Engineered non-stochastic cellular materials show promising characteristics on the laboratory scale, with nearly ideal specific stiffness and strength scaling at ultralight mass density. These properties suggest performance benefits in any application with combined stiffness and mass constraints, such as air vehicles. We investigate here the application of re-configurable cellular composite materials and structures to lighter than air vehicles. We describe the properties and applicability of these materials, provide an example analysis of governing loading conditions associated with airships, show an example method for navigating the design space, and describe how recent advances in cellular material manufacturing and reconfiguration enable system performance benefits including new concepts of operation. A key result is that these cellular materials exist in a regime that allow an airship to be designed and manufactured simply as a monocoque cellular solid or very thick shell. Lastly, we propose lighter than air vehicle concept of operations that takes advantage of the resulting manufacturing simplicity by assembling and maintaining large vehicles in-flight, eliminating structural compromises associated with transitional flight modes and ground handling.

Nomenclature

\( \omega_1 \)  
Beam/mechanism first resonant mode

\( \phi_{shell} \)  
Shell slenderness, defined as the ratio of shell thickness to width \( t : w \) constrained to \( 0 < \phi_{shell} < 0.5\% \)

\( \phi_{ship} \)  
Ship slenderness defined as the ratio of width to length \( w : l \) constrained to \( 0 < \phi_{ship} < 1 \)

\( \rho \)  
Mass density

\( \rho_c \)  
Mass density of cellular solid material

\( \rho_H \)  
Mass density of hydrogen gas

\( \rho_s \)  
Mass density of constituent solid material

\( \sigma_{max} \)  
Maximum stress

\( A \)  
Area (cross section of airship)

\( A_{surf} \)  
Surface area

\( C_d \)  
Coefficient of drag

\( d \)  
Diameter

\( d_b \)  
Diameter of beam/mechanism

\( E \)  
Elastic modulus

\( F_d \)  
Drag force

\( I \)  
Second moment of area

\( I_b \)  
Second moment of area of beam/mechanism

\( K_b \)  
Beam/mechanism boundary constraint constant

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\( K_m \)  Beam/mechanism modal boundary constraint constant

\( K_{\text{power}} \)  Propulsion system mass scaling constant

\( l \)  Length (airship length)

\( l_p \)  Beam/mechanism length

\( M \)  Bending moment

\( m_b \)  Beam/mechanism mass

\( m_b \)  Beam/mechanism modal mass

\( m_p \)  Payload mass

\( P \)  Power

\( P_{cr} \)  Critical load

\( t \)  Thickness (shell thickness)

\( u \)  Velocity of free stream (apparent wind velocity of airship)

\( V \)  Volume (volume of airship)

\( w \)  Width (airship width)

\[ E \propto \rho \]

I. Introduction

Cellular materials provide improved performance, in many engineering applications, over conventional solid materials. While this is common knowledge for applications such as thermal insulation and energy absorption, recent work shows engineered cellular materials that exhibit nearly ideal \( E \propto \rho \) specific modulus scaling [1–3] (conventional cellular materials exhibit \( E \propto \rho^2 \) or \( E \propto \rho^3 \) scaling [4]). Together with corresponding results for strength and failure modes, [1] these breakthroughs promise to enable more efficient, lightweight, and therefore cheaper structural systems. This is possible because assembly allows production of geometry that has not been achieved with previous ultralight processes, such as foaming methods. Limited capability has been demonstrated at or below the density of air at standard pressure, but methods to create materials systems - which are orders of magnitude less dense than conventional solid materials at near ideal theoretical specific stiffness and strength scaling - are maturing rapidly.

These materials can be made using a building block approach, by reversibly assembling them from simple, discrete, and reusable parts. [1] The mass density cost of robust connections is low for ultralight assembled materials, [1] where strut member slenderness continues to govern overall mass density scaling, as with previously known methods of making cellular materials. [4] Reversibly Assembled Cellular Composite Materials (RACCM) may be considered as three dimensional fiber composites with mechanical behavior that is tunable by composition from different part types or densities, with overall behavior determined by relative placement. [1]

![Figure 1. Lightweight Reversibly Assembled Cellular Composite Aerostructure [5]](image-url)
Application of this field to aeronautics includes a morphing wing proof of concept, with precisely tunable and actively deformable structures for mass and aerodynamically efficient shape-morphing wing aircraft. [5, 6] Significant additional benefit for industry is provided by an intrinsic economy of design, manufacturing, and service methods provided by the ability to have relatively very few unique part types. While the total number of parts required for a given application is likely to increase by a small factor, the orders of magnitude reduction in the number of types of parts can result in a significant reduction of complexity in the entire process from design and analysis to testing, repair, re-use, and supply chain complexity.

