Phase-Retrieval Wave-Front Sensing for the Hubble and Future Space Telescopes

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• Hubble Space Telescope (HST)
  o Problem needs fixing
  o Phase retrieval algorithms
  o Effect of phase retrieval on the prescription
  o HST Repaired
• James Webb Space Telescope (JWST)
  o Background: foldable, segmented-aperture telescope
  o Phase retrieval will be used for wavefront sensing and control

Jim Fienup was at ERIM, Ann Arbor, MI, during the HST recovery period
Hubble Space Telescope: Great Expectations, Pre-Launch

Sky & Telescope

Note: Pads
First HST Point-Spread Function, 1990

Expected

Actual
Panels and Organizations
Characterizing and Fixing HST

HST Optical System Board of Investigation
(Lew Allen Committee)
How did it happen? Who was to blame?

HST Independent Optical Review Panel (HIORP)
Characterize error to fix WF/PC2
(JRF, IPWG representative)

HST Strategy Panel
How to fix HST in general

HST Image Processing Working Group (IPWG)
Improve imagery by post-detection processing
(JRF, member)

Instrument Teams:
WF/PC, FOC, HRS, FGS, ...
Vested interest

JPL WF/PC2 Team
Build camera and relay optics to fix problem
(JRF, subcontractor)

Space Telescope Science Institute
Process imagery

Hughes Danbury Optical System
Characterize problem, align system
HST Ground & Polished
According to Interferometer Measurements
1.3 mm Mistake in Null Corrector Spacing Causes 2 µm Mistake in Primary Mirror
Can Correct Primary Mirror Error on Secondary of WF/PC2 Relay Telescope

CORRECTION APPROACH

(Not to scale)
Determine HST Aberrations from PSF

Measurements & Constraints:
- Pupil plane: known aperture shape
- Phase error fairly smooth function
- Focal plane: measured PSF intensity

Wavefronts in pupil plane and focal plane are related by a Fourier Transform
Benefits of Phase Retrieval

Knowing aberrations precisely allows for:

- Design correction optics to fix the HST
  - WF/PC II
  - COSTAR
- Optimize alignment of secondary mirror of HST OTA
- Monitor telescope shrinkage (desorption) and focus
- Compute analytic point-spread functions for image deconvolution
  - Noise-free
  - Depends on $\lambda$, $\Delta \lambda$, camera, field position
  - Is highly space-variant for WF/PC
  - Eliminates requirement to measure numerous PSF's

In addition, reconstruction of pupil function allows determination of alignment between OTA and WF/PC.
Phase Retrieval Basics

Focal plane field

Fourier transform: \( F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)e^{-i2\pi(ux + vy)}dxdy \)

\[ = |F(u, v)|e^{i\psi(u, v)} = F[f(x, y)] \]

Focal plane field magnitude

= sqrt(intensity)

Focal plane field phase

Pupil plane field

Inverse transform: \( f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v)e^{i2\pi(ux + vy)}dudv = F^{-1}[F(u, v)] \)

Phase retrieval problem:

Given \(|F(u, v)|\) and some constraints on \(f(x, y)\),
Reconstruct \( f(x, y) \), or equivalently retrieve \( \psi(u, v) \)

Equivalently, reconstruct field \( f(x, y) \) in the pupil
— its phase is the phase error we wish to correct
Is Phase Retrieval Possible?

"Hey, no problem!"
Phase Retrieval for Image Reconstruction from Stellar Speckle Interferometry Data

IMAGING WITH PHASE ERRORS

Object \rightarrow \text{Aberrations} \rightarrow \text{Blurred Images}

\begin{center}
\begin{tikzpicture}
\node[draw] (data) {Data Preprocessing};
\node[draw, right of=data] (fourier) {Fourier Modulus};
\node[draw, right of=fourier] (recon) {Image Reconstruction};
\node[draw, right of=recon] (image) {Image};
\node[draw, below of=image, fill=white] () {Computer};
\node[draw, right of=data, fill=white] () {Image}
\node[draw, right of=data, fill=white] () {Reconstructed Image}
\node[draw, right of=data, fill=white] () {Object}
\node[draw, right of=data, fill=white] () {Blurred Image}
\draw[->] (data) -- (fourier);
\draw[->] (fourier) -- (recon);
\draw[->] (recon) -- (image);
\end{tikzpicture}
\end{center}

