Static Analysis using Abstract Interpretation

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1 Introduction
   - Software development
   - Safety properties
   - Abstract Interpretation

2 IKOS

3 Analyses

4 Miscellaneous

5 Conclusion
Software development

- Software represent more than half of the development cost of an aircraft
- Regulated by international standards (DO-178 rev. B/C)
Software development

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- Tests
  - Expensive because run on a special hardware
  - Can miss bugs
  - Slow
Software development

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- Regulated by international standards (DO-178 rev. B/C)
- Tests
  - Expensive because run on a special hardware
  - Can miss bugs
  - Slow
- Solution : use static analysis
- NASA V&V program
Safety properties

- **Main objectives**: no runtime errors
  - buffer overflow
  - null dereference
  - division by zero
  - integer overflow

- **Harder objectives**: 
  - assertions (pre/post invariants)
  - termination

- **certified** $\implies$ soundness is required

- abstract interpretation is a good candidate

- runtime errors can be security vulnerabilities!
based on the concrete semantics of your program
automatic formal proof
sound approximation of reachable states
Abstract Interpretation

$x(t)$

$\text{semantics}(P)$

Possible trajectories
Forbiden zone

\textit{specification}(P)
Abstract Interpretation

\[ x(t) \]

Forbidden zone

Possible trajectories

\[ \text{semantics}(P) \subseteq \text{specification}(P) \]
Tests

Using testing

Forbidden zone

Error !!!

Test of a few trajectories

Possible trajectories

$x(t)$

$t$
Abstract Interpretation

Abstraction of the trajectories

\textit{abstraction}(P)
Abstract Interpretation

Forbidden zone

Abstraction of the trajectories

\[ \text{abstraction}(P) \subseteq \text{specification}(P) \]
Abstract Interpretation

Forbidden zone

Abstraction of the trajectories

\[ \text{semantics}(P) \subseteq \text{abstraction}(P) \subseteq \text{specification}(P) \]
Thank you Pierre Loïc Garoche
1. Introduction

2. IKOS
   - Project
   - Toolchain
   - Demo
   - Results

3. Analyses

4. Miscellaneous

5. Conclusion
The IKOS project

- Inference Kernel for Open Static Analyzers
- C++ library for abstract interpretation
- C/C++ static analyzer
- Target embedded systems
- Analyses:
  - Buffer overflow
  - Division by zero
  - Null dereference
  - Uninitialized variables
  - Prover

https://ti.arc.nasa.gov/opensource/ikos/
Toolchain

C/C++ code
clang
LLVM IR

ikos-pp
Optimized LLVM IR

LLVM opt command + AR pass (-arbos)
AR in s-expr

ARBOS
{AR parser, analysis plugin framework}

Analysis results

 Abstract Domains
- Interval
- Constants
- Discrete
- Congruence
- Interval + Congruence
- Octagons
- Difference Bounds Matrix
- Pointer Analysis

IKOS

ikos-pp
- Ikos-pp is an executable that embeds the LLVM opt command. It applies several LLVM built-in optimizations + our own optimization passes to produce an intermediate optimized LLVM IR. Using the optimized LLVM IR, we run LLVM opt command with -arbos option to translate the optimized LLVM IR to AR
- ikos-pp does at least the following optimizations before translating to AR: -mem2reg, -loweratomic, -lowerswitch, and -instnamer

AR Plugin Analyzers
- BOA - buffer overflow analysis
- DBZ - Intra-procedural integer division-by-zero analysis
- UVA - Inter-procedural uninitialized variable + array analysis
- NullPtr - Inter-procedural null dereference pointer analysis

Analysis results
- Outputs reports to console
- IKOSView: desktop GUI that queries results stored in SQLite3 database
- Integrated into web services (such as continuous build + bug tracking systems)
  - SonarQube – using sonar_runner
  - CodeDX – import results in cppcheck XML format
  - SWAMP – used in cybersecurity
- Low Level Virtual Machine
- Compiler Infrastructure
- Generic assembly language
- Allow language independent optimization
• Low Level Virtual Machine
• Compiler Infrastructure
• Generic assembly language
• Allow language independent optimization
$ cat test.c

```
#include <stdio.h>

int main(int argc, char** argv) {
    int a[10];
    int i;
    for (i = 0; i < 10; i++) {
        a[i] = i;
    }
    printf("%d\n", a[i - 1]);
    printf("%d\n", a[0]);
    return 0;
}
```
$ clang -c -emit-llvm -O1 -o test.bc test.c
$ opt -S test.bc

define i32 @main(i32, i8** nocapture readnone) local_unnamed_addr #0 {
  %3 = alloca [10 x i32], align 16
  %4 = bitcast [10 x i32]* %3 to i8*
  call void @llvm.lifetime.start(i64 40, i8* %4) #3
  br label %5

; <label>:5: ; preds = %5, %2
  %6 = phi i64 [ 0, %2 ], [ %9, %5 ]
  %7 = getelementptr inbounds [10 x i32], [10 x i32]* %3, i64 0, i64 %6
  %8 = trunc i64 %6 to i32
  store i32 %8, i32* %7, align 4
  %9 = add nuw nsw i64 %6, 1
  %10 = icmp eq i64 %9, 10
  br i1 %10, label %11, label %5

