Orbital Maintenance for the Wide Field Infrared Survey Telescope: Effects of Solar Radiation Pressure and Navigation Accuracies on Stationkeeping

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NAVIGATION & MISSION DESIGN BRANCH
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Outline

- Wide Field Infrared Survey Telescope (WFIRST)
- Force Models and Stationkeeping Strategy
- Modeling Solar Radiation Pressure (SRP)
- SRP Effect on Stationkeeping
- Navigation Errors Effect on Stationkeeping
- Conclusions and Future Work
Wide Field Infrared Survey Telescope

- Scheduled to launch in 2025 to an orbit about the Sun-Earth Libration Point 2 (SEL2)
- 2.4 meter primary mirror along with a Wide Field Instrument (WFI) will be used to scan up to 100x more sky than Hubble
- Coronograph Instrument (CGI) will be used to search for exoplanets

- Mission Objectives:
  - Explore exoplanets
  - Research into dark energy
  - Perform galactic and extragalactic surveys
Goal of the work

- WFIRST will be orbiting at Sun-Earth L2 around a Quasi-Halo orbit. To deal with the instability of the environment, and remain close to its nominal orbit, stationkeeping maneuvers will be performed every 21 days.

- Routine Momentum Unloads (MUs) will be performed to unload the stored momentum in the reaction wheels.

- The effect of Solar Radiation Pressure (SRP) on the stationkeeping $\Delta v$ has been explored using different SRP models.

- The effect of Orbit Determination and Navigation errors, and maneuver execution errors on the stationkeeping $\Delta v$ has also been explored.
Force Models

- **Circular Restricted Three Body Problem (RTBP)**

\[ \ddot{R} + 2\omega \times \dot{R} = \nabla \Omega + a_{srp}, \]

where \( R = (X, Y, Z) \) is the location of the satellite, \( \Omega = \frac{1}{2}(X^2 + Y^2) + \frac{1-\mu}{r_{ps}} + \frac{\mu}{r_{pe}} \) is the gravitational potential, and \( a_{srp} = (a_X, a_Y, a_Z) \) is the SRP acceleration.

- **Point Mass Ephemeris Model**

\[ \dddot{R}_{S,sc} = Gm_S \frac{R_{S,sc}}{R_{S,sc}^3} + Gm_E \left( \frac{R_{E,sc}}{R_{E,sc}^3} - \frac{R_E}{R_E^3} \right) + Gm_M \left( \frac{R_{M,sc}}{R_{M,sc}^3} - \frac{R_M}{R_M^3} \right) + a_{srp}, \]

where \( R = (X, Y, Z) \) is the location of the satellite, \( R_i = (X_i, Y_i, Z_i) \) is the position of the Sun-Earth and Moon \((i = S, E, M)\), and \( m_S, m_E, m_M \) their respective masses.
The baseline trajectory for WFIRST has been computed with the ATD Module developed by Dr. Natasha Bosanac [1].

Main Steps:
1. Select candidate Halo orbit in CR3BP.
2. Find transfer from LEO to Halo in CR3BP.
4. Export to GMAT.

We use information from the natural dynamics around a Halo orbit to determine the stationkeeping maneuver.

*Stationkeeping Maneuver Process*

- State Transition Matrix (STM) Calculated from state vector
- State Vector from Last Momentum Unload (MU)
- \( \Delta v \) Calculated to reach RLP XZ crossings based on reference orbit
- \( \Delta v \) Optimized using stable eigenvector direction from STM

*Process Repeats Every 21 Days*
Solar Radiation Pressure Models

- **Cannonball Model** (simple) the satellite’s shape is approximated by a sphere:

\[
a_{srp} = -\frac{P_{srp} C_r A_{sat}}{m_{sat}} r_s.
\]

- **N-plate Model** (intermediate) the satellite’s shape is approximated by a set of flat plates, each one with different reflectivity properties:

\[
a_{srp} = -\frac{P_{srp}}{m_{sat}} \sum_{i=1}^{N} \left( A_i \langle n_i, r_s \rangle \left[ (1 - \rho_s^i) n_i + 2 \left( \rho_s^i \langle n_i, r_s \rangle + \frac{\rho_d^i}{3} \right) r_s \right] H(\theta_i) \right).
\]

- **Finite Element Model** (high-fidelity) a CAD model is used to approximate the satellite’s shape and ray-tracing techniques are used to approximate the SRP acceleration:

\[
a_{srp} = -\frac{P_{srp}}{m_{sat}} \int_{\partial \Omega} A \langle n, r_s \rangle \left[ (1 - \rho_s) n + 2 \left( \rho_s \langle n, r_s \rangle + \frac{\rho_d}{3} \right) r_s \right] d\Omega.
\]

Comparison between SRP models

- The main difference between the Cannonball and the N-plate model is that it does not account for the satellite’s attitude.