Example:

Iterative Transform Algorithm

START:
Initial Estimate

Constraints:
Support,
(Nonnegativity)
(aperture function)

Form New Input
Using Image
Constraints

G = |G| e^{i\phi}

Satisfy Fourier
Domain
Constraints

Measured Data:
Magnitude, IFI
(blurred image
of a star)

Enforcing magnitude constraints in both domains
is the "Gerchberg-Saxton" algorithm
Sources of Obscurations in HST

RELAY
SEC secondy 
and SPIDERS

SPIDERS

SECONDARY MIRROR OBSCURATION

CCD/Wide field/planetary camera
Primary mirror
Secondary mirror
WF/PC Camera Details

- Relay telescope secondary obscuration appears to translate vs. field angle
Phase Retrieval Techniques

Minimize error metric by

- Cut & try [Jon Holtzman (Lowell Observatory)]
- Iterative transform algorithm (Gerchberg-Saxton/Misell/Fienup)
- Gradient search (steepest descent, conjugate gradient, ...)
- Damped least squares (Newton-Raphson)
- Neural network [Todd Barrett & David Sandler (Thermo Electron)]
- Linear programming
- Prescription retrieval [David Redding (Draper Lab)]
- Phase diversity
- etc. (intensity transport, tracking zero sheets, simulated annealing, ...)

- Other groups doing phase retrieval
  - Rick Lyon et al. Hughes Danbury Optical Systems
  - Chris Burrows (Space Telescope Science Institute)
  - Mike Shao et. al. (JPL)
  - Francois Roddier (U. Hawaii), ...
Phase Retrieval by Optimization

- Model optical system
  - Known parameters (constraints)
  - Unknown parameters (to retrieve)
- Compute model of data
- Compare model of data with actual measured data
  - Compute error metric
- Minimize error metric over space of unknown parameters
  - Using nonlinear optimization algorithms
Error Metrics

Detector plane: 

- Computed
- Measured

\[ E_1^2 = \sum_u W(u) \left[ |G(u)| - |F(u)| \right]^2 \]

\[ E_2^2 = \sum_u W(u) \left[ |G(u)|^2 - |F(u)|^2 \right]^2 \]

**Maximum likelihood for additive Gaussian noise**

\[ E_3^2 = \sum_u W(u) \left[ |G(u)| - |F(u)| - |G(u)| \ln \left( \frac{|F(u)|}{|G(u)|} \right) \right]^2 \]

\[ L = -\sum_u W(u) |G(u)|^2 + \sum_u W(u) |F(u)|^2 \ln \left( |G(u)|^2 \right) \]

**Maximum likelihood for Poisson noise**

Aperture plane:

- Treating detector-plane phase as optimization parameters

\[ E_{ap}^2 = \sum_x \left[ |g(x)| - |f(x)| \right]^2 \]

etc. 

- Computed
- Known Pupil
Techniques Employing Gradients

Minimize Error Metric, e.g.: \( E = \sum_{u} W(u)[|G(u)| - |F(u)|]^2 \)

Contour Plot of Error Metric

Repeat three steps:
1. Compute gradient:
   \[ \frac{\partial E}{\partial p_1}, \frac{\partial E}{\partial p_2}, \ldots \]
2. Compute direction of search
3. Perform line search

Gradient methods:
(Steepest Descent)
Conjugate Gradient
BFGS/Quasi-Newton

...
Analytic Gradients of $E = \sum_u W(u)[|G(u)| - |F(u)|]^2$

Pupil: $g(x) = m_\circ(x)e^{i\theta(x)}$  
Detector plane: $G(u) = P[g(x)]$

$g^W(x) = P^\dagger[G^W(u)]$  
$G^W(u) = W(u)[|F(u)|G(u) - G(u)]$

Derivative w.r.t. general parameter: $\frac{\partial E}{\partial p} = -2\text{Re}\left[\sum_x \frac{\partial g(x)}{\partial p} g^W_*(x)\right]$  

