Static Analysis using Abstract Interpretation

Maxime Arthaud

NASA Ames Research Center, California
1 Introduction
- Software development
- Safety properties
- Abstract Interpretation

2 IKOS

3 Analyses

4 Miscellaneous

5 Conclusion
Software development

- Software represent more than half of the development cost of an aircraft
- Regulated by international standards (DO-178 rev. B/C)
Software represent more than half of the development cost of an aircraft.


Tests
- Expensive because run on a special hardware
- Can miss bugs
- Slow
Software development

- Software represent more than half of the development cost of an aircraft
- Regulated by international standards (DO-178 rev. B/C)
- Tests
  - Expensive because run on a special hardware
  - Can miss bugs
  - Slow
- Solution: use static analysis
- NASA V&V program
Safety properties

Main objectives: no runtime errors
- buffer overflow
- null dereference
- division by zero
- integer overflow

Harder objectives:
- assertions (pre/post invariants)
- termination

certified => soundness is required

abstract interpretation is a good candidate

runtime errors can be security vulnerabilities!
Abstract Interpretation

- based on the concrete semantics of your program
- automatic formal proof
- sound approximation of reachable states
Abstract Interpretation

\[ x(t) \]

\[ t \]

\[ \text{semantics}(P) \]

Possible trajectories
Abstract Interpretation

Forbidden zone

specification(\(P\))
Abstract Interpretation

\[ x(t) \]

Forbidden zone

semantics(\(P\)) \(\subseteq\) specification(\(P\))
Using testing

Forbidden zone

Error !!!

Test of a few trajectories

Possible trajectories

$x(t)$
Abstract Interpretation

\[\text{abstraction}(P)\]

Abstraction of the trajectories
Abstract Interpretation

Forbidden zone

Abstraction of the trajectories

\[ \text{abstraction}(P) \subseteq \text{specification}(P) \]
Abstract Interpretation

Forbidden zone

Abstraction of the trajectories

\[ \text{semantics}(P) \subseteq \text{abstraction}(P) \subseteq \text{specification}(P) \]
Thank you Pierre Loïc Garoche
Syllabus

1. Introduction

2. IKOS
   - Project
   - Toolchain
   - Demo
   - Results

3. Analyses

4. Miscellaneous

5. Conclusion
The IKOS project

- Inference Kernel for Open Static Analyzers
- C++ library for abstract interpretation
- C/C++ static analyzer
- Target embedded systems
- Analyses:
  - Buffer overflow
  - Division by zero
  - Null dereference
  - Uninitialized variables
  - Prover
- https://ti.arc.nasa.gov/opensource/ikos/
Toolchain

- C/C++ code
- clang
- LLVM IR

ikos-pp

Optimized LLVM IR

LLVM opt command + AR pass (-arbos)

AR in s-expr

ARBOS
{AR parser, analysis plugin framework}

Analysis results

Abstract Domains
- Interval
- Constants
- Discrete
- Congruence
- Interval + Congruence
- Octagons
- Difference Bounds Matrix
- Pointer Analysis

IKOS

ikos-pp
- Ikos-pp is an executable that embeds the LLVM opt command. It applies several LLVM built-in optimizations + our own optimization passes to produce an intermediate optimized LLVM IR. Using the optimized LLVM IR, we run LLVM opt command with -arbos option to translate the optimized LLVM IR to AR
- ikos-pp does at least the following optimizations before translating to AR: -mem2reg, -loweratomic, -lowerswitch, and -instnamer

AR Plugin Analyzers
- BOA - buffer overflow analysis
- DBZ - Intra-procedural integer division-by-zero analysis
- UVA - Inter-procedural uninitialized variable + array analysis
- NullPtr - Inter-procedural null dereference pointer analysis

Analysis results
- Outputs reports to console
- IKOSView: desktop GUI that queries results stored in SQLite3 database
- Integrated into web services (such as continuous build + bug tracking systems)
  - SonarQube – using sonar_runner
  - CodeDX – import results in cppcheck XML format
  - SWAMP – used in cybersecurity
- Low Level Virtual Machine
- Compiler Infrastructure
- Generic assembly language
- Allow language independent optimization
- Low Level Virtual Machine
- Compiler Infrastructure
- Generic assembly language
- Allow language independent optimization

```
LLVM bitcode
C     C++   Fortran   Ada
x86   PowerPC  ARM    AR
```
$ cat test.c

```
#include <stdio.h>

int main(int argc, char** argv) {
  int a[10];
  int i;
  for (i = 0; i < 10; i++) {
    a[i] = i;
  }
  printf("%d\n", a[i - 1]);
  printf("%d\n", a[0]);
  return 0;
}
```
%2:
  %3 = alloca [10 x i32], align 16
  %4 = bitcast [10 x i32]* %3 to i8*
  call void @llvm.lifetime.start(i64 40, i8* %4) #3
  br label %5