; <label>:11: ; preds = %5
  %12 = getelementptr inbounds [10 x i32], [10 x i32]* %3, i64 0, i64 9
  %13 = load i32, i32* %12, align 4
  %14 = tail call i32 (i32, i8*, ...) @printf_chk(1, 
  i8* getelementptr inbounds ([4 x i8], [4 x i8]* @.str, i64 0, i64 0), i32 %13) #3
  %15 = getelementptr inbounds [10 x i32], [10 x i32]* %3, i64 0, i64 0
  %16 = load i32, i32* %15, align 16
  %17 = tail call i32 (i32, i8*, ...) @printf_chk(1, 
  i8* getelementptr inbounds ([4 x i8], [4 x i8]* @.str, i64 0, i64 0), i32 %16) #3
  call void @llvm.lifetime.end(i64 40, i8* nonnull %4) #3
  ret i32 0
}
%2:
%3 = alloca [10 x i32], align 16
%4 = bitcast [10 x i32]* %3 to i8*
call void @llvm.lifetime.start(i64 40, i8* %4) #3
br label %5

%5:
%6 = phi i64 [ 0, %2 ], [ %9, %5 ]
%7 = getelementptr inbounds [10 x i32], [10 x i32]* %3, i64 0, i64 %6
%8 = trunc i64 %6 to i32
store i32 %8, i32* %7, align 4, !tbaa !3
%9 = add nuw nsw i64 %6, 1
%10 = icmp eq i64 %9, 10
br i1 %10, label %11, label %5

%11:
%12 = getelementptr inbounds [10 x i32], [10 x i32]* %3, i64 0, i64 9
%13 = load i32, i32* %12, align 4, !tbaa !3
%14 = tail call i32 (i32, i8*, ...) @printf_chk(i32 1, i8* getelementptr ...
inbounds ([4 x i8], [4 x i8]* @.str, i64 0, i64 0), i32 %13) #3
%15 = getelementptr inbounds [10 x i32], [10 x i32]* %3, i64 0, i64 0
%16 = load i32, i32* %15, align 16, !tbaa !3
%17 = tail call i32 (i32, i8*, ...) @printf_chk(i32 1, i8* getelementptr ...
inbounds ([4 x i8], [4 x i8]* @.str, i64 0, i64 0), i32 %16) #3
call void @llvm.lifetime.end(i64 40, i8* nonnull %4) #3
ret i32 0
IKOS-PP

- IKOS pre-processor
- Run llvm optimization passes:
  - mem2reg: SSA Form
  - globaldce: Dead Code Elimination
  - globalopt: Global Variable Optimizer
  - simplifycfg: Control Flow Graph Optimizer
  - scalarrepl: Scalar Replacement of Aggregates
  - sccp: Sparse Conditional Constant Propagation
  - loop-simplify: Canonical Form for Loops
  - lcssa: Loop Closed SSA Form
  - loop-deletion: Dead Loop Elimination
  - lowerinvoke: Lower Invoke Instructions
  - lowerswitch: Lower Switch Instructions
- Run home made llvm passes:
  - Lower Global Variable Initialization
  - Lower Constant Expressions
  - Lower Select Instructions
  - Name Values
- Abstract Representation
- Major differences with llvm:
  - Branching instructions are translated into assertions
  - Memory instructions are byte oriented
  - Some instructions are removed
- Translation from llvm to AR using a llvm pass
- Text representation using s-expressions
($function
  ($name ($main)) ($ty (!8))
  ($params ($p ($name ($main.arg_1)) ($ty (!9))) ($p ($name ($main.arg_2)) ($ty (!10))))
  ($local-vars ($local_var ($var ($name ($main._1)) ($ty (!11)))))
  ($code
    ($entry ($bb_1)) ($exit ($bb_5)) ($unreachable) ($ehresume)
    ($basicblocks
      ($basicblock ($name ($bb_1))
        ($instructions
          ($allocate ($dest ($cst ($localvariablenref ($name ($main._1)) ($ty (!11)))))
           ($alloca_ty (!12)) ($array_size ($cst ($constantint ($val (#-1)) ($ty (!9)))))
           ($debug ($srcloc ($line (#-1)) ($col (#-1)) ($file (!2)))))
      )
    )
    ($basicblock
      ($name ($*in_bb_1_to_bb_2_phi))
      ($instructions
        ($assign ($lhs ($var ($name ($main.i.0)) ($ty (!9)))))
        ($rhs ($cst ($constantint ($val (#0)) ($ty (!9)))))
        ($debug ($srcloc ($line (#6)) ($col (#10)) ($file (!13)))))
      )
    )
    [...]$
  )
  )
)$trans
  ($edge ($bb_1) ($*in_bb_1_to_bb_2_phi))
  ($edge ($*in_bb_1_to_bb_2_phi) ($bb_2))
  ($edge ($bb_2) ($*out_bb_2_to_bb_3_icmp_true))
  ($edge ($bb_2) ($*out_bb_2_to_bb_5_icmp_false))
  ($edge ($*in_bb_4_to_bb_2_phi) ($bb_2))
  [...]
bb_1:
main._1 = allocate(1, [10 x i32])

*in_bb_1_to_bb_2_phi:
main.i.0 = 0

bb_2:

*in_bb_4_to_bb_2_phi:
main.i.0 = main._14

bb_3:
main._10 = sext main.i.0
__v:7 = mul(4, main._10)
main._11 = ptr_shift(main._1, __v:7)
memory[main._11] = main.i.0

