Invariant Imbedded T-Matrix Method for Axial Symmetric Hydrometeors with Extreme Aspect Ratios

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Introduction

One of the most fundamental and crucial quantities for accurate precipitation retrievals is the single-scattering property (SSP) of each individual hydrometeor in the precipitating volume. Thus, an important step in minimizing retrieval uncertainty is to ensure that the particles used for single-scattering calculations resemble those that occur in nature, allowing the SSPs to be computed precisely with numerical techniques.

Hydrometeors with rotational symmetry (axial symmetry) are most efficiently computed using the T-Matrix methods. For hydrometeors with a uniform dielectric, the Extended Boundary Condition Method (EBCM) is the most efficient. However, for mixed-phase hydrometeors with non-uniform dielectrics EBCM is not applicable. In these cases, we can use the Invariant Imbedded T-Matrix (IITM), which we developed for this purpose. It is also noted that hydrometeors with more extreme aspect ratios can be computed with IITM than EBCM and that such hydrometeors are found in precipitating systems. For hydrometeors without any rotational symmetry, however, volume integral methods like Discrete Dipole Approximation (DDA) and the Method of Moments (MoM) must be employed.

Algorithm Flow-Chart for SSPs

Angular Quadrature—Gauss-Legendre

1. Surface discontinuity leads to significant numerical error.
2. Integration must be broken up: I = \int_{S_1} \int_{S_2}.
3. Gauss-quadrature dominates the run time.
4. Functions are cached for each shell to speed up performance.

Numerical Convergence / Precision

Process: 0 1 2 3 4 5 6

Naïve scaling—no communication. Scales well "inward", the partial wave cutoff.

Mixed-Phase Hydrometeor Spheroids

To test and study the behavior of our IITM implementation, we study spheroids with an outer shell of melt water. We consider a solid ice core as well as "fluffy" mixed air-ice cores with densities 0.1, 0.2, and 0.3 g/cm³. Numerical calculations are carried for both prolate and oblate spheroids with a semi-minor to semi-major ratios equal to 1/2(1) for frequencies 13.6 GHz, 35.6 GHz, and 94 GHz. The corresponding size parameters are ~0.25, ~0.64, and 1.7.

Implementation

The IITM is typically an order of magnitude more computationally expensive than EBCM. To make IITM practical requires an efficient implementation that is optimized and takes advantage of parallelism in the IITM algorithm.

Algorithm Schematic

1. Initialize with Separation of Variables (SOV).
2. Perform quadrature over spherical shell \( r_k \) – Gauss-Legendre quadrature.
3. Using recursion relations to get the T-Matrix at \( r_k \), \( T_\ell \), \( r_k \) – matrix inversion + linear algebra. Romberg integration used.

Circular Cylinder (radial shells = 2048)

Long double precision fails for \( \ell > 96 \)

Double precision fails.

Oblate Spheroid

Extinction efficiency

Scattering efficiency

IITM+SOV (this) = 3.2851 (2)/2(2)

IITM+SOV [1] = 3.2850

Comparison with results from [1] with error estimates associated with partial wave truncation and the number of radial shells (0/4/8), respectively.

References


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