Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA’s scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA’s institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- **TECHNICAL PUBLICATION.** Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA's counterpart of peer-reviewed formal professional papers but has less stringent limitations on manuscript length and extent of graphic presentations.

- **TECHNICAL MEMORANDUM.** Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.

- **CONTRACTOR REPORT.** Scientific and technical findings by NASA-sponsored contractors and grantees.

- **CONFERENCE PUBLICATION.** Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or cosponsored by NASA.

- **SPECIAL PUBLICATION.** Scientific, technical, or historical information from NASA programs, projects, and mission, often concerned with subjects having substantial public interest.

- **TECHNICAL TRANSLATION.** English-language translations of foreign scientific and technical material pertinent to NASA’s mission.

Specialized services that complement the STI Program Office’s diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results…even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI program home page at <http://www.sti.nasa.gov>
- E-mail your question via the Internet to <help@sti.nasa.gov>
- Phone the NASA STI Help Desk at 757–864–9658
- Write to: NASA STI Information Desk Mail Stop 148 NASA Langley Research Center Hampton, VA 23681–2199, USA
# TABLE OF CONTENTS

1. INTRODUCTION ............................................................................................................. 1

2. BACKGROUND ............................................................................................................. 2
   2.1 Jacchia Models ............................................................................................................. 2
   2.2 Marshall Space Flight Center/Jacchia 70 Orbital Atmosphere Model ......................... 2
   2.3 Marshall Engineering Thermosphere Model ............................................................... 2

3. MODEL DESCRIPTION .................................................................................................. 4
   3.1 Overview ..................................................................................................................... 4
   3.2 Model Density Variations Associated With Thermosphere Heating ............................. 5
   3.3 Model Density Seasonal Variations ............................................................................. 7

4. MARSHALL ENGINEERING THERMOSPHERE 2007 MODEL UPDATES .................. 9
   4.1 Corrections to the Marshall Engineering Thermosphere Model Number Densities and Mean Molecular Weight ......................................................................................... 9
   4.2 Minor Improvements Regarding Model Inputs ........................................................... 10

5. PROGRAM USAGE ........................................................................................................ 11
   5.1 Input Parameters ......................................................................................................... 11
   5.2 Output Parameters ....................................................................................................... 13

APPENDIX A—NUMERICAL EXPRESSIONS FOR TEMPERATURE AND DENSITY CALCULATION ...................................................................................................... 14
   A.1 Expressions for the Temperature Profile ..................................................................... 14
   A.2 Density Calculations ................................................................................................... 17

REFERENCES ..................................................................................................................... 21
LIST OF FIGURES

1. Ratios of uncorrected/corrected molecular weight and ratios of uncorrected/corrected number density versus height ................................................................. 10

LIST OF TABLES

1. Input parameters .................................................................................................................................................. 11
2. Output parameters .................................................................................................................................................. 13
LIST OF SYMBOLS AND ACRONYMS

Ar  argon
EUV  extreme ultraviolet
GRAM  Global Reference Atmosphere Model
H  hydrogen
He  helium
J64  Jacchia 1964 (thermosphere model)
J70  Jacchia 1970 (thermosphere model)
J71  Jacchia 1971 (thermosphere model)
JD  Julian date
MET  Marshall Engineering Thermosphere
MET-2.0  Marshall Engineering Thermosphere, version 2.0
MSFC  Marshall Space Flight Center
N\textsubscript{2}  molecular nitrogen
O  atomic oxygen
O\textsubscript{2}  molecular oxygen
SAO  Smithsonian Astrophysical Observatory
UTC  coordinated universal time
NOMENCLATURE

$A$ temperature profile equation parameter variable
$A_p$ daily planetary average geomagnetic index
$a$ temperature profile inflection point equation coefficient
$a_p$ linearized 3-hour geomagnetic index
$B$ temperature profile equation parameter constant
$b$ temperature profile inflection point equation coefficient
$c$ temperature profile inflection point equation coefficient
$c_n$ temperature profile equation coefficient for altitude less than 125 km
$d$ number of days since January 1
$\bar{F}_{10.7}$ 162-day, smoothed 10.7 cm solar radio flux
$F_{10.7}$ 10.7-cm solar radio flux
$G_x$ temperature gradient at inflection altitude of 125 km
$G_{z_0}$ temperature gradient at 90 km altitude
$g$ acceleration of Earth’s gravity
$g_0$ acceleration of Earth’s gravity at sea level
$H$ hour angle of the Sun
$i$ individual atmospheric constituent (index)
$K_p$ logarithmic 3-hour geomagnetic index
$\bar{k}$ temperature profile inflection point equation coefficient
$\bar{M}$ mean molecular weight
$M_i$ constituent molecular weight
$m$ equation parameter constant
$N_{av}$ Avogadro’s number
NOMENCLATURE (Continued)

\[ n \]
- equation parameter constant

\[ n_i \]
- number density of atmosphere constituent

\[ n'_i \]
- corrected number density of atmosphere constituent

\[ n_T \]
- total number density

\[ P \]
- seasonal-latitudinal density equation parameter variable

\[ p \]
- diurnal exospheric temperature equation parameter constant

\[ q_0 \]
- volume fraction of atmospheric constituent

\[ q_0(i) \]
- individual constituent volume fraction

\[ R \]
- universal gas constant, parameter

\[ R_k \]
- diurnal exospheric temperature equation parameter constant

\[ T \]
- local atmospheric temperature

\[ T_0 \]
- temperature at 90 km altitude

\[ T_\infty \]
- exospheric temperature

\[ T_c \]
- global minimum exospheric temperature

\[ T_L \]
- diurnal exospheric temperature

\[ T_x \]
- temperature at inflection point of 125 km altitude

\[ T(z) \]
- temperature as a function of altitude

\[ Y \]
- length of tropical year

\[ z \]
- altitude variable

\[ z_0 \]
- altitude of lower temperature boundary = 90 km

\[ z_x \]
- altitude at temperature inflection point = 125 km

\[ \beta \]
- diurnal exospheric temperature equation parameter constant

\[ \gamma \]
- diurnal exospheric temperature equation parameter constant

\[ \Delta T_G \]
- exospheric temperature change associated with geomagnetic activity

\[ \Delta T_S \]
- exospheric temperature change associated with semiannual variation
NOMENCLATURE (Continued)