- The 14-plate approximation for WFIRST shows good agreement with the Finite Element approximation.

Fig 1: Cannonball vs 1-plate Model.

Fig 2: 1-plate vs 14-plate vs Finite Element
Effect of SRP on LPOs

- The extra acceleration due to SRP essentially displaces the invariant objects toward the Sun.

Table 1: Relationship between the location of L2 and Cr values.

<table>
<thead>
<tr>
<th>$C_r$</th>
<th>$q_{srp}$</th>
<th>$L_2$ location</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_r^0$ = 0.00</td>
<td>0.0</td>
<td>151,105,099.17 km</td>
</tr>
<tr>
<td>$C_r^1$ = 1.25</td>
<td>$5.7799 \times 10^{-5}$</td>
<td>151,104,145.49 km</td>
</tr>
<tr>
<td>$C_r^2$ = 2.00</td>
<td>$9.2472 \times 10^{-5}$</td>
<td>151,103,573.97 km</td>
</tr>
</tbody>
</table>

Table 2: Distance between L2 Halo orbits for different Cr values.

<table>
<thead>
<tr>
<th></th>
<th>$C_r^0 - C_r^1$</th>
<th>$C_r^0 - C_r^2$</th>
<th>$C_r^1 - C_r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RTBP</td>
<td>1,300 km</td>
<td>2,000 km</td>
<td>800 km</td>
</tr>
<tr>
<td>Ephem</td>
<td>1,430 km</td>
<td>2,400 km</td>
<td>930 km</td>
</tr>
</tbody>
</table>
Effect of SRP on Stationkeeping

- Using the ATD module, two different reference orbits have been generated: one with $C_r = 0.0$ and another with $C_r = 2.0$.

- For each reference, orbit 5 simulations for stationkeeping over 5 years have been performed using three different $C_r$ values ($C_r = 0.0, 1.25$ and $2.0$) and different MUs sizes (1.3 mm/s and 13.3 mm/s).

- Results show that following the reference orbit with the same $C_r$ value helps lower the total $\Delta v$ cost.

- Increasing the size of the MUs increases the $\Delta v$ cost, and the accuracy in SRP models is less relevant.

Fig 1. Total $\Delta v$ cost for 5 years stationkeeping simulations using a No SRP reference trajectory for different $C_r$ values.

Fig 2. Total $\Delta v$ cost for 5 years stationkeeping simulations using an SRP reference trajectory for different $C_r$ values.
Effect of SRP on Stationkeeping

- Using a reference trajectory the same as in the previous examples \( (C_r = 2.0 \text{ with cannonball SRP}). \)

- 5 simulations for stationkeeping over 5 years have been performed with different fixed offset angles and MUs sizes (1.3 mm/s and 13.3 mm/s).

- Results show that large offset angles result in larger total \( \Delta v \) cost.

- Increasing the size of the MUs increases the \( \Delta v \) cost, and the accuracy in SRP models is less relevant.

- Explorations with variable attitude will be done in the future.

### Table 3: Total stationkeeping \( \Delta v \) cost with no MUs for a fixed plate offset.

<table>
<thead>
<tr>
<th>Offset Angle</th>
<th>Total ( \Delta v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0^\circ )</td>
<td>0.1287 m/s</td>
</tr>
<tr>
<td>( \alpha = 10^\circ )</td>
<td>0.1355 m/s</td>
</tr>
<tr>
<td>( \alpha = 20^\circ )</td>
<td>0.1848 m/s</td>
</tr>
<tr>
<td>( \alpha = 40^\circ )</td>
<td>0.2754 m/s</td>
</tr>
</tbody>
</table>

**Fig 3.** Total \( \Delta v \) cost for 5 years stationkeeping simulations using 1-plate model for SRP for different fixed offset angles.
Effects of Navigation Errors on Stationkeeping

- Currently NEN and DSN ground stations will be used for orbit determination.
- Having errors of 5 km on the steady-states and 5 cm/s on the velocity estimates.

