Implementation of a Parameterized Interacting Multiple Model Filter on an FPGA for Satellite Communications

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Overview of Research

Diagram showing the satellite TDRS 12, the ScaN Testbed, and ground stations connected by different bands (Ku-band, Ka-band, S-band). Distances are marked: 36,000 km, 2,000 km, 425 km, and 200 km.
Targeted Platform

**JPL Radio**
- 66-MHz SPARC processor
- 2 x Xilinx Virtex-II FPGAs
- S-band transceiver
- L-Band (GPS) receiver

Interacting Multiple Model (IMM) Background

### Interacting Multiple Model (IMM) Background

<table>
<thead>
<tr>
<th>Step Number</th>
<th>Step Type</th>
<th>Iterations Needed</th>
<th>Expression Executed</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIX01</td>
<td>SF</td>
<td>1</td>
<td>$\hat{\mu}(k+1</td>
</tr>
<tr>
<td>MIX02</td>
<td>SF</td>
<td>1</td>
<td>$\pi^T \times \text{diag}(\mu(k))$</td>
</tr>
<tr>
<td>MIX03</td>
<td>SF</td>
<td>1</td>
<td>$\mu(k) = (\pi^T \times \text{diag}(\mu(k)))^T \times (\text{diag}(\mu(k+1</td>
</tr>
<tr>
<td>MIX04</td>
<td>SF</td>
<td>1</td>
<td>$\hat{X}_0(k</td>
</tr>
<tr>
<td>MIX05</td>
<td>DF</td>
<td>$R^2$</td>
<td>$\hat{X}_0(k</td>
</tr>
<tr>
<td>MIX06</td>
<td>DF</td>
<td>$R^2$</td>
<td>$(\hat{X}_0(k</td>
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<tr>
<td>MIX07</td>
<td>DF</td>
<td>$R^2$</td>
<td>$P_{j0}(k</td>
</tr>
<tr>
<td>KF01</td>
<td>DF</td>
<td>$R$</td>
<td>$\hat{X}_j(k+1</td>
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<tr>
<td>KF02</td>
<td>DF</td>
<td>$R$</td>
<td>$F \times P_{j0}(k</td>
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<tr>
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<td>DF</td>
<td>$R$</td>
<td>$P_j(k+1</td>
</tr>
<tr>
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<td>DF</td>
<td>$R$</td>
<td>$P_j(k+1</td>
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<td>$S_j(k+1) = H \times P_j(k+1</td>
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<td>DF</td>
<td>$R$</td>
<td>$K_j(k+1) = \frac{P_j(k+1</td>
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<td>DF</td>
<td>$R$</td>
<td>$P_j(k+1</td>
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<tr>
<td>KF08</td>
<td>DF</td>
<td>$R$</td>
<td>$e_j(k+1) = \frac{1}{2}(k+1) - H \times \hat{X}_j(k+1</td>
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<tr>
<td>KF09</td>
<td>DF</td>
<td>$R$</td>
<td>$\hat{X}_j(k+1</td>
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<td>$R$</td>
<td>$S_j^{-1}(k+1</td>
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<tr>
<td>COM02</td>
<td>SC</td>
<td>$R$</td>
<td>$-\frac{1}{2}e_j^T(k+1) \times S_j^{-1}(k+1</td>
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<tr>
<td>COM03</td>
<td>SC</td>
<td>$R$</td>
<td>$\exp(-\frac{1}{2}e_j^T(k+1) \times S_j^{-1}(k+1</td>
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<tr>
<td>COM04</td>
<td>SC</td>
<td>$R$</td>
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<tr>
<td>COM05</td>
<td>SC</td>
<td>$R$</td>
<td>$</td>
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<tr>
<td>COM06</td>
<td>SC</td>
<td>$R$</td>
<td>$L_j(k+1) = (2\pi)^{\frac{n}{2}}(</td>
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<tr>
<td>COM07</td>
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<td>$\text{diag}(L(k+1)) \times \hat{\mu}(k+1</td>
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<tr>
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<td>SF</td>
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<tr>
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<td>SF</td>
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<td>SF</td>
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<td>$\hat{X}(k+1</td>
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</table>
Implementation Trade Studies

\[
A = \begin{bmatrix}
a_{11} & a_{12} \\
a_{12} & a_{22}
\end{bmatrix} \quad A^{-1} = ?
\]

Integer (16b)

Fraction (32b)

Schur Complement Architecture

Sudarsanam, A. et al., *IET Computers & Digital Techniques*, 2010
## Schur Complement Analysis

<table>
<thead>
<tr>
<th>Multiplier Latency</th>
<th>Addition Latency</th>
<th>Divider Cycles Per Operation</th>
<th>Divider Latency</th>
<th>Occupied Slices</th>
<th>Slice Flip Flops</th>
<th>4-Input LUTs</th>
<th>Max Clock Frequency (MHz)</th>
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<td>14</td>
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<tr>
<td>3</td>
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<td>27</td>
<td>910</td>
<td>1201</td>
<td>1343</td>
<td>175.389</td>
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</tbody>
</table>
IMM Analysis

in_bus
load_en
in_mem_bank_addr
in_sub_mem_bank_addr
start
rst
clk

imm_filter

out_element
data_ready
rdy_to_load

N_BITS_PRECISION
N_X_ELEMENTS
N_KALMAN_FILTERS
in_mem_bank_addr
in_sub_mem_bank_addr
start
rst
clk

IMM Filter
IMM Controller
KF Controller
KF Controller
IMM Mem Bank
KF Mem Bank 1
KF Mem Bank 2
KF Mem Bank 3
KF Mem Bank 4
KF Mem Bank 5
MEM1
MEM2
MEM3
MEM4
MEM5

Architecture Controller

IMM PE Controller
LL PE Controller
IMM PE Controller 1
IMM PE Controller 2
IMM PE Controller 3

N_Z_ELEMENTS
N_X_ELEMENTS
N_KALMAN_FILTERS
N_PES_PER_KF_CONTROLLER
N_PES_PER_IMM_CONTROLLER
N_LUT_PRECISION_EXP_CALC
N_B_TO_I_CELL_BUFF_LENGTH
load_en
in_bus
rdy_to_load

PennState
Example Scenario

**State Model**
Constant Acceleration Model

**Available Measurements**
SNR, SIR

**System Models**
Clear Sky, Rain Fade, Ionospheric Scintillation, Tropospheric Scintillation, Multipath Fading
IMM Analysis

![Graph showing filter rate vs. number of processing elements for different KFs and Z Elements configurations.]

- KFs = 2, X Elements = 2, Z Elements = 3
- KFs = 2, X Elements = 2, Z Elements = 4
- KFs = 2, X Elements = 4, Z Elements = 2
- KFs = 6, X Elements = 2, Z Elements = 3

A, B, C = PE Controllers,
PEs Per KF Controller,
PEs Per IMM Controller
If you’re interested, contact me at: Tim Hackett: tmh5344@psu.edu