For point-by-point phase map, $\theta(x)$, $\frac{\partial E}{\partial \theta(x)} = 2\text{Im}\{g(x) g^W_*(x)\}$

For Zernike polynomial coefficients, $\frac{\partial E}{\partial a_j} = 2\text{Im}\left\{\sum_x g(x) g^W_*(x) Z_j(x)\right\}$

where $\theta(x) = \sum_{j=1}^J a_j Z_j(x)$

Propagator $P[\cdot]$ can be single FFT or multiple-plane Fresnel transforms with phase factors and obscurations

Analytic gradients very fast compared with finite differences

System Modeling — Propagation

- Simple Fourier propagation
  - All obscurations, phase errors in same plane
  - Phase errors in two mirrors, wavefront translates with field angle

- Fresnel propagation, using multiple planes of diffraction
  - Obscurations planes, phase error planes

Thin lens model of HST + PC
More System Modeling Considerations

- Multi-plane propagation including vignetting or multiple aberration planes
- Jitter in telescope pointing during exposure time
- Exclude bad pixels from error metric (dust/saturation/cosmic rays)
- Finite spectral bandwidth
- Shifted WF/PC obscurations vs. field position
- Correct plate scale (depends on field position)
- CCD pixel integration, sampling (undersampling/aliasing)
- Include model of noise (photon, readout)
- Higher-order Zernike's and micro-roughness
- Effect of aberrations in OTA secondary, in WF/PC cameras
- Design aberrations versus field position
- Possibility of non-point-like star
Dust Artifacts and Glitches in WF/PC Images

Raw image (point source)  Sharpened image
Hubble Telescope Retrieval Approach

- Pupil (support constraint) was known imperfectly
- Phase was relatively smooth and dominated by low-order Zernike's
  - Use boot-strapping approach

1. With initial guess for pupil, fit **Zernike polynomial** coefficients
2. With initial guess for Zernike polynomials, estimate **pupil** by ITA

3. Redo steps 1 and 2 until convergence (2 iterations)
Comparison of Actual and Simulated HST Image of a Point Star
ERIM Phase Retrieval Results
Greater Accuracy --> Larger Z11 Values

- Results -- PC6 F889N_P2 data (Δz = -260):

Zernike Coefficients (microns rms of wavefront error):

<table>
<thead>
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<th></th>
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<tbody>
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<td>4</td>
<td>-2.212</td>
<td>-2.227</td>
<td>-2.223</td>
<td>-2.303</td>
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<tr>
<td>5</td>
<td>-0.018</td>
<td>-0.003</td>
<td>0.006</td>
<td>-0.003</td>
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<td>6</td>
<td>-0.025</td>
<td>0.025</td>
<td>0.026</td>
<td>0.031</td>
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<td>7</td>
<td>0.004</td>
<td>0.001</td>
<td>0.005</td>
<td>-0.001</td>
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<tr>
<td>8</td>
<td>0.017</td>
<td>0.010</td>
<td>0.009</td>
<td>0.013</td>
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<tr>
<td>9</td>
<td>-0.022</td>
<td>-0.020</td>
<td>-0.009</td>
<td>-0.021</td>
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<tr>
<td>10</td>
<td>0.002</td>
<td>0.008</td>
<td>0.010</td>
<td>0.005</td>
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<tr>
<td>11</td>
<td>-0.280</td>
<td>-0.292</td>
<td>-0.295</td>
<td>-0.300</td>
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<tr>
<td>12</td>
<td>0.008</td>
<td></td>
<td></td>
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<tr>
<td>16</td>
<td>-0.009</td>
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<tr>
<td>20</td>
<td>0.006</td>
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<tr>
<td>22</td>
<td>0.005</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
</tr>
</tbody>
</table>

conic κ = -1.0144 (-1.0151 -1.0152 -1.01545)
rms err = 0.1583 0.1352 0.1353

New parameters: Plate scale = 0.0442 arcsec/pixel, bq = -0.000059
Other Phase Retrieval Approaches

John Holtzman (Lowell Observatory)
"Cut and Try" = compare images with various computed spherical

Francoise & Claude Roddier
Miselle algorithm

![Diagram showing phase retrieval processes: Gerchberg-Saxton, Miselle, PSF 1, PSF 2, PSF 3, and Pupil.](image)

Fig. 2. Typical PC-6 image showing the points defining the center of the pad diffraction spot and the periphery of the image.

