1.7 km, respectively (Tokay and Bashor 2010). Later, four disdrometers were distributed across the same strip where the minimum and maximum separation distances were 0.4 and 5.0 km, respectively (Schröer 2011). The three-parameter exponential function was tested to determine the spatial variability of DSD and integral rain parameters. Since four disdrometers provide six pairs of correlations across 5 km, the fitted exponential function was subject to noticeable error.
Measurement accuracy is one of the key uncertainties in determining the spatial variability of DSDs quantitatively. Each disdrometer type has its own shortcomings in measuring the DSD, and comparative field studies where different types of disdrometers were collocated, help quantify the uncertainties of the disdrometers (Krajewski et al. 2006, Thurai et al. 2011, Tokay et al. 2013, 2014b). Indeed, the Miriovsky et al. (2004) pioneer study was unable to determine the spatial variability of reflectivity within 1 km² due to uncertainties of four different types of disdrometers. Lee et al. (2009), on the other hand, used four Particle Occurrence Sensor System (POSS) to study the spatial variability of DSD in stratiform rain events. An S-band dual polarization radar scanned over the POSS units and the spatial correlation of rainfall was higher in radar than in POSS especially at 15 and 30 km. This could partly be due to the differences between the sampling volumes of the instruments.

The first comprehensive field study to quantify the spatial variability of the DSD was conducted in Central Spain where eight dual PARSIVEL (PARticle Size VElocity) disdrometers were deployed (Tapiador et al. 2010). The availability of number of disdrometers allowed 28 pairs of correlations where the distances range from 0.2 km to 3.2 km. The dual units were aligned in both North-South and East-West directions, and one of the five events had significant differences in correlations of reflectivity between North-South and East-West aligned disdrometers. Jaffrain et al. (2011), on the other hand, analyzed 53 hours of 16 PARSIVEL observations to determine the spatial variability of DSD in a radar pixel (about 1 x 1 km²) in
Lausanne, Switzerland. They concluded that the coefficient of variations for the mass weighted diameter ($D_{\text{mass}}$), total concentration ($N_T$), and rain rate (RR) were high and could not be explained solely by the uncertainty of the measurement. Jaffrain and Berne (2012, JB12 hereafter) used the same set up of instruments to determine the spatial variability of $D_{\text{mass}}$, $N_T$, and RR using a variogram analysis. They reported that the variability was greater in convective rain than transitional and frontal rain. They also noted a decreasing variability with decreasing temporal resolution. Perhaps, the main issue of these studies is the measurement accuracy of the PARSIVEL disdrometer. The low-cost laser resulted in overestimation of large drops due to an inhomogeneous beam (Thurai et al. 2011, Tokay et al. 2013). The manufacturer upgraded the PARSIVEL disdrometer with PARSIVEL² in 2011 (Tokay et al. 2014b).

In contrast to disdrometers, rain gauges are low-cost, durable, easy to maintain and are frequently deployed to adjust or validate radar rainfall estimates during either two-month long field studies or longer-term field observations. Thus, rain gauge networks are an excellent resource to study the spatial variability of rainfall (Habib and Krajewski 2002, Gebremichael and Krajewski 2004, Ciach and Krajewski 2006, among others). The spatial variability of rainfall is often quantified applying a three-parameter exponential function to the corrections between the paired gauge measurements. The parameters of the exponential function differs from one study to another due to the differences in quality of rain gauge data, inter-gauge distances, sample size, experiment period and location (Villarini et al. 2008). Logistics may
limit the inter-gauge distances, while long-term observations with dual or multiple
gauge at each site has a clear advantage for a larger sample size and continuous
gauge record at a given gauge site. The long-term observations also allow
investigation of season-to-season differences in the spatial variability of rainfall.
Tokay et al. (2014a), for instance, used five years of continuous rain gauge
measurements in the Southern Delmarva Peninsula and showed that weather
systems dominate spatial variability over a season. Nor’easters, for instance,
dominate winter precipitation in the Mid-Atlantic region.

