Planetary Crater Detection and Registration Using Marked Point Processes, Multiple Birth and Death Algorithms, and Region-based Analysis

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Crater Detection
- Marked Point Process Model
- Energy Function
- Multiple Birth and Death Algorithm
- Region-of-Interest Approach
- Experimental Results

Image Registration
- 2-step Approach
- Experimental Results

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Need for automated methods for image registration

Launch of several planetary missions

Design of new and powerful sensors

Large data sets

- Multisensor
- Multitemporal
- Both

Original Data Set

Crater Detection

Crater Map

Registration

Crater Detection

Crater Map

Objective

- **Crater detection** in planetary images
- Development of an **image registration** method based on the extracted features
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Crater detection based on a marked point process (MPP) model

MPP: Stochastic Process \[\text{Realizations}\] Configurations of objects, each described by a marked point

Mathematical Formulation

A point process \(X\), defined over a bounded subset \(P\) of \(\mathbb{R}^2\) maps from a probability space to a configuration of points in \(P\).

Realizations of the process \(X\) are random configurations \(x\) of points, \(x = \{x_1, \ldots, x_n\}\), where \(x_i\) is the location of the \(i^{\text{th}}\) point in the image plane \((x_i \in P)\)

A configuration of an MPP consists of a point process whose points are enriched with additional parameters, called marks and aimed at parameterizing objects linked to the points.

Bayesian approach: Maximum \(a\ posteriori\) (MAP) rule to fit the model to the image is equivalent to minimizing an energy function (computationally challenging)
Marked Point Process for Crater Detection

Craters
- Center: \((x_0, y_0)\)
- Major Axis: \(a\)
- Minor Axis: \(b\)
- Orientation: \(\theta\)

Point Process
\[ x = \{x_1, \ldots, x_7\} \]

Parameters
\[ x_i = (x_{0i}, y_{0i}, a_i, b_i, \theta_i) \]

Modelling
- Crater
- Ellipse
- Distribution of craters on the surface
- Distribution of ellipses on the image plane

Realization Example
Energy function of the configuration \( X = \{x_1, x_2, ..., x_n\} \) wrt the extracted set \( C \) of **contour pixels** (Canny):

\[
U(X|C) = U_P(X) + U_L(C|X)
\]

**Prior**

**Repulsion** coefficient based on the overlapping of the ellipses (overlapping craters are quite unlikely)

\[
U_P(X) = \frac{1}{n} \sum_{x_i \cap x_j > 0} \frac{x_i \cap x_j}{x_i \cup x_j}
\]

\( x_i \cup x_j = \text{area of union of ellipses } x_i \text{ and } x_j \)

\( x_i \cap x_j = \text{area of intersection of } x_i \text{ and } x_j \)

**Likelihood**

Two terms, one based on a **correlation** measure, the other based on a **distance** measure (fit between contours and realization of \( X \))

\[
U_L(C|X) = \sum_{i=1}^{n} \left[ \frac{d_H(x_i^0, C)}{na_i} - \frac{|x_i^0 \cap C|}{|C|} \right]
\]

\( x_i^0 = \text{set of pixels corresponding to ellipse } x_i \text{ in the image plane} \)

\( d_H(x_i^0, C) = \text{Hausdorff distance between ellipse } x_i \text{ and the contours:} \)

\[
d_H(A, B) = \max \left\{ \sup_{\alpha \in A} \inf_{\beta \in B} d(\alpha, \beta); \sup_{\beta \in B} \inf_{\alpha \in A} d(\alpha, \beta) \right\}
\]

Classical distance between sets \( d(A, B) = 0 \)
Markov chain Monte Carlo-type method
Simulated Annealing scheme

- **Initialization**
  - Contour Extraction (Canny) and Parameter Initialization
  - Generation of the Birth Map used to speed up the convergence

- **Birth Step**
  - Computation of the Birth Probabilities for each pixel
  - Birth Phase
  - Energy Computation for all the ellipses

- **Death Step**
  - Computation of Death Probability and Death Phase according to Likelihood
  - Configuration refinement according to Prior

- **Update**
  - Convergence Test

Markov chain sampled by a multiple birth and death (MBD) algorithm
MBD – Birth and Death Steps

Birth Step

For each pixel $s$ in the image, compute the birth probability as $\min\{\delta \cdot B(s), 1\}$, where:

$$B(s) = \frac{b(s)}{\sum_s b(s)}$$

$b(s)$ is the birth map computed from the contour map using generalized Hough transform and Gaussian filtering.