*out_bb_2_to_bb_3_icmp_true:
main.i.0 slt 10
main._8 = -1

*out_bb_2_to_bb_5_icmp_false:
main.i.0 sge 10
main._8 = 0

bb_4:
main._14 = add(main.i.0, 1)

bb_5:
main._17 = sub(main.i.0, 1)
main._18 = sext main._17
__v:10 = mul(4, main._18)
main._19 = ptr_shift(main._1, __v:10)
memory[main._19] = main.i.0
main._20 = memory[main._1]
main._21 = ptr_shift(.str, 0)
main._22 = call printf(main._21, main._20)
main._24 = memory[main._1]
main._25 = ptr_shift(.str, 0)
main._26 = call printf(main._25, main._24)
return 0
Load an Abstract Representation file (.ar) and apply passes
Similar to llvm opt command
IKOS passes:
- ps-opt: Optimize pointer shift statements
- branching-opt: Optimize the Control Flow Graph
- inline-init-gv: Inline initialization of global variables in main
- unify-exit-nodes: Unify exit nodes
- analyzer: Analyzer pass
bb 4:
main.14 = add(main.i.0, 1)
main.i.0 = main.14

*out_bb_2_to_bb_3_icmp_true:
main.i.0 slt 10
main.8 = -1

bb 3:
main.10 = sext main.i.0
__v:7 = mul(4, main.10)
main.11 = ptr_shift(main.1, __v:7)
memory[main.11] = main.i.0

bb 1:
main.1 = allocate(1, [10 x i32])
main.i.0 = 0
main.i.0 slt 10
main.8 = -1

*out_bb_2_to_bb_5_icmp_false:
main.i.0 sge 10
main.8 = 0

bb 5:
main.17 = sub(main.i.0, 1)
main.18 = sext main.17
__v:10 = mul(4, main.18)
main.19 = ptr_shift(main.1, __v:10)
main.20 = memory[main.19]
main.21 = ptr_shift(.str, 0)
main.22 = call printf(main.21, main.20)
main.24 = memory[main.1]
main.25 = ptr_shift(.str, 0)
main.26 = call printf(main.25, main.24)
return 0
Analyzer

- Liveness analysis
- Pointer analysis
- Memory analysis combining:
  - Numerical analysis
  - Pointer analysis
  - Uninitialized variable analysis
  - Null pointer analysis
- Checkers:
  - buffer overflow
  - division by zero
  - null dereference
  - uninitialized variables
  - assertion prover
- Store results in a SQLite database
The toolchain is launched via a python script

Generate reports in different formats:
  - Console (gcc style)
  - JSON
  - XML
  - etc.

Output database reusable (using *ikos-render*)
**Toolchain**

- **C/C++ code**
- **clang**
- **LLVM IR**

**IKOS**
- Abstract Domains
  - Interval
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  - Interval + Congruence
  - Octagons
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**ARBOS**
- (AR parser, analysis plugin framework)
  - Outputs reports to console
  - IKOSView: desktop GUI that queries results stored in SQLite3 database
  - Integrated into web services (such as continuous build + bug tracking systems)
    - SonarQube – using sonar_runner
    - CodeDX – import results in cppcheck XML format
    - SWAMP – used in cybersecurity

**Tool Chain Execution Flow**

- **Optimized LLVM IR**
- **LLVM opt command + AR pass (-arbos)**
- **AR in s-expr**
- **Analysis results**

Maxime Arthaud
Demo.
Aeroquad - The Open Source Quadcopter

- Code size:
  - lines of code: 167k
  - bitcode instructions: 4634

- Time stats:
  - arbos: 1 min 51.888 sec
  - ikos-pp: 0.126 sec
  - llvm-to-ar: 0.898 sec

- Summary:
  - number of checks: 2908
  - number of unreachable checks: 46 (1.6%)
  - number of safe checks: 2688 (92.4%)
  - number of definite unsafe checks: 0
  - number of warnings: 174 (5.9%)
Aeroquad - The Open Source Quadcopter

- Writes at specific addresses:
  
  ```
  *(0x42) = x;
  ```

- False positives on loops with casts:
  ```
  for (byte axis = 0; axis < 3; axis++) {
    accelSample[axis] = 0;
  }
  ```

- Tricky array indexing:
  ```
  static byte receiverPin[6] = 
  {2, 5, 6, 4, 7, 8};
  pinData[receiverPin[channel]].edge = FALLING_EDGE;
  ```
Paparazzi - Autopilot System for UAV

- **Code size**:
  - lines of code: 23k
  - bitcode instructions: 4436

- **Time stats**:
  - arbos: 1 min 2.930 sec
  - ikos-pp: 0.132 sec
  - llvm-to-ar: 1.111 sec

- **Summary**:
  - number of checks: 2372
  - number of unreachable checks: 352 (14.8%)
  - number of safe checks: 2020 (85.2%)
  - number of definite unsafe checks: 0
  - number of warnings: 0
GEN2

- Code size:
  - lines of code: 13k
  - bitcode instructions: 5340

- Time stats:
  - arbos: 2 min 16.161 sec
  - ikos-pp: 0.199 sec
  - llvm-to-ar: 1.358 sec

- Summary:
  - number of checks: 3121
  - number of unreachable checks: 0
  - number of safe checks: 3028 (97.1%)
  - number of definite unsafe checks: 0
  - number of warnings: 93 (2.9%)
MNAV