Recent analysis of current airship developments efforts identifies two areas where significant technology gaps exist across all projects: manufacturing and assembly processes and ground handling. [7] Air vehicle design and operations are generally heavily constrained by traditional manufacturing, operation, and service methods. Emerging robotics capabilities, manufacturing automation processes, dynamic control methods, and structural system optimization can converge to fill the former gaps and remove the latter constraints. What if air vehicles could be efficiently made and maintained in flight, so that they never have to take off, and never have to land?

II. Analysis

Take-off, landing, and ground handling currently present many different governing static and dynamic load conditions for air-vehicles, and result in design compromises. If it is possible to construct and maintain an air vehicle in its primary operating environment (e.g., at cruise altitude), the aforementioned constraints can be separated. Examples of target applications include sustained mission air-vehicles that fully utilize physical scaling advantages to stay aloft and support heavy total cargo loads with very high efficiency, such as lighter than air vehicles. Such air-vehicles will not have to be designed around existing infrastructure such as runways, hangars, and factories. They will also not have to be designed to take off and land; conversely, aircraft that transfer payloads to and from the ground will not have to be designed to cruise. This extends recently updated models for airships as carrier vehicles for smaller air vehicles that perform short duration functions, such as reconnaissance or cargo distribution. [8]

We consider first principles lighter than air vehicle specific structural mass requirements to illustrate RACCM based airship design for different sized applications.

The following parameter constraints are used throughout this paper:

1. Shape - The design space is constrained to cylindrical vessels with spherical end caps.

2. Materials - The density of the cellular solid material, \( \rho_c \), is that of a material documented in the literature: 7.2kg/m\(^3\) [1]. The constituent solid material, \( \rho_s \), is conventional carbon fiber reinforced polymer.

3. Operations - The baseline airship model specifies 100 metric tons of cargo and a cruising speed of 160 km/hr. [9]

A. General Quasi-Static and Dynamic Mode Benefits

It is well accepted in the literature that takeoff and landing conditions provide governing load conditions for aerostructure design, especially for lighter than air vehicles, taking into account weather conditions associated with low altitude flight [10] and relatively discontinuous flow around service and storage facilities. Using dimensional scaling methods we compare the difference between the structural mass required at low altitude cruise, characterized by a high amplitude quasi static and dynamic load environment versus that required by a relatively low load environment characteristic of a high altitude cruise.

Considering an airship structure as a system of beam/column mechanisms that maintain the relative spatial positions of the non-structural components (e.g., cargo, cabins, engines, empennages, energy storage), we find that the structural mass is proportional to the square root of the maximum acceleration for which the system must maintain stability. This relationship is the same when considering the maximum acceleration for which the system components must stay within specific deflections.

For beam/column stability limited mechanisms:

\[
P_{cr} = K_b \frac{EI}{I_b} = m_p a^2 l_b = \frac{d^4}{12} ; m_b = \rho_b d^2 l_b
\]

\[
m_b \propto \rho_b d^2 \sqrt{\frac{m_p a}{E}} ; m_b \propto \sqrt{a}
\]

For beam/column deflection limited mechanisms:

\[
\delta = \frac{P l_b}{3 E I_b} ; P = m_p a^2
\]
We see here that a factor of ten decrease in the low frequency acceleration amplitude can result in about a factor of three decrease in required structural mass. This structural mass effect is even more pronounced when considering dynamic modes, given the typical strategy for mitigating maximum loading conditions by keeping natural resonant frequencies higher than the significant driving frequencies during the governing phase of flight. We find that the structural mass is proportional to the square of the first natural mode, or highest expected environmental driving frequency.

Natural resonant mode limited mechanism:

\[
\omega_1 = \frac{K_b}{I_b} \sqrt{\frac{E I_b}{K_m m_b}}
\]

(5)

\[
m_b = \frac{\omega_1^2 \rho_b^2 L_b^5 + \sqrt{\omega_1^2 \rho_b^2 L_b^5 + 12 \rho_b^2 L_b^5} (12 \rho_b^2 L_b^5 + 12 \rho_b^2 L_b^5)}{6 E}
\]

(6)

\[
m_b \propto \omega_1^2 L_b^5 \rho_b^2 ; m_b \propto \omega_1^2
\]

(7)

We see here that a factor of ten decrease in dynamic input amplitude can result in a hundred-fold decrease in required structural mass. These significant potential structural mass savings depend on the spatial distribution of non-structural mass. Airships, by their nature of having a large proportion of overall mass serving structural functions, can be expected to approach the upper bounds of these estimations. The potential mass savings gained by having dedicated structural configurations at cruise altitude – purely on account of structural mass – is significant, with corresponding expected total system cost savings.