\( \delta_s \)  
Sun declination angle

\( \varepsilon \)  
obliquity of the ecliptic

\( \eta \)  
diurnal exospheric temperature equation parameter variable

\( \theta \)  
diurnal exospheric temperature equation parameter variable

\( \rho \)  
local mass density

\( \tau \)  
diurnal exospheric temperature equation parameter variable

\( \phi \)  
Earth latitude
1. INTRODUCTION

The region of the Earth’s atmosphere between about 90 and 500 km altitude is known as the thermosphere, while the region above about 500 km is known as the exosphere. For space vehicle operations, the neutral atmosphere in these regions is significant. Even at its low density, it produces torques and drags on vehicles and affects orbital lifetimes. The thermosphere density above 100 km altitude also modulates the flux of trapped radiation and orbital debris. Atomic oxygen at orbital altitudes is important because it can erode and chemically change exposed vehicle surfaces.

The NASA Marshall Engineering Thermosphere, version 2007 (MET-2007) is a computer program that provides atmospheric density and temperature for the altitude range of 90 to 2,500 km as a function of latitude, longitude, time, solar flux, and geomagnetic index. Like previous versions of the MET model, MET-2007 is a product of the Natural Environments Branch, NASA Marshall Space Flight Center (MSFC) and is used for defining the space environment’s neutral atmospheric density for vehicle design and mission analysis. MET-2007 retains the capability of the previous version (MET-2.0) but with several improvements. These improvements include a correction to remove the inconsistency between constituent number density and mass density at certain altitudes. Also, minor modifications were made to subroutines using time and date inputs to allow for the use of a continuous variable instead of discreet values.

The sections below provide background on the development of the MET models and a description of the model algorithms including the recent updates that were made.
2. BACKGROUND

2.1 Jacchia Models

In the mid-1960s, personnel at MSFC responsible for predicting the orbital decay of satellites began studies to determine which atmospheric model when combined with the appropriate orbit propagation computer programs would most accurately predict the observed altitude decay of satellites. Since the observed altitude decay histories were already available, a number of available models of the thermosphere were selected for use in appropriate computer programs. Because the satellites had decayed prior to the start of the study, actual values of solar proxy input parameters required by the models were used. These proxy parameters were representative of actual solar conditions that occurred during the decay periods of the satellites. The Jacchia 1964 (J64) thermosphere model when implemented into the orbit propagation programs provided the best performance statistically and, therefore, was selected for use by MSFC.

Almost coincidental with the completion of MSFC’s study, the Smithsonian Astrophysical Observatory (SAO) published the Jacchia 1970 (J70) thermosphere model. The J64 and J70 models were similar in their description of the thermosphere but the J70 model included improvements based on more recent observations. The major difference between the two models was that the J64 model temperature-induced density bulge remained on the equator all year while the J70 model bulge followed the latitudinal excursions of the Sun, which was supported by observations. Follow-on studies showed that the J70 model performed better than the J64 model when implemented in orbit propagation programs; thus, the J70 model became the baseline thermosphere model used by MSFC.

2.2 Marshall Space Flight Center/Jacchia 70 Orbital Atmosphere Model

In 1971, SAO published the Jacchia 1971 (J71) model. Although this model had several new features confirmed by observational data, overall, the model was not as representative of the atmosphere as the J70 model. An independent study by personnel at MSFC showed that the J70 model could be improved by adding the J71 model’s seasonal-latitudinal variations in the lower thermosphere density below 170 km and adding seasonal-latitudinal variations in helium (He) above 500 km altitude while retaining the orbital decay prediction accuracy of the J70 model. This new computerized version of the modified J70 model along with the supporting numerical ephemeris and integration algorithms was referred to as the MSFC/J70 Orbital Atmosphere Model.

2.3 Marshall Engineering Thermosphere Model

In the late 1980s, errors were detected in the output of the MSFC/J70 model and were traced to the implementation of the J71 modifications described above. These modifications introduced step function increases in density requiring fairing routines for their elimination. It was also
determined that there were additional errors attributed to the numerical integration routine. As a result, the computer code was rewritten with higher fidelity routines, made more understandable and user friendly, and upgraded from FORTRAN IV to FORTRAN 77. This new version of the code was named the Marshall Engineering Thermosphere (MET).\textsuperscript{5–7}

While reviewing the 1988 version of MET, it was determined that the model code would not correctly adapt to the change in the date of the approaching new millennium. While performing this update, it was decided to also update the calculation of the solar position to use the more recent standard epoch, J2000, which corresponds to January 1.5, 2000, or Julian date (JD) 2451545.0. An improvement to the model was also achieved in calculating the solar coordinates. A low precision ephemeris for the Sun can be calculated using relatively simple equations. This simpler scheme for computing the solar position requires fewer arithmetic calculations, yet it maintains more than adequate accuracy. This modified version of the MET was referred to as MET-1999.\textsuperscript{8} This version was also incorporated into the MSFC Global Reference Atmosphere Model (GRAM) 1999 version.

A later modification to the MET-1999 version of the model was introduced in the fairing subroutine which controls the transition from the lower altitude density profile to that of the upper altitude region where the seasonal-latitudinal variation of He is significant. The previous version of this subroutine made the transition in a series of small steps. The new scheme introduced in MET, version 2.0 (MET-V2.0)\textsuperscript{9} provides a smooth functional form for the transition.