Death Step

For each ellipse $x_i$ in the configuration, compute the death probability as $d(x_i)$:

$$d(x_i) = \frac{\delta \cdot a(x_i)}{1 + \delta \cdot a(x_i)}$$

$$a(x_i) = \exp[-\beta (U_L(X \setminus \{x_i\} | C) - U_L(X | C))]$$
Crater Detection – Region Based Approach

Region Based Flowchart and Example

**Region-Based Approach**

Why?
- MBD is **computationally heavy**
- Computational burden increases with **image size**

 Initialization

Detection of **regions on interest** based on the birth map

MBD in each **region**

Aggregation
Crater Detection – Data Sets

- 6 THEMIS (Thermal Emission Imaging System) images, TIR, 100m resolution, Mars Odissey mission
- 7 HRSC (High Resolution Stereo Color) images, VIS, ~20m resolution, Mars Express mission
- Image sizes from 1581 × 1827 to 2950 × 5742 pixels

Quantitative Performance Assessment of the crater detection algorithm: Detection Percentage ($D$), Branching Factor ($B$), and Quality Percentage ($Q$)

<table>
<thead>
<tr>
<th>Data</th>
<th>$D = \frac{TP}{TP + FN}$</th>
<th>$B = \frac{FP}{TP}$</th>
<th>$Q = \frac{TP}{TP + FP + FN}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg on all THEMIS</td>
<td>0.91</td>
<td>0.10</td>
<td>0.83</td>
</tr>
<tr>
<td>Avg on all HRSC</td>
<td>0.89</td>
<td>0.06</td>
<td>0.85</td>
</tr>
<tr>
<td>Avg on all images</td>
<td>0.90</td>
<td>0.09</td>
<td>0.84</td>
</tr>
</tbody>
</table>
Crater Detection – Results

Crater geometric properties extracted by the proposed method

<table>
<thead>
<tr>
<th>Crater</th>
<th>$C = (x_0, y_0)$</th>
<th>Semi-axes $(a, b)$</th>
<th>Orientation $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crater 1</td>
<td>(139, 393)</td>
<td>(35, 33)</td>
<td>64°</td>
</tr>
<tr>
<td>Crater 2</td>
<td>(258, 756)</td>
<td>(51, 50)</td>
<td>115°</td>
</tr>
<tr>
<td>Crater 3</td>
<td>(343, 23)</td>
<td>(13, 12)</td>
<td>180°</td>
</tr>
<tr>
<td>Crater 4</td>
<td>(591, 215)</td>
<td>(19, 18)</td>
<td>31°</td>
</tr>
<tr>
<td>Crater 5</td>
<td>(919, 157)</td>
<td>(15, 14)</td>
<td>106°</td>
</tr>
</tbody>
</table>
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Why a 2-step Optimization?

**Feature-based registration**
- Min Hausdorff distance ($d_H$) between extracted craters through genetic algorithm
- Fast but sensitive to accuracy of crater maps

**Area-based registration**
- Max Mutual Information ($MI$) through genetic algorithm
- Highly accurate but computationally heavy
Image Registration – Region of Interest

Transformation found for an interactively selected region of interest \( p_B^* \)
\[
p_B = (t_x, t_y, \theta, k)
\]

Transformation derived for the entire Image \( p_A^* \)
\[
p_A = (T_x, T_y, \beta, \alpha)
\]

\[
p_A^* = \begin{pmatrix}
-k \cos(\theta) x_0 - k \sin(\theta) y_0 + t_x + x_0 \\
-k \sin(\theta) x_0 - k \cos(\theta) y_0 + t_y + y_0 \\
\theta \\
k
\end{pmatrix}
\]

Superposition of Reference and Input
Image Registration – Data Sets

Semi-simulated image pairs

20 pairs composed of one real THEMIS or HRSC image and of an image obtained by applying a synthetic transform and AWGN

Quantitative validation with respect to the true transform (RMSE)

Real multi-temporal image pairs

Real multi-temporal pair of LROC (Lunar Reconnaissance Orbiter Camera) images

100m resolution

Only qualitative visual analysis is available, as no ground truth is available
## Registration Results with Semi-synthetic Data

### Data set vs RMSE [pixel]

<table>
<thead>
<tr>
<th>Data set</th>
<th>RMSE [pixel]</th>
</tr>
</thead>
<tbody>
<tr>
<td>THEMIS (10 data sets)</td>
<td>0.31</td>
</tr>
<tr>
<td>HRSC (10 data sets)</td>
<td>0.22</td>
</tr>
<tr>
<td>Average (20 data sets)</td>
<td>0.26</td>
</tr>
</tbody>
</table>

