On the Minimum Induced Drag of Wings

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Breguet Range Equation

\[ \text{Range} = \eta_{\text{propulsion}} \times \eta_{\text{aero}} \times \log \left( \frac{\text{TOGW}}{\text{LWT}} \right) \times \text{constant} \]
Birds

Wilbur & Orville Wright

- Flying experiments 1899 to 1905
Prandtl Lifting Line Theory

- Prandtl's "vortex ribbons"
- Elliptical spanload for a given span (1920)
- "the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift." \( y = C \)

This is the accepted theory and the standard for the minimum drag of wings. But what is a wing? Is it only aerodynamic? What about the structure?

Wingtip Vortices and Elliptical Spanload
Fundamental Assumptions

For a long time it was difficult to find suitable functions to express the distribution of lift, from which a plausible distribution of $w$ would be obtained by equation (57). After various attempts it was found that a distribution of lift over the span according to a half-ellipse gave the desired solution. According to this, if the origin of coordinates is taken at the center of the wing,

$$
\Gamma = \frac{c}{h} \sqrt{1 - \left(\frac{x}{h}\right)^2}, \text{ hence } \frac{d\Gamma}{dx} = \frac{-c}{h^2} \sqrt{\frac{h^2 - x^2}{x^2}}
$$

(4) The most important of Beta's theorems, from a practical standpoint, furnishes the complete analogy to Munk's theorem concerning the wing system having the least drag, and, corresponding perfectly to the statements in sections 27 and 28, may be expressed thus: The flow behind a propeller having the least loss in energy is as if the screw surfaces passed over by the propeller blades were solidified into a solid figure and this were displaced backward in the nonviscous fluid with a given small velocity. The potential difference between the front and rear sides of a screw surface at one and the same point furnishes, then, again the circulation $\Gamma$ of the corresponding point of the propeller blade.

Birds
Minimum Induced Drag & Bending Moment

- Prandtl (1933)
  Constrain minimum induced drag
  Constrain integrated wing bending moment
  22% increase in span with 11% decrease in induced drag
**Winglets**

- Richard Whitcomb's Winglets
  - induced thrust on wingtips
  - Induced drag decrease is about half of the span "extension"
  - reduced wing root bending stress
Whitcomb's Winglets

- 24 degree leading edge sweep angle
- Chord:
  - root - 15.75 inches
  - tip - 3.875 inches
- Span: 147.8 inches
Twist

Symmetrical Spanloads

- Elevon Trim
- CG Location
Asymmetrical Spanloads

- $C_l\delta_a$ (roll due to aileron)
- $C_n\delta_a$ (yaw due to aileron)
  - induced component
  - profile component
  - change with lift
- $C_n\delta_a/C_l\delta_a$
- $C_l$ (Lift Coefficient)
  - increased lift:
    - increased $C_l\beta$
    - increased $Cn\delta$
  - decreased lift:
    - decreased $C_l\beta$
    - decreased $Cn\delta$

Dr Edward Uden's Results

- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

<table>
<thead>
<tr>
<th>Elevon Config</th>
<th>$C_n\delta_a$</th>
<th>Spanload</th>
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<tr>
<td>I</td>
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<td>XI</td>
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</tbody>
</table>
Kucheman

- Effect on spanloads
  - Increased load at tips
  - Decreased load near centerline
- Upwash due to sweep unaccounted for
- Residual (Kucheman)

Prandtl's Bell Spanload
Prandtl's Spanload

\[ s_l = (1 - x^2)^{3/2} \]

\[ w = 3/2 (x^2 - \frac{1}{4}) \]

\[ \lim_{x \to b/2} L(x) = 0 \]  

\[ \lim_{x \to b/2} \frac{dL(x)}{dx} = 0 \]  

\[ \lim_{x \to b/2} \frac{dW(x)}{dx} = \lim_{x \to b/2} \frac{dW(x)}{dx} \]
Spadding's Gliding Falcon

- Spadding photograph's a gliding falcon's wake with He bubbles
- Vortex cores are 0.76 b apart
- Elliptical spanload is assumed, so the vortex cores are assumed to come from the wingtips
Portugal, et al 2014 (Nature)

