Spacecraft Alignment Determination and Control for Dual Spacecraft Precision Formation Flying

Philip Calhoun, Anne-Marie Novo-Gradac, Neerav Shah

NASA Goddard Space Flight Center
Overview

- VT Concept
  - Astrometric Alignment Concept for Virtual Telescopes (VT)
  - Proposed Missions (MASSIM, New Worlds Observer)
- Stability Requirements and Measurement Models
  - Attitude and Translation Stability Requirements
  - Optical Alignment and Ranging System Measurement Models
- Dynamics and Controls Framework for GN&C Design
  - Dynamics Model Formulation
  - Inertial Measurement Models (IRU, Accelerometers)
- Case Study: GN&C Design for a Heliophysics Mission
  - Navigation Modes for Fine Alignment Acquisition
  - GN&C Architecture Comparison
- Conclusions
VT Concept

- Formation flying missions seek to advance science imaging by utilizing precision dual spacecraft formation flying. ("Virtual" Telescope (VT))
  - Milli-Arc-Second Structure Imager (MASSIM) (Astrophysics X-ray imaging) (Sep ~ 1000 km)
  - New Worlds Observer (NWO) (exoplanet mission) (Sep ~ 25,000 km)
  - Heliophysics concept missions for Solar Coronagraphs and Solar imaging (Sep. 50m – 500m)

Objective: Develop models for a complete GN&C design framework of VT architectures
VT Attitude and Translation Stability Requirements

- Science detector image smear and depth of field stability model
  - Considers detector and optics not co-located with S/C mass center

\[
\begin{bmatrix}
\bar{s}_x \\
\bar{s}_y \\
\bar{d}
\end{bmatrix}
= \begin{bmatrix}
\bar{P}_D + \bar{P}_{DO} + \bar{\delta}_R - \bar{P}_O + R(\bar{\theta}_O)\bar{P}_O + f(R(\bar{\theta}_O))\bar{P}_{OD} \\
\end{bmatrix} - R(\bar{\theta}_D)\bar{P}_D
\]

\[f(R(\bar{\theta}_O)) = [I + ^n\bar{\theta}_O], \quad ^n\bar{\theta}_O = \Phi \bar{\theta}_O, \quad \Phi \text{ is diag matrix, } 0 < \phi < 1\]

\[
\begin{bmatrix}
\bar{\delta}_l \\
\bar{\theta}_l \\
\bar{r}_l
\end{bmatrix}
= \begin{bmatrix}
\bar{P}^x_D \bar{\theta}_D + [\Phi \bar{P}^x_{DO} - \bar{P}^x_O]\bar{\theta}_O + \bar{\delta}_R \\
\end{bmatrix}
\]

- Same model is used for laser centration and ranging measurements
Alignment Camera Measurement Model

- Measurement model for Alignment Camera (AC) to track laser beacon bearing angles.
  - AC used for acquisition of Laser Centration and Ranging elements.

\[
\tilde{P}_{CB} = [\tilde{P}_C + \tilde{P}_{CO} + \delta_R + R(\bar{\theta}_O)\tilde{P}_B] - R(\bar{\theta}_D)\tilde{P}_C
\]

Laser beacon centroids on the camera image

\[
[\theta_x, \theta_y] = \text{atan}\left(\frac{\tilde{P}_{CB}(2)}{\tilde{P}_{CB}(3)}\right), \text{atan}\left(\frac{\tilde{P}_{CB}(1)}{\tilde{P}_{CB}(3)}\right)
\]
VT Dynamics Framework for GN&C Design

- Dual Spacecraft Relative Dynamics (Based on Luquette’s work)
  - Restricted Three-body Framework
  - Mods: Additional gravitational bodies, and express equations in terms of Follower

Differential acceleration, expressed in an inertial frame,

\[
\ddot{x} = \ddot{r}_F - \ddot{r}_L \\
\ddot{r}_F = -\sum_{i=1}^{n} \mu_i \frac{\dot{r}_{iF}}{||\dot{r}_{iF}||^3} + \dddot{f}_{solar,F} + \dddot{f}_{pert,F} + \dddot{u}_{thrust,F} \\
\ddot{r}_L = -\sum_{i=1}^{n} \mu_i \frac{\dot{r}_{iL}}{||\dot{r}_{iL}||^3} + \dddot{f}_{solar,L} + \dddot{f}_{pert,L} + \dddot{u}_{thrust,L}
\]

Can be simplified in terms of follower S/C, following derivation by Luquette.

- Assume \( ||\ddot{x}|| \ll ||\ddot{r}_{iF}|| \) , \( \ddot{x} = \ddot{R}_{ref} + \ddot{\delta}_R \) and remove higher order terms.

\[
\dddot{\delta}_R = \Gamma_{GG} \ddot{\delta}_R + \Gamma_{GG} \dddot{R}_{ref} + \dddot{u}_R \\
\Gamma_{GG} = -\sum_{i=1}^{n} \frac{\mu_i}{||\dot{r}_{iF}||^3} \left( [I] - 3\hat{\dot{r}}_{iF}^{ref} \left[ \dot{r}^{ref}_i \right]^T \right)
\]
VT Dynamics Framework for GN&C Design

• Inertial Measurement Sensor (Accelerometers)