The latest version of the MET model (MET-2007) incorporates changes discussed in section 4. In naming this latest version, it was decided to revert back to the original naming convention of previous versions. Since these latest updates were first implemented in the version of the MET model contained in GRAM, version 2007, the standalone updated version of MET was named MET-2007.
3. MODEL DESCRIPTION

3.1 Overview

The major source of heating in the thermosphere is from the absorption of the solar extreme ultraviolet (EUV) radiation. The temperature in the lower thermosphere increases rapidly with increasing altitude beginning at approximately 90 km. Eventually it becomes altitude independent at upper thermospheric altitudes. At these upper altitudes, this asymptotic temperature, known as the exospheric temperature, becomes constant with altitude due to the extremely short thermal conduction time. The EUV radiation heats only the dayside thermosphere, and although conductive and convective processes act to redistribute some of this energy, a large temperature gradient always exists between the daytime and nighttime thermosphere. An average daytime exospheric temperature is about 1060 K, and an average nighttime exospheric temperature is about 840 K.

An additional heat source for the thermosphere is the interaction of the solar wind with Earth’s magnetic field. This interaction causes energetic particles to penetrate down into the lower thermosphere at high geographic latitudes and directly heat the thermosphere by Joule heating and particle precipitation. These energetic particles are also responsible for the aurora seen at these high latitudes.

Whenever the neutral thermospheric gas is heated, it expands radially outward. Because the undisturbed thermospheric density decreases with increasing altitude, an outward expansion of the gas results in an increase of density at high altitudes. Thus, the daytime thermospheric density is greater than the nighttime density, while during times of geomagnetic storms, the high-latitude density is greater than it is during undisturbed periods. This anisotropic heating leads to the so-called diurnal and polar bulges, which were first inferred from the increased drag experienced by orbiting satellites.

Below the turbopause (located at about 105 km altitude), the atmosphere is well mixed by turbulence so that the composition of the atmosphere does not vary with altitude. Above the turbopause, however, diffusion becomes so rapid that the altitude variation of the various species becomes dependent on molecular weight, with the result that composition varies with altitude. Thus, the number densities of the heavier thermospheric species (molecular nitrogen (N₂) and molecular oxygen (O₂)) decrease with increasing altitude much faster than those of the lighter species (hydrogen (H) and He). This means that the heavier molecular species predominate in the lower thermosphere, while the lighter atomic species predominate in the upper thermosphere. Lifting of the thermosphere will cause the mean molecular weight at a given altitude to increase, while a sinking motion will cause it to decrease.

As stated above, the NASA MET-2007 model is based upon the J70 model with the inclusion of equations from the J71 model that characterize the effects of seasonal-latitudinal density
variations of density below 170 km altitude and that of He above 500 km. Data from satellite drag calculations were used to formulate the empirical functions used to calculate the thermosphere temperature and density. With the proper input parameters, the model calculates an empirically derived temperature profile between 90 and 2,500 km altitude for any location on the Earth. With the temperature profile specified, the density can be calculated for any altitude and location. The model assumes that mixing in the thermosphere prevails to an altitude of 105 km; thus, density between 90 and 105 km is calculated by the integration of the barometric equation which is a function of the temperature profile. For altitudes above 105 km, diffusion among the constituents is assumed to take place; thus, density is determined by the integration of the diffusion equation which is also a function of the temperature profile.

3.2 Density Variations Associated With Thermosphere Heating

As stated above, the temperature profile in the J70 model was not the measured quantity from satellite drag data. The modeled thermosphere temperature profile was an empirical functional fit that, when used in the appropriate density equation (barometric or diffusion equation), yielded densities that agreed with a database of satellite drag derived densities.

The thermosphere temperature as a function of altitude is empirically modelled by requiring the temperature at 90 km altitude to be constant and equal to 183 K. Then, from 90 km, the temperature increases to an inflection point at a fixed altitude equal to 125 km. Above 125 km altitude the temperature increases asymptotically to an exospheric temperature ($T_\infty$). The inflection temperature and temperature gradient at 125 km altitude is a function of $T_\infty$ as well as the shape of the temperature profile. Empirical derived equations using model inputs are used to calculate $T_\infty$. The heating and diurnal variations in thermospheric temperature are modeled by adjusting $T_\infty$. These variations include:

1. Variations with solar EUV radiation.
2. The diurnal variation.
3. Variations with geomagnetic activity.
4. The semiannual variation.

Thus, the exospheric temperature, $T_\infty$, is given by

$$T_\infty = T_L + \Delta T_G + \Delta T_S,$$

where $T_L$ is the exospheric temperature for a given location on the globe and is a function of the EUV heating and diurnal variation, $\Delta T_G$ is the variation in exospheric temperature associated with geomagnetic activity, and $\Delta T_S$ is the variation in exospheric temperature due to the observed semiannual density variation. A description of these variations is given below. The equations used in MET-2007 to calculate the exospheric temperature and its variations as well as the temperature profile as a function of altitude are given in references 1 and 8 and summarized in the appendix.
3.2.1 Solar Extreme Ultraviolet Activity

The solar EUV radiation that heats the thermosphere is modeled as having two components. One component is related to the active regions on the solar disk and the other is related to the overall clear disk itself. The active region component is associated with high-temperature solar regions of highly ionized atoms such as active coronal holes, flares, and eruptions. The active region component varies rapidly from day to day and is modulated by the 27-day rotation period of the Sun. Thus, density variations are often seen to vary with an approximately 27-day periodicity associated with active regions on the Sun rotating into view. Since EUV measurements are not readily measurable from the ground, a surrogate measurement is used that correlates well with solar EUV variations. This index is the 10.7 cm (2,800 MHz) solar radio flux ($F_{10.7}$) which has been measured routinely at ground observatories since 1947 and is an input to the model.

The EUV radiation from the clear disk component comes from less ionized atoms and varies over the course of the 11-year solar cycle. The effect the EUV heating is much less than the active region component. For a given increase in the active region component, the thermosphere temperature increases by a factor of 3 for the same increase in the disk component in EUV. To characterize the disk component of the EUV, $F_{10.7}$ is also used. It was found that the EUV disk component is linearly related to the average or smoothed radio flux over a least six solar rotations (162 days). Thus, the associated long-term density variation observed over a solar cycle can be described quite well by using the smoothed $F_{10.7}$ as an input to the model.