As a comparative study, the spatial variability of rainfall was quantified through X-
band radar and rain gauge network (Moreau et al. 2009). The correlations between
the paired radar pixels at 1 km² resolution were significantly higher than those
between paired gauges underneath the radar pixel up to 8 km. The differences in
correlations are attributed to the differences in sampling volume, while the errors in
radar rainfall estimate and time-height ambiguity also be played a role. Another
comparative study of the spatial variability of rainfall was conducted using S-band
dual-polarization radar and 2DVD measurements during Mid-latitude Continental
Convective Clouds Experiment (MC3E) (Bringi et al. 2015). A good agreement was
found between the radar and disdrometer based spatial variability for RR, median
volume diameter, and logarithmic normalized intercept parameter with respect to
liquid water content (NW) in a long lasting stratiform event. The correlation
distances were lower in a relatively short convective event than the stratiform event
for all three parameters.
This study investigates the pixel-space variability of DSD and integral rain parameters employing a disdrometer network at NASA Wallops Flight Facility (WFF), Wallops Island, Virginia (37.9 degree N and 75.4 degree W). The manuscript is organized as follows: The disdrometer network and the database are presented in Section 2. Section 3 introduces the three-parameter exponential function while the parametric form of the DSD is given in Section 4. The probability and cumulative distributions of the DSD and rain parameters can be found in Section 5. Section 6 depicts the spatial variability of DSD and rain parameters, while the sensitivity studies are in Section 7. The conclusions are given in the last section.

2. Field Study

The disdrometer network consists of six sites where each site had one two dimensional video disdrometer (2DVD) and one PARSIVEL \(^2\) disdrometer. The sites were distributed across WFF where the minimum and maximum separation distances were 0.5 and 2.3 km, respectively (Figure 1a). One of the sites (Pad) hosted a variety of tipping and weighing bucket rain gauges including two pit gauges (Figure 1b). Another site was collocated with the Wallops Automated Surface Observing System (ASOS) as well as additional tipping bucket rain gauges (Figure 1c). Table 1 summarizes the locations and distances between the sites.
PARSIVEL\textsuperscript{2} is the third generation of the laser-optical PARSIVEL disdrometer (Tokay et al. 2014b). It is designed to measure the size and fall velocity of individual hydrometeors across its laser beam and is also present weather sensor. For rain, PARSIVEL outputs drop counts in a 32 x 32 size versus fall velocity matrix at selected time intervals ranging from 10-second to one-minute. The first two size bins are empty due to low signal to noise ratio and the smallest measurable raindrop corresponds to 0.25 mm diameter. The width of the size bins increases with the size of raindrops from 0.125 mm for drops to 1.2 mm, to 0.25 for drops up to 2.5 mm, to 0.5 mm for drops up to 5 mm, and to 1.0 mm for the larger drops. Therefore, there is noticeable uncertainty in determining the maximum drop size of the DSD. Based on 2DVD observations, the largest raindrop ever recorded is 9.7 mm diameter (Gatlin et al. 2015). The 25\textsuperscript{th} bin corresponds to 9-10 mm and was considered as the largest bin for rain. For fall velocity, PARSIVEL has a range from 0.05 to 20.8 m s\textsuperscript{-1}, covering fall speeds of all types of hydrometeors. For rain, the 26\textsuperscript{th} bin corresponds to 8-9.6 m s\textsuperscript{-1}, and covers the expected range for terminal fall speeds of very large raindrops (Beard 1976). However, PARSIVEL\textsuperscript{2} underestimates the fall velocity of raindrops by approximately 1 m s\textsuperscript{-1} with respect to the expected terminal fall speed of raindrops at around 1 mm diameter (Tokay et al. 2014b). The underestimation in fall velocity is also evident for larger drops but the difference between the mean PARSIVEL\textsuperscript{2} and terminal fall speed decreases with increasing size. The manufacturer recognizes this matter as a software error and it is expected that the new generation of instruments will mitigate this issue (Kurt Nemeth, OTT PARSIVEL, personal communication, 2015).
The 2DVDs are the “compact” third generation and are composing of sensor unit and indoor personal computer (Schönhuber et al. 2007). The third generation compact version has the optical components firmly mounted with no need for re-alignment by the user. The distance of the optical slits to the rim of the housing was also reduced aiming to eliminate inhomogeneous filling of the measurement area in windy conditions. The high-speed line-scan cameras provides better matching of falling hydrometeors between the two planes that are nominally 6 mm apart. The 2DVD records the time stamp of each raindrop including its equivalent diameter, fall velocity, and oblateness as well as the sampling area. Common interruption of data occurs often due to non-meteorological items (e.g. leaves) in the sampling cross section, which is the main shortcoming of the 2DVD.