%5:
  %6 = phi i64 [ 0, %2 ], [ %9, %5 ]
  %7 = getelementptr inbounds [10 x i32], [10 x i32]* %3, i64 0, i64 %6
  %8 = trunc i64 %6 to i32
  store i32 %8, i32* %7, align 4, !tbaa !3
  %9 = add nuw nsw i64 %6, 1
  %10 = icmp eq i64 %9, 10
  br i1 %10, label %11, label %5

%11:
  %12 = getelementptr inbounds [10 x i32], [10 x i32]* %3, i64 0, i64 9
  %13 = load i32, i32* %12, align 4, !tbaa !3
  %14 = tail call i32 (i32, i8*, ...) @__printf_chk(i32 1, i8* getelementptr...
  ... inbounds ([4 x i8], [4 x i8]* @.str, i64 0, i64 0), i32 %13) #3
  %15 = getelementptr inbounds [10 x i32], [10 x i32]* %3, i64 0, i64 0
  %16 = load i32, i32* %15, align 16, !tbaa !3
  %17 = tail call i32 (i32, i8*, ...) @__printf_chk(i32 1, i8* getelementptr...
  ... inbounds ([4 x i8], [4 x i8]* @.str, i64 0, i64 0), i32 %16) #3
  call void @llvm.lifetime.end(i64 40, i8* nonnull %4) #3
  ret i32 0
IKOS-PP

- IKOS pre-processor
- Run llvm optimization passes:
  - mem2reg: SSA Form
  - globaldce: Dead Code Elimination
  - globalopt: Global Variable Optimizer
  - simplifycfg: Control Flow Graph Optimizer
  - scalarrepl: Scalar Replacement of Aggregates
  - sccp: Sparse Conditional Constant Propagation
  - loop-simplify: Canonical Form for Loops
  - lcssa: Loop Closed SSA Form
  - loop-deletion: Dead Loop Elimination
  - lowerinvoke: Lower Invoke Instructions
  - lowerswitch: Lower Switch Instructions
- Run home made llvm passes:
  - Lower Global Variable Initialization
  - Lower Constant Expressions
  - Lower Select Instructions
  - Name Values
Abstract Representation

Major differences with llvm:
- Branching instructions are translated into assertions
- Memory instructions are byte oriented
- Some instructions are removed

Translation from llvm to AR using a llvm pass

Text representation using s-expressions
bb_4:
main._14 = add(main.i.0, 1)

*in_bb_4_to_bb_2_phi:
main.i.0 = main._14

bb_2:

*out_bb_2_to_bb_3_icmp_true:
main.i.0 slt 10
main._8 = -1

bb_3:
main._10 = sext main.i.0
_v:7 = mul(4, main._10)
main._11 = ptr_shift(main._1, _v:7)
memory[main._11] = main.i.0

(*out_bb_2_to_bb_3_icmp_true:
main.i.0 slt 10
main._8 = -1

bb_4:
main._14 = add(main.i.0, 1)

*in_bb_4_to_bb_2_phi:
main.i.0 = main._14

bb_5:
main._17 = sub(main.i.0, 1)
main._18 = sext main._17
_v:10 = mul(4, main._18)
main._19 = ptr_shift(main._1, _v:10)
memory[main._19] = main.i.0

(*out_bb_2_to_bb_5_icmp_false:
main.i.0 sge 10
main._8 = 0

bb_1:
main._1 = allocate(1, [10 x i32])

*in_bb_1_to_bb_2_phi:
main.i.0 = 0

*out_bb_2_to_bb_3_icmp_true:
main.i.0 slt 10
main._8 = -1

bb_5:
main._17 = sub(main.i.0, 1)
main._18 = sext main._17
_v:10 = mul(4, main._18)
main._19 = ptr_shift(main._1, _v:10)
memory[main._19] = main.i.0

(*out_bb_2_to_bb_5_icmp_false:
main.i.0 sge 10
main._8 = 0

return 0
Load an Abstract Representation file (.ar) and apply passes

Similar to llvm opt command

IKOS passes:

- ps-opt: Optimize pointer shift statements
- branching-opt: Optimize the Control Flow Graph
- inline-init-gv: Inline initialization of global variables in main
- unify-exit-nodes: Unify exit nodes
- analyzer: Analyzer pass
bb 4:
main. 14 = add(main.i.0, 1)
main.i.0 = main. 14

*out_bb_2_to_bb_3_icmp_true:
main.i.0 slt 10
main. 8 = -1

bb 3:
main. 10 = sext main.i.0
__v:7 = mul(4, main. 10)
main. 11 = ptr_shift(main. 1, __v:7)
memory[main. 11] = main.i.0

bb 1:
main. 1 = allocate(1, [10 x i32])
main.i.0 = 0
main.i.0 slt 10
main. 8 = -1