- Code size :
  - lines of code : 159k
  - bitcode instructions : 2145

- Time stats :
  - arbos : 12.950 sec
  - ikos-pp : 0.056 sec
  - llvm-to-ar : 0.468 sec

- Summary :
  - number of checks : 430
  - number of unreachable checks : 17 (3.9%)
  - number of safe checks : 330 (76.7%)
  - number of definite unsafe checks : 0
  - number of warnings : 83 (19.3%)
CASS

- **Time stats**:
  - arbos : 1 day 2 hour 17.463 sec
  - ikos-pp : 13.234 sec
  - llvm-to-ar : 24.431 sec

- **Summary**:
  - number of checks : 254452
  - number of unreachable checks : 33300 (13.0%)
  - number of safe checks : 172521 (67.8%)
  - number of definite unsafe checks : 0
  - number of warnings : 48631 (19.1%)
Results

FLTz - flight simulator with OpenGL displays

- **Code size:**
  - lines of code: 91k
  - bitcode instructions: 14501

- **Time stats:**
  - arbos: 5 day 9 hour 27 min 41.459 sec
  - ikos-pp: 25.211 sec
  - llvm-to-ar: 1 min 2.661 sec

- **Summary:**
  - number of checks: 1302470
  - number of unreachable checks: 72409 (5.5%)
  - number of safe checks: 153312 (11.7%)
  - number of definite unsafe checks: 19 (0.001%)
  - number of warnings: 1076730 (82.6%)
Introduction

IKOS

Analyses
- Liveness analysis
- Pointer analysis
- Memory analysis
- Property checking

Miscellaneous

Conclusion
Liveness analysis

- Mark *live* and *dead* variables after each basic block
- Dataflow analysis
- Used to clean up variables in the abstract domain
- Problem for relational domains
Liveness analysis - Algorithm

- **Kill - Gen algorithm**
- \( GEN[b] \): set of variables used in \( b \) before any assignment
- \( KILL[b] \): set of variables that are assigned in \( b \)
 Kill - Gen algorithm

\( GEN[b] \) : set of variables used in \( b \) before any assignment

\( KILL[b] \) : set of variables that are assigned in \( b \)

\[ GEN[stmt : y ← f(x_1, \cdots, x_n)] = \{x_1, \ldots, x_n\} \]

\[ KILL[stmt : y ← f(x_1, \cdots, x_n)] = \{y\} \]
Liveness analysis - Algorithm

- Kill - Gen algorithm
- $GEN[b]$ : set of variables used in $b$ before any assignment
- $KILL[b]$ : set of variables that are assigned in $b$

- $GEN[stmt : y \leftarrow f(x_1, \ldots, x_n)] = \{x_1, \ldots, x_n\}$
- $KILL[stmt : y \leftarrow f(x_1, \ldots, x_n)] = \{y\}$

- $LIVE_{in}[b] = GEN[b] \cup (LIVE_{out}[b] - KILL[b])$
- $LIVE_{out}[b] = \bigcup_{p \in succ[b]} LIVE_{in}[p]$
- $LIVE_{out}[final] = \emptyset$
Liveness analysis - Example

```
bb_4:
main._14 = add(main.i.0, 1)
main.i.0 = main._14

*out_bb_2_to_bb_3_icmp_true:
main.i.0 slt 10
main._8 = -1

bb_3:
main._10 = sext main.i.0
_v:7 = mul(4, main._10)
main._11 = ptr_shift(main._1, _v:7)
memory[main._11] = main.i.0

bb_5:
return 0

bb_1:
main._1 = allocate(1, [10 x i32])
main.i.0 = 0
main.i.0 slt 10
main._8 = -1

*out_bb_2_to_bb_5_icmp_false:
main.i.0 sge 10
main._8 = 0
```
Pointer analysis: What memory locations can a pointer expression refer to?

Alias analysis: Are two pointers referring to the same locations?

Intraprocedural vs Interprocedural

Flow sensitive vs Flow insensitive

Context sensitive vs Context insensitive
How to model memory locations?

- Global variables: use symbolic names (e.g., `g`)
- Local variables: use symbolic names (e.g., `main.x`)
- Dynamically allocated memory: use symbolic names?
  - Problem: potentially unbounded locations (think about a loop)
  - Solution: use symbolic names with an instruction counter (e.g., `blk(l, λ)`)
Andersen’s pointer analysis

For each pointer $p$, we call $T_p$ the set of memory locations pointed by $p$

Goal : find $T_p$ for each pointer $p$

Idea : view pointer assignments as subset constraints

Complexity : $O(n^3)$, worst case $O(n^4)$
Andersen’s pointer analysis

For each pointer $p$, we call $T_p$ the set of memory locations pointed by $p$

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Idea : view pointer assignments as subset constraints

Complexity : $O(n^3)$, worst case $O(n^4)$

- $p = \& x \iff T_p \supseteq \{x\}$
- $p = q + o \iff T_p \supseteq T_q$
- $p = *q \iff T_p \supseteq *T_q \iff \forall x \in T_q, T_p \supseteq O(x)$
- $*p = q \iff *T_p \supseteq T_q \iff \forall x \in T_p, O(x) \supseteq T_q$
Andersen’s pointer analysis

