Stability and Control Derivative Estimation for the Bell-Shaped Lift Distribution

Loren J. Newton
The University of California, Berkeley
NASA Armstrong Flight Research Center
Introduction: The Bell-Shaped Lift Distribution

- Ludwig Prandtl, 1933
- Minimum induced drag solution for a wing of constrained mass
- Results:
  - 11% less induced drag, 22% greater span than the elliptical spanload (solution for a wing of defined span)
  - Upwash at the wingtips
  - Proverse yaw & tailless flight
Introduction: PRANDTL-D

• Preliminary Research AerodyNamic Design To Lower Drag
• Uninhabited, unpowered flying wings with the Bell-Shaped Lift Distribution
  • Prandtl-1: Lightly instrumented proof of concept (12.3’ span)
  • Prandtl-2: Flight computer-equipped data acquisition (12.3’ span)
  • Prandtl-3: Pressure/strain data for spanload measurement (25’ span)
Flight Test Procedures

• Edwards AFB lakebeds
• Average flight ~ 90 sec.
• Elastic high-start launch
• Doublet maneuvers: square wave input to control surfaces
  • Pitch
  • Roll
• 2-3 doublets per flight
Flight Dynamics

\[
\dot{V} = -\frac{\ddot{q}s}{m} C_D + g \left( \cos \phi \cos \theta \sin \alpha \cos \beta + \sin \phi \cos \theta \sin \beta - \sin \theta \cos \alpha \cos \beta \right)
\]

\[
\dot{\alpha} = q - \tan \beta \left( p \cos \alpha + r \sin \alpha \right) - \frac{\ddot{q}s R}{mV \cos \beta} C_L + \frac{gR}{V \cos \beta} \left( \cos \theta \cos \phi \cos \alpha + \sin \theta \sin \alpha \right)
\]

\[
\dot{\beta} = p \sin \alpha - r \cos \alpha + \frac{\ddot{q}s R}{mV} C_Y + \frac{gR}{V} \left[ \cos \beta \cos \theta \sin \phi - \sin \beta \left( \cos \theta \cos \phi \sin \alpha - \sin \theta \cos \alpha \right) \right]
\]

\[
I_{x\dot{y}} \dot{p} - I_{xy} \dot{q} - I_{xz} \dot{r} = \ddot{q}s b C_L R + \left[ qr \left( I_{y\dot{y}} - I_{z\dot{z}} \right) + \left( q^2 - r^2 \right) I_{yz} + pq I_{xz} - pr I_{xy} \right] / R
\]

\[
I_{y\dot{y}} \dot{q} - I_{yz} \dot{r} - I_{xy} \dot{p} = \ddot{q}s b C_m R + \left[ pr \left( I_{z\dot{z}} - I_{x\dot{x}} \right) + \left( r^2 - p^2 \right) I_{zx} + qr I_{xy} - pq I_{yz} \right] / R
\]

\[
I_{z\dot{z}} \dot{r} - I_{xz} \dot{p} - I_{yz} \dot{q} = \ddot{q}s b C_n R + \left[ pq \left( I_{x\dot{x}} - I_{y\dot{y}} \right) + \left( p^2 - q^2 \right) I_{xy} + pr I_{yz} - qr I_{xz} \right] / R
\]

\[
\dot{\theta} = q \cos \phi - r \sin \phi
\]

\[
\dot{\phi} = p + \tan \theta \left( r \cos \phi + q \sin \phi \right)
\]
Flight Dynamics

\[ C_A = C_{A_0} + C_{A\alpha} \alpha + \frac{c}{2VR}C_{Aq} q + C_{A\delta e} \delta e \]
\[ C_N = C_{N_0} + C_{N\alpha} \alpha + \frac{c}{2VR}C_{Nq} q + C_{N\delta e} \delta e \]
\[ C_m = C_{m_0} + C_{m\alpha} \alpha + \frac{c}{2VR}C_{m_q} q + C_{m\delta e} \delta e \]

\[ C_Y = C_{Y_0} + C_{Y\beta} \beta + \frac{b}{2VR}(C_{Yp} p + C_{Yr} r) + C_{Y\delta a} \delta a \]
\[ C_l = C_{l_0} + C_{l\beta} \beta + \frac{b}{2VR}(C_{lp} p + C_{lr} r) + C_{l\delta a} \delta a \]
\[ C_n = C_{n_0} + C_{n\beta} \beta + \frac{b}{2VR}(C_{np} p + C_{nr} r) + C_{n\delta a} \delta a \]

\[ C_L = C_N \cos \alpha - C_A \sin \alpha \]
\[ C_D = C_A \cos \alpha + C_N \sin \alpha \]
Parameter Estimation