*Stationkeeping Maneuver Process*

- State Vector from Last Momentum Unload (MU)
- OD Error from Covariance Matrices applied to state vector
- Stationkeeping Maneuver
  - Uses OD Error State as Initial State
  - Applies Optimized $\Delta v$ and adds 5% to the magnitude (maneuver execution error)
- State Transition Matrix (STM) Calculated from state vector
- $\Delta v$ Calculated to reach RLP XZ crossings based on reference orbit
- $\Delta v$ Optimized using stable eigenvector direction from STM

*Process Repeats Every 21 Days*
Effects of Navigation Errors on Stationkeeping

- Four different cases have been analyzed: with no SRP \((C_r = 0.0)\) and SRP \((C_r = 2.0)\), each one taking different MUs sizes (1.3 mm/s and 13.3 mm/s).

- 10 simulations using the cannonball model for SRP have been performed including random OD errors and maneuver execution errors for 1 year of stationkeeping.

<table>
<thead>
<tr>
<th>Analysis Case</th>
<th>(C_r) Value Used in Analysis and Reference Orbit</th>
<th>Momentum Unload Residual (\Delta v) (mm/s)</th>
<th>Maximum Position OD Error (km)</th>
<th>Maximum Velocity OD Error (cm/s)</th>
<th>Average Total Stationkeeping (\Delta v) for 1 Year (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1.33</td>
<td>9.57</td>
<td>3.22</td>
<td>1.12</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>13.33</td>
<td>12.81</td>
<td>3.83</td>
<td>1.20</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1.33</td>
<td>13.72</td>
<td>4.04</td>
<td>1.06</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>13.33</td>
<td>10.75</td>
<td>4.53</td>
<td>1.16</td>
</tr>
</tbody>
</table>
Effects of Navigation Errors on Stationkeeping

- 8 different cases have been analyzed using a 1-plate model for SRP: taking different fixed offset angles, each one taking different MUs sizes (1.3 mm/s and 13.3 mm/s).

Table 5. Stationkeeping $\Delta v$ with Orbit Determination Errors and N-Plate Model.

<table>
<thead>
<tr>
<th>Analysis Case</th>
<th>1-Plate Offset Angle ($^\circ$)</th>
<th>Momentum Unload Residual $\Delta v$ (mm/s)</th>
<th>Maximum Position OD Error (km)</th>
<th>Maximum Velocity OD Error (cm/s)</th>
<th>Average Total Stationkeeping $\Delta v$ for 1 Year (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1.33</td>
<td>8.43</td>
<td>3.14</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>13.33</td>
<td>10.62</td>
<td>4.09</td>
<td>0.99</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>1.33</td>
<td>11.31</td>
<td>4.62</td>
<td>0.91</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>13.33</td>
<td>9.82</td>
<td>3.87</td>
<td>0.97</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>1.33</td>
<td>7.64</td>
<td>4.17</td>
<td>0.89</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>13.33</td>
<td>16.27</td>
<td>4.54</td>
<td>1.02</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>1.33</td>
<td>10.94</td>
<td>3.75</td>
<td>0.88</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>13.33</td>
<td>12.03</td>
<td>4.03</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Conclusions and Future Work

- We have analyzed how SRP acceleration uncertainties, the size of MUs and OD errors affect WFIRST’s total $\Delta v$ for stationkeeping.

- Simulations without OD errors: agreement between the SRP model and reference orbit help lower the $\Delta v$ cost. Moreover, large MUs increase the total $\Delta v$.

- Simulations with OD errors: the OD errors introduced are similar in size to the individual stationkeeping maneuvers. Better navigation errors (either using more tracking or Onboard OD) may reduce the total stationkeeping maneuver $\Delta v$.

- The total stationkeeping $\Delta v$ increased significantly when OD errors were introduced vs. when just looking at SRP and MU sizes.

- In the future, using a variable attitude profile for WFIRST, the effects of SRP can fully be studied as WFIRST moves through its orbit with different orientations.
Questions?

Thank you for your attention