*out_bb_2_to_bb_5_icmp_false:
main.i.0 sge 10
main. 8 = 0

bb 5:
main. 17 = sub(main.i.0, 1)
main. 18 = sext main. 17
__v:10 = mul(4, main. 18)
main. 19 = ptr_shift(main. 1, __v:10)
main. 20 = memory[main. 19]
main. 21 = ptr_shift(.str, 0)
main. 22 = call printf(main. 21, main. 20)
main. 24 = memory[main. 1]
main. 25 = ptr_shift(.str, 0)
main. 26 = call printf(main. 25, main. 24)
return 0
- Liveness analysis
- Pointer analysis
- Memory analysis combining:
  - Numerical analysis
  - Pointer analysis
  - Uninitialized variable analysis
  - Null pointer analysis
- Checkers:
  - buffer overflow
  - division by zero
  - null dereference
  - uninitialized variables
  - assertion prover
- Store results in a SQLite database
- The toolchain is launched via a python script
- Generate reports in different formats:
  - Console (gcc style)
  - JSON
  - XML
  - etc.
- Output database reusable (using *ikos-render*)
Toolchain

- **C/C++ code**
- **clang**
- **LLVM IR**
- **ikos-pp**
- **Optimized LLVM IR**

**LLVM opt command + AR pass (-arbos)**

**AR Plugin Analyzers**
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**ARBOS**
{AR parser, analysis plugin framework}

- Outputs reports to console
- IKOSView: desktop GUI that queries results stored in SQLite3 database
- Integrated into web services (such as continuous build + bug tracking systems)
  - SonarQube - using sonar_runner
  - CodeDX - import results in cppcheck XML format
  - SWAMP - used in cybersecurity

**IKOS**

- *Abstract Domains*
  - Interval
  - Constants
  - Discrete
  - Congruence
  - Interval + Congruence
  - Octagons
  - Difference Bounds Matrix
  - Pointer Analysis

**ikos-pp**
- Ikos-pp is an executable that embeds the LLVM opt command. It applies several LLVM built-in optimizations + our own optimization passes to produce an intermediate optimized LLVM IR. Using the optimized LLVM IR, we run LLVM opt command with `-arbo` option to translate the optimized LLVM IR to AR
- ikos-pp does at least the following optimizations before translating to AR: `-mem2reg`, `-loweratomic`, `-lowerswitch`, and `-instnamer`
Demo.
Aeroquad - The Open Source Quadcopter

- **Code size:**
  - lines of code: 167k
  - bitcode instructions: 4634

- **Time stats:**
  - arbos: 1 min 51.888 sec
  - ikos-pp: 0.126 sec
  - llvm-to-ar: 0.898 sec

- **Summary:**
  - number of checks: 2908
  - number of unreachable checks: 46 (1.6%)
  - number of safe checks: 2688 (92.4%)
  - number of definite unsafe checks: 0
  - number of warnings: 174 (5.9%)
Aeroquad - The Open Source Quadcopter

- Writes at specific addresses:
  
  \[ *(0x42) = x; \]

- False positives on loops with casts:
  
  \[
  \text{for (byte axis = 0; axis < 3; axis++)} \{
    \text{accelSample[axis]} = 0;
  \}
  \]

- Tricky array indexing:
  
  \[
  \text{static byte receiverPin[6] = }
  \{2, 5, 6, 4, 7, 8\};
  \text{pinData[receiverPin[channel]].edge = FALLING_EDGE;}
  \]
Paparazzi - Autopilot System for UAV

- **Code size:**
  - lines of code: 23k
  - bitcode instructions: 4436

- **Time stats:**
  - arbos: 1 min 2.930 sec
  - ikos-pp: 0.132 sec
  - llvm-to-ar: 1.111 sec

- **Summary:**
  - number of checks: 2372
  - number of unreachable checks: 352 (14.8%)
  - number of safe checks: 2020 (85.2%)
  - number of definite unsafe checks: 0
  - number of warnings: 0
GEN2

- **Code size**:
  - lines of code: 13k
  - bitcode instructions: 5340

- **Time stats**:
  - arbos: 2 min 16.161 sec
  - ikos-pp: 0.199 sec
  - llvm-to-ar: 1.358 sec

- **Summary**:
  - number of checks: 3121
  - number of unreachable checks: 0
  - number of safe checks: 3028 (97.1%)
  - number of definite unsafe checks: 0
  - number of warnings: 93 (2.9%)
MNAV

- **Code size:**
  - lines of code: 159k
  - bitcode instructions: 2145

- **Time stats:**
  - arbos: 12.950 sec
  - ikos-pp: 0.056 sec
  - llvm-to-ar: 0.468 sec

- **Summary:**
  - number of checks: 430
  - number of unreachable checks: 17 (3.9%)
  - number of safe checks: 330 (76.7%)
  - number of definite unsafe checks: 0
  - number of warnings: 83 (19.3%)
CASS

- **Time stats:**
  - arbos: 1 day 2 hour 17.463 sec
  - ikos-pp: 13.234 sec
  - llvm-to-ar: 24.431 sec

- **Summary:**
  - number of checks: 254452
  - number of unreachable checks: 33300 (13.0%)
  - number of safe checks: 172521 (67.8%)
  - number of definite unsafe checks: 0
  - number of warnings: 48631 (19.1%)
Results

