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Wavenumber-Frequency Spectra of Pressure Fluctuations on a Generic Space Vehicle Measured via Unsteady Pressure-Sensitive Paint

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Abstract

Time histories of pressure fluctuations on a generic, “hammerhead” space vehicle model were measured using unsteady Pressure-Sensitive Paint (uPSP). The test was conducted in the 11-foot transonic wind tunnel of NASA Ames Research Center over a Mach number range of 0.6 ≤ M ≤ 1.2, and angles of attack of -4° ≤ α ≤ 4°. The model was coated with a porous binder and PtTFPP-based porous polymer paint. An elaborate system of four high-speed cameras, and forty LED lamps was used for image acquisition. Various steps for image registration, reduction of shot noise, photogrammetry procedure to map images from the four cameras on a grid for the model, and finally a calibration procedure to convert the measured fluctuations in light intensity to fluctuating pressure, are discussed in the paper. The calibration process using a set of unsteady pressure sensors mounted on the model, was found to overcome some of the inherent problems of the fast response paint, such as rapid photodegradation, non-linearity in pressure response, and significant temperature sensitivity. Comparison of spectra of pressure fluctuations between UPSP and pressure sensors demonstrated the ability of the paint to faithfully follow fluctuations up to 10 kHz, the maximum attempted. It was also found that the camera bit-depth and the illumination level limited the lowest measurable levels of pressure fluctuations to around 140dB. The large data set exposed various critical transonic flow physics not seen before, such as a coupling of the shock motion on the Payload Fairing (PF) with the separated flow region on the upper stage of the launch vehicle, and upstream convection of pressure fluctuation on PF at certain Mach numbers. The data also confirmed the expectation of a general lowering of the coefficient of pressure fluctuation with Mach number. The availability of the data set on a dense, regularly-spaced, surface grid allowed for the calculation of wavenumber-frequency (k-ω) spectra via straightforward applications of Fourier transform. The k-ω spectra were compared for the separated flow regions on the Second Stage, and the shock-boundary layer interactions on PF. The former showed self-similarity with Mach number while the latter was distinctly different, and confirmed the upstream propagation of pressure fluctuations. The k-ω spectra were dominated by the convected fluctuations; the acoustic domain was not discernable. These data, valuable for the vibro-acoustics analysis of aerospace vehicles, are believed to be the first obtained for the transonic flight regime, and pave the path for application on production models of aerospace vehicles.

I. INTRODUCTION

The subject of “flow induced sound and vibration,” also known as “aero-vibro-acoustics” in the aerospace community, finds a wide swath of applications in space vehicles, airplanes, automobiles and underwater vehicles. In all of these applications the exterior surface is subjected to turbulent pressure fluctuations from the flow of air or water, which is the “aero” part. The panel structure then vibrates and transmits the pressure fluctuations to the interior, which is the “vibro-acoustics” part. The transmitted pressure fluctuations are responsible for the interior acoustic environment of an automobile, the cabin of an aircraft and the payload and crew-compartment of a space vehicle. The panel vibration issue takes significance in higher Mach number flows, particularly experienced by space vehicles, which are subjected to large-amplitude pressure fluctuations from flow separation and shock oscillation. Determination of the vibro-acoustic environment requires knowledge of the external pressure fluctuations and the vibrational properties of the panel structure. For the former, one needs the space-time correlation of the pressure-fluctuations expressed as the wavenumber(frequency) spectrum: \( \Phi_p(k_x, k_y, \omega) \); subscripts x and y denote the axial and transverse component, respectively. For the panel structure one needs the material properties, such as mass distribution, stiffness, acoustic admittance and the spatial mode shapes denoted as \( S_{mn}(k_x, k_y) \). The coupling, expressed as a convolution between the two spectra, is the modal pressure response.
\[
\Phi_{pmn}(\omega) = \iint_{k_{x}k_{y}} \Phi_{p}(k_{x},k_{y},\omega) |S_{mn}(k_{x},k_{y})|^{2} \, dk_{x} \, dk_{y} \tag{1}
\]

The modal pressure response produces different results for different applications. In low-speed flows, structural resonances can filter out a very large part of the external pressure fluctuations, while in high Mach number application the reverse is true. Textbook description of the wavenumber-frequency spectrum for low velocity flows is made of two distinct components: the higher wavenumber, higher energy part arising from the convection of the turbulent eddies, and the lower energy, low wavenumber part due to the radiation of sound waves by such eddies. Measurements in low-speed boundary layers in a quiet wind tunnel have confirmed the presence of these peaks.\textsuperscript{3} For aerospace applications interest is in high-speed flows where measurements of the wavenumber-frequency spectrum of pressure fluctuations are usually unavailable. As the Mach number increases, the convective peak of the wavenumber spectrum is known to strongly drive the structural modes, such as in the aircraft cabins and the payload fairings of space vehicles.\textsuperscript{4,5,6} Typical flight data from space vehicles show the largest response in the transonic and low supersonic Mach number range. There is a distinct need to measure \( k \)-\( \omega \) spectra for the Mach number range of interest for aerospace vehicles.

The advancement of structural dynamics facilitates reliable prediction of the structural mode shapes; however, the same cannot be said for the spectrum of pressure fluctuations. Gross assumptions are involved in the specification of the pressure field, which ultimately are the major contributors to the uncertainties in the vibro-acoustics estimations. While computational fluid dynamics has made significant strides for some types of flows and configurations,\textsuperscript{7} the specification of wavenumber-frequency spectra for large aerospace structures has remained mostly outside its ability. It has been customary to perform model-scale wind-tunnel tests to obtain some measure of the pressure field. Typical wind-tunnel tests employ a few unsteady pressure transducers (whose numbers have increased over the years) to determine spectra of local fluctuations and correlation lengths, which are then employed to approximately model the \( k \)-\( \omega \) spectrum following various models, such as Corcos, Efimtsov, Ffowcs-Williams, Chase etc. (see Blake\textsuperscript{1}, Graham\textsuperscript{2}). A lack of direct measurement of the \( k \)-\( \omega \) spectrum means that these models remain unverified in transonic and supersonic flows.

To directly measure the \( k \)-\( \omega \) spectrum, one needs to measure pressure fluctuations over a very large number of spatial points. The goal of this work is to demonstrate that unsteady Pressure Sensitive Paint (uPSP) is capable of satisfying this data requirement. The uPSP effectively produces distributed “microphones” that can be painted on the surface of a test article. The “microphone density” is related to the pixel numbers of the camera used to measure the photo-luminescence. At the present time, uPSP is a rapidly devolving technology that requires independent validation. A second goal of the present report is to find suitable data analysis and calibration procedures, and finally to validate the uPSP data. The first part of the paper describes the data analysis and calibration procedures; the second part discusses the Mach dependent variations, and various flow features revealed by the large data set; and finally the \( k \)-\( \omega \) spectra is discussed in the third part. The present experiment followed a small-scale test in a smaller wind tunnel; experiences gained from that experiment\textsuperscript{9,10} proved valuable in the present large wind-tunnel setup.

### Ia. Unsteady Pressure Sensitive Paint

Standard PSP has become a matured experimental tool to measure time-averaged surface pressure distribution in model-scale wind-tunnel tests.\textsuperscript{11,12} There has been a steady push to increase the frequency sensitivity of the paint and the binder combination to measure unsteady fluctuations. Gregory et al.\textsuperscript{13} provides a good review of the work. For the present work a PtTFPP-based porous polymer-ceramic paint manufactured by Innovative Scientific Solutions, Inc. (ISSI) was used.\textsuperscript{14,15} Adding porosity to the binder creates a microscopically pitted surface onto which the luminescent molecules can be applied. This allows oxygen to reach the luminescent molecules much faster, resulting in a wider frequency response. In addition, the effective surface area of a porous binder is much larger than its non-porous counterpart, resulting in higher radiative intensity. In previous studies ISSI has shown the frequency response of this paint to be >20 kHz.\textsuperscript{14,15} While uPSP comes with many promising applications, there arise multiple problems with quantitative measurements that need to be acknowledged:
(i) Fundamentally every pressure-sensitive paint is also temperature sensitive. For time-averaged applications many decades of development led to newer paints with very little temperature sensitivity. For example, a binary paint mixture\(^1\) reduced temperature dependence of the PtTFPP/FIB to 0.06% per \(^\circ\)C. Unfortunately, such paints have long time-constants, making them unsuitable for unsteady applications. On the other hand, the highly-porous paint-binder combination, used in the present study, has much faster response to pressure change, and simultaneously has high temperature sensitivity. The PtTFPP based porous paint\(^{13,17}\) has temperature sensitivity between 1.4%/ \(^\circ\)C to 3%/ \(^\circ\)C. The paint is typically applied on conductive surfaces to reduce temperature gradient, still some amount of temperature non-uniformity is expected for models subjected to high Mach number flows.