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How to solve the constraints system? A fix point, of course!
Example:

- $p = \&a$
- $q = \&b$
- $*p = q$
- $r = \&c$
- $s = p$
- $t = *p$
- $*s = r$
Example:

- $p = \& a \iff T_p \supseteq \{a\}$
- $q = \& b \iff T_q \supseteq \{b\}$
- $*p = q \iff *T_p \supseteq T_q$
- $r = \& c \iff T_r \supseteq \{c\}$
- $s = p \iff T_s \supseteq T_p$
- $t = *p \iff T_t \supseteq *T_p$
- $*s = r \iff *T_s \supseteq T_r$

Exercice: solve it!
Solution:

- $T_p = \{a\}$
- $T_q = \{b\}$
- $T_r = \{c\}$
- $T_s = \{a\}$
- $T_t = \{b, c\}$
- $O(a) = \{b, c\}$
- $O(b) = \emptyset$
- $O(c) = \emptyset$
Steensgaard’s pointer analysis

Idea: view pointer assignments as equality constraints

\[
p = \& x \iff T_p \supseteq \{x\}
\]

\[
p = q + o \iff T_p = T_q
\]

\[
p = ^* q \iff T_p = ^* T_q \iff \forall x \in T_q, T_p = O(x)
\]

\[
^* p = q \iff ^* T_p = T_q \iff \forall x \in T_p, O(x) = T_q
\]

Question: Is it more or less precise? Why?

Question: Complexity?
Steensgaard’s pointer analysis
Idea: view pointer assignments as equality constraints

- $p = \& x \iff \mathcal{T}_p \supseteq \{x\}$
- $p = q + o \iff \mathcal{T}_p = \mathcal{T}_q$
- $p = *q \iff \mathcal{T}_p = *\mathcal{T}_q \iff \forall x \in \mathcal{T}_q, \mathcal{T}_p = O(x)$
- $*p = q \iff *\mathcal{T}_p = \mathcal{T}_q \iff \forall x \in \mathcal{T}_p, O(x) = \mathcal{T}_q$
Steensgaard’s pointer analysis

Idea: view pointer assignments as equality constraints

$p = \&x$ $\iff$ $T_p \supseteq \{x\}$

$p = q + o$ $\iff$ $T_p = T_q$

$p = *q$ $\iff$ $T_p = *T_q$ $\iff$ $\forall x \in T_q$, $T_p = O(x)$

$p = *q$ $\iff$ $*T_p = T_q$ $\iff$ $\forall x \in T_p$, $O(x) = T_q$

Question: Is it more or less precise? Why?

Question: Complexity?
- Steensgaard is less precise than Andersen’s algorithm
- Each equality constraint is equivalent to 2 inclusion constraints
- Steensgaard’s constraints system include Andersen’s constraints
- Think fix point: once you reached Andersen’s system fix point solution, you will keep growing to satisfy equality constraints
- Complexity: $O(n\log(n))$ (process each constraint once using union-find)
Solution:

- \( T_p = T_s = \{a\} \)
- \( T_q = T_t = T_r = O(a) = \{b, c\} \)
- \( O(b) = \emptyset \)
- \( O(c) = \emptyset \)
IKOS uses Andersen’s approach

Based on Arnaud Venet’s paper: « A Scalable Nonuniform Pointer Analysis for Embedded Programs », SAS 2004

Compute points-to set (Andersen) and offset (Intervals) for each pointer

\[
D^\# = P \rightarrow (A \cup \{\top\}) \times I
\]

Interprocedural

Flow insensitive

Context insensitive
Memory analysis (also called Value analysis) based on a reduced domain product of:
- Numerical domain for integers (by default, intervals)
- Pointer domain
- Null pointer domain
- Uninitialized variable domain
- Floating points are currently ignored

Based on Antoine Mine’s paper: « Field-Sensitive Value Analysis of Embedded C Programs with Union Types and Pointer Arithmetics », LCTES’06

- Interprocedural
- Context sensitive
Memory analysis - Pointer domain

- Pointer abstract domain
- \( \mathbb{D}_p^\# = \mathbb{V} \to (A \cup \{T\}) \times I \)
- Pointwise order \( \sqsubseteq_p^\# \), Pointwise union \( \sqcup_p^\# \)
- \((\mathbb{D}_p^\#, \sqsubseteq_p^\#, \sqcup_p^\#)\) is a lattice
- Galois connection \((\alpha_p, \gamma_p)\) with the concrete semantics
- Reduction with the previous flow-insensitive pointer analysis
Abstract operations:

- \([p = \&x]#(\rho) = \rho [p \to (\{x\}, [0, 0])]\)
- \([p = q + o]#(\rho) = \rho [p \to (\text{addresses}(\rho(q)), \text{offsets}(\rho(q)) + o)]\)
- \([*p = q]#(\rho) = \rho\)
- \([p = *q]#(\rho) = \rho [p \to (\top, ]-\infty, +\infty[)]\)
Abstract operations:

- \([p = \& x] \#(\rho) = \rho [p \rightarrow (\{x\}, [0, 0])]\)
- \([p = q + o] \#(\rho) = \rho [p \rightarrow (\text{addresses}(\rho(q)), \text{offsets}(\rho(q)) + o)]\)
- \([*p = q] \#(\rho) = \rho\)
- \([p = *q] \#(\rho) = \rho [p \rightarrow (\top, [\infty, +\infty])]\)