### RGB of original non-registered data

#### THEMIS (1396×2334)

#### HRSC (1581×1827)

### RGB of registered data

![RGB of registered data](image)

### Table of Registration Results

<table>
<thead>
<tr>
<th></th>
<th>Left Image</th>
<th>Right Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{GT} )</td>
<td>(7.05, 35.91, 0.18°, 1.071)</td>
<td>(76.59, 19.96, 2.17°, 1.031)</td>
</tr>
<tr>
<td>( p^* )</td>
<td>(7.04, 35.92, 0.19°, 1.071)</td>
<td>(76.41, 20.06, 2.18°, 1.031)</td>
</tr>
<tr>
<td>RMSE 1&lt;sup&gt;st&lt;/sup&gt; Step</td>
<td>0.79</td>
<td>0.51</td>
</tr>
<tr>
<td>RMSE 2&lt;sup&gt;nd&lt;/sup&gt; Step</td>
<td>0.16</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Registration Results with Real Data

Visually accurate matching between reference and registered images in the real multitemporal data set

Checkerboard representation of the registered images (zoom on details)
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Visually accurate matching between reference and registered images in the real multitemporal data set
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Conclusions and Future Developments

Conclusions

• **Accurate** crater maps, useful for both image registration and planetary science, were obtained from data from different sensors.

• **Higher accuracy** as compared to previous work on crater detection (not shown for brevity)

• Reduced time for convergence thanks to a **region-based approach**

• **Sub-pixel accuracy and visual precision** in registration: effectiveness of the proposed 2-step registration method

Future Developments

• Test in conjunction with a **parallel** implementation (e.g. computer cluster)

• Validation with **multi-sensor** real images

• Extension to **other applications** requiring the extraction of ellipsoidal or circular features, e.g. optical Earth observation images or medical images
Short Bibliography

For each pixel in the image compute the Birth Probability as $\min\{\delta \cdot B(s), 1\}$, where:

$$B(s) = \frac{b(s)}{\sum_s b(s)}$$

Being $b(s)$ the Birth Map computed from the Canny Contour Map.
For each ellipse $x_i$ in the configuration compute the Death Probability as $d(x_i)$, where

$$d(x_i) = \frac{\delta \cdot a(x_i)}{1 + \delta \cdot a(x_i)}$$

and

$$a(x_i) = e^{-\beta(U_L(x|I_g) - U_L(x|I_g))} = e^{\beta \cdot U_L(x|I_g)}$$

The complete Flowchart of the Death Step is as follows:

1. Computation of the Energy Terms
2. Computation of Death Prob. based on Likelihood
3. Elimination of ellipses with prob. $d(x_i)$
4. Configuration refinement thanks to Prior term
**Similarity Measures**

**Hausdorff Distance**

\[
\text{Similarity} = \text{mean}_c \left\{ \sum_{i=1}^{N^c} \sum_{t=1}^{P} d_{H}(x_i^c, x_t) \right\}
\]

c = craters in Input Image

\[N^c = \text{sum}(\text{pixels in crater c in Input Image})\]

\[P = \text{sum}(\text{craters' border pixels in Ref Image})\]

\[x_i^c = \text{coord of pixel i in crater c in Input Image}\]

\[x_t = \text{coord of pixel t in Ref Image's craters}\]

\[d_{H}(x_i, x_j)\]

**Mutual Information**

\[
MI(X, Y) = \sum_{x \in X} \sum_{y \in Y} p_{X,Y}(x, y) \log \left( \frac{p_{X,Y}(x, y)}{p_X(x) p_Y(y)} \right)
\]

\(X\): pixel intensity in Reference Image

\(Y\): pixel intensity in Input Image

\(p_X(x)\): probability density function (pdf) of \(X\)

\(p_Y(y)\): probability density function (pdf) of \(Y\)

\(p_{X,Y}(x, y)\): joint pdf of \(X\) and \(Y\)

\(p_X(x)\)

\(p_Y(y)\)

\(p_{X,Y}(x, y)\)

estimated through the corresponding image histograms
RST Transformation

Rotation – Scale – Translation Transformation

Transformation vector

\[ p = (t_x, t_y, \theta, k) \]

\( \{t_x, t_y\} \): Translations in \( x \) and \( y \)

\( \theta \): Rotation angle

\( k \): Scaling Factor

Matrix Formulation

\[ T_p(x, y) = \begin{pmatrix} k \cos(\theta) & k \sin(\theta) \\ -k \sin(\theta) & k \cos(\theta) \end{pmatrix} \begin{pmatrix} t_x \\ t_y \end{pmatrix} \]
**Region of Interest Approach**