(ii) The calibration constants A, B used in the Stern–Volmer formula (which relates the luminescence intensity \(I\) to pressure \(P\), via a reference condition\(^{13}\)) for paints selected for typical steady-state applications are independent of pressure:

\[
\frac{I_{\text{ref}}}{I} = A + B \frac{P}{P_{\text{ref}}} \tag{2}
\]

That is not the case for highly-porous, uPSP. Gregory et al\(^{13}\) writes: “for porous PSPs such as anodized aluminum, however, the linear Stern–Volmer expression is only applicable to higher pressures about ambient. As conditions approach vacuum the pressure sensitivity increases for porous paints, .... This is because the open structure of the paint allows the luminophore molecules to be mostly quenched at the higher pressures, thus decreasing the sensitivity of the PSP at conditions near atmosphere.” An examination of a sample, vendor-provided, calibration for the present paint shows that the gradient of the calibration curves \((P/P_{\text{ref}}\text{ vs } I_{\text{ref}}/I)\) decreases with increasing static pressure. In other words, B decreases with an increase in static pressure and vice-versa; thereby, in effect, making the calibration constants as functions of pressure (and temperature \(T\)):

\[
\frac{I_{\text{ref}}}{I} = A(P,T) + B(P,T) \frac{P}{P_{\text{ref}}} \tag{3}
\]

(iii) The photo-degradation of the thin layer of the porous paint is much faster than its steady-state counterpart (1%/min). Any set of pre-determined calibration values becomes obsolete after half an hour of testing.

All of the above factors make the traditional path of determining calibration constants from a coupon test, and then applying uniformly over the test article difficult. One not only needs to perform an extensive set of calibrations to determine the pressure and temperature dependence, but to apply such calibration curves one needs to know the temperature and static pressure distribution over the test article. Even after all that effort, photo-degradation will introduce an increasing bias error.

For the present work, an in-situ, multiple-point calibration procedure was used, where a limited number of unsteady pressure transducers were distributed over the test articles and the local value of the calibration constants were determined. Note that the Stern–Volmer constant A and B are not independent; therefore, determination of one constant B is adequate to the measurement of pressure:

\[
A + B = 1 \tag{4}
\]

The procedure used for calibration was different from that used for traditional steady-state PSP. To overcome the multiple challenges associated with uPSP, no attempt was made to use the traditional calibration procedure of absolute pressure vs. absolute intensity calibration from a coupon test. Instead the rms values of the intensity fluctuations were equated with the rms of pressure fluctuations measured by the individual unsteady pressure sensors. In effect, these local calibration constants accounted for the local static pressure, static temperature, and the paint degradation. A nearest neighbor based interpolation scheme was used to assign calibration constants to individual pixels. Application of this calibration constant to the fluctuating part of the luminescent intensity produced the desired spatio-temporal distribution of pressure fluctuations. This calibration technique was evaluated during a pilot study performed in a small wind tunnel to gain experience with the uPSP technique.\(^9,10\)

II. EXPERIMENTAL PROCEDURE
The present test was conducted in the 11-foot Unitary Plan, transonic wind tunnel of NASA Ames Research Center over a Mach number range of $0.6 \leq M \leq 1.2$, and angle of attack range of $-4^\circ \leq \alpha \leq 4^\circ$. In addition to uPSP, the launch vehicle model was instrumented with unsteady pressure sensors, accelerometers, strain gauges, and a variety of other instrumentations. A detailed description of the experiment, and all instrumentations can be found in Schuster, et al\textsuperscript{16}. The transonic wind tunnel has a test section of dimension 11-ft (high) X 11-ft (wide) X 22-ft (length) and is capable of simulating static pressure condition at different altitudes (variable density/Reynolds number). The test section walls have lengthwise slots for shock cancellation and for applying wall suction. A particularly desirable feature of the wind tunnel is the optical access on all four walls via segmented windows that are separated by the wall slots. The 40 LED lamps and the four high speed cameras were distributed on the four walls and were placed on various segmented windows (Fig 1).

![Fig 1. Photograph of the generic space vehicle in the test section of 11-foot transonic Wind Tunnel. Flow is from right to left.](image)

**Fig. 2** Schematic of the model.

**IIa. Space Vehicle Model**

The axisymmetric, protuberance-free, hammerhead-shaped, generic model (Fig 2) embodies many of the unsteady aerodynamic issues faced by all space vehicles during transonic and supersonic flights through the thick part of the atmosphere. The model is an exact replica of that of Coe and Nute\textsuperscript{17}. The general hammerhead shape is typical of a large number of current and historical, US and international, launch vehicles, where the larger size of the satellite, telescope or other payloads requires a fairing larger than the diameter of the upper-most stage of the launcher. The Payload Fairing (PF) is needed to protect the cargo from the aerodynamic forces during ascent through the atmosphere; it’s jettisoned once the upper edge of the atmosphere is reached. This geometry was originally investigated by Coe and Nute using schlieren photography, and a row of steady and unsteady pressure transducers. Various dimensions, such as the ratio of the diameters of the PF to that of the Second Stage, the length of the Second Stage to the diameter of the PF, follows the general recommendation of the NASA handbook for a buffet-free configuration.\textsuperscript{18} Various fluid dynamic phenomena, such as flow separation, transonic shock
formation, and unsteady force generation, experienced by this model have attracted interest in the space vehicle community over the past half a century. Recently the configuration was used to validate computational fluid dynamic results.\textsuperscript{18} Only a limited number of pressure measurements were made in the original study by Coe and Nute, which motivated the extensive survey undertaken in the present effort. Two regions of the model are of particular interest: the larger part of the Payload Fairing including the Frustum, and the upper part of the Second Stage marked as "Metric." Attempts were made to measure the fluctuating forces on the Metric part via multiple sets of instrumentations.

The instrumentation sets of particular interest for the present paper are the unsteady pressure sensors and pressure sensitive paint. The model was instrumented with 216 flush-mounted, 0.072 in diameter Kulite\textsuperscript{TM} sensors, 13 of which were found to be unusable. The differential sensors were a mix of 5psi and 15psi range. The reference tubes were connected to a common plenum, and the amplifier outputs were AC-coupled. Output from each sensor was digitized using a 24-bit converter at a sampling rate of 100,000/s, and for some cases 10,000/s.

Fig 3. Locations of the LED lights and the high-speed cameras on (a) ceiling, and (b) south wall of the test section. Similar arrangements were made on the floor and the north wall.

Fig 4. Model co-ordinates and the location of the cameras; the axial x-direction is downstream, along the centerline, and the azimuthal direction θ is clockwise (cardinal angles marked).

Fig 5. A sample uPSP image from one of the cameras (no 3).