3.2.2 Diurnal Variation

Rotation of the Earth induces a diurnal (24-hour period) variation (diurnal tide) in thermospheric temperature and density. Due to a lag in response of the thermosphere to the EUV heat source, density maximizes around 2 p.m. local solar time at orbital altitudes at a latitude approximately equal to the subsolar point. The lag decreases with decreasing altitude. Similarly, minimum density occurs between 2 and 3 a.m. local solar time at about the same latitude in the opposite hemisphere. In the lowest regions of the thermosphere (120 km and below), where characteristic thermal conduction time is on the order of a day or more, the diurnal variation is not a predominant effect.

The diurnal variation in exospheric temperature is modelled in MET by assuming that the daytime maximum temperature occurs at a latitude equal to the Sun’s declination while the nighttime minimum temperature, $T_C$, is diametrically opposed to the maximum temperature at a latitude which is minus the Sun’s declination. The diurnal exospheric temperature, $T_L$, at any location on the globe is modeled as a function of $T_C$, latitude, the hour angle of the Sun, and the Sun’s declination. The temperature effect of the EUV heating is modelled in the calculation of $T_C$ by

$$T_C = 383 + 3.32 F_{10.7} + 1.8 (F_{10.7} - \bar{F}_{10.7}) ,$$

where $\bar{F}_{10.7}$ is the 162-day smoothed (six solar rotations) value of the daily average $F_{10.7}$. 

6
3.2.3 Geomagnetic Activity

Interaction of solar wind with the Earth’s magnetosphere (referred to as geomagnetic activity) leads to a high-latitude heat and momentum source for the thermospheric gases. Energetic particle interactions with the Earth’s magnetosphere dissipates energy in the form of Joule heating and particle precipitation into the lower thermosphere. This high-latitude heat source is effective during both the day and night. Although an intermittent source of energy for the thermosphere, it can at times exceed the global EUV energy absorbed by the thermosphere. In addition, though the energy is deposited at high latitudes (>60°), the disturbance effects are transmitted to lower latitudes through the actions of winds and waves. However, the disturbance effects at low latitudes are significantly smaller than they are at higher latitudes.

The interaction of the solar wind particles with the Earth’s magnetic field is detected as disturbances in the geomagnetic field which can be measured by ground-based magnetometers and are referred to as geomagnetic storms. Geomagnetic activity is observed to vary with the solar cycle, with more intense geomagnetic activity associated with larger solar cycle peaks.

The heating in the thermosphere associated with variations in geomagnetic activity is modelled in the MET model by adjusting the exospheric temperature. The variation is given by two relationships, one for the geomagnetic index $K_p$ and the other for $a_p$ (the linearized representation of $K_p$) which allows the user of the model to use either index as the input. These indices are based on magnetic fluctuation data reported every 3 hours at 12 stations between geomagnetic latitudes 48° and 63° selected to provide good longitude coverage. The daily planetary geomagnetic index ($A_p$) is the average of the eight 3-hour $a_p$ values measured during a day. There is a time lag between changes in the geomagnetic indices and temperature variations which averages approximately 6.7 hours. Thus, the index that is 6–8 hours before the time of the desired density to be calculated should be used in the model. Section 5 describes how to select input values for varying applications.

3.2.4 Semiannual Variation

The semiannual variation is not well understood. The fundamental processes driving the variation are probably a combination of EUV and Joule heating, meridional transport and mixing, variations in lower thermosphere composition, and turbopause height. The variation is observed to be latitudinally independent and is modified by compositional effects. Amplitude of the variation is height dependent and variable from year to year with a primary minimum in July, a primary maximum in October, a secondary minimum in January, and a secondary maximum in April. The magnitude and altitude dependence of the semiannual oscillation varies considerably from one solar cycle to another. This variation is important at orbital altitudes and is modeled in the MET model as a temperature variation by adjusting the exospheric temperature. The variation in temperature is a function of the smoothed $F_{10.7}$ and the day of the year.

3.3 Model Density Seasonal Variations

There are two seasonal-latitudinal variations in density that are not accounted for by the adjustments to the exospheric temperature described above. These are the seasonal-latitudinal variations of the lower thermosphere below 170 km altitude and the seasonal-latitudinal variations
of He above 500 km altitude. These density adjustments are added to the density calculated from the barometric or diffusion equation.

The variations discussed in the previous sections are modelled by adjusting the exospheric temperature and keeping the lower boundary temperature fixed at 183 K at 90 km altitude. In reality, the density at this altitude does vary. The lower thermosphere seasonal-latitudinal variations are driven by the dynamics of the lower atmosphere (mesosphere and below). Amplitude of the variation maximizes in the lower thermosphere between about 105 and 120 km and diminishes to zero around 200 km. Although the temperature oscillation amplitude is quite large, corresponding density oscillation amplitude is small. This variation is not important at orbital altitudes. This variation is modelled as a function of latitude, day number, and altitude below 170 km. The describing equation was adopted from the J71 model.

Seasonal variations in He measured by satellite mass spectrometers have indicated a strong increase in He above the winter pole. Over a year the He number density has been observed to vary by a factor of about 40 at 275 km, 10 at 400 km, and 3 or 4 above 500 km. Formation of the winter He bulge is primarily due to effects of global scale winds that blow from the summer to the winter hemisphere. Amplitude of the bulge decreases with increasing levels of solar activity due to increased effectiveness of exospheric transport above 500 km carrying He back to the summer hemisphere. This variation is modelled as a function of latitude, Sun declination, and altitude above 500 km. The describing equation was also adopted from the J71 model.
4. MARSHALL ENGINEERING THERMOSPHERE 2007 MODEL UPDATES