Question: \([p == q] \#(\rho) = ?\)

Question: \([p \neq q] \#(\rho) = ?\)
Null pointer abstract domain

\[ D_n = \{ \bot, \text{Null}, \text{NonNull}, \top \} \]

\[ \mathbb{D}_n^\# = \mathbb{V} \rightarrow D_n \]

\[ \bot \subseteq_n^\# \text{Null}, \bot \subseteq_n^\# \text{NonNull}, \text{Null} \subseteq_n^\# \top, \text{NonNull} \subseteq_n^\# \top \]

\[ \text{Null} \sqcup_n^\# \text{NonNull} = \top \]

\( (\mathbb{D}_n^\#, \subseteq_n^\#, \sqcup_n^\#) \) is a lattice

Galois connection \((\alpha_n, \gamma_n)\) with the concrete semantics
• Uninitialized variable abstract domain

\[ D_u = \{ \bot, \text{Init}, \text{Uninit}, \top \} \]

\[ D_u^\# = \mathbb{V} \rightarrow D_u \]

\[ \bot \sqsubseteq^u \text{Init}, \bot \sqsubseteq^u \text{Uninit}, \text{Init} \sqsubseteq^u \top, \text{Uninit} \sqsubseteq^u \top \]

\[ \text{Init} \sqcup^u \text{Uninit} = \top \]

\[ (D_u^\#, \sqsubseteq^u, \sqcup^u) \text{ is a lattice} \]

• Galois connection \((\alpha_u, \gamma_u)\) with the concrete semantics
Question: how to model the memory?

LLVM is low level, a byte representation is necessary

The C language is not type safe and is very permissive on casts

By the way, what is $y$'s value?
Question: how to model the memory?
LLVM is low level, a byte representation is necessary
The C language is not type safe and is very permissive on casts

We need to model correctly the following code:

```c
uint64_t x = 1;
uint32_t* p = (uint32_t*)&x;
p += 1;
uint32_t y = *p;
```

By the way, what is y’s value?
Memory model from « Formalizing the LLVM Intermediate Representation for Verified Program Transformations », POPL 2012

Memory cell \( mc = \) | \( mb(\text{size}, \text{byte}) \)
| \( \text{mptr}(\text{blk}, \text{offset}, \text{index}) \)
| \( \text{muinit} \)

- Memory state \( = (N, B, C) \)
- \( N \) : next block id
- \( B = \mathbb{Z}^+ \rightarrow \mathbb{Z}^+ \) : block id to block size (bytes)
- \( C = \mathbb{Z}^+ \times \mathbb{Z}^+ \rightarrow \mathcal{MC} \) : (block id, offset in bytes) to memory cell
Example:

```c
int* p = (int*) malloc(sizeof(int) + sizeof(int*));
*p = 0x01020304;
int** q = (int**)(p + 1);
*q = p + 2;
```
Example:

```c
int* p = (int*) malloc(sizeof(int) + sizeof(int*));
*p = 0x01020304;
int** q = (int**)(p + 1);
*q = p + 2;
```

<table>
<thead>
<tr>
<th>blk id</th>
<th>offset</th>
<th>memory cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>mb(32, 4)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>mb(32, 3)</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>mb(32, 2)</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>mb(32, 1)</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>mptr(l, 8, 0)</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>mptr(l, 8, 1)</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>mptr(l, 8, 2)</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
<td>mptr(l, 8, 3)</td>
</tr>
</tbody>
</table>

By the way, what architecture could it be?
Memory analysis - Memory abstract domain

- Memory abstract domain
- Based on Antoine Mine’s paper: « Field-Sensitive Value Analysis of Embedded C Programs with Union Types and Pointer Arithmetics », LCTES’06
- Idea: abstract memory using cells: \( C(\text{address}, \text{offset}, \text{size}) \)
- Each cell is considered as a variable in the underlying abstract domain
- Cells may overlap
- \( C = A \to \mathbb{Z}^+ \times \mathbb{Z}^+ \)
- \( D_{\text{mem}}^\# = C \times D_{\text{underlying}}^\# \)
- In IKOS, \( D_{\text{underlying}}^\# = D_{\text{num}}^\# \times D_{\text{ptr}}^\# \times D_{\text{null}}^\# \times D_{\text{unini}}^\# \)
- Pointwise partial order, Pointwise union
Abstract operations: forward to $\mathbb{D}_{\text{underlying}}^\#$, except memory read and write.

Memory write:
- set to $\bot$ if $p$ is null or uninitialized
- $(\text{points\_to, offset}) = \rho(p)$
- $\text{cells} = \text{realize\_write(\text{points\_to, offset})}$
- $\forall c \in \text{cells}, \text{strong\_update}(c, rhs)$ or $\text{weak\_update}(c, rhs)$

Memory read:
- set to $\bot$ if $p$ is null or uninitialized
- $(\text{points\_to, offset}) = \rho(p)$
- $\text{cells} = \text{realize\_read(\text{points\_to, offset})}$
- $\forall c \in \text{cells}, \text{strong\_update}(lhs, c)$ or $\text{weak\_update}(lhs, c)$
Example:

```c
int* p = (int*) malloc(sizeof(int) + sizeof(int*));
*p = 0x01020304;
int** q = (int**)(p + 1);
*q = p + 2;
```
Memory analysis - Memory abstract domain

