BENCHMARKING THE COLLOCATION STAND-ALONE LIBRARY AND TOOLKIT (CSALT)

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AGENDA

- Motivation
  - Optimal Control Problems
  - CSALT
  - General Optimal Control Benchmarking Results
  - Low-Thrust Trajectory Design Benchmarking Results
  - Summary/Future Work
Physical systems which involve time varying control decisions can rarely be solved analytically

Low-thrust spacecraft trajectories cannot be designed using intuition

NASA Goddard’s General Mission Analysis Toolkit (GMAT) has limited capability to solve low-thrust problems

Goals:
- Demonstrate that CSALT can solve industry-standard optimal control problems
- Demonstrate that CSALT can solve optimal control low-thrust trajectory problems
- Compare CSALT execution efficiency to other optimal control software packages
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Minimize a cost function of the form:

\[ J = \Phi \left( x(t_o), t_o, x(t_f), t_f \right) + \int_{t_o}^{t_f} \lambda(x(t), u(t), t) dt \]

Subject to the following set of ordinary differential equations:

\[ \dot{x}(t) = a(x(t), u(t), t) \]

Subject to algebraic path constraints:

\[ p(x(t), u(t), t) \leq 0 \]

Subject to boundary conditions:

\[ b(x(t_o), u(t_o), x(t_f), u(t_f), t_o, t_f) \leq 0 \]
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COLLOCATION STAND-ALONE LIBRARY AND TOOLKIT

- 17000 original source lines of code (SLOC) written in C++
- Uses roughly 17000 SLOC from GMAT’s utility code base
- Uses Boost C++ library for sparse matrix arithmetic
- Uses SNOPT for nonlinear programming optimization
- Will be open source released with GMAT eventually (Fall 2017 or Spring 2018)
CURRENT CSALT CAPABILITY

- Multiple collocation transcriptions
  - Trapezoid
  - Hermite-Simpson
  - Lobatto IIIa of order 4, 6 and 8
  - Radau Orthogonal
- Multiple cost-function formulations
  - Mayer
  - Lagrange
  - Bolza
- Algebraic path and point constraints
- Decision vector, cost and constraint scaling
- Analytical collocation derivatives with finite differenced user point and path functions
- Automatic sparsity pattern determination
- Mesh refinement (Radau transcription only)
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HIGH LEVEL RESULTS

- Three comparison tools:
  - SOS - Sparse Optimization Suite written by Dr. John Betts
  - GPOPS-II – Gauss Pseudospectral OPTimization Software written by Dr. Anil Rao
  - PSOPT – PseudoSpectral OPTimization written by Dr. Victor Becerra
- 17 problems selected for objective function comparison
- 3 problems selected for detailed state and control comparison

### Test Case “Truth” Source Difference

<table>
<thead>
<tr>
<th>Test Case</th>
<th>“Truth” Source</th>
<th>Difference</th>
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</thead>
<tbody>
<tr>
<td>Rayleigh (Control Constraint)</td>
<td>SOS</td>
<td>-4.03E-08</td>
</tr>
<tr>
<td>Rayleigh (Control + State Constraint)</td>
<td>SOS</td>
<td>6.59E-08</td>
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<tr>
<td>Goddard Rocket SOS /GPOPS</td>
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<td>-8.57E-09</td>
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<tr>
<td>Hypersensitive SOS /GPOPS</td>
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<td>-1.39E-06</td>
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<tr>
<td>Conway Orbit SOS</td>
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<td>-2.11E-08</td>
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<tr>
<td>Linear Tangent Steering SOS /GPOPS</td>
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<td>-1.19E-09</td>
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<tr>
<td>Brachistichrone SOS /GPOPS</td>
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<td>Bryson Denham GPOPS /PSOPT</td>
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<tr>
<td>Schwartz PSOPT</td>
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<td>4.38E+00</td>
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<tr>
<td>Interior Point PSOPT</td>
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<td>4.53E-08</td>
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<td>Bryson Max Range PSOPT</td>
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<tr>
<td>Obstacle Avoidance PSOPT</td>
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<td>Moon Lander PSOPT</td>
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<td>Rau Automatica Analytic</td>
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<td>4.70E-12</td>
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<td>Hull Problem 9.5 Analytic</td>
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<td>4.00E-12</td>
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### Problem Name Tool Max. Rel. State Error Max. Rel. Control Error