4.1 Corrections to the Marshall Engineering Thermosphere Model Number Densities and Mean Molecular Weight

Output from the MET model includes exospheric temperature, local mass density ($\rho$), temperature ($T$), pressure ($P$), number densities ($n_i$, $i = 1, 6$) for the six MET model constituents ($N_2$, $O_2$, atomic oxygen (O), argon (Ar), He, and H), and mean molecular weight ($\overline{M}$). If $q_0(i)$ denotes the individual constituent volume fractions, then number density ($n_i$) is related to mass density by

$$n_i = q_0(i) \rho \frac{N_{av}}{\overline{M}},$$

where $N_{av}$ is Avogadro’s number. If $M_i$ denotes the individual species molecular weights, then the mean molecular weight is related to species’ molecular weights and number densities by

$$\overline{M} = \sum_{i=1}^{6} M_i \frac{n_i}{\sum_{i=1}^{6} n_i}.$$

In examining the MET model constituent output, it was noticed that there were discrepancies between mass density and number densities as implied by equation (3), or between mean molecular weight and number densities as implied by equation (4). These errors are illustrated in figure 1, which plots ratios of MET model number density divided by correct number density, or MET mean molecular weight divided by correct molecular weight calculated over the globe as a function of altitude. Errors in number density of up to 25% are evident below 170 km altitude, while errors in mean molecular weight of up to 25% are seen above 440 km altitude. The broadening of the curve is due to a dependence of the error on latitude and longitude as well as height.

Number density discrepancies illustrated in figure 1 were traced to MET subroutine SLV, which computes seasonal-latitudinal variation in mass density below 170 km. This subroutine adjusted mass density for these effects, but did not adjust number densities to maintain the proper relationship of equation (3). This error was corrected in MET by replacing the original number densities with new values ($n'_i$) below 170 km, where

$$n'_i = n_i \rho \frac{N_{av}}{\sum_{i=1}^{6} M_i n_i}.$$

Above 440 km, the error was traced to MET subroutine SLVH, which computes seasonal-latitudinal variations in He number density. This subroutine adjusted He number density but did not adjust mean molecular weight to retain consistency required by equation (4). This error was corrected by replacing the original value of mean molecular weight above 440 km with a new value given by equation (4). Note that the mass density from the MET model did not require adjustment due to either of these two errors.
Figure 1. Ratios of uncorrected/corrected molecular weight and ratios of uncorrected/corrected number density versus height.

4.2 Minor Improvements Regarding Model Inputs

With the implementation of MET-2007 into GRAM-2007, small discontinuities in densities at year boundaries, day boundaries, and with time resolutions finer than 1 minute were observed. Modifications made to the model to address these issues are listed as follows:

(1) Treatment of day-of-year as a continuous variable in the semiannual variation term. To alleviate discontinuities of density at day boundaries in this routine, the input of a continuous variable (day plus day fraction) was implemented.

(2) Treatment of year as either 365 or 366 days in length (as appropriate), rather than all years having a length of 365.2422 days.

(3) Allowed continuous variation of time input, rather than limiting time increments to integer minutes.
5. PROGRAM USAGE

5.1 Input Parameters

Four basic types of input parameters are required by the model: time, location, solar activity, and geomagnetic activity. These may be provided to the model using the sample driver program provided, which reads an input data file, puts the data into the INDATA array, and calls the primary subroutine MET. The user can choose to replace the driver program or to integrate the MET model into a larger program to suit the needs of the project. Note that the INDATA array—and therefore all of its elements—is REAL*4. Table 1 presents a summary of the contents of the input array. Note that the year is input as a four-digit number, while the month, day, hour, and minute are all required to be input as two-digit numbers. The first column contains the array element number, the second column identifies the corresponding parameter, and the third column indicates the appropriate range of values, where applicable. Note that INDATA 9 controls whether INDATA 12 is interpreted as $a_p$ or $K_p$.

<table>
<thead>
<tr>
<th>Input Variable</th>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>INDATA (1)</td>
<td>Altitude</td>
<td>90 to 2,500 km</td>
</tr>
<tr>
<td>INDATA (2)</td>
<td>Latitude</td>
<td>-90° to 90°</td>
</tr>
<tr>
<td>INDATA (3)</td>
<td>Longitude</td>
<td>-180° to 180°</td>
</tr>
<tr>
<td>INDATA (4)</td>
<td>Year</td>
<td>1950 to 2050</td>
</tr>
<tr>
<td>INDATA (5)</td>
<td>Month</td>
<td>01 to 12</td>
</tr>
<tr>
<td>INDATA (6)</td>
<td>Day</td>
<td>01 to 31</td>
</tr>
<tr>
<td>INDATA (7)</td>
<td>Hour</td>
<td>00 to 23</td>
</tr>
<tr>
<td>INDATA (8)</td>
<td>Minute</td>
<td>00 to 59</td>
</tr>
<tr>
<td>INDATA (9)</td>
<td>Geomagnetic index type</td>
<td>$a_p$: 2; $K_p$: 1</td>
</tr>
<tr>
<td>INDATA (10)</td>
<td>$F_{10.7}$</td>
<td>0 to $400 \times 10^4$ Jansky</td>
</tr>
<tr>
<td>INDATA (11)</td>
<td>Smoothed $F_{10.7}$</td>
<td>0 to $250 \times 10^4$ Jansky</td>
</tr>
<tr>
<td>INDATA (12)</td>
<td>Geomagnetic index</td>
<td>$a_p$: 0 to 400; $K_p$: 0 to 9</td>
</tr>
</tbody>
</table>

There are three general categories of applications for the MET model: (1) After-the-fact calculations of densities, (2) real-time (or near real-time) calculations of densities, and (3) future applications in which density estimates are required for some time in the future. In each case, the time and location inputs are selected and entered according to the same rules. When entering the time, the year is entered in four-digit form, while the month, day, hour, and minute for which the calculation is to be done are each entered in two-digit form with the hour and minute being the coordinated universal time (UTC). For the position desired, enter the altitude in kilometers and geographic latitude and longitude in decimal degrees of the spacecraft location at the time of
application. Note that the longitude is measured from the Greenwich meridian eastward. The selection of inputs for the solar and geomagnetic activity parameters depends upon which application category applies to the calculation as discussed below.