The acceleration, $\ddot{\delta}_F^m$, at a specific sensor location, $\vec{r}_A$, can be represented as,

$$\ddot{\delta}_F^m = \ddot{\delta}_F + \vec{\omega}_F \times (\vec{\omega}_F \times \vec{r}_A) + \dot{\vec{\omega}}_F \times \vec{r}_A + \vec{b}_A + \vec{v}_A$$

Acceleration can be expressed in terms of forces / torques on S/C

$$\ddot{\delta}_F = \ddot{\bar{u}}_{Ft_0} + \ddot{\delta}u_{FT} + \bar{u}_{FE} \quad \dot{\vec{\omega}}_F = I_F^{-1}(\ddot{T}_{Ft_0} + \ddot{\delta}T_{FT} + \ddot{T}_{FE})$$

And reduced to following linear form,

$$\ddot{\delta}_F^m = ([I] - r_A^x I_F^{-1} r_T^x m_F)\ddot{\bar{u}}_{Ft_0} + ([I] - r_A^x I_F^{-1} r_T^x m_F)\ddot{\delta}u_{FT} + ([I] - r_A^x I_F^{-1} r_E^x m_F)\ddot{\bar{u}}_{FE} + \vec{b}_A + \vec{v}_A$$

• Inertial Reference Unit (Accelerometers)

$$\ddot{\theta} = \ddot{\vec{\omega}}_F^m - \vec{b}_{\vec{\omega}}_F + \vec{v}_{\vec{\omega}}_F$$
Case Study: GN&C Design for Heliophysics Mission

Closed Loop GN&C Simulation Case Study – Photon Sieve
• GN&C design framework applied to an example problem to illustrate trades inherent in PFF for VT.
• Photon Sieve Optics (diffractive optics, ~0.5 m aperture). Solar Imaging at milli-arc-sec level

Table 1, Photon Sieve VT Alignment Requirements and Component Specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Requirement (3s)</th>
<th>Component</th>
<th>Specification (3s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image Smear</td>
<td>6 microns</td>
<td>Laser Centration</td>
<td>30 microns</td>
</tr>
<tr>
<td>Depth of Field</td>
<td>1 mm</td>
<td>Laser Ranging</td>
<td>0.5 cm</td>
</tr>
<tr>
<td>S/C separation</td>
<td>200 m</td>
<td>Microthruster</td>
<td>5 mN-sec (min Impulse)</td>
</tr>
<tr>
<td>Pointing Stability (Optics S/C)</td>
<td>5 milli-arc sec (Sun)</td>
<td>Fine Sun Sensor</td>
<td>30 milli arc-sec</td>
</tr>
<tr>
<td></td>
<td>10 arc-sec (roll)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pointing Stability (Detector S/C)</td>
<td>10 arc-sec</td>
<td>Star Tracker</td>
<td>6 arc-sec (transverse)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>30 arc-sec (boresight)</td>
</tr>
</tbody>
</table>

• State estimation: Extended Kalman Filter, continuous state propagation, discrete measurements
  – Sequential Measurement updates to avoid numerical issues of large matrix inverses
• Separate PID Control for each of 9 DoF, (3) Relative translation, (3) Optics S/C Att, (3) Optics S/C Att
• All Measurement and Actuator models include random + systematic (1st order Markov) errors
Case Study: GN&C Design for Heliophysics Mission

- Simulation of Navigation modes (Leader/Follower) illustrates fine align acquisition

1. Lost-in-space (Radio Ranging)
2. Coarse Align (Alignment Camera)
3. Fine Align (Laser Centration, Laser Ranging)

1. Lost-in-space: Radio Range (60 cm), Radio bearing (9 deg)
2. Coarse Align: Alignment Camera (50 arc-sec), Star Tracker (ST) (6, 6, 30 arc-sec)
3. Fine Align: Laser Centration (30 μm), Laser Ranging: (1 cm), ST(6, 6, 30 arc-sec), Sun Sen (10e-3 arc-sec)
Case Study: GN&C Design for Heliophysics Mission

- Evaluation of Leader/Follower and Partition Architecture illustrates GN&C trades

**Leader/Follower architecture has two possible deficiencies**
- Comm link required to send Optics S/C Attitude to EKF on Detector S/C. Comm delay/timing sync
- Requires Full 6 DoF control on Detector S/C. Thruster coupling may result in poor performance

**Partition architecture:** Control / Estimation is partitioned among the two S/C
- Avoids multi-platform attitude coupling in the measurement process
Case Study: GN&C Design for Heliophysics Mission

- Evaluation of Leader/Follower and Partition Architecture illustrates GN&C trades
  
  - Decoupling of laser centration/ranging measurements from the S/C attitude, (sensor positioning)
    - Improved transverse alignment observability / performance in the Partition architecture.
  
  - Partition architecture performance meets Photon Sieve alignment requirements
    - ~5x error reduction obtained from model-based estimation over laser centration measurements.
    - Total impulse for PFF (5 year) is reduced 35%. Solar pres along VT axis (Optics S/C is ½ mass of Detector S/C)

Figure 5 – Performance results of two representative GN&C architectures for the VT
Conclusions

- Developed General 9 DoF GN&C framework dual S/C PFF for application to VT missions
- GN&C performance assessment for a representative Heliophysics VT imaging mission concept illustrates the potential trade-offs inherent in the choice of system architecture for GN&C design and mission concept.

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