Example:

```
int* p = (int*) malloc(sizeof(int) + sizeof(int*));
*p = 0x01020304;
int** q = (int**)(p + 1);
*q = p + 2;
```

Abstract value at the end:

```
(malloc → {{0, 4}, {4, 4}}))
(C(malloc, 0, 4) → [0x01020304, 0x01020304])
(C(malloc, 4, 4) → (malloc, [8, 8]),
   p → (malloc, [0, 0]),
   q → (malloc, [4, 4]))
(C(malloc, 4, 4) → NonNull, p → NonNull, q → NonNull)
(C(malloc, 0, 4) → Init, C(malloc, 4, 4) → Init, p → Init, q → Init))
```
static union {
    struct { uint8 al, ah, bl, bh, ... } b;
    struct { uint16 ax, bx, ... } w;
} regs;
regs.w.ax = X;    // (1)
if (!regs.b.ah) {    // (2)
    regs.b.bl = regs.b.al;    // (3)
} else {    // (4)
    regs.b.bh = regs.b.al;    // (5)
}
// (6)
regs.b.al = X;    // (7)
Last step: check for properties at each statement location

Checkers:
- buffer overflow: $0 \leq \text{offset}$ and $\text{offset} + \text{read\_size} \leq \text{buffer\_size}$
- division by zero: $\text{divisor} \neq 0$
- null dereference: $p \neq \text{Null}$
- uninitialized variable: $v \neq \text{Uninit}$
- prover: $v \neq 0$
Abstract domains implementation

- Separate domain \((V \rightarrow D)\) are implemented with patricia trees
- Insertion and removal in \(O(\log(n))\)
- Merge in \(O(n)\)
- Transformation in \(O(n)\)
- Very cheap union!
Analyzing C++ is very tricky:

- Heavy chains of function calls because of templates
- The libc++ needs to be modeled
- Need to be precise on pointers for virtual method calls
- Handle exceptions
Analyzing C++ is very tricky:

- Heavy chains of function calls because of templates
- The libc++ needs to be modeled
- Need to be precise on pointers for virtual method calls
- Handle exceptions

Work in progress!
Exception handling

```
bb_1:
  memory[x] = 9
  _Z1fv._2 = call _Z14__ikos_unknownv()

*out_bb_1_to_bb_2_icmp_true:
  _Z1fv._2 ne 0
  _Z1fv._3 = -1

*out_bb_1_to_bb_3_icmp_false:
  _Z1fv._2 eq 0
  _Z1fv._3 = 0

bb_2:
  _Z1fv._5 = call __cxa_allocate_exception(8)
  _Z1fv._6 = bitcast _Z1fv._5
  memory[_Z1fv._6] = $null
  _Z1fv._8 = bitcast _Z1IDn
  __v:6 = call __cxa_throw(_Z1fv._5, _Z1fv._8, $null)
  unreachable

bb_3:
  memory[x] = 0
  return

_unified_exit:
```
Exception handling

bb_8:
main._17 = memory[y]

* bb_8_split_icmp_false:
main._17 ne 0
main._18 = 0

* bb_8_split_icmp_true:
main._17 eq 0
main._18 = -1

*out bb_8_merge_icmp:
__v:17 = invoke_Z13__ikos_assertb(main._18)

bb_10:
landingpad(main._21)
__v:18 = 0
__v:18 = add(__v:18, 0)
main._22 = extract_elem(main._21, __v:18)
__v:19 = 0
__v:19 = add(__v:19, 8)
main._23 = extract_elem(main._21, __v:19)

bb_11:
main._25 = bitcast_ZTIPv
main._26 = call llvm.eh.typeid.for(main._25)

bb_9:
Exception handling

- $\mathbb{D}_{\text{exc}}^\# = \mathbb{D}^\# \times \mathbb{D}^\#$
- $\llbracket \text{throw}(e) \rrbracket^\#(N, E) = (\bot, N \cup E)$
- $\llbracket \text{landingpad}(e) \rrbracket^\#(N, E) = (E, \bot)$
- $\llbracket v = x \rrbracket^\#(N, E) = (\llbracket v = x \rrbracket^\#(N), E)$
- $(N_1, E_1) \sqcup^\# (N_2, E_2) = (N_1 \cup N_2, E_1 \cup E_2)$
Intervals are very imprecise for loops with a non-deterministic bound

Solution: use a weakly-relational domain, such as the DBM domain

Difference-Bound Matrices

- Difference-Bound Matrices
- Weakly-relational abstract domain

\[
\begin{bmatrix}
0 & m_{0,1} & m_{0,2} & \ldots & m_{0,n} \\
m_{1,0} & 0 & m_{1,2} & \ldots & m_{1,n} \\
m_{2,0} & m_{2,1} & 0 & \ldots & m_{2,n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
m_{n,0} & m_{n,1} & m_{n,2} & \ldots & 0
\end{bmatrix}
\]

- \( m_{i,j} \in \mathbb{Z} \cup \{+\infty\} \)
- \( v_i - v_j \leq m_{j,i} \)
- \( v_0 = 0 \), thus \( v_i \in [-m_{i,0}, m_{0,i}] \)

Abstract operations require normalization

- normalization:
  \( v_i - v_k \leq m_{k,i} \) and
  \( v_k - v_j \leq m_{j,k} \) \( \Rightarrow \)
  \( v_i - v_j \leq m_{k,i} + m_{j,k} \)
- cost \( O(n^3) \), \( n \) number of variables
Variable packing

- Idea: keep a list of DBMs, where each DBM contains variables that are related to each other.
- Union-Find structure to dynamically infer relations among variables.
- Normalization cost $O(n)$, $n$ number of DBMs.

