APPLYING GRAPH THEORY TO PROBLEMS IN AIR TRAFFIC MANAGEMENT

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• Introduction and motivation
• Background
• Three ATM Problems
  • Airspace sectorization problem
  • Minimum delay scheduling in traffic flow management
  • Maximum dependent set of an aircraft in arrival scheduling
• Summary
• Conclusion
INTRODUCTION & MOTIVATION

• Use known problems to learn about new problem
• Use graph theoretic problems as a suitable substrate
• Bridging isolated islands of knowledge
• Gaining insights about inherent difficulty of new problems
• Solving new problems efficiently using what is known about related problems
• Reap the benefits of development in other technical domains
INTRODUCTION & MOTIVATION

• Learn about new problems by
  • Linking them to known problems
  • Polynomial “transformation” or “reduction”

• Three examples from graph theory to ATM
  • Airspace sectorization problem (ASP)
  • Minimum delay scheduling (MDS)
  • Maximum set of dependent aircraft (MSDA)

If we can transform \( Q_1 \) to \( P_1 \)
And we know \( Q_1 \) is hard to solve
Then \( P_1 \) must be hard to solve

If we can transform \( P_3 \) to \( Q_3 \)
And we know how to solve \( Q_3 \)
To solve \( P_3 \): Transform it to \( Q_3 \), then solve
Problems have different inherent difficulty

**Example:** Sorting an array of $n$ distinct integers

One correct solution among $n!$ permutations

Know an $O(n \log n)$ time algorithm

Naive algorithm:
- Search among all permutations
- $O(n!)$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$n!$</th>
<th>Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100!</td>
<td>$10^{-6}$ sec.</td>
</tr>
<tr>
<td>102</td>
<td>101!</td>
<td>&gt; 0.01 sec.</td>
</tr>
<tr>
<td>104</td>
<td>102!</td>
<td>&gt; 1.8 min.</td>
</tr>
<tr>
<td>106</td>
<td>106!</td>
<td>&gt; 14 days</td>
</tr>
<tr>
<td>108</td>
<td>108!</td>
<td>&gt; 45 decades</td>
</tr>
<tr>
<td>110</td>
<td>110!</td>
<td>&gt; 5000 millennia</td>
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</tbody>
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Increase in problem size: 10%

Ridiculously longer runtime
Computational complexity: Classify problems based on their inherent difficulty

- **P**: Set of problems that can be solved in polynomial time
- **NP**: Set of problems whose solution can be verified in polynomial time
- **NP-complete (NPC)**: The hardest problems in **NP**
- **NP-hard (NPH)**: Problems at least as hard as the hardest problems in **NP**
- **Known**: $P \subseteq NP$
- **Unknown**: $P \neq NP$

Two possibilities

- $P = NP$
- $P \neq NP$
ATM PROBLEM 1: AIRSPACE SECTORIZATION PROBLEM (ASP)

Airspace
Flight paths
Airport

Partition the airspace into sectors such that:

- Workload in each partition is $\leq c_w$
- The number of sectors is $\leq c_s$

Maximum sector workload
Maximum sector count
ATM PROBLEM 1: AIRSPACE SECTORIZATION PROBLEM (ASP)

- Known problem: PLANAR-P3(6):
  - Given a planar graph
  - Each node connected to no more than 6 other nodes
  - Question: Can we partition the nodes into sets of 3, such that nodes in each set form a triangle)
- In this example the answer is YES
- Known to be NP-complete
ATM PROBLEM 1: AIRSPACE SECTORIZATION PROBLEM (ASP)

• It is known that ASP is **NP-complete** if sectors are required to be axis-aligned rectangles [Sabhnani, et al 2008]

• We transform PLANAR-P3(6) to ASP

• **Theorem 1:** ASP is **NP-complete** under several workload models, in general, even if the flight paths form a planar graph, and no more than 6 flights originate or terminate at each airport.
ATM PROBLEM 2: MINIMUM DELAY SCHEDULING (MDS) IN TRAFFIC FLOW MANAGEMENT

Airspace

Partitioned into sectors $s_1, s_2, s_3, \ldots$

Each sector $s_i$ has a capacity function $c_i(t)$

Each flight $f$ has a schedule $[t_0, (s_1, 4), (s_2, 6), (s_3, 5), (s_7, 3)]$

Assign delays to each flight on the ground or along its path to meet:

- Sector capacity constraints
- Airport arrival departure rate constraints (if any)

Objective: Minimize the sum of all delays imposed
ATM PROBLEM 2: MINIMUM DELAY SCHEDULING (MDS) IN TRAFFIC FLOW MANAGEMENT

• Known problem: Maximum Independent Set (MIS) in graphs
  • Given a graph
  • Find its largest independent set
    Subset of nodes, such that none of them is connected by an edge to any other node in the set
  • Known to be NP-hard
  • Known that unless $P = NP$, the problem cannot be approximated within polynomial time to within a factor $n^{1-\varepsilon}$ for any $\varepsilon > 0$
• It is known that the MDS problem is **NP-hard** [Bertsimas et al 1998]

• We transform graph MIS problem to a simplified version of MDS problem

• **Theorem 2:** Unless $P = NP$, the MDS problem cannot be approximated in polynomial time to within a factor $n^{1-\varepsilon}$ for any $\varepsilon > 0$, where $n$ is the number of aircraft in the problem instance, even if all the delays are to be taken on the ground prior to takeoff.
ATM PROBLEM 3:
MAXIMUM SET OF DEPENDENT AIRCRAFT (MSDA)
IN PRECISION ARRIVAL SCHEDULING

• Consider a set of aircraft \( a, b, c, \ldots \)
• Flying along their arrival routes following their prescribed schedules
• To land on their designated runways at the airport
• Due to off-nominal conditions, an aircraft may not meet its scheduled time slot and needs to be rescheduled
• We need to identify only the set of dependent aircraft who need to be rescheduled along with it

arrival runways
ATM PROBLEM 3:
MAXIMUM SET OF DEPENDENT AIRCRAFT (MSDA) IN PRECISION ARRIVAL SCHEDULING

• Known graph algorithm: Graph reachability (e.g. Breadth-First Search, Depth-First Search)

• Given a directed graph \( G = (V, E) \) and a node \( v \in V \) find the set of all nodes reachable from \( v \)

• Can be solved in \( O(|V| + |E|) \)
ATM PROBLEM 3: MAXIMUM SET OF DEPENDENT AIRCRAFT (MSDA) IN PRECISION ARRIVAL SCHEDULING

- Transformed MSDA into graph reachability and solved it
- Monte Carlo Simulation
- 3 Airports: DAL, LAX, PHX
- 100 random scenarios
- 10, 20, 40, 80, 160 aircraft
- randomly chosen target
- Run-time < 0.5 sec.
SUMMARY

• Studied three problems arising in ATM:
  • Airspace Sectorization Problem (ASP): Showed it is NP-complete
  • Min Delay Scheduling (MDS): Showed unless $P = NP$ the problem cannot be approximated in polynomial time
  • Maximum Set of Dependent Aircraft (MSDA) in arrival scheduling: Solved using a very efficient algorithm
CONCLUSION

• Graph theory is a natural abstraction for many ATM problems
• Used known graph problems to learn about ATM problem
• Polynomial transformation can be used to
  • Gain insights about inherent difficulty of new problems
  • Solve new problems efficiently
• Linking problems allows:
  • Reap the benefits of earlier or future development
  • Fertilization across different technical disciplines
Questions?
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APPLYING GRAPH THEORY TO ATM