Budget of Turbulent Kinetic Energy in a Shock Wave/Boundary-layer Interaction

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June 7, 2016
Introduction
  Motivation
  Approach
  Simulation details

Results
  Logistics
  Validation of upstream boundary layer
  Budget within shock wave/boundary-layer interaction

Conclusions

References
The problem...

- Unsteadiness associated with incoming turbulent boundary layer, separation bubble, shear layer, and corner separation
- Incipiently separated flow correlates with incoming turbulent boundary layer and reflected shock/separation bubble correlates with large separation
- Inhomogeneous and anisotropic
State of the art in turbulence modeling

- one-equation models: linear, quadratic constitutive relation
- two-equation models: linear, non-linear constitutive relations corrections
- Reynolds-stress models: non-linear algebraic, solving for Reynolds-stress—expensive
- For Mach = 2.25 and $\theta = 8.0$ [1], turbulence intensity: $\sqrt{u'{}^2}/U_\infty^2$
State of the art...continued

- For Mach = 2.25 and $\theta = 8.0$, Reynolds shear stress: $-\overline{u'v'}/U_\infty^2$

- Is there a need for improvement in modeling?
- NASA’s Revolutionary Computational Aerosciences subproject in Transformational Tools and Technology Project seeks 40% reduction in predictive error
Study the transport of turbulent kinetic energy

The transport equation:

\[
\frac{\partial (\bar{\rho}k)}{\partial t} + \frac{\partial (\bar{\rho} \bar{u}_j k)}{\partial x_j} = P + T + D_\nu - \bar{\rho} \epsilon + D_p + \Pi + M \tag{1}
\]

- **production**: the rate of transfer of kinetic energy from the mean flow to the turbulence
- **turbulent transport**: propagation of the turbulent kinetic energy
- **molecular diffusion**: viscous transport of the turbulent kinetic energy
- **dissipation**: conversion of turbulent kinetic energy to thermal internal energy
- **pressure diffusion**: transport due to pressure and velocity-gradient interaction
- **compressible terms**: pressure dilatation and mass flux
A closer look...

\[ P = - \bar{\rho} \tilde{u}_i'' u_j'' \tilde{u}_{i,j} \]
\[ T = - \left( \frac{\rho}{2} u_j'' \frac{1}{2} u_i'' u_i'' \right)_{,j} \]
\[ D_\nu = \left( t_{ij} u_i'' \right)_{,j} \]
\[ \bar{\rho} \epsilon = t_{ij} u''_{i,j} \]
\[ D_p = - \left( p' u_i'' \right)_{,i} \]
\[ \Pi = p' u''_{i,i} \]
\[ M = u''_i \left( t_{ij,j} - \bar{p},_i \right) \]

Production (2)
Turbulent Transport (3)
Molecular Diffusion (4)
Dissipation (5)
Pressure Diffusion (6)
Pressure Dilatation (7)
Mass Flux (8)
Solver

FDL3DI [2]

- Implicit large-eddy simulation (ILES), i.e., no explicit subgrid scale model
- Fifth-order bandwidth- and order-optimized weighted essentially non-oscillatory (WENO) scheme
- Roe for inviscid fluxes and viscous fluxes were computed using sixth-order compact scheme
- Implicit time integration with Beam-Warming using two sub-iterations and approximate factorization
- Counterflow force model used to trip the boundary layer
  1. Originally developed by Shyy et al. [4] to model the effect of dielectric barrier discharge (DBD) actuator
  2. Used here to trip the boundary layer from laminar to turbulent
  3. Required a long streamwise domain to accommodate the transition process
Flow and boundary conditions

<table>
<thead>
<tr>
<th>Property</th>
<th>Experiment</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\infty$</td>
<td>2.33</td>
<td>2.33</td>
</tr>
<tr>
<td>$U_\infty$, m/s</td>
<td>556.0</td>
<td>556.0</td>
</tr>
<tr>
<td>$P_\infty$, Pa</td>
<td>23,511.0</td>
<td>2351.1</td>
</tr>
<tr>
<td>$T_0$, K</td>
<td>295.6</td>
<td>295.6</td>
</tr>
<tr>
<td>$T_w$, K</td>
<td>269.7</td>
<td>269.7</td>
</tr>
<tr>
<td>$\delta_{99}$, m</td>
<td>$5.3 \times 10^{-3}$</td>
<td>$5.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>$Re_\delta$</td>
<td>175,202.0</td>
<td>17,520.2</td>
</tr>
</tbody>
</table>

