Shape Sensing for Wings with Spars and Ribs using Simulated Strain

Chan-gi Pak, Ph.D.
Structural Dynamics Group, Aerostructures Branch (Code 560)
NASA Armstrong Flight Research Center
Active Shape Control

  - Had an over-damped response to external loads before mishap
    - Recommendation 11: develop a method to measure wing dihedral in real-time with a visual display available to the test crew.
    - Recommendation 12: develop manual and/or automatic techniques to control wing dihedral in flight.

- NASA Low Boom Flight Demonstration aircraft (2021)
  - Minimize trim shape error: use “Jig shape optimization”
    - The major issue with this jig shape optimization is that the updated jig shape is optimum only at the design flight condition.
    - To overcome this limitation, an active trim shape control technique can be used to minimize error between the target and the actual trim shapes during flight.
Aircraft Shape Sensing from Strain Data

- **Tessler and Spangler: 2003**
  - Inverse finite element method
    - Create simplified 3D structural model
    - Need a finite element model
    - Use numerical optimization technique to minimize strain error at strain gage locations
    - Off-line method

- **Ko: 2007**
  - Use closed-form equation for deformation computation
    - Deformation along a line is available during flight; On-line method
    - Don’t need a finite element model
    - Pitch slope is not available.

- **Pak: 2016**
  - Use two-step approach; On-line method; Based on 3D structure
    - Step 1: deformation along a line (don’t need a finite element model)
    - Step 2: expand deformation along the sensor lines to a 3D structure (Need a finite element model)
  - Deformation sensing
  - Velocity, Acceleration, and Load sensing
Definition of Curvature $\kappa$

- Upper strain due to pure bending
  - $\epsilon_u - x = \epsilon_u - \frac{\epsilon_u + \epsilon_l}{2} = \frac{\epsilon_u - \epsilon_l}{2}$
  - $\kappa = -\frac{(\epsilon_u - \epsilon_l)/2}{h/2} = -\frac{\epsilon_u - \epsilon_l}{h}$

- Lower strain due to pure bending
  - $\epsilon_l - x = \epsilon_l - \frac{\epsilon_u + \epsilon_l}{2} = -\frac{\epsilon_u - \epsilon_l}{2}$

$$\epsilon_u - x = -(\epsilon_l - x)$$

$$\epsilon_u - x = x - \epsilon_l$$

$$2x = \epsilon_u + \epsilon_l$$

$$x = \frac{\epsilon_u + \epsilon_l}{2} : \text{Strain due to in-plane loading}$$
Definition of coordinate systems for deformation computations

- Each fiber element
- Axial strain
- Rotation \( \theta(\bar{s}) \) and translation \( w(\bar{s}) \) in “Fiber coordinate”
  - Use linear assumption
  - \( \theta(\bar{s}) \): integrate curvature with respect to \( \bar{s} \)
  - \( w(\bar{s}) \): integrate \( \theta(\bar{s}) \) with respect to \( \bar{s} \)

- Effect of dihedral/anhedral and/or taper, \( \alpha \)
- Curved fiber element, \( \beta \)

![Diagram of coordinate systems](image)

Fig. 2

- (a) Fiber coordinate \((\bar{s}, \bar{z})\)
- (b) Local coordinate \((s, z)\)
- (c) Global coordinate \((x, y, z)\)
Mathematical Background of the Two-step Theory

**Compute curvature along sensor lines**

\[ \kappa(s) \equiv -\frac{(\epsilon_u - \epsilon_l)/2}{h/2} = -\frac{\epsilon_u(s) - \epsilon_l(s)}{h(s)} \]

**Step 1: Compute wing deflection along sensor lines**

\[ \frac{d^2w(s)}{ds^2} = \kappa(s) \]

\[ \frac{dw(s)}{ds} = w(s) \]

\[ \{q_M(t)\} = [T]\{q_M(t)\} \]

**Step 2: Expand wing deflection**

**Model independent**

\[ \text{Measure strain} \]

\[ \begin{cases} \epsilon_u \end{cases} \]

\[ \begin{cases} \epsilon_l \end{cases} \]

**Model dependent**

\[ \text{Distributed strain sensor} \]
Low-Boom Flight Demonstration aircraft
**Strain, curvature, & deformation**

- Trim load under Mach 1.42 flight condition.
- Differences at wing tip
  - Slope: -11.2%
  - Deflection: -19.8%
- Issue
  - Curvature definition
    - Looks fine
    \[ \kappa = -\frac{\epsilon_u - \epsilon_l}{h} \]
  - FE structural model
    - NASTRAN slope?