Iib. uPSP application and hardware
The model was painted with uPSP (Turbo FIP™ manufactured by ISSI). Two layers of paint were applied, the base layer, made of proprietary FIB polymer, was applied during the preparatory phase of the model; it contained a porous polymer-ceramic binder. The second, active layer, containing the luminescent molecule Platinum Tera Pentfluorophenyl Porphine (PtTFPP), was applied just prior to the test. For the uPSP application “Intensity-based” mode of operation is preferred, where the model is continuously illuminated and high-speed cameras acquire a continuous sequence of images over a long time duration. The paint was excited by continuous light at a nominal 400nm wavelength, produced from 40 Light Emitting Diode (LED) units (Fig 3), supplied by ISSI. The luminescent light emitted by the PSP was measured with four Phantom™ high-speed cameras (Fig 4), equipped with a 1280 x 800 pixel, 12-bit resolution CMOS chip. In an effort to reduce the data volume, the images were compressed and stored with a 10-bit resolution, which in retrospect, should have been avoided. A total of 62,000 images were recorded at frame rate of 5,000/s. A few sets of data points were acquired at a higher frame rate of 10,000/s and 20,000/s. The collected light was at first filtered by an optical band-pass filter in 570nm to 700nm band before imaging by the camera. The intensity of light produced by the lamp showed some oscillation just after powering up. Therefore, a delay generator was used to turn the lamps on a few seconds before starting the image acquisition. The light intensity fell slightly over the duration of image collection. This drop in the intensity had to be accounted for in the image data processing. No attempt was made to time synchronize the PSP and pressure sensor data, although IRIG signals were recorded by both data acquisition systems. A second set of cameras was used to capture time-averaged PSP data. Such data will not be presented in this paper. Fig. 5 is a sample camera image. The dark spots are from the unsteady pressure sensors, and registration marks.

IIc. uPSP data reduction and calibration procedure:

The camera images provide time history of light intensity distribution over the test article, I(X, Y, t). Images were taken at several “wind-off” static pressure conditions before and after the air flow. An average of a set of images was used as the reference image for the present data analysis. As the test section was brought to the desired Mach number, and static pressure conditions the static temperature fell. To minimize the impact of temperature non-uniformity image acquisition was delayed for several minutes to reach a steady surface temperature. The high-speed cameras produced high volume of data: each camera produced 50GB to 60GB of images for each test condition. After each test point data had to be transferred from the camera buffer to external computers over high-speed Ethernet connections. Processing such high volume of data posed several challenges on the computing resources, which needed to be sorted out. Data presented in the present paper was analyzed in a fast, multi-core PC based server, which typically took several days for each data point. However, “quick-look” data to check the image quality could be done as soon as the images were transferred from the camera.

Step 1: Image registration

To reduce model motion, a stiff sting was used to hold the model via the facility strut. Nevertheless, the cantilever model showed small vibration. The cameras were mounted rigidly on the outside wall of the test section. Nonetheless slight relative motion between the two was expected. Registering 62000 images for each of the four cameras (total images to be registered was 248000 for every measurement point) was found to be the most computationally intensive part of the data analysis effort. The Matlab™ routine for the intensity based registration “imregister” was used towards that end. Corrections were limited to image translation only. Subpixel level corrections were found to be necessary to account for the model motion.

Step 2. Patching over registration marks and sensor locations:

Prior to the application of PSP the model was instrumented with flush-mounted unsteady pressure sensors, whose sensing end was covered by small adhesive dots. The dark spots from these dots, and the larger registration marks (used by other analysts) had to patched by the values from the neighboring pixels. Slight smearing of the paint underneath the adhesive dots made identification of the outer boundary of each sensor somewhat tricky. Also small difference between the wind-off and wind-on images created sufficient changes in \( l_{ref} / l \) to create small local spikes in \( C_{prms} \) data presented later in the text. Nearly 4% of the pixels had to be patched over to create smooth variation in the \( C_{rms} \) distribution, still a few number of spurious, single point, spikes were found to be unavoidable, especially since spatial filtering was not used.
Step 3: Noise floor reduction via averaging over increasing number of pixel

Shot noise is an unavoidable reality in all measurements of light intensity. For time-averaged PSP measurements the impact of the shot noise is minimized by averaging over a large number of images. Such a path, however, is unavailable for unsteady applications. In unsteady measurement shot noise creates a noise floor in the spectra and limits the lowest resolvable amplitude of pressure fluctuations. The intensity fluctuations measured by each pixel is due to a sum of the luminescent fluctuations associated with unsteady pressure $\sigma_{i,p}$ and fluctuations associated with shot noise $\sigma_{\text{shot}}$ (and other secondary sources, such as read, thermal, 1/f etc). The variance of the measured intensity can be expressed as the following:

$$\sigma_I^2 = \sigma_{\text{shot}}^2 + \sigma_{i,p}^2$$  \hspace{1cm} (5)

This relationship is independent of the number of camera frames, as long as a sufficient number of frames is used to obtain converged statistics. Since the variance of the shot noise is equal to the total intensity $I$, while that due to the pressure fluctuations is proportional to $I^2$, the signal-to-noise ratio (SNR) in the in uPSP data can be expressed as the following:

$$\text{SNR} = \frac{\sigma_{i,p}}{\sigma_{\text{shot}}} \sim \sqrt{I}.$$  \hspace{1cm} (6)

In other words, SNR can be increased by increasing the total intensity $I$ measured by each pixel. This can be achieved by increasing lamp brightness, using faster optics, using a higher quantum efficiency camera, or by increasing the paint luminescence (by conducting the experiment at lower static pressure). For a given optical setup where all of the above values are fixed, the available path is to add (or more commonly average) intensity levels from adjacent pixels to create super-pixels, thereby increasing the total effective intensity. Note that this path leads to sacrificing spatial resolution to achieve higher SNR.

Figure 6 shows the reduction in the shot noise when adjacent pixels are averaged to create “super-pixels.” Data from a wind-on test condition (M 1.1, camera 3) was used for this study. The standard deviation values were calculated from the time history of light intensity (counts) measured by each super-pixel. A comparison of such plot for wind-off and wind-on conditions can be found in Panda. For the wind-on condition of Fig 6 the averaging over adjacent pixels diminished the shot-noise contribution and made the standard deviation to approach the true value, i.e., $\sigma_i \rightarrow \sigma_{i,p}$. Beside the loss of resolution another important impact of increasing the size of super-pixels is an expansion of spatial filtering of high-frequency fluctuations. The lowering of the standard deviation even beyond the 9-pixel averaging (Fig. 6) may be attributed to the increased filtering. For the present work, a factor of three improvements in the signal to noise obtained via averaging 3x3 original pixels was deemed to be the right balance between increasing SNR, and decreasing spatial resolution. It is to be noted that non-overlapping blocks of 3x3 pixels in the original image had to be used, as much as possible, to create each super-pixel. Use of overlapped blocks of pixels would have artificially increased the spatial correlation among adjacent grid points. There is an important difference between processing data for steady-state pressure and fluctuating pressure. Filtering simply smears sharp gradient in steady-state data, while for the unsteady measurements important information on spatial correlation becomes corrupted.
Step 4: Selection of the wire-mesh grid of the model:

Intensity levels from each of the super-pixels, created by averaging 3x3 original pixels, were assigned to a grid point (Fig. 7). The progressively decreasing azimuthal resolution from the camera normal direction created small overlap for small number of grid points. Therefore, only the center strip of images, covering ±45° from the camera normal direction was used to project to the grid. Figure 8 shows the grid used for the present work. Note that the grid excluded a part of the nose, the aft part of the Second stage, and the Booster section where the intensity of the luminescent light was found to be too dim to provide good measurements. The names and locations of the unsteady pressure sensors are annotated on the grid. The need to avoid overlapped block of pixels, as much as possible, ultimately dictated the grid size of 300 points in the axial x-direction, and 240 points in the circumferential θ direction. In the following discussion subscript i denotes grid indices along the axial (x) direction, j denotes the indices in the azimuthal (θ) direction, and k denotes the frame number, i.e., time dimension.