Upwash and Wing Beats

Portugal 2014

Halesworth 1968

Catts & Speckman 1994

Speckman & Banks 1996
Pointy Wings and Wing Stall

Local Cl

Proverse Yaw

- ...until June 26th, 2013
- Roll and Yaw are the same sign
- From Uden: Cnda is +ve
- uncertainty

Inertias; configuration changes, turbulence, and slope of Cnda
Revalidated Proverse Yaw

Elliptical and Bell shaped Spanloads
**Prandtl 3**
- The aerodynamic testbed
  - wing pressures
  - FOSS

**Wind Tunnel Test**
- NASA Langley
- 12-ft low speed tunnel
- 52 runs, 6 component force-moment
The Future: Prandtl-M on Mars?

- Deploy cubesat from large Mars rover
- Use Exobrake parachute for Mars atmosphere entry
- Deploy Prandtl-m (2 lbm, 2 ft span) from cubesat at 15,000 ft agl
- Glide 5 mins, 22 miles, Mach 0.6
- Crash land on Mars
- Transmit images & data back

The Mars Mission
Prandtl Propulsion

- Propulsion systems currently use "minimum induced loss"
- What if we switched to minimum torque for a given thrust? +15.4%

Prandtl Fan

- Same voltage
- Same diam
- 24% increase in flow
- 88% reduction in noise
2017 & 2018 Interns

NASA Summer Interns 2017

NASA Summer Interns 2018

NASA Fall Interns 2017

PRANDTL-D

- Videos
  - TEDxNASA 2011
    http://www.youtube.com/watch?v=223OmaQ9uLY
  - NASA Aero Academy 2013
    http://www.youtube.com/watch?v=Hr0I6wBFGpY

Red Jensen: pilot, engineer

Red Jensen, Justin Hall, & David Abramson
Control of Yaw

• You Have Three Choices:

• 1/ drag a vertical tail around with you all the time to create a yawing moment

• 2/ manipulate drag at the wing tips to control yaw

-OR-

• 3/ manipulate THRUST at the wing tips to control yaw

• Biological vs Mechanical Flight

Biological Flight

• Mechanical Flight (110 yrs)

• Vertabrate Flight (128 My)
**Efficiency**

- Efficiency: 12.5% increase in wing efficiency
- 20-30% increase in efficiency by eliminating the tail
- 15.4% increase in propulsive efficiency
- TOTAL EFFICIENCY INCREASE: 69%

- CY2011: world jet fuel consumption $134B
  - $55B in jet fuel saved
- CY2011 World GDP: $69.7T
- World power production: $12.0T
- $1.85T savings in world power production

**Prandtl P3**

- The aerodynamic testbed
  - wing pressures
  - FOSS
Prandtl P3c

cFOSS System

cFOSS computer
Prandtl 3c cFOSS

cFOSS Results

cFOSS compressory/tension
cFOSS moment
cFOSS shear
cFOSS spanload
F-8U Supercritical Wing (SCW)
Electronic Pressure Measurement

EPM

Deborah Jackson, Abby Westphal, Teri Heidbreder, Rachel Itzler, Lynda Hamada, Josh Edwards
NASA Interns & Others

References

- Prandtl, Ludwig: "Uber Tiefzufuhr Kleinste Induzierter Widerstande"; Zeitschrift für Flugtechnik und Motorluftschiffahrt, Jg. 24 Nr. 11 pg 925-928, 1933; Munich, Deutschland.
- Horst, Reimar; and Belting, Peter; with Scott, Jan (translator): "Nurflugel: the Story of Horst Flying Wings 1913 - 1980"; Weltbild Verlag; Graz, Austria; 1985.
- Horst, Reimar; unpublished personal notes.
- Kehr, Armin and Viessmann, Bathy, "Approximate Solution for Minimum Induced Drag of Wing with a Given Structural Weight"; Journal of Aircraft, Feb 1971, Vol 12 No 2, AIAA.
- Kuhler, Carl, "California Condor"; Audubon Special Report No 4, 1950, Dover, NY.
If you want to build a ship, don't drum up people to collect wood and don't assign them tasks and work, but rather teach them to long for the endless immensity of the sea...

- Antoine de Saint-Exupery