**LBFD aircraft using sensor lines 1 & 2 data**

**Fig. 4**

- (a) Strain on the upper and lower skin
- (b) Curvature
- (c) Slope in roll direction
- (d) Deflection
Deformation of LBFD aircraft integrated from 18 inch using sensor lines 1 & 2 data

- Don’t include wing root to 18 inch
- Differences at wing tip
  - Slope: -11.2% --> -2.02%
  - Deflection: -19.8% --> -4.97%
- Issue
  - Curvature definition
  - FE structural model
    - NASTRAN slope

Fig. 5

(a) Slope in roll direction

(b) Deflection
C607 Wing

C607 Model

- Stiffener
- Rib 1
- Sensor line 1
- Sensor line 2
- Sensor line 3
- Sensor line 4

- Dihedral
- Anhedral
Strain, curvature, & deformation

- **NASTRAN slope** near wing root area becomes better
  - Stiffening structure effect??
- Differences at wing tip
  - Slope: **-1.91%**
  - Deflection: **-6.63%**
- Issue
  - Curvature definition
    - Looks fine
  - FE structural model
    - **NASTRAN slope**

LBFD aircraft using sensor lines 3 & 4 data

Fig. 6
Tapered Wing
Tapered wing with coarse and fine meshes

- Aluminum
- Eight ribs
  - Every 50 inch
- Four spars
  - 4, 22, 40, 58 inches from LE of wing root section
- Root chord: 70 inch
- Tip chord: 35 inch
- Half span length = 400 inch
- Results are based on sensor lines 1 and 2
- External load: 1 G loading

Fig. 7

Wing tip
Front view
Wing root

Coarse mesh

Rib 1
Rib 2

Sensor line 1
Sensor line 2

Fine mesh

Rib 1
Rib 2
Strain, curvature, & deformation

- Differences at wing tip
  - Slope: **-0.019%**
  - Deflection: **-0.046%**

- Curvatures computed from the two-step theory and the MSC/NASTRAN code are excellent matching between root chord and the first two ribs.

- A fine FE mesh gives excellent results

Tapered wing with fine mesh

(a) Strain on the upper and lower skin

(b) Curvature

(c) Slope in roll direction

(d) Deflection
The upper and lower strains between span stations of 372.5 inch and 395.5 inch are both compressions.

Curvature definition, \( \kappa = -\frac{\epsilon_u - \epsilon_l}{h} \), is accurate.
Strain, curvature, & deformation

- Differences at wing tip
  - Slope: \(-0.036\%\)
  - Deflection: \(-0.066\%\)

- Curvatures obtained from the two-step theory and the MSC/NASTRAN code are good matching between root chord and the first two rib.
  - Rib effect??

- A coarse FE mesh also gives excellent results. (Why??)

**Tapered wing with coarse mesh**

**Fig. 9**

**a) Strain on the upper and lower skin**

**b) Curvature**

**c) Slope in roll direction**

**d) Deflection**
Tapered Wing with Dihedral/Anhedral
Dihedral/anhedral wing with coarse and fine meshes

- Same material and properties with the tapered wing
- **Wing has “Dihedral” & “Anhedral” effects.**
  - The LBFD aircraft wing also has dihedral and anhedral effects.
Strain, curvature, & deformation

- Differences at wing tip
  - Slope: -0.603%
  - Deflection: -0.779%
- Bigger difference than tapered wing case.
  - Strains near the rib location are not continuous.
  - Needs more fine mesh near rib location
- Curvatures from two-step theory and MSC/NASTRAN are excellent matching between root chord and the first two ribs.
- A fine FE mesh gives good results

Dihedral/anhedral wing with fine mesh
Strain, curvature, & deformation

- Differences at wing tip
  - Slope: -0.548%
  - Deflection: -0.794%
- Deflection difference is bigger than fine mesh.
- Curvatures from two-step theory and MSC/NASTRAN are good matching between root chord and the first two rib.
  - Rib effect??
- A coarse FE mesh also gives good results. (Why??)

Dihedral/anhedral wing with coarse mesh

Fig. 12
Comparison of strain and curvature results using coarse and fine meshes

- Strain values from the coarse mesh are close to the average values of the strain values obtained from the fine mesh.
- Therefore, curvature values computed from the coarse and fine meshes have similar behavior.
- However, the fine mesh is needed to have accurate curvature distribution.
- In general, deformation results obtained from the coarse mesh are good. (why??)

(a) Strain on the upper and lower skin

(b) Curvature from two-step theory
Tapered Wing with Dihedral/Anhedral and Wing Root Stiffener
Stiffened dihedral/anhedral wing with coarse, intermediate, and fine meshes

- Same material and properties with dihedral/anhedral wing
- **Investigate stiffener effect**
  - LBFD also has a stiffening structure near FOSS 1 & 2 root.
Strain, curvature, & deformation

- Differences at wing tip
  - Slope: -7.27%
  - Deflection: -10.2%
- Similar prediction error with LBFD case is obtained.
  - Mainly caused by curvature error near wing root area
- Curvatures from two-step theory and MSC/NASTRAN are not matching between root chord and the first two ribs.
  - Rib effect??