FLTz - flight simulator with OpenGL displays

- Code size:
  - lines of code: 91k
  - bitcode instructions: 14501

- Time stats:
  - arbos: 5 day 9 hour 27 min 41.459 sec
  - ikos-pp: 25.211 sec
  - llvm-to-ar: 1 min 2.661 sec

- Summary:
  - number of checks: 1302470
  - number of unreachable checks: 72409 (5.5%)
  - number of safe checks: 153312 (11.7%)
  - number of definite unsafe checks: 19 (0.001%)
  - number of warnings: 1076730 (82.6%)
1 Introduction

2 IKOS

3 Analyses
   - Liveness analysis
   - Pointer analysis
   - Memory analysis
   - Property checking

4 Miscellaneous

5 Conclusion
Liveness analysis

- Mark *live* and *dead* variables after each basic block
- Dataflow analysis
- Used to clean up variables in the abstract domain
- Problem for relationnal domains
Liveness analysis - Algorithm

- Kill - Gen algorithm
- $GEN[b]$ : set of variables used in $b$ before any assignment
- $KILL[b]$ : set of variables that are assigned in $b$
Liveness analysis - Algorithm

- Kill - Gen algorithm
- $GEN[b]$ : set of variables used in $b$ before any assignment
- $KILL[b]$ : set of variables that are assigned in $b$

- $GEN[stmt : y ← f(x_1, \cdots, x_n)] = \{x_1, \ldots, x_n\}$
- $KILL[stmt : y ← f(x_1, \cdots, x_n)] = \{y\}$
Liveness analysis - Algorithm

- **Kill - Gen algorithm**
- $GEN[b]$: set of variables used in $b$ before any assignment
- $KILL[b]$: set of variables that are assigned in $b$

- $GEN[stmt : y \leftarrow f(x_1, \cdots, x_n)] = \{x_1, \ldots, x_n\}$
- $KILL[stmt : y \leftarrow f(x_1, \cdots, x_n)] = \{y\}$

- $LIVE_{in}[b] = GEN[b] \cup (LIVE_{out}[b] - KILL[b])$
- $LIVE_{out}[b] = \bigcup_{p \in succ[b]} LIVE_{in}[p]$
- $LIVE_{out}[final] = \emptyset$
Liveness analysis - Example

bb 4:
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main.i.0 = main. 14

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main.i.0 slt 10
main. 8 = -1

bb 1:
main. 1 = allocate(1, [10 x i32])
main.i.0 = 0
main.i.0 slt 10
main. 8 = -1

*out_bb 2_to_bb_5_icmp_false:
main.i.0 sge 10
main. 8 = 0

bb 3:
main. 10 = sext main.i.0
_v:7 = mul(4, main. 10)
main. 11 = ptr_shift(main. 1, _v:7)
memory[main. 11] = main.i.0

bb 5:
return 0
- Pointer analysis: What memory locations can a pointer expression refer to?
- Alias analysis: Are two pointers referring to the same locations?
- Intraprocedural vs Interprocedural
- Flow sensitive vs Flow insensitive
- Context sensitive vs Context insensitive
How to model memory locations?

- Global variables: use symbolic names (e.g., $g$)
- Local variables: use symbolic names (e.g., $main.x$)
- Dynamically allocated memory: use symbolic names?
  - Problem: potentially unbounded locations (think about a loop)
  - Solution: use symbolic names with an instruction counter (e.g., $blk(l, \lambda)$)
Andersen’s pointer analysis

For each pointer \( p \), we call \( T_p \) the set of memory locations pointed by \( p \)

Goal : find \( T_p \) for each pointer \( p \)

Idea : view pointer assignments as subset constraints

Complexity : \( O(n^3) \), worst case \( O(n^4) \)
Pointer analysis - Andersen’s algorithm

- Andersen’s pointer analysis
- For each pointer \( p \), we call \( T_p \) the set of memory locations pointed by \( p \)
- Goal : find \( T_p \) for each pointer \( p \)
- Idea : view pointer assignments as subset constraints
- Complexity : \( O(n^3) \), worst case \( O(n^4) \)

- \( p = \& x \iff T_p \supseteq \{ x \} \)
- \( p = q + o \iff T_p \supseteq T_q \)
- \( p = *q \iff T_p \supseteq *T_q \iff \forall x \in T_q, \ T_p \supseteq O(x) \)
- \( *p = q \iff *T_p \supseteq T_q \iff \forall x \in T_p, \ O(x) \supseteq T_q \)
Andersen’s pointer analysis

For each pointer $p$, we call $T_p$ the set of memory locations pointed by $p$

Goal: find $T_p$ for each pointer $p$

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Complexity: $O(n^3)$, worst case $O(n^4)$

- $p = \&x \iff T_p \supseteq \{x\}$
- $p = q + o \iff T_p \supseteq T_q$
- $p = *q \iff T_p \supseteq *T_q \iff \forall x \in T_q, T_p \supseteq O(x)$
- $*p = q \iff *T_p \supseteq T_q \iff \forall x \in T_p, O(x) \supseteq T_q$

How to solve the constraints system? A fix point, of course!
Example:

- $p = \&a$
- $q = \&b$
- $*p = q$
- $r = \&c$
- $s = p$
- $t = *p$
- $*s = r$
Example:

- \( p = &a \iff T_p \supseteq \{a\} \)
- \( q = &b \iff T_q \supseteq \{b\} \)
- \( *p = q \iff *T_p \supseteq T_q \)
- \( r = &c \iff T_r \supseteq \{c\} \)
- \( s = p \iff T_s \supseteq T_p \)
- \( t = *p \iff T_t \supseteq *T_p \)
- \( *s = r \iff *T_s \supseteq T_r \)

Exercice: solve it!
Solution:

- $T_p = \{a\}$
- $T_q = \{b\}$
- $T_r = \{c\}$
- $T_s = \{a\}$
- $T_t = \{b, c\}$
- $O(a) = \{b, c\}$
- $O(b) = \emptyset$
- $O(c) = \emptyset$
Steensgaard’s pointer analysis

Idea: view pointer assignments as equality constraints
Steensgaard’s pointer analysis

Idea: view pointer assignments as equality constraints

- $p = \&x \iff T_p \supseteq \{x\}$
- $p = q + o \iff T_p = T_q$
- $p = *q \iff T_p = *T_q \iff \forall x \in T_q, T_p = O(x)$
- $*p = q \iff *T_p = T_q \iff \forall x \in T_p, O(x) = T_q$
Steensgaard’s pointer analysis

Idea: view pointer assignments as equality constraints

- \( p = \&x \iff T_p \supseteq \{x\} \)
- \( p = q + o \iff T_p = T_q \)
- \( p = *q \iff T_p = *T_q \iff \forall x \in T_q, T_p = O(x) \)
- \( *p = q \iff *T_p = T_q \iff \forall x \in T_p, O(x) = T_q \)

Question: Is it more or less precise? Why?

Question: Complexity?
- Steensgaard is less precise than Andersen’s algorithm
- Each equality constraint is equivalent to 2 inclusion constraints
- Steensgaard’s constraints system include Andersen’s constraints
- Think fix point: once you reached Andersen’s system fix point solution, you will keep growing to satisfy equality constraints
- Complexity: $O(n \log(n))$ (process each constraint once using union-find)
Solution:

- $T_p = T_s = \{a\}$
- $T_q = T_t = T_r = O(a) = \{b, c\}$
- $O(b) = \emptyset$
- $O(c) = \emptyset$
IKOS uses Andersen’s approach

Based on Arnaud Venet’s paper: « A Scalable Nonuniform Pointer Analysis for Embedded Programs », SAS 2004

Compute points-to set (Andersen) and offset (Intervals) for each pointer

\[ D^# = P \rightarrow (A \cup \{T\}) \times I \]

Interprocedural

Flow insensitive

Context insensitive
Memory analysis (also called Value analysis) based on a reduced domain product of:

- Numerical domain for integers (by default, intervals)
- Pointer domain
- Null pointer domain
- Uninitialized variable domain
- Floating points are currently ignored

Based on Antoine Mine’s paper: « Field-Sensitive Value Analysis of Embedded C Programs with Union Types and Pointer Arithmetics », LCTES’06

- Interprocedural
- Context sensitive
Memory analysis - Pointer domain

- Pointer abstract domain
  \[ \mathbb{D}_p^\# = \mathbb{V} \rightarrow (\mathbb{A} \cup \{\top\}) \times \mathbb{I} \]
- Pointwise order \( \sqsubseteq^p \), Pointwise union \( \sqcup^p \)
- \( (\mathbb{D}_p^\#, \sqsubseteq^p, \sqcup^p) \) is a lattice
- Galois connection \( (\alpha_p, \gamma_p) \) with the concrete semantics
- Reduction with the previous flow-insensitive pointer analysis
Abstract operations:

- $[p = \& x](\rho) = \rho [p \rightarrow (\{x\}, [0, 0])]$
- $[p = q + o](\rho) = \rho [p \rightarrow (\text{addresses}(\rho(q)), \text{offsets}(\rho(q)) + o)]$
- $[*p = q](\rho) = \rho$
- $[p = *q](\rho) = \rho [p \rightarrow (\top, ]-\infty, +\infty[)]$
Abstract operations:

- \([p = \& x] #(\rho) = \rho \left[ p \rightarrow (\{x\}, [0, 0]) \right]\)
- \([p = q + o] #(\rho) = \rho \left[ p \rightarrow \text{addresses}(\rho(q)), \text{offsets}(\rho(q)) + o \right]\)
- \([*p = q] #(\rho) = \rho\)
- \([p = *q] #(\rho) = \rho \left[ p \rightarrow (\top, ]-\infty, +\infty[) \right]\)

Question: \([p == q] #(\rho) = ?\)

