Forbidden mass ranges for shower meteoroids

Althea Moorhead
NASA Meteoroid Environment Office, MSFC

49th Annual Division for Planetary Sciences Meeting
October 16, 2017
Eccentric comets approach $v_{\text{esc}}$ at perihelion

\[
v_{\text{peri}}^2 = \mu \left( \frac{2}{q} - \frac{1}{a} \right)
\]

\[
v_{\text{esc}}^2 = \mu \frac{2}{q}
\]
Small particles are subject to radiation pressure

- Radiation pressure follows inverse square law
- Reduces central potential by $\beta$:
  \[ \beta = \frac{F_r}{F_g} \]
- Effect is inversely proportional to size (and density):
  \[ \beta \propto \frac{1}{s} \]
Meteoroids can be ejected directly onto escape trajectories.

\[ v_{esc}^2 = \mu (1 - \beta)^2 q \]

- For \( \beta \geq 1 \), there are no bound orbits.
- For \( \beta < 1 \), \( v_{esc} \) is reduced.
- Comet’s velocity alone exceeds \( v_{esc} \) for:

\[ \beta > \frac{1 - e}{2} \]

Burns, Lamy, & Soter (1979)
Ejection speed can give meteoroids a boost

- Meteoroids ejected in the direction of the comet’s motion get a boost; trailing particles the opposite.
- For large particles:
  \[ \Delta v = v_0 \sqrt{\beta} \]
  (Whipple, 1951; Jones, 1995; etc.)
- The value of \( \beta \) above which particles are unbound has an analytical solution. For leading particles:
  \[
  y = \sin^{-1} \left( \frac{v_{peri}}{\sqrt{v_0^2 + v_{esc}^2}} \right) - \text{atan2}(v_{esc}, v_0)
  \]
  \[ \beta_L = \sin^2 y \]
- A similar equation exists for trailing particles
The only thing left to do is calculate $\beta$:

$$\beta = 5.7 \times 10^{-4} \text{ kg m}^{-2} \times (Q_{pr}/\rho s)$$

Geometric optics: $Q_{pr} = 1$

But there are some complications ...
What about small particles?

- For small particles:

\[ \Delta v \propto \sqrt{\beta} \]

- Instead, we must numerically integrate (see Jones, 1995):

\[
\frac{d^2x}{dt^2} = \frac{\Lambda \Gamma}{2} m^{-1/3} \rho_d^{-2/3} \rho_{\text{gas}}(x) \left[ v_{\text{gas}}(x) - \frac{dx}{dt} \right]^2
\]

- Then:

\[
\Delta v = \left. \frac{dx}{dt} \right|_{t \to \infty}
\]
What about small particles?

- $\Delta v$ has no analytic form, but is very close to:

$$\Delta v \simeq v_{gas,0} \left( 0.38532 + 0.50341 \cdot \xi^{-1.054} \right)^{-0.949}$$

$$\xi = \frac{\Delta \Gamma}{2} m^{-1/3} \rho_d^{-2/3} \rho_{gas,0} x_c$$

Ugly, but easy to code up.

- Calculating $\beta$ is another matter.
Calculating $\beta$ for small particles and real materials

$$\beta = 5.7 \times 10^{-4} \text{ kg m}^{-2} \times (Q_{pr}/\rho s)$$

- **Geometric optics**: $Q_{pr} = 1$
- **“Ideal material”**: $Q_{pr} = 1$ for $\lambda < 2\pi s$, 0 otherwise
- **Real materials**: Calculate $Q_{pr}$ using Mie theory
  (Python code available from Navarro & Werts, 2012)
Calculating $\beta$ for real materials

I’ll compare the “ideal material” case with one real material

Tholins are a reddish brown polymer found on icy bodies
Perseids

Ideal material

Tholin

$v$ (km s$^{-1}$)

$m$ (g)

Leading ejecta

Trailing ejecta

Escape velocity

Ejecta lost
Forbidden mass ranges for 10 major showers

Ideal material

Tholin

CAP
DRA
GEM
LEO
LYR
NTA
ORI
PER
QUA
URS

$m (g)$

$10^{-18}$ $10^{-14}$ $10^{-10}$ $10^{-6}$ $10^{-14}$ $10^{-10}$ $10^{-6}$
Small meteoroids originating from eccentric comets may be on unbound orbits. We’ve extended this to handle the ejection velocity imparted by the sublimation process:

- Analytic solution for $\beta$ limit for large particles
- Semi-numerical solution for $\Delta v$ (and thus $\beta$ limit) for all particles
- New $\Delta v$ equation also useful for stream modeling

We’ve calculated $\beta$ for small particles/real materials:

- Ideal material: very small particles may remain in stream
- Tholins: small particles do not remain in stream

Large comets: some small particles can still be ejected

Eccentric comets: excluded range can be large: no Lyrids smaller than $4 \times 10^{-7} \text{ g}$