5.1.1 After-The-Fact Calculations

This type of calculation is required for times more than 81 days in the past, such as the analysis of data from a space mission or testing model performance using observed orbital parameters for one or more spacecraft. In this case, solar activity is specified using the previous day’s value of the \( F_{10.7} \) and the centered (about the day for which the calculation is to be done) average of the \( \overline{F}_{10.7} \) over six solar rotations (162 days), referred to as the 162-day smoothed value. The geomagnetic index, either \( a_p \) or \( K_p \) is the 3-hour index value from 6 to 7 hours prior to the time of application.

5.1.2 Real-Time Calculations

When the application requires that the density calculations be made for the current time, or less than 81 days in the past or future, averaging the observed \( F_{10.7} \) over six solar rotations, as described above, is clearly not possible. In this case, the previous day’s value of the \( F_{10.7} \) is still used for the daily \( F_{10.7} \) value. For the smoothed value, observed values are used where available, and the estimated future smoothed value of the \( \overline{F}_{10.7} \) (typically a 13-month smoothed value) is used as an estimate for the balance of the time period where observed values are not available. These estimated future values of \( F_{10.7} \) are published by the Natural Environments Branch at MSFC and are available electronically via the internet (<http://sail.msfc.nasa.gov/>). The observed \( a_p \) or \( K_p \) from 6 to 7 hours prior to the time of application is still appropriate for this application category except for the time period up to 81 days in the future, when the estimated future smoothed value of \( a_p \) is used.

5.1.3 Future Calculations

When the calculations of densities must be done for a date more than 81 days in the future; e.g., for estimating orbital lifetimes, the problem becomes more dependent upon estimates of future average values for the \( F_{10.7} \) and the average daily planetary geomagnetic index, \( A_p \). Neither the daily nor the 162-day average \( \overline{F}_{10.7} \) is known from observations, nor is the 3-hour geomagnetic activity. In this case, the future estimate of the smoothed value of \( F_{10.7} \) is used as an estimate of both the previous day’s value and of the 162-day average value. Similarly, for the input, a future estimate of the daily or smoothed index is used as an estimate of the 3-hour geomagnetic index input.
For future neutral density calculations, the accuracy of the calculation depends primarily upon two factors:

(1) The ability of the thermospheric model to represent the observed neutral density using the observed values of solar radio flux (proxy for solar EUV heating) and geomagnetic activity used in the development of the model.

(2) The accuracy with which future solar radio flux and geomagnetic activity can be estimated for use as inputs to the thermospheric model.

5.2 Output Parameters

Upon completion of the calculation, the MET model passes the results back to the calling routine through the OUTDATA array which is REAL*4. These calculated results include the exospheric temperature, temperature at the input altitude ($z$), number densities for each constituent, the mean molecular weight, total mass density and its logarithm, and total pressure. All of these parameters are expressed in mks units. Table 2 details the output array. The first column identifies which element of the output array contains the data for the parameter in the second column. The total mass density, temperature, and individual species number densities all have the same phase variation in the MET model.

Table 2. Output parameters.

<table>
<thead>
<tr>
<th>Output Variable</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUTDATA (1)</td>
<td>Exospheric temperature (K)</td>
</tr>
<tr>
<td>OUTDATA (2)</td>
<td>Temperature (K)</td>
</tr>
<tr>
<td>OUTDATA (3)</td>
<td>$N_2$ number density (m$^{-3}$)</td>
</tr>
<tr>
<td>OUTDATA (4)</td>
<td>$O_2$ number density (m$^{-3}$)</td>
</tr>
<tr>
<td>OUTDATA (5)</td>
<td>$O$ number density (m$^{-3}$)</td>
</tr>
<tr>
<td>OUTDATA (6)</td>
<td>$Ar$ number density (m$^{-3}$)</td>
</tr>
<tr>
<td>OUTDATA (7)</td>
<td>$He$ number density (m$^{-3}$)</td>
</tr>
<tr>
<td>OUTDATA (8)</td>
<td>$H$ number density (m$^{-3}$)</td>
</tr>
<tr>
<td>OUTDATA (9)</td>
<td>Average molecular weight</td>
</tr>
<tr>
<td>OUTDATA (10)</td>
<td>Total mass density (kg/m$^3$)</td>
</tr>
<tr>
<td>OUTDATA (11)</td>
<td>Log$_{10}$ mass density (kg/m$^3$)</td>
</tr>
<tr>
<td>OUTDATA (12)</td>
<td>Total pressure (Pa)</td>
</tr>
</tbody>
</table>
APPENDIX A—NUMERICAL EXPRESSIONS FOR TEMPERATURE AND DENSITY CALCULATION

A.1 Expressions for the Temperature Profile

A.1.1 Temperature Profile Boundary Conditions

The temperature as a function of altitude $T(z)$ is defined empirically by requiring the temperature ($T_0$) at altitude $z_0 = 90$ km to be constant. From 90 km, the temperature increases to an inflection point at a fixed altitude $z_x = 125$ km, having a temperature $T_x$ and a gradient of $G_x$. Above 125 km altitude, the temperature increases asymptotically to an exospheric temperature, $T_\infty$. The temperature $T_x$ is a function of $T_\infty$ and the gradient $G_x$ is a function of $T_x$; thus, also of $T_\infty$. These boundary values are defined as

$$T_0 = 183 \text{ K}$$  \hspace{1cm} (6)

and

$$T_x = a + b T_\infty + c \exp\left(\bar{k} T_\infty\right),$$  \hspace{1cm} (7)

where

- $a = 444.3807$
- $b = 0.02385$
- $c = -392.8292$
- $\bar{k} = -0.0021357$.