$I_A(X,Y), I_B(x,y)$: Two Images

$I_B$: sub-image of $I_A$ such that $I_B(0,0) = I_A(x_0,y_0)$

$p_A = (T_x, T_y, \beta, \alpha)$: RST transformation vector transforming $I_A$ into $I_A^{tr}$

$p_B = (t_x, t_y, \theta, k)$: RST transformation vector transforming $I_B$ into $I_B^{tr}$

$I_B^{tr}(0,0) = I_A^{tr}(x_0,y_0)$

Given:

Transformation: $p_B$

Reference of Region: $(x_0,y_0)$

Find: Transformation: $p_A$

From the image

\begin{align*}
X &= x + x_0 \\
Y &= y + y_0
\end{align*}

Expressing the transformation in Matrix Form

\[
T_{p_A} = \begin{pmatrix}
\alpha \cos(\beta) & \alpha \sin(\beta) & T_x \\
-\alpha \sin(\beta) & \alpha \cos(\beta) & T_y
\end{pmatrix}, \quad T_{p_A}(X,Y) = (X',Y')
\]

\[
T_{p_B} = \begin{pmatrix}
k \cos(\theta) & k \sin(\theta) & t_x \\
-k \sin(\theta) & k \cos(\theta) & t_y
\end{pmatrix}, \quad T_{p_B}(x,y) = (x',y')
\]

This should also hold

\[
T_{p_A}(x+x_0,y+y_0) = (x'+x_0,y'+y_0)
\]

Plugging $T_{p_A}$ into this equation and replacing $x'$ and $y'$ according to $T_{p_B}$

\[
\begin{align*}
k \cos(\theta) x + k \sin(\theta) y + t_x + x_0 &= \\
\alpha \cos(\beta)(x + x_0) + \alpha \sin(\beta)(y + y_0) + T_x
\end{align*}
\]

Knowing $\alpha = k$ and solving in $P_1 = (0,0)$ and $P_2 = (-x_0,-y_0)$

\[
p_A = \begin{pmatrix}
-k \cos(\theta) x_0 - k \sin(\theta) y_0 + t_x + x_0 \\
k \sin(\theta) x_0 - k \cos(\theta) y_0 + t_y + y_0 \\
\theta \\
k
\end{pmatrix}
\]
RMS Error Computation

**Ground Truth Transformation**

\[ p_{GT} = (tx_1, ty_1, \theta_1, k_1) \rightarrow T_{p_{GT}}(x, y) = Q_{p_{GT}} \cdot [x, y, 1]^T \]

**Computed Transformation**

\[ p = (tx, ty, \theta, k) \rightarrow T_p(x, y) = Q_p \cdot [x, y, 1]^T \]

\[(x, y) \in \text{Image}, [x', y', 1]^T = Q_{p_e} \cdot [x, y, 1]^T \]

**Error Transformation**

\[ p_e = (tx_e, ty_e, \theta_e, k_e) \rightarrow Q_{p_e} = Q_p \cdot Q_{p_{GT}}^{-1} \]

\[ \begin{align*}
    k_e &= \frac{k_2}{k_1}, & \theta_e &= \theta_2 - \theta_1 \\
    tx_e &= tx_2 - k_e(tx_1 \cos(\theta_e) + ty_1 \sin(\theta_e)) \\
    ty_e &= ty_2 - k_e(ty_1 \cos(\theta_e) - tx_1 \sin(\theta_e))
\end{align*} \]

\[ [x'] = k_e \begin{pmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{pmatrix} [x] + [tx_e]

\[ [y'] = k_e \begin{pmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{pmatrix} [y] + [ty_e] \]

**RMS Error**

\[ E(p_e) = \sqrt{\frac{1}{AB} \int_0^A \int_0^B (x' - x)^2 + (y' - y)^2 \, dx \, dy}, \quad \alpha = A^2 + B^2 \]

\[ E^2(p_e) = \frac{1}{AB} \int_0^A \int_0^B (k_e \cos(\theta_e) x + k_e \sin(\theta_e) y + tx_e - x)^2 + (-k_e \sin(\theta_e) x + k_e \cos(\theta_e) y + ty_e - y)^2 \, dx \, dy \]

\[ E^2(p_e) = \frac{\alpha}{3} (k_e^2 - 2k_e \cos(\theta_e) + 1) + (t_{xe}^2 + t_{ye}^2) - (At_{xe}^2 + Bt_{ye}^2)(1 - k_e \cos(\theta_e)) - k_e(At_{ye} - Bt_{xe}) \sin(\theta_e) \]