**B. Buoyancy Design Impact**

For a buoyancy analysis, we consider the airship to be an open cellular material shell with a sealed outer skin. The lifting gas is taken to be hydrogen. The calculation for buoyancy takes into account the masses of the shell, skin, propulsion system, and cargo. The skin material density, \(\rho_{\text{skin}}\), is taken to be 1460 kg/m\(^3\) based on prior work on the mechanical properties of airship envelopes [11]. The mass associated with the propulsion system is calculated as a function of its power. Power is calculated as proportional to the drag on the airship where the coefficient of drag is approximated using the airfoil design software, Xfoil [12]. Singular value decomposition was used to fit a line to a set of coefficients of drag, generated from a sample of custom airfoil cross sections.

\[m_{\text{airship}} = m_{\text{shell}} + m_{\text{skin}} + m_{\text{powersystem}}\]

(8)

\[m_{\text{shell}} = V_{\text{shell}} \rho_c\]

(9)

\[m_{\text{skin}} = A_{\text{skin}} \rho_{\text{skin}}\]

(10)

\[P = C_{d \text{\text{air}}} A_{\text{surface}} u^3\]

(11)

\[m_{\text{powersystem}} = K_{\text{powermass}} P\]

(12)

\[m_{\text{buoyancy}} = (\rho_{\text{air}} - \rho_{\text{H}}) V_{\text{envelope}}\]

(13)

\[m_{\text{cargo}} = m_{\text{buoyancy}} - m_{\text{airship}}\]

(14)

The equation for \(m_{\text{shell}}\) was kept as a function of thickness and the resulting third order polynomial was solved for varying length, thickness and cargo mass. Figure 2 shows the minimum permissible thickness to maintain buoyancy for a given length and width.
C. Bending Stiffness Design Impact

For stress analysis, we consider the airship as a loaded beam. It is subjected to bending moments, longitudinal gas pressure forces, shear, and moments due to the distribution of mass, buoyancy, and aerodynamic forces acting along the longitudinal axis of the hull. The most critical structural aerodynamic forces are generated normal to the longitudinal axis. They include forces from the rudders and elevators that control the airship direction, the airship height, and also the compensation of any buoyancy inequality. Additionally, they include forces from any wind gusts acting on the air vehicle hull. [13] [14] The literature suggests that the governing loading conditions are provided by forces that result in first order bending of the entire structure, and that the governing load condition is provided by actuation of control surfaces. [15]

Considering the airship as a simple cantilever beam, we estimate the magnitude of this bending moment in worst case quasi-static aerodynamic conditions as resulting from the dynamic pressure on a typically sized and fully actuated control surface in a direction orthogonal to the longitudinal axis of the airship acting on the larger remaining length of the airship (forward of the control surface) virtually fixtured at the distal end by the aforementioned aerodynamic conditions. With $A_c =$ effective area of control surface orthogonal to longitudinal plane in full deflection and $l_c =$ longitudinal position of control surfaces:

$$M = qA_c l_c; \quad q = \frac{\rho_{air} u^2}{2} \quad (15)$$

$$M = \frac{\rho_{air} u^2 A_c l_c}{2} \quad (16)$$

$$A_c = awl; \quad l_c = bl; \quad K_c = \frac{ab}{2} \quad (17)$$

$$M = K_c \rho_{air} u^2 l_c^2 \quad (18)$$

Variables $a$ and $b$ are design parameters relating to the size and position of control surfaces. We can intuitively understand $K_c$ to be a measure of the desired authority via control surfaces. Considering typical historical airship designs, $A_c \approx lw/64$ and the position of the control surface $l_c \approx 63l/64$, giving $K_c = 0.007$, we predict the following relationship for conventional rigid airship design:

$$M_{Conventional} = 0.007 \rho_{air} u^2 l_c^2 w \quad (19)$$

This is similar to the Naatz equation, which was “developed through extensive experimentation in stormy conditions over the Baltic Ocean” with conventional airship design. [16] The Naatz equation determines the bending moment on an airship given the velocity, density, volume and length of the vehicle. Relative to the simple beam based estimation above, the difference in relationships is that this has a lesser contribution of the airship length and greater contribution of the airship width.
\[ M_{Naatz} = 0.005 \rho_{air} u^2 V^2 l \]  