Question: \([p \neq q] #(\rho) = ?\)
Null pointer abstract domain

\[ D_n = \{ \bot, \text{Null}, \text{NonNull}, \top \} \]

\[ \mathbb{D}_n^\# = \mathbb{V} \to D_n \]

\[ \bot \sqsubseteq_n \text{Null}, \bot \sqsubseteq_n \text{NonNull}, \text{Null} \sqsubseteq_n \top, \text{NonNull} \sqsubseteq_n \top \]

\[ \text{Null} \sqcup_n \text{NonNull} = \top \]

\[ (\mathbb{D}_n^\#, \sqsubseteq_n, \sqcup_n) \text{ is a lattice} \]

Galois connection \((\alpha_n, \gamma_n)\) with the concrete semantics
• Uninitialized variable abstract domain

- $D_u = \{ \bot, \text{Init}, \text{Uninit}, \top \}$
- $\mathbb{D}_u^\# = \mathbb{V} \rightarrow D_u$
- $\bot \sqsubseteq_u^\# \text{Init}, \bot \sqsubseteq_u^\# \text{Uninit}, \text{Init} \sqsubseteq_u^\# \top, \text{Uninit} \sqsubseteq_u^\# \top$
- $\text{Init} \sqcup_u^\# \text{Uninit} = \top$
- $(\mathbb{D}_u^\#, \sqsubseteq_u^\#, \sqcup_u^\#)$ is a lattice

• Galois connection $(\alpha_u, \gamma_u)$ with the concrete semantics
Question: how to model the memory?

LLVM is low level, a byte representation is necessary

The C language is not type safe and is very permissive on casts

We need to model correctly the following code:

```c
uint64_t x = 1;
uint32_t* p = (uint32_t*)&x;
p += 1;
uint32_t y = *p;
```

By the way, what is `y`'s value?
Question: how to model the memory?
LLVM is low level, a byte representation is necessary
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We need to model correctly the following code:

```c
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```

By the way, what is y’s value?
Memory model from « Formalizing the LLVM Intermediate Representation for Verified Program Transformations », POPL 2012

Memory cell $mc = \mid mb(size, \text{byte}) \mid mptr(blk, offset, index) \mid muinit$

- Memory state $= (N, B, C)$
- $N$ : next block id
- $B = \mathbb{Z}^+ \rightarrow \mathbb{Z}^+ :$ block id to block size (bytes)
- $C = \mathbb{Z}^+ \times \mathbb{Z}^+ \rightarrow MC :$ (block id, offset in bytes) to memory cell
Example:

```c
int* p = (int*) malloc(sizeof(int) + sizeof(int*));
*p = 0x01020304;
int** q = (int**)(p + 1);
*q = p + 2;
```
Example:

```c
int* p = (int*) malloc(sizeof(int) + sizeof(int*));
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int** q = (int**)(p + 1);
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```

<table>
<thead>
<tr>
<th>blk id</th>
<th>offset</th>
<th>memory cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>mb(32, 4)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>mb(32, 3)</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>mb(32, 2)</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>mb(32, 1)</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>mptr(l, 8, 0)</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>mptr(l, 8, 1)</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>mptr(l, 8, 2)</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
<td>mptr(l, 8, 3)</td>
</tr>
</tbody>
</table>

By the way, what architecture could it be?
Memory abstract domain

Based on Antoine Mine’s paper: « Field-Sensitive Value Analysis of Embedded C Programs with Union Types and Pointer Arithmetics », LCTES’06

Idea: abstract memory using cells: \( C(\text{address}, \text{offset}, \text{size}) \)

Each cell is considered as a variable in the underlying abstract domain

Cells may overlap

\[ C = A \rightarrow \mathbb{Z}^+ \times \mathbb{Z}^+ \]

\[ D^\#_{\text{mem}} = C \times D^\#_{\text{underlying}} \]

In IKOS, \( D^\#_{\text{underlying}} = D^\#_{\text{num}} \times D^\#_{\text{ptr}} \times D^\#_{\text{null}} \times D^\#_{\text{unini}} \)

Pointwise partial order, Pointwise union
Abstract operations: forward to $\mathbb{D}^\#_{\text{underlying}}$, except memory read and write.

Memory write:
- set to $\bot$ if $p$ is null or uninitialized
- $(\text{points}_\text{to}, \text{offset}) = \rho(p)$
- $\text{cells} = \text{realize}_\text{write}(\text{points}_\text{to}, \text{offset})$
- $\forall c \in \text{cells}, \text{strong}_\text{update}(c, \text{rhs})$ or $\text{weak}_\text{update}(c, \text{rhs})$

Memory read:
- set to $\bot$ if $p$ is null or uninitialized
- $(\text{points}_\text{to}, \text{offset}) = \rho(p)$
- $\text{cells} = \text{realize}_\text{read}(\text{points}_\text{to}, \text{offset})$
- $\forall c \in \text{cells}, \text{strong}_\text{update}(\text{lhs}, c)$ or $\text{weak}_\text{update}(\text{lhs}, c)$
Example:

```c
int* p = (int*) malloc(sizeof(int) + sizeof(int*));
*p = 0x01020304;
int** q = (int**)(p + 1);
*q = p + 2;
```
Example:

```c
int* p = (int*) malloc(sizeof(int) + sizeof(int*));
*p = 0x01020304;
int** q = (int**)(p + 1);
*q = p + 2;
```