Step 5. Photogrammetry: Mapping images from four cameras to a wire-mesh grid:

Once the desired grid was created, the next step was to identify which pixel in which camera corresponded to the individual grid points. This required determination of the photogrammetric constants for each camera. When a camera creates a two-dimensional image of a three-dimensional object, the pixel co-ordinates (X, Y) in the image plane are related to the physical co-ordinates (x, y, z), via a variety of parameters such as the camera location, camera pointing angles, principal distances and the lens distortion factors.20, 21, 22 The Direct Linear Transformation (DLT), developed by Abdel-Aziz and Karara22, establishes a linear relation between the above two co-ordinates via 11 calibration constants L₁ through L₁₁ that account for all imaging parameters. Neglecting the
effect of lens distortion (the high quality lens used with the cameras precludes this need), the DLT equations are
the following:

\[ \begin{align*}
X &= \frac{L_1 x + L_2 y + L_3 z + L_4}{L_9 x + L_{10} y + L_{11} z + 1} \\
Y &= \frac{L_5 x + L_6 y + L_7 z + L_8}{L_9 x + L_{10} y + L_{11} z + 1}
\end{align*} \]

The coefficients \( L_1 \) through \( L_{11} \) can be determined from location of the known \( n \) no of marks on the model whose
physical coordinates are known, and can be clearly identified in the pixel co-ordinates:

\[ \begin{bmatrix}
x_1 & y_1 & z_1 & 1 & 0 & 0 & 0 & 0 & x_1 x_1 & x_1 y_1 & x_1 z_1 \\
x_2 & y_2 & z_2 & 1 & 0 & 0 & 0 & 0 & x_2 x_2 & x_2 y_2 & x_2 z_2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_n & x_n & x_n & 1 & 0 & 0 & 0 & 0 & x_n x_n & x_n y_n & x_n z_n \\
0 & 0 & 0 & 0 & x_1 & y_1 & z_1 & 1 & Y_1 x_1 & Y_1 y_1 & Y_1 z_1 \\
0 & 0 & 0 & 0 & x_2 & y_2 & z_2 & 1 & Y_2 x_2 & Y_2 y_2 & Y_2 z_2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_n & x_n & x_n & 1 & 0 & 0 & 0 & 0 & x_n x_n & x_n y_n & x_n z_n
\end{bmatrix} \begin{bmatrix}
L_1 \\
L_2 \\
L_3 \\
L_4 \\
L_5 \\
L_6 \\
L_7 \\
L_8 \\
L_9 \\
L_{10} \\
L_{11}
\end{bmatrix} = \begin{bmatrix}
-X_1 \\
-X_2 \\
-L_3 \\
-X_4 \\
-Y_1 \\
-Y_2 \\
-Y_3 \\
-Y_4 \\
-Y_5 \\
-Y_6 \\
-Y_7
\end{bmatrix} \]

Since each calibration point contribute two equations to the above set, at least six targets are required to solve
for the eleven coefficients. Towards that end, pixels corresponding to the centers of six unsteady pressure sensors
were identified from each camera. Since the \((x, y, z)\) locations of these sensors on the grid were known, the above
equations could be solved in a least-square sense by singular value decomposition. If the above equation is
written in the following compact form:

\[ \mathbf{AL} = \mathbf{p}, \]

and the inverse of \( \mathbf{A} \) is calculated via singular value decomposition (Matlab™ routine svd was used for this purpose)
as the following:

\[ \mathbf{A}^{-1} = \mathbf{V} \left[ \text{diag}(\sigma_1^{-1}, \sigma_2^{-1}, ..., \sigma_{11}^{-1}) \right] \mathbf{U}^T, \text{ where } \sigma_1, ..., \sigma_{11} \text{ are the singular values of } \mathbf{A}, \]

then the least-square solution of \( \mathbf{L} \) is obtained by avoiding inverses of zero singular values: for each singular
value for which \( \sigma_i = 0 \), \( \sigma_i^{-1} = 0 \). The least-square solution is the following.

\[ \mathbf{L} = \mathbf{V} \left[ \text{diag}(\sigma_1^{-1}, \sigma_2^{-1}, ..., \sigma_{11}^{-1}) \right] \left( \mathbf{U}^T \mathbf{p} \right) \]

The vector \( \mathbf{L} \) with 11 calibration constants can be determined from wind-off images prior to the test, as soon as
the camera positions are fixed. Such constants were calculated for each attitude angles, and for all four cameras.
Applications of these photogrammetric constants via equation 7 above created a composite image by stitching
strips from the individual cameras. Figure 9 shows one such stitched, unwrapped image. The figure shows non-
uniformity of the luminescent light due to streaks in the paint, non-uniform illumination, sharp changes in the
model cross-section, and from various joints on the model surface. Such non-uniformities were mostly accounted
for via the normalization procedure described below. Images from the four cameras for each of the 62000 frames
in a given test point had to be stitched to create time histories at every grid point. The grid points were specified
in both Cartesian and Polar coordinates. The latter is used for the subsequent discussion: \( I(x_i, \theta_i, t_k) \).

Step 6. Normalization and calibration:

The first two necessary corrections to the light intensity distribution \( I(x_i, \theta_i, t_k) \) were to account for the spatial
non-uniformity of the paint, and the temporal variation in lamp intensity. The former was accomplished via a
wind-off Image, \( I_{\text{wind-off}} \).

\[ \frac{I_{\text{ref}}}{I} = \frac{I_{\text{wind-off}}}{I} \]

A 6th order polynomial was fitted to the data and the smooth function was then applied to every video frame to
adjust for the time-varying illumination. Finally, the time history from each pixel was de-trended to determine the
fluctuating part of the normalized intensity:

\[ \left( \frac{I_{\text{ref}}}{I} \right)^{\text{fluct}} = \frac{I_{\text{ref}}}{I} - \left( \frac{I_{\text{ref}}}{I} \right)^{\text{trend}} \]

In the next step, the normalized intensity was converted into engineering units (Pascal) via a calibration process.
The intensity of the luminescent light is known to be related to static pressure by the Stern-Volmer formula.
The present interest is in the dynamic, fluctuating part of the pressure which is much smaller than the absolute pressure. Therefore, applying Reynolds decomposition:

\[ P = \bar{P} + p \]  

(also by noting the following:

\[ \left( \frac{I_{\text{ref}}}{I} \right) = A + B \frac{p}{P_{\text{ref}}} \]  

the following simple relationship between the fluctuating (normalized) intensity and the fluctuating pressure is reached:

\[ p = \frac{1}{B} P_{\text{ref}} \left( \frac{I_{\text{ref}}}{I} \right)' \].

The calibration factor could be easily determined by equating rms (standard deviation) values:

\[ c = \frac{1}{B} = \frac{p_{\text{rms}} / P_{\text{ref}}}{I_{\text{ref}} / I_{\text{rms}}} \].

Specifically, grid points next to each unsteady pressure sensor were identified, and the rms of the fluctuations in the frequency interval of \(100\text{Hz} \leq f \leq 1\text{kHz}\) were equated to find the calibration factors (Fig. 10). The lower frequency limit was selected to avoid occasional discrepancies due to the model vibration, and the higher limit was selected to avoid aliased range, seen in some low frame rate uPSP data. The calibration factors were determined for each of the test points, M-\(\alpha\) condition, which accounted for the photo-degradation of the paint. There were 187 unsteady pressure sensors over the PSP gridded region, which provided the same number of calibration factors and accounted for the non-uniform sensitivity of the paint due to variations of the local temperature and static pressure. Therefore, all three concerns with the applications of uPSP, mentioned earlier in section 1a, were addressed via this calibration procedure. To account for the spatial variations in the calibration factors a “nearest-neighbor” based approach was followed, where the calibration factor for a particular grid point was selected from the nearest unsteady pressure sensor. Application of the calibration factors leads to the desired space-time distribution of the pressure fluctuations: \(p(x, \theta, t)\). The instantaneous pressure values, for some plots shown in this paper, were normalized to coefficient of fluctuating pressure via free-stream dynamic pressure \(q\): \(C_p = p/q\). Also, the standard deviation of this coefficient is expressed as \(C_{p\text{rms}} = p_{\text{rms}}/q\).