The maximum bending stress is then calculated using classical thick cylinder bending stress equations.

\[ \sigma_{\text{max}} = \frac{Mw}{I} = \frac{Mw}{2I} \]  

\[ I = \frac{\pi (\frac{w}{2})^4}{4} - \frac{\pi (\frac{w}{2} - t)^4}{4} \]  

Solving for \( t \), where the moment, \( M \), is taken as equation (18):

\[ t = \frac{w}{2} - \frac{1}{2} \sqrt{w^4 - \frac{32K_c l^2 \rho_{air} u^2 w^2}{\pi \sigma_{\text{max}}}} \]  

We use this equation for \( t \) in terms of \( w \) and \( l \) to plot the required thickness as width and length varies. Figure 3 therefore represents the minimum thickness required to ensure that the cellular material thick shell does not experience a bending stress greater than 73.4 kPa. Crucially, many of the minimum thicknesses shown on this graph are less than the maximum thickness required for buoyancy. Therefore, these two pieces of analysis demonstrate a feasible design region for digital cellular material airships structures.

We define \( l > w \), and see from these relationships that:

\[ K_c < \frac{\pi}{32 \rho_{air} u^2} \sigma_{\text{max}} \phi_{\text{ship}}^2 \]  

We can use this inequality to describe the design space as dictated by the maximum bending stress of the material. In order to obtain more control authority, one must increase either \( \sigma_{\text{max}} \) or \( \phi_{\text{ship}} \), where increasing \( \phi_{\text{ship}} \) will have a greater impact.

### III. Application

In order to demonstrate the utility of this method we apply the analytical methods developed in this paper to an example of a transportation cycler.

#### A. Transportation Cycler

The "aerocycler" is envisioned here to operate as a high altitude warehouse that flies a circuit above a large supply and service area. Supplies are transferred to and from the ground via smaller air-vehicles as necessary. We characterize this scenario based on a generic sales revenue of a major online retailer in the North America region (75 billion USD across 12M km²). An average value of goods of 100 USD per kg results in a monthly delivery mass of 62,500 metric
tons of phone cases, collectible coins, sports memorabilia, sugar free gummi bears, and frog statues. We assume that each airship could serve an area with rotorcraft for package delivery - 1000km diameter given typical helicopter range: 785,000 km². This results in a fleet of about 16 airships each supplying about 4,000 kg of cargo per month each. At 20 kph, each airship will pass over any manufacturing facility for resupply and any delivery location within the same month. This fleet would remain permanently at altitude, forming a mobile network of permanently airborne warehouses.

From this description we apply the following inputs to equations described above. Velocity = 5.5 m/s and cargo mass = 4,000 kg.

![Graph](image)

**Figure 4.** Design space for transportation cycler

The graphs above show that the minimum structural thickness based on the bending stress requirement is far less than the maximum thickness permitted for buoyancy, thereby presenting a wide design space for a transportation cycler with these requirements. With allowable shell thickness on the order of ten meters for overall airship dimensions of a few hundred meters, we consider this to be a promising method to for stable monocoque airship design.

### IV. Conclusion

Engineered non-stochastic cellular material properties suggest performance benefits in lighter than air vehicles due to stiffness and mass constraints that are intrinsic to the airship design problem. Recent advances in cellular material manufacturing and reconfiguration enable system performance benefits including new concepts of operation, such as lighter than air vehicles that are assembled and maintained in-flight, eliminating structural compromises associated with transitional flight modes and ground handling. Existing engineered cellular materials display properties allowing large large scale airships design as monocoque cellular solids. Inevitable improvements in cellular material properties and manufacturing will improve feasibility even further. Given the suggestion that the two most significant technology gaps exist across all current airship projects are manufacturing and assembly processes and ground handling [7], a strategy that encompasses construction and maintenance in flight could provide critical rephrasing of the system design problem through these new concepts of operation. Refactoring of traditional manufacturing, operation, and service process constraints could extend to other domains in aerospace systems and manufacturing in general.

In future work, the complexity of the design task would benefit from a form of optimization in order to find the most suitable geometry for a chosen application. For example, the Sequential Least SQares Programming (SLSQP) function from within the SciPy Minimize library is a multiobjective constrained optimization method that has been applied to fixed wing aircraft design. [17] In this situation it would allow for several objective functions such as drag, bending stiffness, buoyancy and cost of transport to be incorporated into a composite objective function.

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Figure 5. Lighter than Air Transportation Cycler Concept

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