The dataset for this study spanned from December 2013 through March 2014. While PARSIVEL² disdrometers operated nearly continuous during the experiment period, a number of 2DVD units failed to operate in a few rain events. The raw outputs of PARSIVEL² and 2DVD observations were integrated to one-minute after screening secondary and mismatched drops that fell outside ±50% of their terminal fall speed (Tokay et al. 2013). Several rain/no-rain thresholds were then applied to the one-minute observations. All thresholds required a minimum of 10 drops. The minimum RR of 0.1 mm h⁻¹ resulted in 447 one-minute samples when all twelve disdrometers reported rainfall. Considering the minimum detectable reflectivity of the GPM DPR, the Ku-band reflectivity ($Z_{Ku}$) of 18 dB and Ka-band reflectivity ($Z_{Ka}$)
of 12 dB (Seto et al. 2013) are the other two thresholds used in this study. The minimum RR was 0.11 and 0.08 mm h$^{-1}$ for $Z_{Ka} > 12$ dB and 0.16 and 0.30 mm h$^{-1}$ for $Z_{Ku} > 18$ dB for 2DVD and PARSIVEL$^2$, respectively. The sample sizes were 445 and 278 for $Z_{Ka} > 12$ dB and $Z_{Ku} > 18$ dB, respectively. The sample of $Z_{Ka} > 12$ included twelve one-minute spectra that were not in the RR > 0.1 mm h$^{-1}$ based sample.

3. Methodology

A three-parameter exponential function is employed to quantify the spatial variability of the DSD. It has been widely used to determine the spatial variability of rainfall through rain gauge and radar rainfall studies (Habib and Krajewski 2002, Gebremichael and Krajewski 2004, Ciach and Krajewski 2006, Villarini et al. 2008, Moreau et al. 2009, Tokay et al. 2014a). The Pearson correlation coefficient, $r$, is applied to the selected DSD and rain parameters that are derived from disdrometer measurements at distance $d$. The parametric form of exponential function is then expressed as

$$r = r_0 \exp \left[ (-d/d_0)^{s_0} \right]$$

(1)

where $r_0$ is the correlation of a selected parameter derived from collocated instruments and is known as the nugget parameter and ideally should be one. However, collocated gauge and disdrometer observations show that the nugget
Parameter ranges between 0.90 and 0.99. The other parameters of the exponential function are not sensitive to changes in $r_0$ within this range (Schröer 2011, Tokay and Öztürk, 2012). In this study, $r_0$ was set to 0.99. The correlation distance $d_0$, and the shape parameter $s_0$, are then derived through minimizing the root mean square error between the observed and derived correlations. As an initial guess, $d_0$ values of 1 to 300 km and $s_0$ values of 0.1 to 2.0 were given.

The $d_0$ decreases with increasing $s_0$ when the correlation coefficient is high (Figure 2). When the variability is investigated over a relatively small domain as in this study, the correlations of the selected DSD or rain parameters may not have a decreasing trend with distance. If the correlations were high, the best fit results in very high $d_0$ and this should not be interpreted independent of other exponential function parameters. The parameters of exponential function are valid within the maximum distance of the study domain (Tokay and Öztürk 2012).

4. Raindrop Size Distribution

The normalized gamma distribution function is adopted to determine the spatial variability of DSD parameters. The normalization was done with respect to $N_T$ and liquid water content $W$ (Tokay and Bashor 2010). The normalized intercept parameters with respect to total concentration, $N_T^*$ and liquid water content, $N_W$ are expressed as
\[ N_T^* = \frac{N_T}{D_{\text{mass}}} \]  
(2)

\[ N_w = \frac{256 \times 10^3 W}{\pi \rho_w D_{\text{mass}}^4} \]  
(3)

where \( \rho_w \) is the density of water. \( D_{\text{mass}} \) is related to the slope, \( \Lambda \) and shape parameter, \( m \) of the complete gamma distribution as;

\[ D_{\text{mass}} = \frac{4 + m}{\Lambda} \]  
(4)

