Measurement of linear coefficient of thermal expansion and temperature-dependent refractive index using interferometric system

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Abstract: A system combining an interferometer with an environmental chamber for measuring both coefficient of thermal expansion (CTE) and temperature-dependent refractive index (dn/dT) simultaneously is presented. The operation and measurement results of this instrument are discussed.

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1. Introduction

The thermal response of an optical element is described mathematically by a number of material parameters. First amongst these is the linear coefficient of thermal expansion (CTE). As a material, optical or otherwise, is heated or cooled, it changes its physical size in keeping with this value. This is represented mathematically by

\[ L' = L(1 + \alpha \Delta T), \]  

where \( L \) and \( L' \) are length of the sample before and after a temperature change of \( \Delta T \) while \( \alpha \) is the CTE [1]. Thus, exposing a material with a positive CTE (as the vast majority do have) to a temperature increase will cause that sample to expand in each dimension.

The second thermal parameter of interest is the temperature-dependent refractive index (dn/dT). This value determines how the index of refraction of an element will change with a given \( \Delta T \) and is represented mathematically according to

\[ N_{00}' = N_{00} + \left( \frac{dn}{dT} \right) \Delta T, \]

where \( N_{00} \) and \( N_{00}' \) are the sample’s index of refraction before and after a temperature change of \( \Delta T \), respectively [2]. While dn/dT is most often a positive value, for certain materials such as CaF\(_2\) and various polymers, these values are negative. Note that each of these parameters call back to one of the two quantities that make up the expression for optical path: physical thickness and index of refraction.

Knowledge of both the CTE and dn/dT values of all materials present in an optical design is necessary to determine how that system will behave when exposed to a varying environment. Under certain circumstances, a lens system can be passively athermalized so that no focus adjustment is required for different operating temperatures; otherwise, active athermalization is accomplished through the use of a moving compensator, most likely the detector plane. Traditionally, CTE has been measured with a dilatometer while dn/dT can be determined using a refractometer to measure index at different temperatures and taking the derivative with respect to temperature to find dn/dT. This paper explores means of measuring these quantities interferometrically.

2. Measurement process

Previously, a number of institutions have demonstrated the ability to measure CTE and dn/dT individually and together interferometrically [3-5]. Most commonly, the interferometer is built so that both the test and reference arms are subjected to the same environment. Additionally, the measurements are often in vacuum. In this way, the measured optical path difference is not affected at all by the refractive index of air changing with temperature. However, the vacuum pressure also distorts the mechanical size of the optic, which will ultimately affect the measured CTE and dn/dT values. This paper explores a method to measure both CTE and dn/dT simultaneously with the two arms of the interferometer in different environments.

Fabry-Perot thermal systems have previously been reported [6-7]. A Twyman-Green configuration was instead chosen for this work because of its reduced cost and added benefit of not having differing cavity finesse when used to measure with different wavelengths [8]. The tradeoff to this, when compared to the Fabry-Perot, is that the limited size of the Espec BTX-475 environmental chamber being used requires the reference arm be outside of the
environmental chamber while the test arm down extends inside of it. The resulting interferograms are sensitive to mechanical vibration and thermal drift. A photograph and diagram of the system are shown in Figure 1.

![Figure 1: (a) Photograph of thermal interferometer above environmental chamber. (b) Schematic of interferometer system](image)

Measurements are performed by placing a sample, half-coated with gold or another reflective material, atop the test mirror in the interferometer. This causes three separate interferograms to form between the reference mirror and (1) the test mirror, $dOPD_1$, (2) the reflection from the top of the sample, $dOPD_2$, and (3) the reflection from the test mirror having transmitted through the sample, $dOPD_3$. These three beam paths are indicated in Figure 2.

![Figure 2: Side view of sample resting on test mirror inside of environmental chamber.](image)

The reference mirror is mounted on a piezo stage, making the system capable of phase-shifting. As the temperature is cycled over the range of interest, phase-maps are continuously generated. An example is shown in Figure 3 where a coated sample is seen side-by-side with a computed phasemap. By unwrapping these phasemaps as a function of time and therefore temperature, one can determine the accumulated piston over the course of the measurement for each pixel in each of the three regions of interest. Comparing the amounts of piston accumulated in paths $dOPD_1$ and $dOPD_2$ to one another, the CTE of the sample is determined while the addition of path $dOPD_3$ yields $dn/dT$.

![Figure 3. (a) Photograph of sample half-coated with gold resting on test mirror for measurement (b) Computed phase-map. Both (a) and (b) show three beam paths of interest for determining CTE and $dn/dT$](image)

3. Results

A number of optical materials have previously been measured using the instrument including steel and CaF$_2$ [8]. Additional measurements on the CTE and $dn/dT$ of the polymers polymethyl methacrylate (PMMA) and polystyrene
at a wavelength of 632.8 nm have been carried out between 5 and 35 °C as shown in Figure 4. The results are consistent with the large range of values reported by other sources for these polymers [9-12].

![Graphs showing measured changes in thickness and index of refraction for PMMA and polystyrene samples at different temperatures.]

Figure 4: Measured change in (a) thickness and (b) index of refraction for PMMA sample ($t_0 = 2.522$ mm, $n_0 = 1.490$). Measured change in (c) thickness and (d) index of refraction for polystyrene sample ($t_0 = 2.508$ mm, $n_0 = 1.585$).

4. References