The temperature gradient is defined as

$$G_{z_0} = 0$$  \hspace{1cm} (8)

and

$$G_x = 1.90 \frac{T_x - T_0}{z_x - z_0}. $$  \hspace{1cm} (9)

The exospheric temperature is a function of the EUV heating and diurnal variation ($T_L$), the variation in geomagnetic activity ($\Delta T_G$), and the semiannual seasonal variation ($\Delta T_S$), and described as

$$T_\infty = T_L + \Delta T_G + \Delta T_S.$$

Each of these contributions are described in sections A.1.1.1 through A.1.1.3.
A.1.1.1 Diurnal Variation in Exospheric Temperature. The global diurnal exospheric minimum temperature is a function of the EUV heating in the upper atmosphere. $F_{10.7}$ is used as a proxy index for the EUV variations:

$$T_C = 383 + 3.32F_{10.7} + 1.8\left(F_{10.7} - \bar{F}_{10.7}\right). \quad (11)$$

The exospheric temperature due to the diurnal variation for a given latitude $\phi$ and longitude (expressed in terms of the local hour angle $H$) on the Earth is given by

$$T_L = T_c\left(1 + R_k \sin^m \theta\right) 1 + R_k \frac{\cos^m \eta - \sin^m \theta}{1 + R_k \sin^m \theta} \cos^n (\tau / 2), \quad (12)$$

where

$$\eta = \frac{1}{2}\left|\phi - \delta_s\right|$$

$$\theta = \frac{1}{2}\left|\phi + \delta_s\right|$$

$$\tau = H + \beta + p \sin(H + \gamma)$$

and

$$R_k = 0.31$$

$$m = 2.5$$

$$n = 3$$

$$\delta_s = \text{declination of the Sun (degrees)}$$

$$H = \text{hour angle of the Sun (degrees)}$$

$$\beta = -3.7^\circ$$

$$p = 6^\circ$$

$$\gamma = 43^\circ.$$  

A.1.1.2 Geomagnetic Variation in Exospheric Temperature. The adjustment to exospheric temperature associated with geomagnetic activity is given by two equations, one as a function of $K_p$ and the other as a function of $a_p$ depending on which geomagnetic index is used as a model input:

$$\Delta T_G = 28K_p + 0.03 \exp(K_p) \quad (13)$$

and

$$\Delta T_G = a_p + 100\left[1 - \exp(-0.08a_p)\right]. \quad (14)$$
where $K_p$ is the 3-hour logarithmic geomagnetic index and $a_p$ is the 3-hour linear geomagnetic index. There is a time lag of 6.7 hours between changes in the geomagnetic indices and temperature.

**A.1.1.3 Semiannual Variation in Exospheric Temperature.** The adjustment to the exospheric temperature due to the semiannual variation is given by

$$\Delta T_S = 2.41 + F_{10.7} \left[ 0.349 + 0.206(\sin(360^\circ \tau + 226.5^\circ)) \right] \sin(720^\circ \tau + 247.6^\circ),$$

(15)

where

$$\tau = \frac{d}{Y} + 0.1145 \left[ 0.5 \left( 1 + \sin \left( \frac{360d}{Y} + 342.3^\circ \right) \right)^{2.16} - 0.5 \right] (d = \text{number of days since January 1 and } Y = \text{length of the tropical year (365.2422 days).})$$

**A.1.2 Temperature Profile**

The temperature profile from $90 < z < 125$ km is defined as

$$T(z) = T_x + \sum_{n=1}^{4} c_n (z - z_x)^n,$$

(16)

where $T_x$ is given by equation (7) and the coefficients $c_n$ are given by:

$$c_1 = 1.9 \frac{(T_x - T_0)}{(z_x - z_0)},$$

$$c_2 = 0,$$

$$c_3 = -1.7 \frac{(T_x - T_0)}{(z_x - z_0)^3},$$

and

$$c_4 = -0.8 \frac{(T_x - T_0)}{(z_x - z_0)^4}.$$  

The temperature profile for $z > 125$ km is given by:

$$T(z) = T_x + A \tan^{-1} \left( \frac{G_x}{A} (z - z_x) \left[ 1 + B (z - z_x)^n \right] \right),$$

(17)
where
\[ G_x \text{ is given by equation (9)}, \]
\[ A = \frac{2}{\pi} (T_\infty - T_x) \]
\[ B = 4.5 \times 10^{-6} \text{ (for } z \text{ in km)} \]
\[ n = 2.5. \]

A.2 Density Calculations

A.2.1 Density Between 90 and 105 km Altitude

Density \( \rho(z) \) at altitudes \( z \) between 90 and 105 km is calculated by integrating the barometric equation:

\[
\rho(z) = \rho(z_0) \frac{T(z_0)}{\bar{M}(z_0)} \frac{T(z)}{\bar{M}(z)} \exp \left( -\int_{z_0}^{z} \frac{\bar{M}(z)g(z)}{RT(z)}\,dz \right),
\]

where
\[ \bar{M} = \text{mean molecular weight (function of altitude } z) \]
\[ T = \text{local temperature (function of altitude } z) \]
\[ R = \text{universal gas constant (8.31432 J K}^{-1} \text{ mole}^{-1}). \]

The values for the parameters at the lower boundary \( z = z_0 = 90 \) km are:
\[
\rho(z_0) = 3.46 \times 10^{-9} \text{ g/cm}^3
\]
\[ T(z_0) = 183 \text{ K} \]
\[ \bar{M}(z_0) = 28.878 \text{ g/mole}. \]

The value of \( g \) is calculated by the expression

\[
g(z) = \frac{g_0}{\left(1 + \frac{z}{R_E}\right)^2},
\]

where \( g_0 = \text{acceleration of gravity at sea level, 9.80665 m/s}^2 \), and \( R_E = \text{is the Earth radius, 6.356766} \times 10^6 \text{ m.} \)
The profile for the mean molecular weight ($\bar{M}$) is given by the empirical equation

$$\bar{M}(z) = \sum_{n=0}^{6} c_n (z-100)^n,$$

(20)

where the coefficients $c_n$ are:

- $c_0 = 28.15204$
- $c_1 = -0.085586$
- $c_2 = 1.2840 \times 10^{-4}$
- $c_3 = -1.0056 \times 10^{-5}$
- $c_4 = -1.0210 \times 10^{-5}$
- $c_5 = 1.5044 \times 10^{-6}$
- $c_6 = 9.9826 \times 10^{-8}$.

The total number density ($n_T$) for altitudes between 90 and 105 km is calculated from the total mass density $\rho$ by

$$n_T = \rho \frac{N_{av}}{\bar{M}},$$

(21)

where $N_{av}$ is Avogadro’s number.