The normalized intercept parameters can then be calculated from observed spectra as well. The corresponding normalized gamma fitted distributions are expressed as:

\[ N(D) = N_T^* f_1(m) \left( \frac{D}{D_{\text{mass}}} \right)^m \exp\left[ -(4 + m) \frac{D}{D_{\text{mass}}} \right] \]  
(5)

\[ N(D) = N_w f_2(m) \left( \frac{D}{D_{\text{mass}}} \right)^m \exp\left[ -(4 + m) \frac{D}{D_{\text{mass}}} \right] \]  
(6)

where \( f_1(m) \) and \( f_2(m) \) are given as;

\[ f_1(m) = \frac{(4 + m)^{m+1}}{\Gamma(m+1)} \]  
(7)
To extract the shape parameter, $m(N_T)$, and $m(N_W)$, the rain rates that are calculated from observed and fitted gamma distributions (e.g. equations 5 and 6) are minimized. The formulations presented above follows the complete gamma function where minimum and maximum drop size is assumed zero and infinity, respectively. In reality, there is a minimum and maximum drop size ($D_{\text{max}}$) in a population of drops and the incomplete gamma function is more appropriate especially if the size spectra do not contain the large drops (e.g. narrow distribution), but this is the beyond the scope of this study.

The GPM DPR algorithm has adopted the normalized gamma function as in equation 6 (Seto et al. 2013). Since the DPR algorithm employs dual frequency reflectivity measurements to determine the three parameters of gamma function, there is an interest in finding relations between the derived parameters. Williams et al. (2014) suggested a power law relation between the standard deviation of $D_{\text{mass}}$ ($\sigma_{\text{mass}}$) and $D_{\text{mass}}$. Both variables are directly calculated from observed DSD and their ratio is a sole function of shape parameter, $m(\sigma_{\text{mass}})$ following complete gamma function.

$$\frac{\sigma_{\text{mass}}}{D_{\text{mass}}} = \frac{1}{(4 + m)^{0.5}}$$
5. Probability and Cumulative Distributions

This study investigates the spatial variability of seven DSD and eight integral rain parameters. The DSD parameters include $D_{\text{mass}}, D_{\text{max}}, N_{T^*}, N_W, m(N_{T^*}), m(N_W),$ and $m(\sigma_{\text{mass}})$, while the rain parameters were $W, RR$, dual polarization parameters of horizontal reflectivity ($Z_H$), and differential reflectivity ($Z_{\text{dr}}$), reflectivity at $W$-band ($Z_W$), $Z_{\text{Ku}}, Z_{\text{Ka}}$, and dual frequency ratio ($\text{DFR} = Z_{\text{Ku}}/Z_{\text{Ka}}$). The $\text{Ka}$-, $\text{Ku}$-, and $W$-band reflectivities are calculated using Mie scattering for spherical particles, while dual-polarization parameters are calculated for S-band radar following Tokay et al. (2002). Table 2 presents the mean, standard deviation, median, and 5th and 95th percentage of 15 physical parameters. Both $N_{T^*}$ and $N_W$ have ranges of several order of magnitude and, hence logarithmic values of these two parameters are used in constructing probability and cumulative distributions.

Among many factors listed in the introduction, knowledge of the characteristics of the DSD and rainfall is essential for quantifying the spatial variability. Observed DSD lacking large drops but with abundant small drops will incur different spatial variability than a DSD with numerous large drops but fewer small drops. The spatial variability of rainfall, on the other hand, differs in the presence and absence of heavy rain. The probability and cumulative distributions of the DSD and rain parameters provide an insight on the characteristics of DSD and rainfall, and these
distributions should be included if a similar study is conducted with a different dataset.

Figure 3 presents the probability and cumulative distributions of fifteen DSD and integral rain parameters based on 2DVD and PARSIVEL\(^2\) measurements, when all twelve units were reporting rainfall. A very good agreement in midsize range (diameter 1-3 mm) of 2DVD and PARSIVEL\(^2\) size spectra reflects an excellent agreement in probability and cumulative distributions of \(D_{\text{mass}}\) between the two disdrometers (Figure 2a). \(D_{\text{mass}}\) mostly resided between 0.8 and 1.5 mm, peaking at 1 mm (Table 2). The binning of drop counts results in quantization error in size measurements in PARSIVEL\(^2\), which is quite significant for \(D_{\text{max}}\) since the larger bins have wider widths. The probability distribution of \(D_{\text{max}}\) in PARSIVEL\(^2\) therefore had multiple modes, while the probability distribution of \(D_{\text{max}}\) in 2DVD was a unimodal (Figure 3b). The cumulative distributions of \(D_{\text{max}}\) agreed well between the two types of disdrometers and the median \(D_{\text{max}}\) was around 1.9 mm for both (Table 2).