![Image](image.png)

**Fig 10.** Typical distribution of the calibration factor on model.

Figure 10 show significant variations in the calibration factors. The large sensitivity of the paint for local temperature and pressure changes associated with shock waves and local separation/reattachments have contributed to the variations. Additionally, it is believed that the different contribution of shot noise to \((I_{\text{ref}}/I)_{\text{rms}}\) in equation 17 above has played a significant role:

\[ \frac{I_{\text{ref}}}{I_{\text{rms}}} = \frac{I_{\text{ref}}}{\sigma_I} = \frac{I_{\text{ref}}}{\sqrt{\sigma_{\text{shot}}^2 + \sigma_{I,p}^2}} \]  

In the quiet part of the flow where \(p_{\text{rms}}\) decreases, luminescent fluctuations associated with unsteady pressure \(\sigma_{ip}\) also decreases, while that due to shot noise remains constant. This makes \((I_{\text{ref}}/I)_{\text{rms}}\) approach a constant level of \((I_{\text{ref}}/\sigma_{\text{shot}})\), which in turn reduces the value of the calibration factor \(c\) (equation 17). In the limiting case of laminar flow where \(p_{\text{rms}} \to 0\), the calibration constant \(c \to 0\). Note that shot noise can never be eliminated from an optical measurement; the averaging over 3X3 pixels merely reduced its impact.
A fundamental question is what is the lowest amplitude of pressure fluctuations that can be measured from a given paint and optical setup? The answer depends on two factors: electronic shot noise, and the resolution of the camera employed. Depending on the setup, either of them may be responsible for the noise floor. The measurement floor can be lowered by increasing the camera bit resolution and by lowering the relative shot noise contribution. The calibration factor embodies both of these influences and can be used as a marker to segregate useable and non-useable regions. This will be discussed in the next section.

III. RESULTS AND DISCUSSION

The central flow feature of the hammerhead model is a flow separation at the Frustum part of the Payload Fairing. The separated shear layer reattaches some distance downstream, on the Second Stage (middle of the Metric part) creating large fluctuations in local static pressure (Fig 11). In the transonic Mach range of $0.8 \leq M \leq 0.95$ various shock-waves are developed on the PF. Such shocks go away when the free-stream flow is supersonic.

![Fig. 11](a) location of a few selected sensors which will be used to compare spectral response, (b) shadowgraph showing local shock waves and the separated shear layer, (c) comparison of $C_{prms}$ distribution along $\theta = 0^\circ$ (M= 1.1, $\alpha=4^\circ$).

IIIa. Validation of uPSP measured spectra:

Fig 11(c) shows good comparison in $C_{prms}$ levels between uPSP and transducers; however, an examination of the power spectra of pressure fluctuations provides a mixed story. Figs 12 and 13 show comparisons of the spectra measured by unsteady pressure sensors and by uPSP at grid points adjacent to the sensor location. The sensors chosen to show this comparison are marked in Fig 11(a). In general, it can be said that the spectra measured by the uPSP closely replicated those measured by pressure sensors. Data from the higher camera frame rate allowed frequency resolution up to 10 kHz. Figure 11 shows close agreement in the uPSP-measured and sensor-measured spectra over the entire frequency band. The differences, can be attributed to multiple causes:

(i) *Floor created by shot noise and camera bit-depth*: This floor manifested in regions where pressure fluctuations were small. It’s difficult to assign a fixed value. Nominally when the overall fluctuations level fall below $\sim 140$dB the calibration process tried to determine a calibration constant mostly based on the noise floor and the resulting comparison became poor (Fig 12a, 13a).

(ii) *Elevated low-frequency (<150Hz) noise floor*: Figs 12(a) and 13(a) show excessive energy in the uPSP spectra, which may be attributed to small relative motion between the model and the cameras. Once again, this floor manifested in the quieter part of the model.
(iii) *Aliasing at the high-frequency end:* The question of aliasing from the unresolved, higher frequency fluctuations needs to be addressed. The pressure sensor data were sampled at twice the camera rate, with anti-aliasing filtering, and was free from aliasing. No such anti-aliasing process could be used for the uPSP images. For example, for the 5000/s frame rate imaging, the camera shutter was kept opened for nearly the entire 200 micro-s duration of each frame, followed by a fast readout, and a new exposure. This photon accumulation process created an integrate-and-dump filter. Panda and Seasholtz\textsuperscript{20} showed that such a process creates a low pass filter with a slow roll-off at the Nyquist frequency and sharp drops around frequencies which are integer multiples of the frame rate. A primary feature of all spectra measured by the unsteady pressure sensor is the hump around 2900 Hz that was attributed to the tunnel background, not a feature of the flow over the model. The higher frame rate data in Fig 12 mostly resolved this peak while the low frame rate data suffered from some amount of aliasing. The naturally present low-pass filtering of the photon-accumulation process reduced the extent of aliasing; however, when the spectral energy was substantial in the unresolved high-frequency part (Fig 13b) aliasing became noticeably higher. Like every other transonic wind tunnel the level of free-stream pressure fluctuations in the 11-foot tunnel are relatively high and are marked by specific peak frequencies that can be identified in empty test-section runs. To reduce shock reflection and excessive blockage, the walls of test section have longitudinal slots which are the source of these peaks. In the quiet part of the model the background hump is clearly identifiable, while in the energetic part of the model (K13, K14, K16) the humps are dwarfed by the broadband spectra.

![Figure 12](image)

**Fig 12.** Comparison of power-spectra of pressure fluctuations measured by uPSP and unsteady pressure sensors; camera frame rate: 20000/s. The sensor locations are indicated in Fig 11(a) above. $M = 1.1$, $\alpha = 4^\circ$. 
Fig 13. Comparison of power-spectra of pressure fluctuations measured by uPSP and unsteady pressure sensors; camera frame rate: 5000/s. The sensor locations are indicated in Fig 11(a) above. \( M = 0.8, \alpha = 0^\circ \).

Returning to the topic of the impact of shot noise on the calibration factor, Table I shows calibration factors calculated from each plot of Figs 12 and 13. An examination of the Table shows clear association of the noise floor with a low value of the calibration constant as predicted earlier. It can be said that for the present optical setup when the calibration factor fell below 0.7, uPSP data was dominated by noise and was mostly unusable. The corresponding overall fluctuation level was around 140dB. Regions of the model where the factor was lower than this threshold were not used for further analysis. Fig. 10 shows that the PF mostly fell in this category except at the Mach numbers when shock-waves were present.

Table I. Variation of the uPSP calibration factor with local overall fluctuating pressure level (OAFPL) measured by indicated sensor. Spectral data shown in the indicated figure numbers.

<table>
<thead>
<tr>
<th>Kulite Name</th>
<th>Fig. 12</th>
<th>Fig 13</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OAFPL(uPSP)</td>
<td>Calib const, c</td>
</tr>
<tr>
<td>K04-01</td>
<td>132.6</td>
<td>0.2865</td>
</tr>
<tr>
<td>K09-01</td>
<td>144.1</td>
<td>0.9166</td>
</tr>
<tr>
<td>K11-01</td>
<td>145.9</td>
<td>0.9363</td>
</tr>
<tr>
<td>K13-01</td>
<td>154.4</td>
<td>1.1568</td>
</tr>
<tr>
<td>K14-01</td>
<td>156.9</td>
<td>1.3807</td>
</tr>
<tr>
<td>K16-01</td>
<td>152.7</td>
<td>1.3464</td>
</tr>
</tbody>
</table>

To investigate the phase and amplitude response of the paint itself, two sensors and the adjacent PSP grid points were located from the middle of the metric section where fluctuations levels were high. The coherence and the phase of the fluctuations were calculated via cross-spectral analysis and are shown in Fig 14. The excellent match between uPSP and the pressure sensor generated data shows that the paint was indeed responsive to the pressure fluctuations up to the maximum tested frequency of 10 kHz. An examination of the power-spectra of Fig. 14(c) shows a slight deviation in the amplitude response of the paint for \( f > 3 \) kHz. At the low frequency end \( f < 150 \) Hz the uPSP-derived power-spectrum shows increased level, which is ascribed to small registration error. It needs to be pointed out that such coherence and phase plots measured from the quieter part of the model (overall level
<140 dB) show far poorer comparison, which once again underlines the unreliability of uPSP data from such regions.

Fig 14. Comparison of (a) cross-spectral coherence, (b) cross-spectral phase measured by a pair of unsteady pressure sensors and corresponding PSP grid points; (c) auto-spectra from one of the sensors and corresponding grid point; camera frame rate: 20000/s. The sensor pairs, and the PSP grid points were located at the middle of the Metric section. M = 1.1, α = 0°.