The number density for the constituents $N_2$, Ar, and He is given by

$$n_i = q_0(i) \frac{\bar{M}}{\bar{M}(z_0)} n_T,$$

(22)

and for O and $O_2$, respectfully,

$$n(O) = 2n_T \left(1 - \frac{\bar{M}}{\bar{M}(z_0)}\right),$$

(23)

and

$$n(O_2) = n_T \left[\frac{\bar{M}}{\bar{M}(z_0)}(1 + q_0(O_2))\right]^{-1},$$

(24)

where the volume fraction $q_0(i)$ of the constituents at $z = z_0 = 90$ km is given as:

- $N_2 = 0.78110$
- $O_2 = 0.20955$
- $Ar = 0.00934$
- $He = 1.289 \times 10^{-5}$. 
A.2.2 Density Above 105 km Altitude

The density above 105 km is obtained by integrating the diffusion equation to the altitude of interest \((z)\) to obtain the number densities of the individual atmospheric constituents, \(n_i\). The diffusion equation is given by

\[
n_i(z) = n_i(105) \left( \frac{T(105)}{T(z)} \right)^{1+a} \exp \left( -\frac{M_i}{R} \int_{105}^{z} \frac{g(z)}{T(z)} \, dz \right),
\]

(25)

where \(i\) denotes the individual constituents \((\text{N}_2, \text{O}_2, \text{O}, \text{Ar}, \text{He}, \text{or} \text{H})\). The parameter \(a\) is the thermal diffusion factor which is zero for all species except for He.

For He, the value of \(-0.380\) is used. The parameter \(M_i\) is the molecular weight of the constituent that is being calculated.

The number density of H is assumed to be negligible below 500 km. Thus, for H, the diffusion equation is integrated from a lower boundary of 500 km to the altitude of interest. The H number density at 500 km is given by:

\[
\log_{10} n_H(500) = 73.13 - 39.40 \log_{10} T_\infty + 5.5 \left( \log_{10} T_\infty \right)^2,
\]

(26)

where \(T_\infty\) is the exospheric temperature as defined in equation (10). The total mass density is obtained from the number densities of the constituents by the following:

\[
\rho(z) = \sum_{i=1}^{6} \frac{M_i(z) n_i(z)}{N_{av}},
\]

(27)

thus, the mean molecular weight is given by

\[
\bar{M}(z) = \frac{\rho(z) N_{av}}{\sum_{i=1}^{6} n_i(z)}.
\]

(28)

A.2.3 Seasonal-Latitudinal Density Variations

The lower thermosphere density adjustment for altitudes below 170 km from the J71 model is given by

\[
\Delta \log_{10} \rho(z) = S(z) \frac{\phi}{\phi_0} P \sin^2 \phi,
\]

(29)

where

\[
S(z) = 0.014(z - 90) \exp \left[ -0.0013(z - 90)^2 \right] \quad \text{and} \quad P = \sin \left[ \frac{2\pi}{Y} (d + 100) \right].
\]
As before, \( d \) is the number of days since January 1 and \( Y \) is the length of the tropical year (365.2422 days).

The He number density adjustment for altitudes above 500 km from the J71 model is given by

\[
\Delta \log_{10} n(\text{He}) = 0.65 \left( \frac{\delta_S}{\epsilon} \right)^3 \left[ \sin^3 \left( \frac{\pi}{4} - \frac{\phi}{2} \frac{\delta_S}{\delta_S} \right) - \sin^3 \frac{\pi}{4} \right],
\]

where \( \epsilon \) is the obliquity of the ecliptic, \( \delta_S \) is the Sun declination, and \( \phi \) is the latitude.
REFERENCES


**R.J. Suggs and R.M. Suggs**

George C. Marshall Space Flight Center
Huntsville, AL 35812

National Aeronautics and Space Administration
Washington, DC 20546–0001

Unclassified-Unlimited
Subject Category 46
Availability: NASA STI Information Desk (757–864–9658)

Prepared by the Spacecraft & Vehicle Systems Department, Engineering Directorate

This Technical Memorandum documents the latest version of the Marshall Engineering Thermosphere (MET) computer program, version 2007. Background about the thermosphere model including the numerical equations that make up the computer code and the evolution of the model from previous versions to the current version is given. Also, the use of the model is described including the input and output parameters needed to execute the computer program.

### Marshall Engineering Thermosphere model, MET model, thermosphere model

<table>
<thead>
<tr>
<th>16. SECURITY CLASSIFICATION OF:</th>
<th>17. LIMITATION OF ABSTRACT</th>
<th>18. NUMBER OF PAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. REPORT U</td>
<td>U</td>
<td>34</td>
</tr>
<tr>
<td>b. ABSTRACT U</td>
<td>U</td>
<td></td>
</tr>
<tr>
<td>c. THIS PAGE U</td>
<td>U</td>
<td></td>
</tr>
</tbody>
</table>

STI Help Desk at email: help@sti.nasa.gov
STI Help Desk at: 757–864–9658

**STI Help Desk at email: help@sti.nasa.gov**
**STI Help Desk at: 757–864–9658**
Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- **TECHNICAL PUBLICATION.** Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA's counterpart of peer-reviewed formal professional papers but has less stringent limitations on manuscript length and extent of graphic presentations.

- **TECHNICAL MEMORANDUM.** Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.

- **CONTRACTOR REPORT.** Scientific and technical findings by NASA-sponsored contractors and grantees.

- **CONFERENCE PUBLICATION.** Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or cosponsored by NASA.

- **SPECIAL PUBLICATION.** Scientific, technical, or historical information from NASA programs, projects, and mission, often concerned with subjects having substantial public interest.

- **TECHNICAL TRANSLATION.** English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office's diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results...even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI program home page at <http://www.sti.nasa.gov>

- E-mail your question via the Internet to <help@sti.nasa.gov>

- Phone the NASA STI Help Desk at 757–864–9658

- Write to:
  NASA STI Information Desk
  Mail Stop 148
  NASA Langley Research Center
  Hampton, VA 23681–2199, USA