PARSIVEL\(^2\) is relatively more sensitive to small drops less than 0.5 mm diameter than the 2DVD (Tokay et al. 2013, 2014b). \(N_T^*\) is very sensitive to the number of small drops since their concentrations are much higher than larger drops. The probability distribution of \(\log(N_T^*)\) of 2DVD and PARSIVEL\(^2\) show a wide range, but the PARSIVEL\(^2\) based distribution was shifted to the larger concentrations, consistent with its sensitivity to small drops (Figure 3c). \(N_W\) is more sensitive to midsize drops and therefore an agreement is expected between the 2DVD and
PARSIVEL\textsuperscript{2} distributions. A general agreement is evident between the distributions of \(\log(N_w)\) with a median 3.55, but the PARSIVEL\textsuperscript{2} also had low percentages of \(\log(N_w)\) less than 2.5 (Figure 3d).

The shape parameter of the gamma distribution is very sensitive to the method of derivation and has a wide range but was mostly between 0 and 14 with mean and median around 5 (Table 2). While the agreement between 2DVD and PARSIVEL\textsuperscript{2} derived distributions of shape parameters were reasonable for \(m(N_T)\) \(m(N_W)\), and \(m(\sigma_{\text{mass}})\), the probability distribution of \(m(N_T)\) was shifted toward larger values in PARSIVEL\textsuperscript{2} and \(m(N_W)\) and \(m(\sigma_{\text{mass}})\) had slightly larger values in 2DVD (Figures 3e-g). The differences in probability and cumulative distributions are more pronounced in \(m(N_T)\) reflecting the differences in distributions of \(\log(N_T)\).

An excellent agreement was evident in the distributions of \(W\), and RR between the two different types of disdrometers (Figures 3h-i). Both \(W\) and RR are primarily sensitive to midsize drops, while the upper end of the small drops (0.8-1.0 mm) and the lower end of the large drops (3.0-4.0 mm) contribute to \(W\) and RR significantly in the presence of light and heavy rainfall, respectively. The mean and median RR is 1.1 and 0.9 mm h\textsuperscript{-1}, respectively (Table 2), indicating the dominance of light rain. Good agreement was also evident in distributions of \(Z_{H}, Z_{dr}, Z_{W}, Z_{Ku}, Z_{Ka}\), and DFR between the two types of disdrometers (Figures 3j-o). The median \(Z_{H}, Z_{Ku}\) and \(Z_{Ka}\) were in the 22-23 dB range, while 95\textsuperscript{th} percentiles were in the 31-32 dB range (Table 2) in the presence of widespread frontal rainfall.
The correlations of $D_{\text{mass}}$ and $D_{\text{max}}$ are higher for 2DVD than for PARSIVEL² at a given distance (Figures 4a-b). PARSIVEL² based correlations were 0.55-0.7 for five paired observations resulting in a higher RMSE. The quantization error due to binning contributed to the low correlations in PARSIVEL² based $D_{\text{max}}$. The correlations of $N_T^*$ and $N_W$ were higher than 0.9 for all distances for the 2DVD and decreased gradually with distance (Figures 4c-d). PARSIVEL² based correlations have a wider range than 0.1 at a given distance but remained higher than 0.84 and the RMSE was quite low for fitted exponential function. The correlations of shape parameters decreased with distance from 0.8 to 0.5 for $m(N_T^*)$ and from 0.8 to 0.4 for $m(N_W)$ and $m(\sigma_{\text{mass}})$ for both 2DVD and PARSIVEL² (Figures 4e-g). The range of correlations was less than 0.1 at a given distance for both disdrometers resulting in low RMSE. The sample size was 74% and 77% of the database for $m(N_T^*)$ and $m(N_W)$, respectively due to the fact that the shape parameter was outside the expected range of -4 to 20 for a number of size spectrum when normalized gamma function was fitted minimizing rain rate. Similarly, the sample size was 76% of the database for $m(\sigma_{\text{mass}})$ due to presence of shape parameters larger than 20. The large shape parameters results from narrow size spectrum in the absence of large drops. The shape of size spectrum exhibits more than a single peak with a plateau in midsize regime for collisional break-up dominated DSD (D’Adderio et al. 2015).
The gamma distribution is not the best mathematical fit in these conditions, but this is beyond scope of this paper.