IIIb. Comparative study of Mach dependent flow features

The large PSP dataset for each of the M-α point allowed for interesting comparative studies. To set the stage a set of shadowgraph images is shown in Fig 15. The uPSP-generated data sets were used to calculate $C_{prms}$ and are shown in Figs. 16 and 17. At the lowest $M=0.6$ no shock waves are formed. Trip dots were applied near the nose of the model to ensure turbulent boundary layer flow over the Payload Fairing. The sharp boat tail angle at the Frustum pinned the separated shear layer which impinged on the Second Stage creating high levels of pressure fluctuations. As the free-stream Mach number was increased to 0.8, local supersonic zones set in at the two locations where area increases on the PF, each terminated by a normal shock. The shock-boundary layer interaction leads to the large spikes in pressure fluctuations seen in Figs 16 and 17. An examination of the $C_{prms}$ plots shows that at $M=0.8$ the second shock wave created a higher level of pressure fluctuation than its predecessor. An advantage of the uPSP data is that the very high spatial resolution allowed capturing of the local maxima which the sensor data had missed. Note that in the prior test on a similar model, Coe and Nute\textsuperscript{17} made painstaking effort in operating the wind tunnel in small increments to allow their transducers to measure the local maxima. Unsteady PSP obviates such operations, and makes peak-capturing far easier. The same is true with other local peaks seen in Fig 16 and 17. A small increase in the free-stream Mach number to $M=0.85$ caused the supersonic bubble to grow and pushed the normal shock downstream on the Fairing. At $M = 0.92$, supersonic flow covers almost the entire PF except for the Frustum; the normal shocks coalesced into one and stood at the upstream end of this neck-down.

The central flow feature of a separated shear layer from the top of the Payload Fairing impinging on the Upper Stage is seen in the shadowgraph images at all Mach numbers (Fig 15). A closer look into the $C_{prms}$ distribution shows a couple of interesting features. First, the peak $C_{prms}$ captured by the uPSP is progressively lowered with increasing Mach number (Fig 16); this is attributed to the compressibility effect which lowers the turbulent fluctuations with increasing $M$. The second is the location of the peak fluctuations on the Upper Stage. The peak moved downstream until $M=0.92$ and then shifted back upstream in $M = 1.03$ and 1.1. An examination of the shadowgraph photographs, particularly Fig 15(d), revealed the cause of this movement. The local shock waves that formed on the top of the Fairing lifted the separating shear layer on the Frustum. The lift is the most prominent at $M = 0.92$. Such a lifting of the shear layer increased the extent of the separated flow and moved the reattachment point downstream. Since the location of the peak fluctuations is associated with the reattachment point, the maxima in fluctuations are also pushed downstream. When the local shocks disappeared with an increase in Mach number, so did the upward curving of the shear layer, and the reattachment point, the peak location retracted back upstream.

The 3-dimensional distribution on the unwrapped surface (Fig 17) shows mostly axisymmetric flow at nominally 0° angle of attack. Some circumferential non-uniformity may be attributed to small local differences on the model and also perhaps in the tunnel free-stream flow. A point on measurement uncertainty at the end of the
Metric section at x=29.4” is worth noting. The Metric section had thinner wall, separated from the rest of the model via flexible seals and was used for direct measurement of forces via strain-gauges and accelerometers. The heat generated by these sensors was found to elevate temperature of this part by ~2.5°C. The calibration procedure is expected to account for changes in temperature; however, the pressure sensors were mounted somewhat upstream and downstream of the discontinuity. Data from a narrow strip along this location carried higher level of uncertainty, and showed up in some of the subsequent figures.

Fig 15. Shadowgraph images of the upper half of the model at indicated Mach numbers, α = 4°.

Fig 16. Distribution of the \( C_{prms} \) along \( \theta = 0° \) via uPSP (red) and Kulite (black) at indicated Mach number, \( \alpha = 0° \).

Fig 17. Distribution of the \( C_{prms} \) on unwrapped surface of the model, measured via uPSP, at indicated Mach number, \( \alpha = 0° \).
Fig 18. (d1–g2) Instantaneous $C_p$ fluctuations at indicated time (referenced from the first image), (c1, c2) $C_{prms}$ levels on unwrapped surface of the model for $M = 0.8$ and (left column) $\alpha = 0^\circ$ (right column) $\alpha = 4^\circ$; (b) shadowgraph image.
The large data set of instantaneous pressures generated by uPSP was used to create video animations that provided interesting insight into the turbulent eddies and shock-turbulence interactions occurring on the model. Figure 18 shows selected time instances from two animations at $M = 0.8$ and at $\alpha = 0^\circ$ and $4^\circ$. Corresponding $C_{prms}$ distributions are shown above the instantaneous plots. A shadowgraph photograph for $\alpha = 4^\circ$ case is shown in Fig 18(b); no shadowgraph was collected for $\alpha = 0^\circ$. A comparison of the $C_{prms}$ distributions (Fig 18-c1 vs. 18-c2) shows the expected stronger asymmetry for the $4^\circ$ case where lower level of fluctuations occurs on the leeward side of the Metric Section. The highest fluctuations occur at approximately $35^\circ$ off of windward direction on both sides of the vehicle. The positive angle of attack is expected to create axial vortices on both sides of the vehicle, which modified the impinging shear layer on the Second Stage and created localized patches of high $C_{prms}$ levels away from the vehicle centerline.

The $C_p$ plots of Figs. 18-d1 through 18-g2 are instantaneous snapshots of the fluctuating component of the pressure from every 20$^{th}$ camera frame starting with the first. The pressure levels fluctuate between positive and negative values. These plots show a complex pattern of pressure fluctuations that become energetic at the foot of the two shock waves and then again at the reattachment zone. The strong negative pressure troughs and positive pressure crests at the two shock locations appear to occur intermittently. Intriguingly, they also appear to be in opposite phase: a negative pressure in the first shock is accompanied by a positive pressure at the second shock and vice versa. Perhaps the most intriguing observation is that the entire field - from the first shock till the end of the measurement region - seem to be inter-related. Not only the fluctuations at the two shock locations on PF are interconnected but certain fluctuations on the reattachment region on the Second Stage appear to be connected with those occurring on the PF. This is in contradiction to the initial expectation that the pressure fluctuations on the PF are independent of those on the Second Stage. This aspect will be explored in more details in the following discussion of correlation and $k$-$\omega$ spectra.

**Fig 19.** Correlation of pressure fluctuations between $x=23.4''$ (middle of Metric section) and points separated by indicated distances, along $\theta=0^\circ$; $\alpha = 0^\circ$ and (a) $M = 0.8$, (b) $M = 1.1$.

**Fig 20.** Correlation along circumferential direction at $x=23.4''$ (middle of Metric section) and points separated by indicated angles, $M = 0.8$, $\alpha = 0^\circ$. 

17
IIIc. Space-time correlation

Before discussing k-ω spectra it’s worthwhile to observe the correlation of pressure fluctuations. The normalized correlation coefficient \( r \) between two spatial points separated by \( \Delta x \) and \( \Delta \theta \) is given as:

\[
r(\Delta x, \Delta \theta, \tau) = \frac{1}{\text{prms}(x, \theta) \text{prms}(x + \Delta x, \theta + \Delta \theta)} \int_0^T p(x, \theta, t)p(x + \Delta x, \theta + \Delta \theta, t) \, dt
\]

(19)

Here \( T \) is the time duration of the signal. Figure 19 shows two sets of space-time correlations, both along the axial direction, measured on the Metric part of the model. Each correlation plot has a local peak that appears at a delay time \( \tau \) which can be used as a measure of convection velocity of pressure fluctuations over the vehicle surface.

For a fixed separation \( \Delta x \), the time delay, \( \tau \), that produced the peak in correlation \( dr/d\tau = 0 \) provides the convection velocity \( U_c = \Delta x/\tau \). For the lower \( M=0.8 \) (Fig 19a) an average convection velocity, normalized by the free-stream velocity is \( U_c/U = 0.51 \), and for the higher \( M=1.1 \) (Fig 19b) \( U_c/U = 0.57 \). The relatively slower frame rate of 5000/s produces higher uncertainty in the convective velocity calculation for these high velocity flows. The magnitude of the correlation peak can be used to measure correlation-length (also called the integral length scale). A comparison between the two Mach number cases in Fig 19 shows a general similarity of the correlation curves, except for an increase in the convection velocity and a lowering of correlation length for the higher Mach case. The latter is an expected outcome of the compressibility effect with increasing Mach number. Another interesting feature is the negative value of the peak correlation at large negative \( \Delta x \). This indicates that the pressure fluctuations on the upstream part of the separated zone are in opposite phase to that near the impingement point.