Abstract value at the end:

```
(malloc → \{\{0, 4\}, \{4, 4\}\})
(C(malloc, 0, 4) → [0x01020304, 0x01020304])
(C(malloc, 4, 4) → (malloc, [8, 8]),
  p → (malloc, [0, 0]),
  q → (malloc, [4, 4]))
(C(malloc, 4, 4) → NonNull, p → NonNull, q → NonNull)
(C(malloc, 0, 4) → Init, C(malloc, 4, 4) → Init, p → Init, q → Init))
```
static union {
    struct { uint8 al, ah, bl, bh, ... } b;
    struct { uint16 ax, bx, ... } w;
} regs;
regs.w.ax = X; // (1)
if (!regs.b.ah) { // (2)
    regs.b.bl = regs.b.al; // (3)
} else { // (4)
    regs.b.bh = regs.b.al; // (5)
}
// (6)
regs.b.al = X; // (7)
Last step: check for properties at each statement location

Checkers:

- buffer overflow: $0 \leq offset$ and $offset + read\_size \leq buffer\_size$
- division by zero: $divisor \neq 0$
- null dereference: $p \neq Null$
- uninitialized variable: $v \neq Uninit$
- prover: $v \neq 0$
Syllabus

1 Introduction

2 IKOS

3 Analyses

4 Miscellaneous
   - Abstract domains implementation
   - Analyzing C++
   - Exception handling
   - Relational abstract domains
   - Function summarization
   - Integer overflow

5 Conclusion
Abstract domains implementation

- Separate domain ($\forall \to \mathbb{D}$) are implemented with patricia trees
- Insertion and removal in $O(\log(n))$
- Merge in $O(n)$
- Transformation in $O(n)$
- Very cheap union!
Analyzing C++ is very tricky:

- Heavy chains of function calls because of templates
- The libc++ needs to be modeled
- Need to be precise on pointers for virtual method calls
- Handle exceptions
Analyzing C++ is very tricky:
- Heavy chains of function calls because of templates
- The libc++ needs to be modeled
- Need to be precise on pointers for virtual method calls
- Handle exceptions

Work in progress!
Exception handling

bb_1:
memory[x] = 9
_Z1fv._2 = call _Z14__ikos_unknownv()

*out_bb_1_to_bb_2_icmp_true:
_Z1fv._2 ne 0
_Z1fv._3 = -1

*out_bb_1_to_bb_3_icmp_false:
_Z1fv._2 eq 0
_Z1fv._3 = 0

bb_2:
_Z1fv._5 = call __cxa_allocate_exception(8)
_Z1fv._6 = bitcast _Z1fv._5
memory[_Z1fv._6] = $null
_Z1fv._8 = bitcast _ZTIDn
__v:6 = call __cxa_throw(_Z1fv._5, _Z1fv._8, $null)
unreachable

bb_3:
memory[x] = 0
return

_unified_exit:
\[ \mathbb{D}_{\text{exc}}^\# = \mathbb{D}^\# \times \mathbb{D}^\# \]

\[
\llbracket \text{throw}(e) \rrbracket^\#(N, E) = (\perp, N \cup E)
\]

\[
\llbracket \text{landingpad}(e) \rrbracket^\#(N, E) = (E, \perp)
\]

\[
\llbracket v = x \rrbracket^\#(N, E) = (\llbracket v = x \rrbracket^\#(N), E)
\]

\[
(N_1, E_1) \sqcup^\# (N_2, E_2) = (N_1 \cup N_2, E_1 \cup E_2)
\]
Intervals are very imprecise for loops with a non-deterministic bound

Solution: use a weakly-relational domain, such as the DBM domain

Difference-Bound Matrices

- Difference-Bound Matrices
- Weakly-relational abstract domain

\[
\begin{bmatrix}
0 & m_{0,1} & m_{0,2} & \ldots & m_{0,n} \\
m_{1,0} & 0 & m_{1,2} & \ldots & m_{1,n} \\
m_{2,0} & m_{2,1} & 0 & \ldots & m_{2,n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
m_{n,0} & m_{n,1} & m_{n,2} & \ldots & 0
\end{bmatrix}
\]

- \( m_{i,j} \in \mathbb{Z} \cup \{+\infty\} \)
- \( v_i - v_j \leq m_{j,i} \)
- \( v_0 = 0 \), thus \( v_i \in [-m_{i,0}, m_{0,i}] \)
- Abstract operations require normalization
  - normalization:
    - \( v_i - v_k \leq m_{k,i} \) and
    - \( v_k - v_j \leq m_{j,k} \) \( \Rightarrow \)
    - \( v_i - v_j \leq m_{k,i} + m_{j,k} \)
- cost \( O(n^3) \), \( n \) number of variables
• Idea: keep a list of DBMs, where each DBM contains variables that are related to each other.
• Union-Find structure to dynamically infer relations among variables
• Normalization cost $O(n)$, $n$ number of DBMs
Idea: keep a list of DBMs, where each DBM contains variables that are related to each other.