<table>
<thead>
<tr>
<th>Problem Name</th>
<th>Tool</th>
<th>Max. Rel. State Error</th>
<th>Max. Rel. Control Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goddard</td>
<td>SOS</td>
<td>6.863e-07</td>
<td>2.191e-08</td>
</tr>
<tr>
<td>Hypers.</td>
<td>GPOPSII</td>
<td>4.305e-02</td>
<td>2.911e-01</td>
</tr>
<tr>
<td>Conway</td>
<td>SOS</td>
<td>9.814e-06</td>
<td>1.610e-02</td>
</tr>
</tbody>
</table>
CONWAY ORBIT

- Classic problem in the literature
- Finite-thrust orbit raising
- Optimal solutions match perfectly to the scale of the graphics
- Maximum relative error in state is better than control
  - $9.841\times10^{-6}$ in state
  - $1.610\times10^{-2}$ in control
- Multi-phase problem
- Control in optimal solution is discontinuous and has bang-off structure
- Control varies in the middle phase to maintain constant terminal velocity
- CSALT solution matches very well with reference software in terms of relative error
  - 6.863e-7 in state
  - 2.191e-8 in control

![Graphs showing altitude, velocity, and thrust over time for CSALT and SOS models.](image-url)
HYPERSENSITIVE PROBLEM

- Stressing case to the mesh refinement algorithm
- Dynamics rapidly change near the boundary conditions, but are nearly constant in between
- Relative errors are larger than desirable
  - $4.305 \times 10^{-2}$ in state
  - $2.911 \times 10^{-1}$ in control
- Believed to be due to interpolation to common discretization times (and away from the collocation points)
- This is supported by the relative error between SOS and GPOPS-II ($3.54 \times 10^{-1}$ and $1.037 \times 10^{-1}$ in state and control respectively)
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MARS TRANSFER

- CSALT was compared to solutions presented by Horsewood and Dankanich*
- Direct transfer from Earth to Mars using 2 ion engines
- Qualitatively identical solutions were found using all 3 software tools
- The final solutions all match to the expected margin of error of the modeling techniques

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Malto</th>
<th>HILTOP</th>
<th>CSALT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch date</td>
<td>3/29/22</td>
<td>4/7/22</td>
<td>4/4/22</td>
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<tr>
<td>Launch C₃ (km³/s²)</td>
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<td>Launch declination (deg)</td>
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<td>Launch mass (kg)</td>
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<td>Cruise flight time (days)</td>
<td>458.2</td>
<td>455.7</td>
<td>452.0</td>
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<td>Cruise propellant (kg)</td>
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<td>338.7</td>
<td>336.9</td>
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<td>Mars Arrival date</td>
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<td>7/6/23</td>
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</table>

CSALT was compared to solutions presented by Horsewood and Dankanich*

Repeat of the Dawn mission
- Earth to Mars low-thrust transfer
- Mars flyby
- Rendezvous with Vesta
- Low-thrust transfer to Ceres

CSALT found a qualitatively improved solution compared to the other two tools
- At a cost of ~220 days of additional flight time, 160 extra kg of dry mass could be delivered to Ceres using less propellant

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<tr>
<td><strong>Leg 1 Earth-Mars</strong></td>
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<tr>
<td>Launch $C_3$ ($\text{km}^2/\text{s}^2$)</td>
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<td>510</td>
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<td>Propellant used (kg)</td>
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<td><strong>Leg 2 Mars-Vesta</strong></td>
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<td>Swingby $v_w$ (km/s)</td>
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<td>Passage altitude (km)</td>
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<td><strong>Leg 3 Vesta-Ceres</strong></td>
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<td>Propellant used (kg)</td>
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<tr>
<td><strong>Total Propellant (kg)</strong></td>
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<td><strong>Mission duration (days)</strong></td>
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<td>2,711</td>
<td>2,930</td>
</tr>
</tbody>
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SUMMARY AND FUTURE WORK

- CSALT is a mature software package capable of solving a variety of optimization problems with high accuracy.
- CSALT has been successfully benchmarked against industry standard tools in both general engineering problems and for the solution of low-thrust trajectories.
- Future Work
  - Static and integral decision parameters and integral constraints
  - Second derivatives of the collocation
  - Improve performance
  - Full integration into GMAT