On the Minimum Induced Drag of Wings

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Breguet Range Equation

Range = $\eta_{propulsion} \cdot \eta_{aero} \cdot \log \left( \frac{TOGW}{LWT} \right) \cdot \text{constant}$
Birds

Wilbur & Orville Wright

- Flying experiments 1899 to 1905
Prandtl Lifting Line Theory

- Prandtl's "vortex ribbons"
- Elliptical spanload for a given span (1920)
- "the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift." \( y = c \)

Wingtip Vortices and Elliptical Spanload

This is the accepted theory and the standard for the minimum drag of wings. But what is a wing? Is it only aerodynamic? What about the structure?
Fundamental Assumptions

For a long time it was difficult to find suitable functions to express the distribution of lift, from which a plausible distribution of $w$ would be obtained by equation (87). After various attempts it was found that a distribution of lift over the span according to a half ellipse gave the desired solution. According to this, if the origin of coordinates is taken at the center of the wing,

$$
\Gamma = \Gamma_0 \sqrt{1 - \left(\frac{x}{a}\right)^2}, \quad \text{where} \quad \frac{\partial \Gamma}{\partial x} = \frac{\Gamma_0}{b} \sqrt{1 - \left(\frac{x}{a}\right)^2}
$$

(4) The most important of Beta's theorems, from a practical standpoint, furnishes the complete analogy to Munk's theorem concerning the wing system having the least drag, and, corresponding perfectly to the statements in sections 37 and 38, may be expressed thus: The flow behind a propeller having the least loss in energy is as if the screw surfaces passed over by the propeller blades were solidified into a solid figure and this were displaced backward in the nonviscous fluid with a given small velocity. The potential difference between the front and rear sides of a screw surface at one and the same point furnishes, then, again the circulation $\Gamma$ of the corresponding point of the propeller blade.

Birds
Minimum Induced Drag & Bending Moment

- Prandtl (1933)
  Constrain minimum induced drag
  Constrain integrated wing bending moment
  22% increase in span with 11% decrease in induced drag
Winglets

- Richard Whitcomb's Winglets
- Induced thrust on wingtips
- Induced drag decrease is about half of the span "extension"
- Reduced wing root bending stress

![Winglet Diagram]
Whitcomb's Winglets

- 24 degree leading edge sweep angle
- Chord:
  - root - 15.75 inches
  - tip - 3.875 inches
- Span: 147.6 inches
Asymmetrical Spanloads

- $C_{\delta a}$ (roll due to aileron)
- $C_{n\alpha}$ (yaw due to aileron)
- Induced component
- Profile component change with lift
- $C_{n\alpha}/C_{\delta a}$
- $CL$ (Lift Coefficient)
  - Increased lift:
    - Increased $C_{\delta a}$
    - Increased $C_{n\alpha}$
  - Decreased lift:
    - Decreased $C_{\delta a}$
    - Decreased $C_{n\alpha}$

Dr Edward Uden's Results

- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

<table>
<thead>
<tr>
<th>Elevon Config</th>
<th>$C_{n\alpha}$</th>
<th>Spanload</th>
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</tbody>
</table>
Kucheman

- Effect on spanloads
  - Increased load at tips
  - Decreased load near centerline
- Upwash due to sweep unaccounted for
- Residual (Kucheman)

Prandtl's Bell Spanload
Prandtl's Spanload

\[ \sigma_l = (1 - x^2)^{3/2} \]
\[ w = 3/2 (x^2 - \frac{1}{4}) \]

\[ \lim_{x \to b/2} L(x) = 0 \quad (1) \]

\[ \lim_{x \to b/2} \frac{dL(x)}{dx} = 0 \quad (2) \]

\[ \lim_{x \to b/2} \frac{dDW(x)}{dx} = \lim_{x \to b/2} \frac{dDW(x)}{dx} \quad (3) \]
Spadding’s Gliding Falcon

- Spedding photograph's a gliding falcon's wake with He bubbles
- Vortex cores are 0.76 b apart
- Elliptical spanload is assumed, so the vortex cores are assumed to come from the wingtips
Portugal, et al 2014 (Nature)

Upwash and Wing Beats
Pointy Wings and Wing Stall

Pointy wings and wing stall are critical in aerodynamic stability. Important factors include local Cl (lift coefficient) and flow velocity. The diagram illustrates the relationship between these variables, emphasizing the importance of understanding lift coefficients in various flight conditions.