Fig. 5. Best fit of measured points to the Schulte lines defines the paraxial focus as being at $-120 \, \mu m$ and the rim image diameter as being $6.15 \pm 0.1$ arcsec. This leads to an apparent conic constant of $-1.01429 \pm 0.0002$. 
Discrepancy with Phase Retrieval Caused NASA to Look for Additional Errors

HST FOSSIL AND IMAGE INVERSION DATA USE TO DETERMINE K1
THE PRIMARY MIRROR CONIC CONSTANT

Smaller sph. aber

Larger sph. aber

Fossil Data 1

Fossil Data 2

WF/PC2 Corrected

Final ERIM P.R. allowing for PC6

COSTAR Corrected

Early ERIM P.R.

Later ERIM P.R.

Allowance for Z11 in PC6

John Mangus Feb 26, 1991

JRF 1/11-32
## Sources of Estimates of Spherical Aberration

<table>
<thead>
<tr>
<th>DATA SOURCE</th>
<th>CONIC CONSTANT</th>
<th>ERROR BARS</th>
<th>WFE (µ rms)</th>
<th>ERROR BARS (µ rms)</th>
</tr>
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<tbody>
<tr>
<td>1 WETHERELL: RNC, note 1</td>
<td>-1.01276 ±</td>
<td></td>
<td>-0.2405 ±</td>
<td></td>
</tr>
<tr>
<td>2 MANGUS: INC, note 2</td>
<td>-1.01280 ± 0.0008</td>
<td>-0.2415 ± 0.0183</td>
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<tr>
<td>3 FUREY: RvNC, note 3</td>
<td>-1.01288 ±</td>
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<td>-0.2433 ±</td>
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<tr>
<td>4 FUREY: RNC, note 1</td>
<td>-1.01290 ± 0.0002</td>
<td>-0.2437 ± 0.0046</td>
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<tr>
<td>5 MANGUS: RvNC, note 4</td>
<td>-1.01326 ± 0.0008</td>
<td>-0.2520 ± 0.0183</td>
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<td></td>
</tr>
<tr>
<td>6 MIENELS': PAD LOCATION</td>
<td>-1.01341 ±</td>
<td></td>
<td>-0.2554 ±</td>
<td></td>
</tr>
<tr>
<td>7 MIENELS': RIM IMAGE</td>
<td>-1.01342 ±</td>
<td></td>
<td>-0.2556 ±</td>
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<tr>
<td>8 FUREY: RNC, note 5</td>
<td>-1.01349 ± 0.0006</td>
<td>-0.2571 ± 0.0137</td>
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<td></td>
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<tr>
<td>9 LYONS: HDOS-FOC, HARP I</td>
<td>-1.01357 ±</td>
<td></td>
<td>-0.2590 ± 0.0005</td>
<td></td>
</tr>
<tr>
<td>10 BURROWS: Sci-FOC, HARP I</td>
<td>-1.01368 ± 0.0008</td>
<td>-0.2615 ± 0.0183</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 FABER/HOLTZMANN : WF/PC-PC</td>
<td>-1.01420 ±</td>
<td></td>
<td>-0.2734 ± 0.0000</td>
<td></td>
</tr>
<tr>
<td>12 LYONS: HDOS-PC, HARP I</td>
<td>-1.01430 ± 0.0005</td>
<td>-0.2757 ± 0.0114</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 LYONS: HDOS-WF</td>
<td>-1.01440 ± 0.0009</td>
<td>-0.2780 ± 0.0205</td>
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<td></td>
</tr>
<tr>
<td>14 RODIER: PC, HARP I</td>
<td>-1.01450 ±</td>
<td></td>
<td>-0.2802 ± 0.0000</td>
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</tr>
<tr>
<td>15 BURROWS : HST Sci Inst-PC,HARP I</td>
<td>-1.01480 ± 0.0003</td>
<td>-0.2871 ± 0.0068</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16 VAUGHN : PAD LOCATION</td>
<td>-1.01484 ± 0.0003</td>
<td>-0.2881 ± 0.0068</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17 FIENUP : ERIM - PC, HARP I</td>
<td>-1.01510 ± 0.0007</td>
<td>-0.2939 ± 0.0160</td>
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<tr>
<td>18 SHAO : JPL-PC, HARP I</td>
<td>-1.01520 ±</td>
<td></td>
<td>-0.2962 ± 0.0000</td>
<td></td>
</tr>
</tbody>
</table>