(a) Strain on the upper and lower skin

(b) Curvature

(c) Slope in roll direction

(d) Deflection

Stiffened dihedral/anhedral wing with coarse mesh

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Fig. 14
Strain, curvature, & deformation

- Differences at wing tip
  - Slope: -0.535%
  - Deflection: -0.694%
- A medium FE mesh gives good results.
- Curvature values computed from the medium mesh is similar to the NASTRAN results.
  - Rib effects exist

(a) Strain on the upper and lower skin

(b) Curvature

(c) Slope in roll direction

(d) Deflection

Stiffened dihedral/anhedral wing with intermediate mesh

MSC/NASTRAN: Two-step theory

Structural Dynamics Group
Strain, curvature, & deformation

- Differences at wing tip
  - Slope: \(-0.564\%\)
  - Deflection: \(-0.796\%\)
- A fine FE mesh gives good results.
- Curvatures from two-step theory and MSC/NASTRAN are excellent matching between root chord and the first two rib.
- The fine mesh is needed to have accurate curvature distribution
  - Numerical derivatives are used for the computation of curvatures from MSC/NASTRAN

Stiffened dihedral/anhedral wing with fine mesh

**Fig. 16**

- **(a) Strain on the upper and lower skin**
- **(b) Curvature**
- **(c) Slope in roll direction**
- **(d) Deflection**
Chord-wise deformation of Stiffened dihedral/anhedral wing

- Step 2 is the FE model dependent procedure.
- Expand measured master DOF to master and slave DOF
  \[
  \{q(t)\} = \begin{bmatrix} q_M(t) \\ q_S(t) \end{bmatrix} = \begin{bmatrix} \Phi_M(\Phi_M^T\Phi_M)^{-1}\Phi_M^T \\ \Phi_S(\Phi_S^T\Phi_S)^{-1}\Phi_S^T \end{bmatrix} \{\tilde{q}_M(t)\} \]
  Values are based on the fine mesh.

- DOF of \{\tilde{q}_M(t)\} = coarse mesh (51); intermediate mesh (972); & fine mesh (2403)
- The first six flexible mode shapes are selected as the basis functions.
- Eigen-matrices, \(\Phi_M\) & \(\Phi_S\), are computed based on the FE model with coarse or intermediate meshes. (computer speed and memory issue with fine mesh)

- DOF of coarse mesh = 1,356 \(\Phi_M(51 \times 6)\); \(\Phi_M^T(6 \times 6)\); \(\Phi_S(1305 \times 6)\)
- DOF of intermediate mesh = 21,192 \(\Phi_M(972 \times 6)\); \(\Phi_M^T(6 \times 6)\); \(\Phi_S(20220 \times 6)\)
- DOF of fine mesh = 2,240,442 \(\Phi_M(2403 \times 6)\); \(\Phi_M^T(6 \times 6)\); \(\Phi_S(2238039 \times 6)\)

Results are based on strains along the sensor lines 1 and 2.

Table 3. Deformation of stiffened dihedral/anhedral wing at wing-tip section

<table>
<thead>
<tr>
<th>Deformation</th>
<th>Target</th>
<th>Step 1 (fine mesh)</th>
<th>Step 2 (coarse mesh)</th>
<th>Step 2 (intermediate mesh)</th>
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<tr>
<td></td>
<td>Value</td>
<td>% difference</td>
<td>Value</td>
<td>% difference</td>
</tr>
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<td>1.601E-4</td>
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</tr>
</tbody>
</table>

Least squares surface fitting technique (SEREP)

Fig. 17

(a) At typical section

(b) At wing-tip section
Deformed shape of stiffened dihedral/anhedral wing after step 2

(a) Use coarse mesh for step 2

(b) Use intermediate mesh for step 2
Conclusion

A finite element structural model with **a fine mesh is desired** to have **accurate curvature distributions** during a pre-test analysis for the wing shape sensing of a wing with ribs and spars.

In case of a finite element (FE) model with **a regular rib configuration**, such as the tapered wing and the dihedral/anhedral wing in this study, even the FE models with **coarse mesh give acceptable** strain data and slope and deflection information.

- However, there’s no guarantee that the strain data obtained from the coarse mesh is acceptable.
- A FE model with a fine mesh may be needed to have accurate curvature distribution.
- A FE model with **a fine mesh is needed for the pre-test analysis of the LBFD aircraft**.

It is proved that **the two-step theory** used in this study **works excellent** for the wing shape sensing of the tapered wing, the dihedral/anhedral wing, and the stiffened dihedral/anhedral wing.

- The curvature equation based on the decomposition of the in-plane strain and pure bending strain was successfully applied to the wing with spars and ribs.
Questions?