The correlations of RR and W decreased with distance from 0.9 to 0.55 and from 0.92 to 0.64, respectively (Figures 4h-i). The spread in correlation between the same and different types of disdrometers was much less than 0.1 at a given distance resulting in very low RMSE. Thus, the exponential fits were nearly identical for both types of disdrometers. The correlations of reflectivities at different wavelengths decreased with distance but there were noticeable differences and similarities among them. The correlations of ZH and ZKu were about the same at a given distance for a given disdrometer while the spread in PARSIVEL\(^2\) derived correlations of ZKa were less resulting in lower RMSE (Figures 4j, 4m-n). The spread in correlations of ZW were much less (less than 0.05) at a given distance for both disdrometer types resulting in an excellent fit (Figure 4l). For Zdr, there were significantly lower correlations (0.35-0.55) for five PARSIVEL\(^2\) pairs resulting in high RMSE (0.14) (Figure 4k). For 2DVD, Zdr decreased with distance and had a spread of 0.1 at a given distance resulting in a reasonable fit with low RMSE. DFR, which is another reflectivity ratio, had lower correlations even at short distances and the spread in correlations were as high as 0.3 at a given distance in PARSIVEL\(^2\) resulting in relatively poor fit with high RMSE (0.12) (Figure 4o). For 2DVD, the fit was better but the spread in correlation was 0.2 at a given distance.
The correlation distances were about the same for RR and W but higher in 2DVD than in PARSIVEL$^2$ for the rest of rain and DSD parameters (Figure 5a). For $N_r$ and $N_w$, $d_0$ was at around upper limit of initial guess due to high correlations for all distances. The $d_0$ was 4.2 and 4.5 km in RR in PARSIVEL$^2$ and 2DVD, respectively. These correlation distances correspond to high spatial variability following gauge-based studies (Ciach and Krajewski 2006). However, the correlations remained high (> 0.7) in this study (Figure 4i). The gauge studies were conducted for 5-minute or longer integration periods while high temporal scale in this study results in higher variability even in the presence of lighter rain. The shape parameter, on the other hand, remained between 0.4 and 1.0 for most of the fits (Figure 5b). The RMSE was less than 0.08 for almost all parameters showing the goodness of fit (Figure 5c). It should be emphasized that three-parameter exponential function is applied for the first time for a number of DSD and rain parameters following feasibility study (Schröer 2011). Therefore, there is no direct comparison available for these parameters of exponential function with any other studies.

7. Sensitivity Studies

a) Disdrometer network availability

It is not uncommon that one or more disdrometers fail to operate throughout a field campaign. For spatial variability, it is important to assess the goodness of fit of the three-parameter exponential function when one of the sites is not available. In that
regard, five out of six sites were employed to quantify the sensitivity of parameters of exponential function to $D_{\text{mass}}$ and RR. The elimination of a site reduced paired correlations from 15 to 10 and if site 5 or site 6 in Table 1 were not functioning, the maximum separation distance would be reduced to 1.84 km.

Fits were very good for 2DVD derived $D_{\text{mass}}$ and RR regardless of the choice of elimination of paired correlations. The $d_0$ had a very narrow range of 4.2-4.7 km for RR but remained mostly within 11-21 km for $D_{\text{mass}}$ (Figure 6a). For PARSIVEL², fits were good for RR but quite poor for $D_{\text{mass}}$ repeating previous finding in Figure 4a. The $d_0$ had a wide range in PARSIVEL² derived $D_{\text{mass}}$ but was not sensitive to the elimination of any paired correlation in RR. The $s_0$ was less than 1.0 and highly variable due to elimination of paired correlation in $D_{\text{mass}}$ but remained mostly above 1.0 with almost no sensitivity to the elimination of paired correlation in RR (Figure 6b). The RMSE was 0.03 or less for 2DVD for both $D_{\text{mass}}$ and RR but was higher than 0.1 for four trials for PARSIVEL² derived $D_{\text{mass}}$ (Figure 6c).

b) Rain/No-rain threshold

The rain/no-rain threshold results in differences in the sample size and the distribution of the DSD and rain parameters. The higher sensitivity DPR footprint where $Z_{K_a}$ is larger than 12 dB is expected to have less spatial variability with higher $d_0$ than the lower sensitivity DPR footprint where $Z_{K_u}$ is larger than 18 dB. As shown in the gauge based study of Tokay and Özturk (2012), the higher thresholds
eliminate the light rain or low reflectivity samples and results in lower correlations, especially if there is no precondition where both disdrometers must report rainfall.