- Laminar boundary-layer profile imposed at the inflow
- No slip wall with expected adiabatic wall temperature to freestream static temperature set to 1.95
- Shock imposed using Rankine-Hugoniot relations by specifying pre- and post-shock conditions
- X, Y, and Z = 95, 25, and 5 non-dimensionalized by $\delta_{99}$
- $1301 \times 251 \times 201$ for a total of approximately 66M grid points
- 1000 points in the constant area section
- Shock imposed at $x = 23.2$
Data averaging?

- How long should the case be run before collecting statistics? - 6 flow-through times (FTS)
- Flow-through times necessary to converge the high-order statistics? - 3 and 6 flow-through times
- Centerline versus span-averaged statistics

![Graph showing turbulence budget and production](image-url)
### Key stations

<table>
<thead>
<tr>
<th>Station</th>
<th>Location</th>
<th>$x$</th>
<th>$Re_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Incoming flat plate boundary layer</td>
<td>56.0</td>
<td>3270</td>
</tr>
<tr>
<td>2</td>
<td>Upstream of the reflected shock</td>
<td>56.6</td>
<td>3280</td>
</tr>
<tr>
<td>3</td>
<td>Downstream of the reflected shock</td>
<td>57.5</td>
<td>3300</td>
</tr>
<tr>
<td>4</td>
<td>Separation bubble</td>
<td>59.5</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>Downstream of the impinging shock</td>
<td>63.4</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>Recovered flat plate boundary layer</td>
<td>70.0</td>
<td>-</td>
</tr>
</tbody>
</table>
Iso-surface of Q-criterion

Q = 0.5

Mean u

0.0  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9  1.0

National Aeronautics and Space Administration
Beyond $x = 60$, centerline skin friction oscillates about the theoretical turbulent skin friction, but span-averaged was marginally lower.

An even finer mesh should help provide a better match, but at a higher computational cost.
Density scaling necessary

Boundary layer has not reached an equilibrium state at \( Re_\theta = 3270 \)

\[ -\overline{uv^+} = 0.0008y^+ \] holds true, which was shown by Patel et al. [3]
Turbulent kinetic energy budget

Station 1

- Similar trends in comparison with the incompressible DNS, but current simulations show larger magnitude
- Can be attributed to the lack of mesh resolution at the simulated Reynolds number—finer mesh or lowering simulated Reynolds number will improve comparison
The production and dissipation terms increased by an order of magnitude aft of the reflected shock.

The trough in the buffer layer of the turbulent transport term moved closer to the wall and a new peak developed at the beginning of the log layer.
In the separation bubble, the peak in the production term moved away from the wall and into the beginning of the log layer.

The peak in the turbulent transport term also shifted away from the wall.

Aft of the impinging shock budget was reminiscent of the boundary layer forward of the interaction region.
A return to undisturbed pre-shock budget profiles

Production and turbulent transport terms remained active in the log and outer layer regions ($10^2 < y^+ < 10^3$) with secondary peaks and troughs.
Ratio of production-to-dissipation

- $\frac{P}{\epsilon}$ ratio does not become unity in the log layer
- But, a good match with the incompressible DNS in the viscous sublayer
Iso-surfaces of the production and dissipation
Conclusions

- Budgets of the turbulent kinetic energy calculated using the ILES framework
- Current fine mesh was inadequate to resolve the log and outer layer regions even at a reduced Reynolds number
- The station upstream of the interaction did not reach a complete equilibrium
- Matching trends with that of the incompressible DNS
- Improve spatial characterization of the turbulent kinetic energy in modeled form
- Current work: improved inflow and a finer mesh
Questions?
References