While Figs 19 shows data for separations along the axial directions, Fig 20 presents a set of plots for circumferential separation. Here all correlation peaks appear at \( \tau = 0 \), implying a perfectly axisymmetric flow (\( U_c = \infty \)). Large values of the correlation coefficient even at \( \pm 18^\circ \) separation, imply long correlation lengths in the \( \theta \) direction.

Figure 21 shows a set of correlation plots obtained from the location of the second shock on the Payload Fairing at \( M = 0.8 \). A striking feature of the correlation is that the peaks occur in negative delay times for positive spatial separations and vice versa. Note that this trend is exactly opposite of that seen in Fig 19. Physically this will mean that the local pressure fluctuations propagate upstream on the body surface. To further investigate this phenomenon upstream propagation fluctuations at the reattachment point on the second stage (Metric section) was correlated with those occurring progressively upstream, all the way to the location of the first shock wave (Fig
Although the correlation coefficient became progressively weaker with increased separation, still the non-zero levels confirm the earlier observation that some fluctuations on the Payload fairing, including those at the shock locations, are coupled with the separated flow present on the Second stage. Such a coupling is capable of generating large forces and structural deflection of the entire stack. It is worth recalling that nearly half a century ago the hammerhead family of space vehicles were scrutinized to mitigate excessive buffet forces encountered by similar configurations.\textsuperscript{17,18,25} Recommendations for the Frustum angle, ratio of the Payload diameter to that of the Second Stage, length of the Second Stage to the Fairing diameter, and other similar parameters were generated to avoid the excessive unsteady forces. These parameters are widely used in the present vehicle designs. The current model configuration is one that was recommended by those studies. The present observation indicates that for the other buffet-prone hammerhead geometries, an even stronger coupling could have been present between the Second Stage and the Payload Fairing, which would have created an excessive sidewise force at small angles of attack.

**IIIId. Wavenumber-frequency (k-\omega) spectra:**

The availability of pressure fluctuation data on a dense, regularly spaced, grid points paved the path for the calculation of k-\omega spectra. First the instantaneous pressure values were converted to coefficient of pressure via free-stream dynamic pressure q, \( C_{p} = p/q \). The calculation procedure involved three applications of discrete Fourier transform: first, along the axial x direction repeated for every \( \theta \) and time dimensions; second, along the \( \theta \)-direction repeated for every x and time; finally, along the time dimension repeated for every x and \( \theta \). The application along the axial direction over \( \eta \), number of grid points resulted in the complex axial transform \( \psi \):

\[
\psi(k_{x},\theta,t) = \mathcal{F}\{C_{p}(x,\theta,t)\}, \quad i = -\left(\frac{\eta_{x}}{2} - 1\right), 0,1,2,\ldots,\frac{\eta_{x}}{2} \quad (20)
\]

The axial wavenumbers were calculated based on the grid spacing \( \Delta x \) and the total length of the region of interest L:

\[
k_{x} = \frac{2\pi}{\Delta x} = \frac{2\pi i}{L} \quad (21)
\]

These wavenumbers were normalized by diameter D. While the Metric part had a constant diameter of \( \frac{1}{2} \) ft, that for the Payload Fairing varied. Therefore, an area weighted diameter of 0.74 ft was used for the PF. In the past, most measurements of the k-\omega spectra were made in attached turbulent boundary layers on simple geometries, and \( k_{s} \) was normalized by the boundary layer thickness.\textsuperscript{5} The large separated flow region covering the axisymmetric geometry of the present model makes such normalization irrelevant; no effort was made to measure the boundary layer thickness.

In the next step, the complex numbers along the \( \theta \)-direction were Fourier transformed for each of the axial and time dimensions:

\[
\psi(k_{x},k_{\theta},t) = \mathcal{F}\{\psi(k_{x},\theta,t)\}, \quad j = -\left(\frac{\eta_{\theta}}{2} - 1\right), 0,1,2,\ldots,\frac{\eta_{\theta}}{2} \quad (22)
\]

\( \eta_{\theta} \) is the total number of grid points (240) along the circumferential direction. The corresponding wavenumbers were calculated similarly using grid spacing \( \Delta \theta \), and the total angle \( 2\pi \):

\[
k_{\theta} = \frac{2\pi}{\Delta \theta} = \frac{2\pi j}{2\pi} = j \quad (23)
\]

Finally, at each grid point, a Fourier transform was applied in the time dimension. To improve convergence, the time series was segmented into N overlapped sets, each \( S = 1024 \) long. Discrete Fourier transform was applied to each set, providing a realization in the statistical ensemble:

\[
\psi(k_{x},k_{\theta},\omega_{\ell}) = \mathcal{F}\{\psi(k_{x},k_{\theta},t)\}, \quad \ell = -\left(\frac{S}{2} - 1\right), 0,1,2,\ldots,\frac{S}{2} \quad (24)
\]

Where \( \omega_{\ell} \) are the discrete frequencies:

\[
\omega_{\ell} = \frac{2\pi \ell}{T} \quad (25)
\]

In the above equation \( T \) is the time duration for the 1024 data points used in the individual realization. The circular frequency \( \omega \) was normalized by diameter D, and the free-stream velocity U: \( \omega D/U \). Similar to that used with the axial wavenumbers, D used for the Payload Fairing was the area-weighted value of 0.74 ft. The power...
spectral density of the wavenumber-frequency spectrum was calculated by multiplying the complex spectra, for each realization, by its conjugate, and then taking an average over all \( N \) sets.

\[
\phi(k_x, k_\theta, \omega_\ell) = \frac{1}{N} \sum_{s=1}^{N} \{ \psi(k_x, k_\theta, \omega_\ell) \psi^*(k_x, k_\theta, \omega_\ell) \}
\]

An important property of Fourier transform is the equality in the variance of the time-series data and the integrated power spectrum. This requires \( \Phi \) to be scaled such that the following holds:

\[
\sum_{x, \theta, \ell} C_x^2 = \sum_{i, j, \ell} \Phi(k_{x,i}, k_{\theta,j}, \omega_\ell) \Delta k_x \Delta k_\theta \Delta \omega
\]  

Fig 23. Auto-spectra of (top row) \( \phi(\omega) \), (mid row) axial wave-number \( \phi(k_\theta) \), (bottom row) circumferential wavenumber \( \phi(k_\theta) \) for: (left column) Metric section, \( M = 0.8, \alpha = 0^\circ \); (middle column) Metric section, \( M = 1.1, \alpha = 0^\circ \); (right column) Payload Fairing, \( M = 0.8, \alpha = 0^\circ \). The red chain lines are least square fits.

Fig 24. Contour plots of \( \phi(k_x, \omega) \) for the same three conditions of the previous figure: (a1) Metric section, \( M = 0.8, \alpha = 0^\circ \); (a2) Metric section, \( M = 1.1, \alpha = 0^\circ \); (a3) Payload Fairing, \( M = 0.8, \alpha = 0^\circ \). The red chain line is a least square fit through the local maxima, used to calculate convection velocity.
Figures 23, 24 and 25 show various aspects of the k-ω spectra for three conditions: the first two cover the Metric part of the Second Stage for the two Mach numbers of M=0.8 and 1.1, and the third covers the Payload Fairing at M=0.8. Recall that at M=0.8 strong shock-waves are formed on PF, so the latter covers the shock-boundary layer interaction, upstream feedback and other physics. The former two show the Mach number dependence of the separation-reattachment zone. Note that space-time correlations for the same three cases were presented earlier.