Union-Find structure to dynamically infer relations among variables

Normalization cost $O(n)$, $n$ number of DBMs
## Variable packing - Tests

Pointer analysis using function summarization.

<table>
<thead>
<tr>
<th>File</th>
<th>DBMs</th>
<th>Size</th>
<th>Var Packing</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>astree-ex</td>
<td>1.01s</td>
<td>36</td>
<td>0.13s</td>
<td>7</td>
</tr>
<tr>
<td>test-1</td>
<td>0.13s</td>
<td>27</td>
<td>0.03s</td>
<td>4</td>
</tr>
<tr>
<td>test-1-unsafe</td>
<td>0.13s</td>
<td>27</td>
<td>0.02s</td>
<td>4</td>
</tr>
<tr>
<td>test-10</td>
<td>0.03s</td>
<td>10</td>
<td>0.02s</td>
<td>4</td>
</tr>
<tr>
<td>test-10-unsafe</td>
<td>0.03s</td>
<td>11</td>
<td>0.02s</td>
<td>4</td>
</tr>
<tr>
<td>paparazzi-microjet</td>
<td>3241.14s</td>
<td>611</td>
<td>158.50s</td>
<td>88</td>
</tr>
<tr>
<td>gen2</td>
<td>&gt; 5h</td>
<td>?</td>
<td>7817.42s</td>
<td>367</td>
</tr>
<tr>
<td>aeroquad-servo</td>
<td>78.12s</td>
<td>71</td>
<td>1.33s</td>
<td>14</td>
</tr>
<tr>
<td>aeroquad-new</td>
<td>86.18s</td>
<td>65</td>
<td>0.76s</td>
<td>5</td>
</tr>
<tr>
<td>cornell</td>
<td>447.06s</td>
<td>226</td>
<td>2.64s</td>
<td>6</td>
</tr>
<tr>
<td>sporesate2-spore-pl</td>
<td>895.45s</td>
<td>?</td>
<td>10.29s</td>
<td>19</td>
</tr>
</tbody>
</table>
Other ideas

- Group variables depending on heuristics
- Use the gauge domain
Other ideas

- Group variables depending on heuristics
- Use the gauge domain

Work in progress!
IKOS uses dynamic inlining

Idea: analyse each function only once to build a summary

\[ \text{main} \rightarrow \text{f} \rightarrow \text{g} \]
\[ \text{main} \rightarrow \text{h} \rightarrow \text{w} \]
Problem: call graph cycle

```
main
f

f

h

h

w

w

g
```

Maxime Arthaud
Problem: call graph cycle

Strongly connected component analysis
Topological order
Bottom-up analysis (from the leaves to the root)
Top-down analysis (from the root to the leaves)
Need a way to express the effect of a function call on the memory

More particularly on global variables and pointer parameters

Relation between the input memory state and the output memory state

Idea: Introduce *input cells* and *output cells*

\[ x = x + 1 \iff \text{Cell}\{x, 0, 4, Out\} = \text{Cell}\{x, 0, 4, In\} + 1 \]
Buffer overflow analysis using function summarization

<table>
<thead>
<tr>
<th>File</th>
<th>Inlining</th>
<th>Summaries</th>
<th>Warnings</th>
<th>Errors</th>
<th>Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>astree-ex</td>
<td>0.36s</td>
<td>0.57s</td>
<td>2/2</td>
<td>0/0</td>
<td>22 (1)</td>
</tr>
<tr>
<td>test-1</td>
<td>0.14s</td>
<td>0.16s</td>
<td>0/0</td>
<td>0/0</td>
<td>22 (1)</td>
</tr>
<tr>
<td>test-1-unsafe</td>
<td>0.13s</td>
<td>0.18s</td>
<td>0/0</td>
<td>2/2</td>
<td>22 (1)</td>
</tr>
<tr>
<td>test-10</td>
<td>0.10s</td>
<td>0.13s</td>
<td>0/2</td>
<td>0/0</td>
<td>20 (3)</td>
</tr>
<tr>
<td>paparazzi</td>
<td>154.03s</td>
<td>110.09s</td>
<td>0/0</td>
<td>0/0</td>
<td>24650 (199)</td>
</tr>
<tr>
<td>gen2</td>
<td>307.66s</td>
<td>&gt; 3h</td>
<td>195/ ?</td>
<td>0/ ?</td>
<td>22030 (82)</td>
</tr>
</tbody>
</table>
Problem: LLVM integer types are signedness agnostic

Because most instructions are signedness agnostic: add, sub, mul, etc.

How to be sound and precise?

- Intervals with infinite precision: imprecise or unsound
- Suppose integers are unsigned: imprecise
- Suppose integers are signed: imprecise
- Wrapped intervals: Jorge Navas’s paper « Signedness-Agnostic Program Analysis: Precise Integer Bounds for Low-Level Code »
- Domain product: unsigned and signed
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Thank you. Questions?