Based on 2DVD observations, $d_0$ of $D_{\text{mass}}$ was the lowest and the highest for $Z_{\text{Ku}}$ and RR thresholds, respectively (Figure 7a). The $d_0$ of RR and $W$ was the same for RR and $Z_{\text{Ka}}$ thresholds and slightly lower for $Z_{\text{Ku}}$ threshold. The $d_0$ of $Z_{\text{Ka}}$, $Z_{\text{Ku}}$ and $Z_{\text{H}}$ were about the same for $Z_{\text{Ka}}$ and $Z_{\text{Ku}}$ thresholds, and were lower than RR threshold. The $d_0$ of DFR was the same for all three thresholds, while the other DSD and rain parameters showed different trends. The shape parameter of exponential function showed little or no variability when $d_0$ was about the same for given thresholds (Figure 7b). The RMSE did not show significant differences between the different thresholds for most of the DSD and rain parameters except for $Z_{\text{dr}}$ and DFR where RMSE was noticeably higher for $Z_{\text{Ku}}$ threshold (Figure 7c).

c) Rain coverage

Partial coverage of satellite footprint or radar pixel by rain is one of the sources of NUBF and contributes significantly to the spatial variability. To quantify the spatial variability in the presence of partial coverage, PARSIVEL\textsuperscript{2} observations were reprocessed when all units were reporting rainfall and when at least one unit reporting rainfall. The latter is more commonly observed in nature. The sample size is 4,645 one-minute size distributions for the latter condition, which makes it more than 2.6 times of the sample size of the former condition. The difference is
mainly due to the rain intermittence. The sample size when all six PARSIVEL$^2$
reported rainfall was 4 times larger than that when all twelve disdrometers
reported rainfall.

The correlation distance was distinctly lower for almost all physical parameters
when one or more PARSIVEL$^2$ reported rainfall (Figure 8a). It should also be noted
that $d_0$ was different from earlier findings when all six PARSIVEL$^2$ were reporting
rainfall. The bigger sample included more uniform rainfall across the study domain
and $d_0$ of RR was 14 km. The $s_0$ was higher when one or more PARSIVEL$^2$ reported
rainfall (Figure 8b). Fits were good for both conditions for physical parameters
except for normalized intercept and shape parameter of the gamma distribution
where fit was relatively poor with RMSE of 0.1 or higher (Figure 8c).

d) Rain gauge simulation

Tipping bucket rain gauges are widely used in precipitation studies but suffer from
significant sampling errors over short integration periods depending on the bucket
size (Habib et al 2001). A bucket size of 0.01 inch (0.254 mm) is used in ASOS
network but the gauge manufacturers also provide gauges with bucket resolutions
of 0.1 mm and 0.2 mm. The gauge based spatial variability of rainfall studies
employs 5-minute or longer integration periods to mitigate the sampling errors.
PARSIVEL$^2$ and 2DVD one-minute rain rate time series were employed to simulate the time of tip for 0.1 mm, 0.2 mm, and 0.254 mm bucket resolutions. The simulation study ignored any disdrometer malfunctions throughout the experiment period. The 5-minute rainfall was then calculated from one-minute disdrometer and three different simulated gauge datasets. The nugget parameter of 0.99 was considered in all simulations. The correlations decrease noticeably with the size of the bucket at a given distance in both PARSIVEL$^2$ and 2DVD based simulations (Figure 9a-b). The decrease in correlations was gradual for disdrometer but quite sharp for coarser relation bucket size based simulations from nugget to short separation distances. At 2 km separation distance, the differences in correlations were approximately 0.2 between disdrometer and 0.254 mm bucket simulations of 2DVD and PARSIVEL$^2$.

8. Conclusions

The pixel-scale variability of seven DSD and eight integral rain parameters was investigated through a unique set of disdrometer observations where collocated 2DVD and PARSIVEL$^2$ were operated at six sites across the main base of NASA/WFF. A three-parameter exponential function was employed to quantify the spatial variability. The $r_0$, or nugget parameter, was fixed to 0.99 and $d_0$ and $s_0$ were retrieved minimizing RMSE following fitting an exponential function to the correlations that were derived from 2DVD or PARSIVEL$^2$ observations. The correlations were calculated during periods when all twelve disdrometers reported
rainfall and a minimum RR threshold of 0.1 mm h⁻¹ was satisfied with a minimum of
10 drops occurring in one-minute.