- e 1: assumes M2 to FL, M1 to M2 and CORI to M1 errors are real
- e 2: assumes as built errors had correct spacing to correct for element fab error
- e 3: assumes reticle in and EPI to NL distance adjusted by +.68 mm
- e 5: assumes only FLPE as real, other spacing measurements as
Hubble Fixed
Greatly Improved Imagery

Small residual blurring noticeable only for bright point sources

Gaseous Pillars • M16

HST • WFPC2

PRC95-44a • ST ScI OPO • November 2, 1995
J. Hester and P. Scowen (AZ State Univ.), NASA
Phase-retrieval analysis of pre- and post-repair Hubble Space Telescope images

John E. Krist and Christopher J. Burrows

Phase-retrieval measurements of point-spread functions from the pre- and post-repair Hubble Space Telescope are presented. The primary goal was to determine the aberrations present in the second wide-field and planetary camera (WFPC2) to align and validate its corrective optics. With both parametric model-fitting techniques and iterative (Gerchberg–Saxton) methods, accurate measurements have been obtained of the WFPC2 and Hubble Space Telescope optics, including improved maps of the zonal errors in the mirrors. Additional phase-retrieval results were obtained for the aberrated, prerepair cameras and the corrected faint-object camera. The information has been used to improve models produced by point-spread-function simulation programs. On the basis of the measurements, a conic constant for the primary mirror of $\kappa = -1.0144$ has been derived.

WF/PC2 corrected to
$\kappa = -1.0135 \ (Z_{11} = -0.254 \ \mu m)$
COSTAR corrected to
$\kappa = -1.0139 \ (Z_{11} = -0.263 \ \mu m)$
Fienup 1991 (after $-0.013 \ \mu m$ PC)
$\kappa = -1.0144 \ (Z_{11} = -0.276 \ \mu m)$
Krist & Burrows 1995 agrees

Our results for WFPC2 indicated that the compromise conic constant derived by the HIORP underestimated the spherical aberration by a small but measurable amount.

The Institute of OPTICS

James Webb Space Telescope
(Next Generation Space Telescope)

See farther back towards the beginnings of the universe
Light is red-shifted into infrared

http://ngst.gsfc.nasa.gov/
James Webb Space Telescope (JWST)

- See red-shifted light from early universe:
  - 0.6 µm to 28 µm
  - L2 orbit for passive cooling, avoiding light from sun and earth
  - 6.6 m diameter primary mirror
    - Deployable, segmented optics
    - Phase retrieval to align segments
Segmented-Mirror Deployment
Phase Retrieval for JWST

NASA has chosen phase retrieval as the fine phasing approach for JWST.
WFS at U of R
WaveFront Sensing Improvements

- Develop improved WFS (phase retrieval) algorithms
  - Faster, converge more reliably, less sensitive to noise, $2\pi$ jumps
  - Work with larger aberrations, broadband illumination, jitter
    - Refining iterative transform, gradient search algorithms
    - Increase robustness and accuracy
  - Extended objects
    - Phase diversity
    - Phase retrieval performance

- Experiments with U of R telescope laboratory simulator
  - Adaptive optics MEMS deformable mirror
  - Interferometer measure wavefront independently
  - Put in misalignment, reconstruct wavefronts, compare with interferometer "truth"

- 61 piston/tip/tilt hexagonal mirrors
- 497 µm diam. (polysilicon), 500 µm center-center
- 27 µm stroke
- 99% fill factor (polysilicon), 96% (metal)
Conclusion

- Phase Retrieval found the correct prescription for HST
  - Getting all the physics into the algorithm key to accuracy
  - Was not fully trusted by NASA
  - Hubble repair successful

- NASA has chosen phase retrieval for fine phasing of JWST
  - Key component in the system operational concept
Questions?