DBM 1
{x, y, u}

DBM 2
{z}

DBM 3
{v, w}

Diagram of DBMs with variables x, y, u, z, v, w.
Variable packing

- Idea: keep a list of DBMs, where each DBM contains variables that are related to each other.
- Union-Find structure to dynamically infer relations among variables
- Normalization cost $O(n)$, $n$ number of DBMs
Pointer analysis using function summarization.

<table>
<thead>
<tr>
<th>File</th>
<th>DBMs</th>
<th>Size</th>
<th>Var Packing</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>astree-ex</td>
<td>1.01s</td>
<td>36</td>
<td>0.13s</td>
<td>7</td>
</tr>
<tr>
<td>test-1</td>
<td>0.13s</td>
<td>27</td>
<td>0.03s</td>
<td>4</td>
</tr>
<tr>
<td>test-1-unsafe</td>
<td>0.13s</td>
<td>27</td>
<td>0.02s</td>
<td>4</td>
</tr>
<tr>
<td>test-10</td>
<td>0.03s</td>
<td>10</td>
<td>0.02s</td>
<td>4</td>
</tr>
<tr>
<td>test-10-unsafe</td>
<td>0.03s</td>
<td>11</td>
<td>0.02s</td>
<td>4</td>
</tr>
<tr>
<td>paparazzi-microjet</td>
<td>3241.14s</td>
<td>611</td>
<td>158.50s</td>
<td>88</td>
</tr>
<tr>
<td>gen2</td>
<td>&gt; 5h</td>
<td>?</td>
<td>7817.42s</td>
<td>367</td>
</tr>
<tr>
<td>aeroquad-servo</td>
<td>78.12s</td>
<td>71</td>
<td>1.33s</td>
<td>14</td>
</tr>
<tr>
<td>aeroquad-new</td>
<td>86.18s</td>
<td>65</td>
<td>0.76s</td>
<td>5</td>
</tr>
<tr>
<td>cornell</td>
<td>447.06s</td>
<td>226</td>
<td>2.64s</td>
<td>6</td>
</tr>
<tr>
<td>sporesate2-spore-pl</td>
<td>895.45s</td>
<td>?</td>
<td>10.29s</td>
<td>19</td>
</tr>
</tbody>
</table>
Other ideas

- Group variables depending on heuristics
- Use the gauge domain
Other ideas

- Group variables depending on heuristics
- Use the gauge domain

Work in progress!
IKOS uses dynamic inlining

Idea: analyse each function only once to build a summary
Problem: call graph cycle
Function summarization: Call graph analysis

- Problem: call graph cycle

```
main
f     g
h     w
```

- Strongly connected component analysis
- Topological order
- Bottom-up analysis (from the leaves to the root)
- Top-down analysis (from the root to the leaves)
Function summarization: Memory analysis

- Need a way to express the effect of a function call on the memory
- More particularly on global variables and pointer parameters
- Relation between the input memory state and the output memory state
- Idea: Introduce input cells and output cells

\[ x = x + 1 \Leftrightarrow \text{Cell}\{x, 0, 4, \text{Out}\} = \text{Cell}\{x, 0, 4, \text{In}\} + 1 \]
Buffer overflow analysis using function summarization

<table>
<thead>
<tr>
<th>File</th>
<th>Inlining</th>
<th>Summaries</th>
<th>Warnings</th>
<th>Errors</th>
<th>Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>astree-ex</td>
<td>0.36s</td>
<td>0.57s</td>
<td>2/2</td>
<td>0/0</td>
<td>22 (1)</td>
</tr>
<tr>
<td>test-1</td>
<td>0.14s</td>
<td>0.16s</td>
<td>0/0</td>
<td>0/0</td>
<td>22 (1)</td>
</tr>
<tr>
<td>test-1-unsafe</td>
<td>0.13s</td>
<td>0.18s</td>
<td>0/0</td>
<td>2/2</td>
<td>22 (1)</td>
</tr>
<tr>
<td>test-10</td>
<td>0.10s</td>
<td>0.13s</td>
<td>0/2</td>
<td>0/0</td>
<td>20 (3)</td>
</tr>
<tr>
<td>paparazzi</td>
<td>154.03s</td>
<td>110.09s</td>
<td>0/0</td>
<td>0/0</td>
<td>24650 (199)</td>
</tr>
<tr>
<td>gen2</td>
<td>307.66s</td>
<td>&gt; 3h</td>
<td>195/7</td>
<td>0/?</td>
<td>22030 (82)</td>
</tr>
</tbody>
</table>
Problem: LLVM integer types are signedness agnostic

Because most instructions are signedness agnostic: add, sub, mul, etc.

How to be sound and precise?

- Intervals with infinite precision: imprecise or unsound
- Suppose integers are unsigned: imprecise
- Suppose integers are signed: imprecise
- Wrapped intervals: Jorge Navas’s paper « Signedness-Agnostic Program Analysis: Precise Integer Bounds for Low-Level Code »
- Domain product: unsigned and signed
Thank you. Questions?