The there parameter exponential function is a simple mathematical form that
determines the observed correlations well. It converged rapidly for almost all
physical parameters where RMSE was mostly less than 0.08. Very good agreement
between the 2DVD and PARSIVEL² derived correlations boosted our confidence in
the quality of the observations. The 2DVD outperformed the PARSIVEL² for
sampling of D_max, while the reverse was true for N_r*. This is likely due to the
difference in the sensitivity of the respective disdrometers to the small and large
drop end of the size spectrum.

Given the confidence in measurement quality and fitting method, d₀ and s₀ should
mainly be sensitive to the characteristics of the DSD. The exclusion of a site did not
change the distribution of rainfall resulting in insignificant changes in d₀ and s₀ of
RR. The retrieved parameters of exponential function fits quite different in
PARSIVEL² derived D_mass when one site was removed but this is attributed to the
poor fitting. The d₀ and s₀ were sensitive to the different rain/no-rain thresholds.
They were also sensitive to the differences in conditions where one or more and all
six PARSIVEL² were reporting rainfall. The differences in the sample of
observations play a key role for differences in d₀ and s₀. The disdrometers have a
clear advantage to the rain gauges when spatial variability was studied for short
time intervals. The larger the bucket size the lower the correlations when the
disdrometer observations were used to simulate rain gauge observations at different bucket size.

This is the first study where the three-parameter exponential function has been used for estimating the variability of 15 physical parameters. Previously, JB12 used the exponential function for RR. Their study was more robust in terms of the number of available instruments and a longer period of observation. The correlations decreased more rapidly for the first 500 m reaching 0.8 in JB12 study. This study did not have any paired correlation in this short distance regime and the correlation of RR was about 0.9 at 500 m. The differences in correlations are attributed to the nature of rain. JB12 study included higher rainfall intensities than this study.

The logistics often dominate the experiment set up and therefore the minimum and maximum distances in the study domain. The sample size and type of rainfall play a crucial role in quantifying the spatial variability. WFF is ideal site since rainfall from multiple weather systems falls in all around year. In this study, light rain was persistent throughout field study where mean RR was 1.1 mm h\(^{-1}\) and the 95\(^{th}\) percentile of \(D_{\text{max}}\) was 3.1 mm (2DVD). For future studies, the larger domain where the maximum disdrometer distance exceeds the DPR footprint and the combination of light and heavy rain is desirable for quantification of spatial variability of DSD. The limited but unique data set from Mid-latitude Continental Convective Clouds
experiment from Northern Oklahoma may prove to be a great resource for such a follow-up study.

Acknowledgements

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References


Figure Captions:

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Figure 5. (a) Correlation distance and (b) shape parameter of the three-parameter exponential function and (c) root-mean-square error for fifteen physical parameters based on 2DVD (blue dots) and PARSIVEL$^2$ (red stars) observations. Several correlation distances were higher than y-axis range and are marked with their values.

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Table 1. NASA Wallops Flight Facility Precipitation Data Acquisition Network. The distances between the six sites are given in km.

<table>
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<tr>
<th>Site #</th>
<th>Site name</th>
<th>Site 1</th>
<th>Site 2</th>
<th>Site 3</th>
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<td></td>
<td>Site name</td>
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<table>
<thead>
<tr>
<th>DSD and integral rain parameters</th>
<th>mean</th>
<th>standard deviation</th>
<th>median</th>
<th>5th percentile</th>
<th>95th percentile</th>
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<td>2DVD</td>
<td>Par(^2)</td>
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<td>(D_{\text{max}}) (mm)</td>
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<td>(N_T^*) (m(^3)mm(^{-1}))</td>
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<td>951</td>
<td>394</td>
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<td>27.7</td>
<td>18.5</td>
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<td>21.9</td>
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<tr>
<td>(Z_{Ku}) (dB)</td>
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<td>26.2</td>
<td>5.8</td>
<td>6.3</td>
<td>23.4</td>
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<td>(DFR)</td>
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<td>-0.86</td>
<td>1.19</td>
<td>1.11</td>
<td>-1.27</td>
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</table>
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