- Method for determining stability and control derivatives from flight data
- Derivatives are varied in the aircraft equations of state until the mathematical model matches recorded flight data
- NASA Dryden code: MATLAB pEst MX.96
Flight Data Conversion

- Isolate doublets in data time histories
- Adjust units to pEst convention
- Correct axes and signs to flight control/pEst convention
- Define constants (geometry, mass properties) to pEst
• Different pEst scripts for lateral and longitudinal maneuvers
  • Lateral: estimated $\beta$, $p$, $r$, $a_y$ signals
  • Longitudinal: estimated $\alpha$, $q$, $a_n$ signals
• User input: selecting estimating weights $W_{ii}$ for each signal $i$
• Algorithm minimizes cost function: summed squared difference between flight data and model estimate, scaled by $W$

$$J = \frac{1}{2n_z n_t} \sum_{i=1}^{n_t} [z(t_i) - \tilde{z}(t_i)]^T W [z(t_i) - \tilde{z}(t_i)]$$
Stability & Control Derivative Maps

• Prandtl-2 flew entirely in the linear regime
• Linear regressions were created for each S&C derivative with respect to $\alpha$
• Data points were weighted by the inverse of the Cramer-Rao bound error estimated by pEst
• Applicable to lookup tables in simulation
Results: Unique Flight Dynamics

- $C_{n\delta a}$: nondimensional yawing moment due to aileron deflection
  - Quantifies how the aircraft responds in yaw due to a roll command
  - Sign specifies nature of yaw/roll coupling
Results: Algorithmic User Input

- Demonstrated algorithmic weight selection to accelerate analysis
- Normalize square of signals by $W_{ii} = \left[\text{range}(i)\right]^{-2}$
  - Cost function distributes error evenly as a percent error of each signal
- 2 data analysis teams: 1 algorithmic weight selection, 1 iterative “trial and error” weight selection
Results: Algorithmic User Input

Prandtl-2 $C_{n\theta a}$ vs. $\alpha$

- Algorithmic Weighting Data
- Iterative Weighting Data
- Algorithmic Weighting Linear Regression
- Iterative Weighting Linear Regression
- Vortex Lattice Estimate
Conclusions & Future Steps

• Prandtl-2 flight testing returned sufficient flight data to quantify the flight dynamics of the Bell-Shaped Lift Distribution equipped vehicle.

• Parameter Estimation was used to determine, from flight data, the characteristic stability and control derivatives. Two different teams with different weight selection schemes produced agreeing results.

• A positive $C_{n\delta a}$ provided quantifiable evidence of proverse yaw.

• Potential future steps: Prandtl-3 spanload measurements, PRANDTL-D flight dynamics simulator, autopilot development
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Thank you!
Questions?


• Prandtl, L., “Regarding Wings with Minimum Induced Drag,” 1933.
Appendix: Nomenclature

- $A = \text{axial force}$
- $b = \text{reference span}$
- $C_i = \text{nondimensional coefficient of force or moment } i$
- $C_{mn} = \text{nondimensional stability/control derivative: coefficient of } m \text{ due to } n$
- $c = \text{reference chord}$
- $D = \text{drag force}$
- $g = \text{gravitational acceleration}$
- $I_{jk} = \text{moment of inertia}$
- $L = \text{lift force}$
- $l = \text{rolling moment}$
- $M = \text{vehicle mass}$
- $m = \text{pitching moment}$
- $N = \text{normal force}$
- $n = \text{yawing moment}$
- $n_t = \text{number of time steps}$
- $n_z = \text{number of signals}$
- $o = \text{coefficient bias}$
- $p = \text{roll rate}$
- $q = \text{pitch rate}$
- $\bar{q} = \text{dynamic pressure}$
- $R = \text{conversion parameter: } 180/\pi$
- $r = \text{yaw rate}$
- $s = \text{reference area}$
- $V = \text{equivalent airspeed}$
- $W = \text{weighting matrix}$
- $Y = \text{side force}$
- $z = \text{measured signal}$
- $\tilde{z} = \text{estimated signal}$
- $\alpha = \text{angle of attack}$
- $\beta = \text{angle of sideslip}$
- $\xi = \text{set of signal/estimate pairs}$
- $\varphi = \text{roll angle}$
- $\theta = \text{pitch angle}$
- $\psi = \text{yaw angle}$
- $\delta e = \text{elevator deflection}$
- $\delta a = \text{aileron deflection}$