Proverse Yaw

...until June 26th, 2013

Roll and Yaw are the same sign
- From Uden: Cnda is +ve
- Uncertainty

Inertias, configuration changes, turbulence, and slope of Cnda are key factors in understanding yaw behavior. The diagram highlights the interplay between these elements, underscoring the complexity of maintaining stable yaw in flight conditions.
Revalidated Proverse Yaw

Elliptical and Bell shaped Spanloads
Prandtl 3

- The aerodynamic testbed
  - wing pressures
  - FOSS

Wind Tunnel Test

- NASA Langley
- 12-ft low speed tunnel
- 52 runs, 6 component force-moment
The Future: Prandtl-M on Mars?

- Deploy cubesat from large Mars rover
- Use Exobrake parachute for Mars atmosphere entry
- Deploy Prandtl-m (2 lrm, 2 ft span) from cubesat at 15,000 ft agl
- Glide 5 mins, 22 miles, Mach 0.6
- Crash land on Mars
- Transmit images & data back

The Mars Mission
Prandtl Propulsion

- Propulsion systems currently use "minimum induced loss"
- What if we switched to minimum torque for a given thrust? +15.4%
Prandtl Prop 1

Prandtl Prop 2
2017 & 2018 Interns

PRANDTL-D

- Videos
  - TEDxNASA 2011
    http://www.youtube.com/watch?v=223OmeQ9uLY
  - NASA Aero Academy 2013
    http://www.youtube.com/watch?v=Hr0I6wBFQpY
Control of Yaw

- You Have Three Choices:
  1/ drag a vertical tail around with you all the time to create a yawing moment
  2/ manipulate drag at the wing tips to control yaw
  -OR-
  3/ manipulate thrust at the wing tips to control yaw

Biological vs Mechanical Flight

Biological Flight

- Mechanical Flight (110 yrs)
- Vertabrate Flight (128 My)
Efficiency

- Efficiency: 12.5% increase in wing efficiency
- 20-30% increase in efficiency by eliminating the tail
- 15.4% increase in propulsive efficiency
- TOTAL EFFICIENCY INCREASE: 69%

- CY2011: world jet fuel consumption $134B
- $55B in jet fuel saved

- CY2011 World GDP: $69.7T
- World power production: $12.0T
- $1.85T savings in world power production

Prandtl P3

- The aerodynamic testbed
  - wing pressures
  - FOSS
Prandtl 3c cFOSS

cFOSS Results

cFOSS compressive/tension
cFOSS moment
cFOSS shear
cFOSS spanload
F-8U Supercritical Wing (SCW)
Electronic Pressure Measurement

Transducers 1-82
Pressure Distribution on EPM1

Transducers 83-84
Pressure Distribution on EPM2

Transducers 85-86
Pressure Distribution on EPM3

EPM

Deborah Jackson, Abby Westfall, Teri Hewitt, Rachel Arden, Lynda Hartman, Noah Edwards
Spanload

NASA Interns & Others

References

- Prandtl, Ludwig: "Über Treppflügel kleinster Induktionswiderstände"; Zeitschrift für Flugtechnik und Motorluftfahrt, Jr 24 Nr 11 pg 525-528 1933; Munich, Deutschland.
- Horst, Reinmar; unpublished personal notes.
- Kaai, Armin and Viswanathan, Sathy: "Approximate Solution for Minimum Induced Drag of Wings with a Given Structural Weight"; Journal of Aircraft, May 1979, Vol 12 No 2, AIAA.
- Lee, Russell: "Only the Wing; Reinmar Horst's Epic Quest to Stabilize and Control the All-Wing Aircraft," Smithsonian Institution Scholarly Press (Rowman & Littlefield), Washington D.C., 2011.
If you want to build a ship, don't drum up people to collect wood and don't assign them tasks and work, but rather teach them to long for the endless immensity of the sea...

- Antoine de Saint-Exupery