Visualization of the three-dimensional wavenumber-frequency spectrum requires creation of degenerated one-dimensional and two-dimensional spectra that can be easily plotted. The 1-D power spectra \( \Phi (k, \omega) \) were obtained by summing \( \Phi (k_D, k_\theta, \omega D/U) \) over the other two variables. Similarly, the 2-D power spectrum: \( \Phi (k, \omega D/U) \) was calculated by adding the \( k_\theta \) dimension. All spectra presented are normalized such that their integral was equal to the mean-square \( C_p \) fluctuation. It needs to be pointed out that unlike data from a few point-measurements, each spectrum represents an average over the entire surface, and is more appropriate to create forcing functions for vibro-acoustics analysis. The 1-D power spectra are shown in Fig. 23. The \( \Phi (\omega D/U) \) spectra show large energy at very low frequencies which is typical of both separation-reattachment flow and shock-vortex interaction. Similarly, the \( \Phi (k, D) \) spectra also showed an exponential decay from the lowest wavenumbers. The decay rate was much smaller in the axial direction than in the circumferential direction, indicating the expected longer correlation of the flow fluctuations in the axial direction. The sharp peaks in Fig 23(a3) were identified as the tunnel background tones centered at \( f \sim 2900 \)Hz and at its first harmonic (shown earlier in Fig. 12).

The two-dimensional \( \Phi (k, \omega D/U) \) spectra, presented as contour plots in fig. 24, have the shape expected due to the convected hydrodynamic fluctuations. The contours at spectral levels of \( 0.25 \times 10^{-6} \), \( 0.5 \times 10^{-6} \), \( 1 \times 10^{-6} \), \( 2 \times 10^{-6} \), \( 4 \times 10^{-6} \) are lower than the peak values that occurred at the lowest frequency band: \( \omega D/U = 0 \). The staircase like shape of the contours is due to poor resolution of the \( k_x \) wavenumbers arising from the limited spatial extent \( L \) of each zone. According to Wills\(^{24}\) the \( k_x, \omega \) spectrum can be used to obtain a measure of convective velocity that is a function of the wavenumber:

\[
\frac{U_c(k_x)}{U} = \frac{\omega_c}{K_x}(28)
\]

Where for every wavenumber the frequency for the local peaks are identified:

\[
\left( \frac{\partial \Phi(k_x, \omega)}{\partial \omega} \right)_{\omega = \omega_c} = 0 \quad (29)
\]

The chain lines in fig. 24 are least square fits through all such maxima providing an average convection velocity quoted in the figure. Note that for the Metric part the calculated values are comparable to those estimated from the space-time correlation plots shown earlier. However, unlike the space-time data, the numbers shown in Fig 24 are representative of an average over the entire panel structure of the Metric and PF.

The \( \Phi(k_x, \omega) \) spectrum for the Payload Fairing (Fig 24-a3) stands in stark contrast to those measured on the Metric part. The negative slope of the fitted chain line confirms negative convection velocity as suspected in the earlier discussion of the instantaneous pressure fields and the space-time correlation. The concentration of the contours at \( \omega D/U = \pm 4.2 \) corresponds to the tunnel background fluctuations and therefore, should be ignored. The concentration of the contours close to the origin, in each plots of Fig 24, correspond to the very long time scale associated with separation-reattachment flow and shock-boundary layer interactions.
Fig 25. $\Phi(K_x, K_\theta)$ spectra for the indicated frequencies for (left column, a1-d1) Metric section, $M = 0.8$, $\alpha = 0^\circ$; (mid column, a2-d2) Metric section, $M = 1.1$, $\alpha = 0^\circ$; (right column, a3-d3) Payload Fairing, $M = 0.8$, $\alpha = 0^\circ$.

Figure 25 shows cuts through the 3-D, $\Phi(k_x, k_\theta, \omega D/U)$ spectra along four fixed values of the frequency $\omega D/U$. The expected shapes of such spectra in very low speed attached boundary layers have been discussed and measured by many (see textbook by Blake1). In such flows $\Phi(k_x, k_\theta)$ spectrum is expected to have two peaks, one for the radiated acoustic fluctuations centered at the origin and the second for the convected pressure fluctuations centered at $k_x = \omega/U_c$. For the transonic and supersonic Mach numbers of present interest, clear separation may be impossible since the convection velocity of the turbulent eddies is comparable to that of the local sound speed. In addition, the hydrodynamic pressure fluctuations can easily swamp fluctuations from the radiated acoustic emission. Fig. 25 shows asymmetry about the positive and negative values of $k_x$. At very low frequencies the contours are clustered around the origin. An increase in frequency progressively moves the cluster towards the convection direction. While for the plots for the Metric section the peak is found in positive $k_x$,
indicating a downstream convection of pressure fluctuations; the opposite is true for the plots from the Payload Fairing, confirming the upstream propagation discussed earlier. These plots also convey information about convection, dispersion and length scales in a concise manner. As described in the introduction, should there be a need to perform vibro-acoustics analysis of a space-vehicle with the geometry of the present model, then the calculated $\Phi(k_3D, k_9, \omega D/U)$ spectra would give a complete description of the unsteady forcing. There will be no need to resort to any modeling.

IV. SUMMARY

The progression of the PSP technology from time-averaged pressure measurement to the measurement of unsteady pressure fluctuations comes with many promises and challenges. Reliable measurement of the unsteady pressure field, particularly at transonic and low supersonic Mach numbers, provides critical data for vibro-acoustic and buffet environments of all aero-space vehicles. The present paper demonstrates that the high-resolution data obtainable from the application of uPSP can bring unprecedented insights, albeit for a limited frequency range of 10 kHz, that in many ways are difficult to obtain from the traditional unsteady pressure sensors. The test article was a generic hammerhead shaped space vehicle that was studied during the 1960’s and carried similarity with many present and past vehicles used for Payload delivery to Earth orbits. The model was coated with a porous binder and PtTFPP-based polymer paint supplied by a commercial vendor (ISSI corporation). The elaborate system of high-speed cameras, and UV lamps was set up around the optical access panels of the transonic wind-tunnel. The first part of this paper describes the steps to convert the large number of camera images to pressure histories on a gridded model of the test article, and is followed by a validation of the data. The second part describes various transonic flow physics revealed by the uPSP data, and descriptions of the wavenumber-frequency spectra for different regions of the models.

The first part describes the details of every step needed to process the camera images, while emphasizing the differences between steady-state applications and unsteady application of uPSP. The paint used for the latter had orders of magnitude higher sensitivity to temperature, non-linear dependence on pressure, and was subjected to fast photo-degradation. All of these difficulties were overcome via in-situ calibration using a set of unsteady pressure sensors distributed over the model surface. While the steady-state applications required registration of a few camera images, nearly 250,000 images had to be registered for each test point in the present application. While electronic shot-noise can be reduced via averaging a number of images in steady-state PSP application, no such path was available for unsteady application. In fact, the shot noise contribution was demonstrated to limit measurable pressure fluctuations. It was demonstrated that the calibration factor provided the telltale sign of this limit. In general, the paint was found to faithfully reproduce the phase relation among pressure fluctuations up to the highest attempted frequency range of 0-10 kHz. The amplitude response was found to deviate slightly above about 3kHz. Spatial filtering, a common tool applied to steady-state data to smooth out pixel-to-pixel variations could not be used with unsteady data (to avoid artificially increased correlation), which led to an increased random noise in the uPSP data.

The detailed time histories were used to explore various aspects of the transonic flow in the second part of the paper. The Mach number dependent physics of the two primary flow phenomena - shock-boundary layer interactions on the Payload Fairing, and the separation-reattachment flow on the Second Stage - were explored via video animation of pressure histories, and $C_{p_{rms}}$, space-time correlation and wavenumber-frequency analyses. It was observed, somewhat unexpectedly, that the two phenomena were coupled. Any stronger coupling would have excessively increased the unsteady forces experienced by such vehicles. The hammerhead shape of the present model was an outcome of past studies to reduce aerodynamic forces on similarly shaped vehicles\textsuperscript{25}. The present result indicates that the buffet-prone bodies might have stronger than the presently measured coupling between the shock waves on the Payload Fairing and the separated flow zones on the Second Stage. It was also observed that the pressure fluctuations were convected in the upstream direction when shock waves were present on the Payload Fairing. The measured $k$-$\omega$ spectra contain all information of convection velocity, length-scales and spatio-temporal correlations in a succinct manner and are the desired input for vibro-acoustic analysis.
The present effort demonstrates, for the first time, that such information can be obtained from realistic test models, and Mach numbers using uPSP.

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Reference:


