Mach 10 Bow Shock Unsteadiness Modeled by Linear Combination of Two Mechanisms

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Abstract

This manuscript presents mechanisms to explain and mathematics to model time-averaged spatially-resolved amplitude observations of number density and number density unsteadiness in a Mach 10 flow as it transitions from the freestream, through a bow shock wave, and into the gas cap created by a blunt-body model. The primary driver for bow shock unsteadiness is freestream unsteadiness or “tunnel noise”. The primary unsteadiness is bow shock oscillation. It scales spatially with number density first derivative and is modeled using a $\text{sech}^2(z)$ term. Secondary weaker unsteadiness begins as freestream unsteadiness and increases linearly in direct proportion to gas number density across the bow shock and into the gas cap. This is the well-known amplification of freestream turbulent kinetic energy mechanism and is modeled using a $\tanh(z)$ term. Total unsteadiness (fit using $\tanh(z)$ term + $\text{sech}^2(z)$ term) is expressed as number density standard deviation and modeled as a linear combination of the latter two independent, simultaneous, and nonlinear unsteadiness mechanisms. Relationships between mechanism coefficients and various flow field and wind tunnel parameters are discussed. For example, bow shock and gas cap oscillation amplitudes are linearly correlated with stagnation pressure and by deduction freestream unsteadiness.

I. Introduction

In 1914, Wieselsberger [1] concluded that the variation in drag values obtained in different subsonic wind tunnels could be explained by differences in free stream turbulence. Since that time, experiments have shown that the freestream of hypersonic facilities contains high frequency unsteady pressure oscillations or “tunnel noise” at frequencies up to 1 MHz [2, 3]. Unsteadiness can begin in the stagnation chamber [4], nozzle throat, or turbulent boundary layer on the nozzle wall [5] and enter the freestream [6]. Unsteadiness can be an order-of-magnitude above what is encountered in flight [6]. Unsteadiness can interact with and significantly perturb the observed offbody flow field around test articles [7]. This process is called receptivity and creates a disturbance with a given frequency, amplitude, and phase [8]. The resulting flow field instability can influence laminar and turbulent transition, affect surface heat transfer, and increase stagnation point heating. This in turn can influence model downstream

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vehicle aerodynamics / aerothermodynamics including lift, drag, pitching moment, transition onset, and turbulence [7]. Due to limited understanding of the instability mechanisms and transition processes inherent in a given facility, computational fluid dynamic (CFD) codes have difficulty computing aerodynamic quantities in transitional and turbulent flows. These problems are exacerbated by the aerothermodynamic considerations present in hypersonic flows. After a century of study, the present manuscript seeks to increase modestly the understanding of these phenomena by measuring the effects of hypersonic freestream unsteadiness as it propagates through and induces oscillations in a Mach 10 bow shock and gas cap (boundary layer instability) ahead of a blunt body. For clarity, a single flow field state variable density is measured.

Researchers typically focus on measuring the frequency content of unsteadiness. Instrumentation capable of valid measurements of unsteadiness at high frequencies is limited. Problems with traditional physical probes capable of Pitot pressure measurements and hot-wire anemometry have been reviewed [9]. Schlieren imaging is attractive due to its nonintrusive nature and capacity for both frequency and amplitude measurements but lacks spatial resolution [10-11]. Focused laser differential interferometry (FLDI) has many desirable characteristics and has demonstrated 1 MHz freestream measurements [9]. FLDI is a point method limited to a 20 mm (z-axis) spatial resolution. Laser Rayleigh scattering (LRS) has demonstrated freestream density measurements at two points in space and 50 KHz [12] using a continuous-wave laser at 532 nm and atmospheric pressure. This approach suffers from particle interference and produces insufficient signals in low-density hypersonic flows.

There are a small but growing number of experimental studies of hypersonic bow shock oscillations induced by freestream disturbances [10, 11, 13, and 14]. Marineau and Hornung [10] measured bow-shock wave motion using a 7-inch diameter Apollo-shaped capsule in the T5 hypervelocity shock tunnel. Shock position oscillation amplitudes up to ~ 5% of standoff distance were observed. Bow-shock motion displayed power spectral frequencies of ~ 10 kHz. Fujii et al. [11] observed bow shock motion created by a blunt body capsule at ~ Mach 9.5. Vashishtha et al. [13] observed high frequency, small amplitude shock oscillations in front of a convex hemispherical shell and circular flat plate at Mach 7. Balla [14] observed bow shock oscillations ahead of a 4-inch Apollo-shaped capsule at Mach 10. Pulsed laser Rayleigh scattering along a 38 mm line at 30 Hz repetition rates measured amplitude unsteadiness. There have been several mathematical analysis and computational studies on freestream-unsteadiness-induced bow shock oscillations [15-21].

The present manuscript is the culmination of a series of studies designed to demonstrate the effectiveness of pulsed laser Rayleigh scattering (LRS) for quantitative density and spatial unsteadiness measurements in Mach 10 flows. Pulsed LRS has several attractive traits relevant to unsteadiness measurements including high temporal resolution (20 ns laser pulse - frozen flow), offbody, seedless, nonintrusive, high spatial resolution (~200 µm in each dimension) and the capability of linear/planar measurements. Using a standard deviation approach, the latter provide spatial correlations of relative flow unsteadiness with
high spatial resolution. Previous hypersonic results include measurements in the freestream [22-24], supersaturation region [25], wake [26], and gas cap [23, 26] of a blunt body. Using planar LRS at 10 Hz repetition rates, Shirinzadeh et al. [27] demonstrated that density standard deviation and density standard deviation divided by density mean (referred to as relative standard deviation or fluctuating component) are a measure of flow unsteadiness and can be related to flow field turbulence.

The NASA Langley 31-Inch Mach 10 (31M10) facility has several unique characteristics relevant to this manuscript. Quantitative LRS freestream density measurements indicate clusters are not detected in this facility [24]. LRS quantitative density measurements in the supersaturation region indicate freestream “frozen” vibrational nonequilibrium population are effectively removed by interaction with clusters [25]. By varying stagnation pressure at constant stagnation temperature, entropy fluctuations per unit stagnation pressure are presumed constant. All these variables are removed as possible contributors to the observed unsteadiness.

In a previous LRS manuscript, Balla [14] presented a unique Mach 10 bow shock and gas cap unsteadiness dataset. Using sampling rates of 30 Hz, LRS measured time-averaged disturbance amplitudes along a 38.7 mm line. The time-averaged spatial profile of standard deviation unsteadiness as flow transitions from the freestream, through the bow shock, and into the gas cap produced a distinct and unexpected profile. The gas cap is defined as the shock-heated stagnation zone ahead of a blunt-body model. Assuming amplification of freestream turbulent kinetic energy, unsteadiness was expected to increase monotonically from the freestream, across the bow shock wave, and end with maximum unsteadiness in the gas cap. This was not observed. Instead unsteadiness was minimum in the freestream, rises to a maximum at densities and a spatial location which are halfway between the freestream and gas cap (~50% of density maximum or first derivative of bow shock density profile), and then decreases to levels expected in the gas cap assuming turbulent kinetic energy amplification. Results clearly demonstrate that two unsteadiness mechanisms are operative. The first and minor unsteadiness is the well-known amplification of freestream turbulent kinetic energy (unsteadiness) across a shock wave. The second is bow shock oscillation. It was concluded that tunnel-noise-induced bow shock oscillation not only exists but also is the dominant oscillation.

The present manuscript builds on the previous manuscript by analyzing the spatially-resolved unsteadiness results. It proposes a linear combination of two simultaneous, nonlinear, and independent mechanisms which spatially have a total shape which can be represented as a sum of a tanh(z) function and a sech²(z) function to explain the spatially-resolved standard deviation unsteadiness amplitude results. It provides equations to calculate each mechanism. It correlates spatially-averaged flow unsteadiness observed along each line with facility conditions and suggests possible fluid dynamics responsible for creating the observed unsteadiness.
II. Experimental

Complete descriptions of various components of this experimental setup have been described [14, 22-27]. A brief unifying synopsis is presented.

A. Facility

LRS is performed during Test 466 in the NASA Langley 31-Inch Mach 10 (31M10) Air Wind Tunnel. This facility is a unique 12.5-MW electrically-heated blowdown-mode facility [28]. The test gas (air) is dried using an activated alumina dryer, which provides a dew point temperature of approximately 213 K (−60 °C) at a pressure of 34.48 MPa (5000 psi). Air is expanded using a three-dimensional contoured nozzle to minimize centerline disturbances characteristic of axisymmetric contoured nozzles. The result is a highly uniform core flow. The nozzle throat is 2.72 cm square and the test section is 78.7 cm square. Core flow is 25% of the test section or ±10 cm about the centerline. Optical access is provided by three uncoated Corning 7980 windows with transmission > 85% at 193 nm which form three orthogonal walls of the test section. Models are positioned on the facility centerline in less than 0.6 seconds using a hydraulically-operated injection system mounted on the rear sidewall.

Maximum facility test times are 60 seconds. A low-pressure preheat of the nozzle walls is performed prior to each run. The facility was operated at a single stagnation reservoir temperature $T_t = 990 \pm 11$ K to prevent air liquefaction. Facility stagnation reservoir pressures $P_t$ were varied from 2.41 MPa (350 psia) to 10.0 MPa (1454 psia). These operating pressures correspond to freestream unit Reynolds numbers of 1.71 million/m and 6.73 million/m (0.52 million/ft and 2.05 million/ft), respectively. Flow conditions are calculated using the GASPROPS code [29].

B. Laser, Optics, and Electronics Setup

For the light source, a low-pulse-energy Lambda Physik OpTexPro excimer laser was selected to operate in broadband mode on argon fluoride near 193 nm at 30 Hz. The unpolarized output passes through approximately 1 meter of air before it enters the facility. It is turned 90 degrees with a mirror and focused 0.71 meters downstream from the nozzle exit along a line normal to the flow near the center of the facility using an uncoated 600 mm focal length Suprasil lens. The LRS signal generated along a 38.7 mm line in the freestream and gas cap is imaged using a gated, double-intensified ITT Model F4577 CCD camera. The detector has 240 vertical (27 µm) and 754 horizontal (11.5 µm) pixels. The resulting RS170 video signal is digitized at 30 frames per second using an EPIX model PIXCI SV5 frame grabber card and stored in a computer. The frame grabber was modified to output a field index pulse at 30 Hz for synchronizing the laser, charge-coupled-device (CCD) camera gate, and frame grabber.
For data collection, the following procedure was used. First, the computer acquired a freestream dataset consisting of 371 images (64 Megabytes of memory) during a 12.4 second time interval. Six seconds are required to store the data. Next, the CEV model is injected, flow allowed to stabilize, and CCD camera gain adjusted (5 sec). Next, a series of postshock datasets are acquired and stored. This process continues until pressure in the vacuum spheres rises to a critical level and affects freestream conditions. Typically, one freestream and two postshock datasets can be acquired during a single windtunnel run.

Experiments were performed using 1.25 mJ of laser energy focused inside the test section. Shot-to-shot variation in laser energy is < 3%. Laser pulse width is 10 ns. Focal spot size is ∼200 μm and Rayleigh length is ∼65 cm. During any given dataset, energy decrease was < 5%. Ambient stray light was minimized using a 5 μsec intensifier gate and extinguishing the room lights. Laser timing jitter was ± 100 nsec.

The calibration procedure for each pixel along the laser line has been described [22]. Briefly, the facility was evacuated to the pressure range of interest and Rayleigh data acquired over a range of known static pressures. Pressures were converted to density using a thermocouple in contact with the sidewall of the facility. For each of the 240 pixels along the 38.7 mm line of each averaged image, a linear plot of the Rayleigh signal as a function of the air density was obtained. Using a linear least-squares fitting routine, the slope, intercept, and their associated uncertainties were calculated and stored in a file. The flow-field signals were converted to density using this calibration file. This procedure removes the systematic nonuniformities in the laser-camera system. Errors associated with applying calibration results to static facility data yielded results similar to those discussed previously.

C. MPCV Model

Figure 1 is a composite representation of three images constructed using a virtual diagnostics interface or ViDI. ViDI is a NASA Langley software tool for interactive 3D display of the facility, test article, data and CFD prediction [30]. The first image is a digital ViDI reconstruction of a MultiPurpose Crew Vehicle (MPCV) model with sting at 28 degree angle of attack. The second image is an overlay of a 6-inch diameter time-averaged Schlieren image of the bow shock wave. The third image is an overlay of a 371 image averaged LRS density image within the CCD camera field-of-view (darkened rectangular region). Stagnation conditions are Po = 350 psi and To = 990K. Bowshock and laser standoff on the sting centerline are 10.6 and 9.3 mm respectively.

The MPCV model is an early version of the Crew Exploration Vehicle (CEV), or Orion Spacecraft that is based on the Apollo geometry. It has been renamed a MultiPurpose Crew Vehicle (MPCV). It is 4 inches in diameter (2% scale). The heat shield radius is 1.2 times the diameter, the corner radius is 0.05 times the diameter, and the back-shell angle is 33 deg. The model is attached to a sting at a 28 degree angle, which was approximately the entry angle of attack of Apollo. An earlier publication [26] provides additional model detail. This was a model of convenience used to create a shock jump and to evaluate other measurement possibilities. This simple model produces a well-known laminar flow field that is easily calculated using existing theories and current computational methods.

Laser position was a compromise between being as far as possible behind the shock to accurately sample the gas cap while preventing scattered laser light from the model surface from interfering with LRS signals. Laser scatter from the model surface has been removed digitally. Although Schlieren images (31M10 facility, test 400, run 51) and LRS data were acquired during different wind tunnel tests, stagnation conditions
varied < 1%. The LRS image is 38.7 mm high. It begins near the freestream, traverses the near normal shock at the top of the images, spans the gas cap, traverses the oblique shock near the bottom of the images, and intersects the freestream. The LRS image consists of 11 columns. Line results presented below use the center column since it has the highest signal-to-noise ratio.

At this high angle of attack, fig. 1 shows the bow shock standoff distance is large near the top of the model and monotonically decreases towards the bottom of the model. Beyond this point, shadowgraphs provided by Walpot et al. (Fig. 16 in [31]) and Fujii et al. (Figs. 3-5 in [11]) show shock standoff distance decreases rapidly, the shock turns rapidly, and becomes more oblique. In this manuscript, the location above sting centerline where the laser beam intersects the shock will be referred to as a near normal shock while the location below will be referred to as an oblique shock.

D. Model Motion

Heating of the sting, strut, and model causes the model to move downstream and towards the camera during each run [26]. For a single set of 371 images, streamwise motion ranges from 2 pixels (137 um) at Po = 350 psi to 4 pixels (275 um) at 1454 psi. For 2 sets of data at 1454 psi including data storage time, maximum total motion is 10 pixels (686.9 um). Trigonometry dictates spanwise motion toward the camera will be 2.748x the downstream motion. Evidence that this motion has a minimal effect on results during a single dataset has been presented [26]. This manuscript uses this motion as a convenient and passive method of sampling different spatial locations in the bow shock wave as a function of facility run time.

![ViDI composite image- MPCV model, bow-shock Schlieren Image, and LRS image.](image-url)
III. Results and Discussion

A. Textbook Hypersonic Bow Shock Wave Density Profile

As hypersonic flow transitions between the low-temperature high-velocity freestream and the high-temperature low-velocity gas cap ahead of a blunt body, macroscopic properties of the gas such as translational, rotational, and particularly vibrational temperature are required to change by an order-of-magnitude over a few molecular mean free paths. The resulting nonequilibrium conditions in the interior of a hypersonic shock wave are equilibrated by kinetic processes which require a finite time and hence a finite distance to occur. The result is a thickening of the shock wave [32]. This is in contrast to supersonic flows where equilibration typically requires ~10 collisions, equilibration distances are greatly reduced, and shocks can be treated as simple discontinuities.

The density profile as hypersonic flow begins in the freestream, traverses a hypersonic bow shock wave, and ends in the gas cap ahead of a blunt body is shown schematically in Fig. 2 [33]. Here, $\delta_s$ is the shock wave thickness derived from the maximum slope of the density profile and $\lambda_1$ is the freestream mean free path. Normalized bow shock thickness is express as $\delta_s / \lambda_1$ and is ~3.3 for a Mach 10 nitrogen shock [34]. At $P_o = 350$ psi, $\lambda_1 = 0.023$ mm and $\delta_s$ is ~ 0.077 mm. At $P_o = 1450$ psi, $\lambda_1 = 0.0055$ mm and $\delta_s$ is ~ 0.018 mm. $\bar{n} = 0.5$ is the density midpoint or median between freestream and gas cap. Convention correlates $\bar{n} = 0.5$ with $x / \lambda_1 = 0$. Hereafter, for brevity, $\delta_s$ will be written as $\delta$.

Fig. 2. Hypersonic bow shock wave density profile.
The spatial density profile in fig 2 is described using a hyperbolic tangent function (equation 93.12 in [35]).

\[ \rho(x) = a + b^* \tanh \left( \frac{x}{\delta} \right) \quad (1) \]

Values for a and b allow the \( \tanh(x) \) function to map its maximum and minimum values of +1 and -1 onto the gas cap and freestream densities respectively such that \( a+b \) is gas cap density and \( a-b \) is freestream density. The constant \( \delta \) is the shock thickness in mm. It allows for a finite shock thickness (\( \delta \)) which may vary with freestream pressure or spatial variations in local flow field Mach number.

**B. LRS Time-averaged Density and Density Standard Deviation Spatial Profiles**

Number density measurements are obtained along the laser beam (z-axis) spanning \( \sim 38.7 \) mm at locations shown in Fig. 1. For brevity, they are labeled in the figures as density measurements. Results begin in the freestream, traverse the near normal shock and gas cap and oblique shock and end near the bottom of the model in the freestream. Fig. 3 shows line measurements (371 image average) of average density (\( \rho \)) at five stagnation pressures and two spatial locations within the shock (black and red curves). They were discussed previously [26]. Spatial oscillations at \( \text{Po} = 350 \) psi (run a) in the post shock number density (\( \pm 0.25 \) out of \( 5.25 \) or \( \pm 5\% \)) are primarily measurement uncertainty. The dip near \( z = +2 \) mm for all \( \text{Po} > 350 \) psi may be flow spatial nonuniformity. Higher fidelity measurements are required to support this speculation. Lines are color coded with black corresponding to the first dataset taken at the first spatial location and red for the second dataset taken at the second spatial location. Except for the single \( \text{Po} = 350 \) psi case, datasets were recorded and stored in \( \sim 18 \) second intervals during each facility run (\( 12 \) second data acquisition and 6 second storage time). Figures are arranged vertically as a function of stagnation pressure for rapid visual inspection and comparison of a single measured parameter at different stagnation pressures and spatial locations during each run.

Figure 4 shows the standard deviation (\( \sigma \)) for each corresponding image in fig 3. Results in Fig. 4 are calculated using 371 instantaneous images used to generate the average data in fig. 3. In fig. 4e, maximum unsteadiness indicates \( \sigma \sim 5 \) near \( z = +12 \) mm. For simplicity, values for standard deviation (\( \sigma \)) in this manuscript are written and plotted without the exponent of \( 1 \times 10^{17} \). By comparison with density at the \( z = +12 \) mm location in fig. 3e, time-averaged flow field unsteadiness is \( 5 \) out of \( 10 \) or \( 50\% \) (\( \pm 1\sigma \)) density unsteadiness. Near the edges of these figures (freestream conditions), \( \sigma \) approaches 1. This is primarily instrument unsteadiness. True freestream unsteadiness is a fraction of this value. Instrument unsteadiness has not been subtracted from Fig. 4 images. Its effect will be computed and discussed following the data analysis below. Unfortunately, locations of interest for many of the unsteadiness phenomena discovered in this manuscript were not known during the design phase of this experiment. One result is that the length of the imaged line is not sufficiently long to observe all unsteadiness behavior of interest at all stagnation conditions and flow field locations. Line images are not normalized by pulse-to-pulse laser energy variation (< 3%); this effect is negligible. Model motion weakly broadens Fig. 3 and Fig. 4 profiles. Therefore time-averaged results are a lower limit to true unsteadiness.
Run a) Po = 350 psi
Run b) Po = 651 psi
Run c) Po = 902 psi
Run d) Po = 1254 psi
Run e) Po = 1454 psi

Fig. 3. LRS bow shock and gas-cap density $\rho(z)$ profiles.
Fig. 4. LRS density standard deviation $\sigma(z)$ profiles.
C. Analysis of Time-Averaged Spatial Density Profiles

1. Tanh(z) Fits and Coefficients

The average density data in fig. 3d at Po = 1254 psi and the second dataset (red datapoints at t = 18 sec) is shown in Fig. 5. This profile is split into data (solid black circles) spanning the freestream, near normal shock, and gas cap in fig. 5a and data spanning the freestream, oblique shock, and gas cap in fig. 5b. Each profile has been shifted based on visual inspection (-12 to -18 mm for the normal shock and +12 to +18 mm for the oblique shock) so the median density (\( \bar{n} = 0.5 \) in fig. 1) is centered near z = 0 to conform to a tanh(z) fit. Near normal shock z-axis coordinates are inverted. The meaning of \( \Delta Z \) inserts are discussed in section III.D.2.A.

Visual inspection suggests \( \rho(z) \) in fig. 5 and all data in fig. 3 along the laser line in fig. 1 are a stretched version of \( \rho(x) \) in fig. 2. To validate this, data in fig. 5 and all data in fig. 3 are fit to equation 2 using a nonlinear least squares algorithm.

\[
\rho(z) \sim \rho(x) = a_0 + b_0 \ast \tanh \left( \frac{(z-c)}{\delta_0} \right) \tag{2}
\]

Image length was insufficient to provide data over the entire 30 mm in fig. 5 required for fit convergence. To compensate, previously measured average freestream densities (Fig. 3 in [24]) measured at different runtimes during the same facility run were inserted as red circles between z = -10 to -15 mm. Each of these experimental densities agrees to better than 6% with GASPROPS computed freestream values. The constant c allows the fit to refine the shift guess. Solid lines in fig. 5 are nonlinear least squares fits. Fit parameters \( a_0 \) and \( b_0 \) (x 10^{17} /cm^3) and \( \delta_0 \) (mm) for all datasets in fig. 3 are given in Table 1. Sequential Po listings in Table 1 correspond to sequential data sets acquired during each run (t = 0 and t = 18 sec). Fig. 3a contains insufficient data for fit convergence for near normal shock data at Po = 350 psi.

Correlation coefficient (R^2) values for all fits in table 1 are not listed but are 99%. Shift guess values for each profile and c-coefficient fit parameters are not listed. All c values were < 1 mm indicating good initial shift guesses. Values for \( a_0 + b_0 \) and \( a_0 - b_0 \) are not listed but agree within 3% and 10% respectively with gas cap and freestream densities computed using the GASPROPS code. All these results strengthen the validity of the fits.

With a single CCD pixel flow field resolution of \( \sim 70 \mu m \), imaged bow shock thickness (~ 20 \mu m) cannot be resolved along the streamwise direction i.e. \( \rho(x) \). To compensate and as dictated by the experimental optical constraints imposed by the 31M10 facility, vertical line images are acquired normal to the freestream flow (fig. 1). Comparing x-axis streamwise (~ 0.02 mm) to z-axis fit shock thickness (3.5 mm normal shock and 2.5 mm oblique shock) in Table 1, vertical imaging “stretches” the imaged shock thickness by a factor of ~ 175 and ~125 for the normal and oblique shocks respectively. This allows observation of bow shock phenomena using the spatial resolution of typical CCD cameras.

It is concluded in this section and assumed for the remainder of this manuscript that all \( \rho(z) \) data along the laser line shown in fig. 1 is a spatially stretched version of \( \rho(x) \) in fig. 2. Stated another way, \( \rho(z) \) is proportional to or maps onto \( \rho(x) \) for near normal and oblique shocks for the blunt body configuration in fig. 1.
a) Near Normal Shock Wave

b) Oblique Shock Wave

Fig. 5. LRS Density Data and tanh(z) Fit
Near Normal Shock | Oblique Shock
---|---
P₀ (psi) | a₀ | b₀ | δ₀ | P₀ (psi) | a₀ | b₀ | δ₀
350 | - | - | - | 350 | 3.066 | 2.211 | 2.6831
651 | 5.498 | 4.052 | 3.5398 | 651 | 5.726 | 4.165 | 2.2366
651 | 5.559 | 4.014 | 3.6390 | 651 | 5.692 | 4.145 | 2.5278
902 | 8.196 | 5.849 | 3.9277 | 902 | 8.159 | 5.772 | 2.4913
902 | 8.001 | 5.651 | 3.5448 | 902 | 7.943 | 5.604 | 2.5511
1254 | 10.75 | 7.652 | 3.4483 | 1254 | 10.64 | 7.553 | 2.5833
1254 | 10.71 | 7.672 | 3.4758 | 1254 | 10.62 | 7.544 | 2.6281
1454 | 11.68 | 8.116 | 3.0497 | 1454 | 11.41 | 7.864 | 2.4201
1454 | 11.36 | 7.967 | 3.3014 | 1454 | 10.96 | 7.723 | 2.5947

Table 1. Near Normal and Oblique Shock Wave fit coefficients to tanh(z) function.

2. Effects of shock angle and stagnation pressure on z-axis shock thickness

Figure 6 plots computed shock thickness (δ₀ coefficient in equation 2 and Table 1) as a function of stagnation pressure for both the normal (black circles) and oblique (red squares) shocks. Based on results spanning 350-1254 psi, average δ₀ (normal shock) = 3.5959 ± 0.2537 mm (±1σ) and average δ₀ (oblique shock) = 2.5288 ± 0.1322 mm (±1σ). Data at P₀ = 1454 psi are plotted but not included in the averages since model motion effects discussed in section II.D affect these results. Straight lines at the average values are shown in Fig. 6. Results show δ₀ (normal) / δ₀ (oblique) = 1.42. Visual inspection of fig. 5 confirms δ₀ (normal shock) > δ₀ (oblique shock).

Shock thickness varies as the inverse shock pressure ratio i.e. 1 / (P₂-P₁) (equation 93.13 in [35]). For a constant freestream, P₂-P₁ is maximum for a normal shock and decreases as shock angle decreases (obliqueness increases). The relevant Mach number is normal to the shock(x-axis). For the locations probed in fig. 1, the shock is predicted to be thinnest where it is normal to the freestream velocity (in front of the model). As the bow shock curves backwards (away from and below from the model), shock angle increases, the normal Mach number decreases, the pressure difference decreases, and the shock thickness increases. If data in Fig. 6 were measured along the x-axis, oblique shock thickness would be greater than normal shock thickness.

Figure 6 data are measured along the z-axis and the opposite effect is observed. It is concluded that as shock obliqueness increases, the shock length along the z-axis is reduced. The latter effect dominates over the 1 / (P₂-P₁) argument. Unfortunately, Fig. 1 contains no data on the oblique shock angle to
verify this. Z-axis shock thickness at a given location on the shock is independent of stagnation pressure.

As Po increases from 350 to 1454 psi, the tunnel wall boundary-layer thickness decreases considerably. This changes the fluid dynamic shape of the nozzle. The Mach number increases by 2.7% from 9.71 to 9.97 [25]. If $P_1 = 1$ Torr, $P_2 - P_1$ changes from 109 to 115 i.e. 5.5%. This causes the local shock thickness to decrease. With errors of ~ 15%, data in Fig. 6 from 350-1250 psi are essentially independent of stagnation pressure and hence the corresponding Mach number. It is concluded that fit uncertainties (7%, ±1σ) are too large to observe the effect of Mach number variation on near normal or oblique shock thickness. Since this approach can detect differences in the z-axis shock thickness at different locations on the shock, results are useful for comparison with CFD predictions.

![Figure 6. Z-axis shock thickness ($\delta_0$ coefficient) versus stagnation pressure.](image)

D. Analysis of Standard Deviation Unsteadiness Spatial Profiles

1. Unsteadiness Mechanism 1

The time-averaged spatially-resolved unsteadiness profile from Fig. 4d at $P_0 = 1254$ psi (second dataset, red, $t = 18$ sec) spanning the freestream, near normal shock, and gas cap from $z = 0$ to $+20$ mm is shown in Fig. 7 (connected data points). Results are expressed as density standard deviation. For continuity,
this dataset will be analyzed and discussed in subsequent subsections. As with the data in fig. 5, it has been shifted using previous shift guesses and computed c-coefficient fits from equation 2. This results in a standard deviation maximum centered at z = 0.

Since no known single mechanism can create the spatial profile of fig. 7 and all data in fig. 4., it was concluded two unsteadiness mechanisms are operative [14]. To deconvolve these mechanisms and calculate their spatially-dependent relative magnitudes, four assumptions are required.

All data in this manuscript are time-averaged and spatially resolved. For the first assumption, time-averaged freestream unsteadiness under all stagnation conditions is spatially isotropic.

Freestream turbulent kinetic energy (TKE) or unsteadiness is amplified across a shock wave [21]. The physical mechanism is energy transfer from kinetic to potential modes of turbulence energy through acoustic fluctuations. Given a weak freestream disturbance (assumed 1% in [14]), a short distance for disturbance growth (< 3 mm on centerline in Fig. 1), and the low densities in a Mach 10 freestream, Ribner's linear interaction analysis (LIA) [15] can be applied to model post shock unsteadiness amplification for any given shock strength, incidence angle, and unsteadiness amplitude [18]. For the second assumption, total unsteadiness can be written as a linear combination of each individual unsteadiness mechanism. In other words, each mechanism operates simultaneously, has spatial profiles which are independent of each other, and are linearly additive.

Previous data show that gas cap unsteadiness (combined instrument and TKE) scales linearly with gas cap density (Fig. 10 in [14]) and hence freestream density and stagnation pressure. Therefore, LIA is known to apply to TKE unsteadiness passing through the post shock gas cap [21]. For the third assumption, LIA is assumed to apply to TKE unsteadiness passing through the bow shock. Therefore, as freestream unsteadiness is transported from the freestream, thru the bow shock, and into the gas cap, TKE unsteadiness grows in direct linear proportion to gas density in these regions. Figure 5 and Table 1 show the freestream, bow shock wave, and gas cap spatial density can be modelled as a tanh function (equation 2). Therefore, the spatial dependence of TKE amplification in the bowshock can also be modelled as a tanh function. As a consequence of this assumption, all freestream TKE unsteadiness (z ~ -8 to -10 mm), bow shock TKE unsteadiness (z ~ -8 to +8 mm) and gas cap TKE unsteadiness (z ~ 8 to 15 mm) in Fig. 7 is the result of a single unsteadiness mechanism and designated unsteadiness mechanism 1.

Based on these three assumptions, the spatial dependence of unsteadiness mechanism 1 created by TKE amplification is designated $\sigma_1(z)$ and can be written as follows.

$$\sigma_1(z) \sim \rho(z) = a_1 + b_1 \times \tanh \left( \frac{z}{\delta_1} \right)$$

Data in Fig. 7 contains insufficient information for fit convergence to equation 3 (solid line fit) with an accurate result for $\delta_1$. This problem is solved as follows. First, to compensate for a limited image length in Fig. 7 which inadequately samples freestream unsteadiness, previously measured spatially-averaged freestream unsteadiness data using LRS (fig. 2B in [14]) were inserted as red datapoints between z = -7.5 to -10 mm. These results are dominated by instrument noise and hence are independent of stagnation pressure. All values are 1.05. Errors are ± 5% based on the average of 60 spatial datapoints centered around the facility centerline [14]. Imaged length is
sufficiently long for a handful of datapoints near the edges of fig. 4e to achieve this value. Data in fig. 7 and all other data in fig. 4a-d approach it.

Since fig. 7 is a superposition of mechanisms 1 and 2, there is insufficient data between $z = -5$ to $+5$ mm for fits using equation 3 to extract an accurate representation of $\delta_1$. For the fourth assumption, unsteadiness width for mechanism 1 equals density shock thickness ($\delta_1 = \delta_0$). Since, in equation 3, $\sigma_1(z) \sim \rho(z)$, $\delta_0$ coefficient values computed from equation 2 fits and listed in Table 1 are used as fixed $\delta_1$ coefficient values in equation 3.

The solid line in Fig. 7 is a fit to tanh(z) using equation 3. The $a_1$ and $b_1$ coefficients map the tanh(z) function onto freestream ($z = -5$ to $-10$ mm) and gas cap ($z = +10$ to $+15$ mm) unsteadiness with fixed $\delta_1$ coefficient. In this fit, the $\delta_1$ coefficient represents unsteadiness width associated with $\sigma_1(z)$. Results of $a_1$ and $b_1$ fit parameters, reproduced values of $\delta_0$ from table 1, and $R^2$ values for all datasets in fig. 4 for which fit convergence were obtained are given in table 2. As in Table 1, sequential Po values represent data taken at sequential times ($t = 0$ and $t = 18$ sec) during a single facility run. Data in fig. 4 for $t=0$ data for $Po = 651$ and 902 psi and the near normal shock contained insufficient information for fit convergence. Typical correlation coefficient ($R^2$) values are $\geq 0.86$. 

![Fig. 7. Total Unsteadiness data and mechanism 1 $\sigma_1(z)$ fit (TKE amplification).](image)

Fig. 7. Total Unsteadiness data and mechanism 1 $\sigma_1(z)$ fit (TKE amplification).
Near Normal Shock | Oblique Shock
---|---
P0 (PSI) | a₁ | b₁ | δ₁ | R² | a₁ | b₁ | δ₁ | R²
651 | - | - | 3.54 | - | 1.19 | 0.14 | 2.24 | 0.67
651 | 1.14 | 0.094 | 3.64 | 0.68 | 1.14 | 0.09 | 2.53 | 0.68
902 | - | - | 3.93 | - | 1.43 | 0.38 | 2.49 | 0.88
902 | 1.42 | 0.419 | 3.54 | 0.91 | 1.44 | 0.39 | 2.55 | 0.87
1254 | 1.56 | 0.511 | 3.45 | 0.92 | 1.56 | 0.51 | 2.58 | 0.86
1254 | 1.56 | 0.521 | 3.48 | 0.98 | 1.57 | 0.43 | 2.63 | 0.94
1454 | 1.66 | 0.529 | 3.05 | 0.92 | 1.58 | 0.47 | 2.42 | 0.84
1454 | 1.62 | 0.574 | 3.30 | 0.88 | 1.57 | 0.53 | 2.59 | 0.91

Table 2. Fit parameters for total σ₁(z).

Using fit parameters in Table 2, results at P0 = 1250 psi (2nd dataset, t=18 sec) for the near normal shock (Fig. 4d) are presented as the black curve in Fig. 8 (total σ₁(z) from equation 3). This is identical to the fit line in Fig. 7. Since all Table 2 results are a combination of freestream TKE amplification (bow shock and gas cap) and instrument unsteadiness, results in Table 2 are designated total σ₁(z). A separation procedure is presented.

Coefficients from Table 1 are used to calculate the corresponding spatial static gas density curve for Fig. 8 at at P0 = 1250 psi. A previous manuscript [14] has shown the equation \( \sigma_{(instrument)} = 0.0626 \times \rho_{(static \ gas)} + 0.60 \) converts static gas density to instrument unsteadiness (red curve in fig. 8 labeled instrument σ₁(z)). Statistical analysis dictates standard deviations cannot be subtracted. Each curve (total σ₁(z) and instrument σ₁(z)) are first converted to their respective variances, the variances are subtracted, and the result is converted to standard deviation. Using a nonlinear least square fit, individual data points are fit to a tanh(z) function using equation 3 and presented as the green curve in fig. 8 (deconvolved σ₁(z)). The latter represents the deconvolved freestream TKE amplification across this shock wave. Since gas cap and freestream unsteadiness at any given stagnation pressure in Fig. 4 is essentially equal for both shocks, only one set is analyzed using normal shock data. Since sequential datasets for a given Po are similar, they are used to test reproducibility. Results are presented in Table 3. Correlation coefficient (R²) values are ≥ 0.9. Since total σ₁(z) at z = -5 mm is within 1.5% of freestream total σ₁(z) at z= -10 mm (Fig. 7), this small correction to the results was ignored.

Since tanh (z > 5 mm) = 1, a₁+b₁ column in Table 3 represents gas cap unsteadiness with the instrument unsteadiness removed. Values for a+b increase ~10% over the range of stagnation pressures (Po ~50% increase) listed. Errors in a+b values are estimated at 17% at P0 = 902 psi and decrease slightly with increasing P0. Although not plotted, deconvolved σ₁(z) are linear with P0. For any given pressure, a+b increases ~3% for successive data (t=0 vs t=18 sec). This is attributed to model motion causing the laser to probe closer to
bow shock and sample higher unsteadiness due to bow shock oscillation as shown in Fig. 7.

Fig. 8. Total, instrument, and total-instrument unsteadiness amplification for $\sigma_1(z)$.

<table>
<thead>
<tr>
<th>$P_0$ (PSI)</th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$\delta_1$</th>
<th>$R^2$</th>
<th>$a_1+b_1$</th>
<th>$a_1-b_1$</th>
<th>$(a_1+b_1)/(a_1-b_1)$</th>
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</thead>
<tbody>
<tr>
<td>902</td>
<td>0.905</td>
<td>0.162</td>
<td>2.49</td>
<td>0.90</td>
<td>1.07</td>
<td>0.743</td>
<td>1.44</td>
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<td>902</td>
<td>0.901</td>
<td>0.171</td>
<td>3.54</td>
<td>0.99</td>
<td>1.12</td>
<td>0.684</td>
<td>1.63</td>
</tr>
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<td>0.99</td>
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<td>0.690</td>
<td>1.60</td>
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<td>3.46</td>
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<td>1.13</td>
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</tr>
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<td>0.198</td>
<td>3.05</td>
<td>0.99</td>
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</tr>
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<td>0.944</td>
<td>0.290</td>
<td>3.30</td>
<td>0.99</td>
<td>1.23</td>
<td>0.654</td>
<td>1.89</td>
</tr>
</tbody>
</table>

Table 3. $\sigma_1(z)$ unsteadiness amplification coefficients (total minus instrument).
Values for \(a_1-b_1\) represent deconvolved freestream unsteadiness. Since instrument errors contribute a large fraction of total \(\sigma_1(z)\), errors in deconvolved freestream turbulence are estimated at 25%.

Values for \((a_1+b_1)/(a_1-b_1)\) in Table 3 represent the amplification of TKE across the bow shock. McKenzie and Wesphal [17] computed a value of 2.9 for the ratio of unsteadiness amplitudes across a density jump for a sound wave at Mach 11.3 and normal incidence. Average results in Table 3 are 1.62 ± 0.16 or approximately half the expected value of 2.9. Since Mach number varies < 1.5% [25] over this Po range, Mach number variation cannot explain the discrepancy in ratios. The discrepancy in ratios is attributed to the freestream unsteadiness measurement \((a_1-b_1)\) being dominated by instrument noise. If true freestream unsteadiness is ~ 0.4 versus the Table 3 result of ~ 0.7, good agreement with the expected value would be obtained. Hence, Table 3 ratios represent a lower limit to unsteadiness amplification across this shock.

In essence, Table 3 shows freestream unsteadiness amplification across a shock wave can be measured using LRS. This exploratory investigation is used to show the potential of LRS instrumentation in what is believed to be the first such molecular measurement of the spatial profile of TKE amplification across a shock. Data with higher signal-to-noise ratio (reduced instrument unsteadiness) will be required for more accurate results.

2. Unsteadiness Mechanism 2

A. Mechanism

Inspection of fig. 7 indicates a second unsteadiness mechanism \(\sigma_2(z)\) is active. To determine the spatial dependence of \(\sigma_2(z)\), the computed function for total \(\sigma_1(z)\) using table 2 is subtracted from the total spatial unsteadiness data \((\sigma_{\text{total}}(z)\) in Fig. 7 at Po = 1254 psi at t = 18 sec) using equation 4. Procedures for subtracting standard deviations were detailed in section III.D.1. Resulting datapoints \(\sigma_2(z)\) are presented in Fig. 9.

\[
\sigma_2(z) = \sigma_{\text{total}}(z) - \sigma_1(z)
\]

To determine the mathematics responsible for fig. 9, the following observations are presented. Fig. 9 shows that \(\sigma_2(z)\) is maximum at the infection point of tanh\((z)\) in Fig. 7 and all data in Fig. 4 where d \((p(z)) / dz\) is maximum. Also, \(\sigma_2(z)\) is both minimal and constant in the gas cap and freestream where d \((p(z)) / dz = 0\) and d \((\sigma_1(z)) / dz = 0\). LRS measures number density. Number density spatial derivative is the only known variable capable of producing Fig. 9 results.

A simplistic visual explanation using fig. 5b can explain the relative spatial unsteadiness in fig. 9. Given that bow shock number density is described by tanh, assume that a time averaged freestream disturbance impinging on the bow shock causes the entire tanh spatial profile to oscillate over a fixed small distance \((\Delta z)\) at all
spatial locations. Consider three locations in fig. 5b to include the gas cap at \( z = 10 \) mm, density second derivative at \( z = 3 \) mm and density first derivative at \( z = 0 \) mm.

For this flow field, it is assumed that \( \Delta z \) is small and small when compared to the abscissa range of this tanh function. For a blunt-body model with convex surface at Mach 7, Vashishtha et al. [13] observed \( \Delta z \) to be typically small; large \( \Delta z \) was observed in \( \sim 1\% \) of the data. Large \( \Delta z \) were correlated with and attributed to particle strikes. For the current Mach 10 flow, no particles were observed. LRS observed \( \Delta z \) is small for > 95\% of the results[14]. Large \( \Delta z \) variations observed were attributed to infrequent large-scale freestream unsteadiness.

Since \( \Delta z \) is small, density unsteadiness for the tanh function in figure 5 will scale as the local spatial density derivative \( \frac{dp}{dz} \). In the gas cap at \( z = 10 \) mm, density is nearly spatially invariant producing \( \frac{dp}{dz} \sim 0 \) and hence minimum unsteadiness. For a given small \( \Delta z \), the slope at the first derivative (\( z = 0 \) mm) is greater than the slope at the second derivative (\( z = 3 \) mm) is greater than the slope in the gas cap. Since tanh is continuous and can be differentiated at all \( z \), unsteadiness will decrease continuously from the inflection point to the gas cap. By symmetry, it will also decrease from the inflection point towards the freestream. These results are observed for all stagnation conditions, at all shock locations measured in fig. 4, and in detail in fig. 7.

Therefore, the following physical mechanism is responsible for Fig. 9 results. Hypersonic freestream disturbances impact the bow shock and momentum transfer causes the stagnation-region bowshock density described by \( \tanh(z) \) in equation 2 to oscillate upstream and downstream. The regions of highest density gradient produce the regions of highest density unsteadiness. Therefore, the second unsteadiness mechanism designated \( \sigma_2(z) \) in Fig. 9 is attributed to bow shock wave motion. The spatial dependence of \( \sigma_2(z) \) is fit to the first derivative of \( \rho(z) \) from equation 2 or \( \frac{d}{dz} \left( \tanh(z) \right) \). This produces a \( \sech^2(z) \) function in equation 5.

\[
\sigma_2(z) \sim \frac{d \rho(z)}{dz} = a_2 + b_2 \cdot \sech^2 \left[ \frac{z}{\delta_2} \right]
\]

By strict definition, \( \frac{d \rho(z)}{dz} \) would produce \( a_2 = 0 \). In equation 5, \( a_2 \) is added to allow the \( \sigma_2(z) \) fits to account for possible unsteadiness in the freestream and gas-cap not accounted for by \( \sigma_1(z) \). The \( b_2 \)-coefficient is peak unsteadiness at the maximum density gradient i.e., inflection point at \( z = 0 \) mm. In this fit, the \( \delta_2 \) coefficient represents unsteadiness width (full width at half maximum) associated with \( \sigma_2(z) \). The resulting nonlinear least squares fit is shown as the solid line in Fig. 9. Correlation coefficient (R²) value for fig. 9 is 0.99. Results of fitting all data in Fig. 4 using this approach are presented in Table 4. All R² results in Table 4 are ≥ 0.93.

All \( a_2 \) coefficients are effectively zero. This represents the density gradient induced by unsteadiness far from the bow shock density inflection point (\( z=0 \)) in both the freestream and the gas cap which is expected to be essentially zero. Hence \( \sigma_2(z) \) unsteadiness is confined to the bow shock wave. \( \sigma_2(z) \) makes no contribution to freestream (\(< -10 \) mm) or gas cap (\( > +10 \) mm) unsteadiness where the spatial density derivative is essentially zero. Primary unsteadiness contributions to the freestream and gas cap are contained within the \( \sigma_1(z) \) mechanism. This was one of the assumptions in section III.D.1 above. High R² values in Table 4 and lack of any visually-apparent spatially-unfitted data trends in Fig.9 indicate no third unsteadiness mechanism is active.
Fig. 9. Unsteadiness mechanism 2 data and computed sech²(z) fit (bow-shock oscillation)

Table 4: Fit parameters for σ₂(z).

<table>
<thead>
<tr>
<th>Po (PSI)</th>
<th>Near Normal Shock</th>
<th>Oblique Shock</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a₂</td>
<td>b₂</td>
<td>δ₂</td>
</tr>
<tr>
<td>651</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>651</td>
<td>-0.018</td>
<td>2.32</td>
<td>4.14</td>
</tr>
<tr>
<td>902</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>902</td>
<td>0.000</td>
<td>3.24</td>
<td>4.04</td>
</tr>
<tr>
<td>1254</td>
<td>0.055</td>
<td>4.21</td>
<td>3.81</td>
</tr>
<tr>
<td>1254</td>
<td>0.010</td>
<td>4.25</td>
<td>4.31</td>
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<td>0.017</td>
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</tr>
<tr>
<td>1454</td>
<td>-0.135</td>
<td>4.67</td>
<td>5.25</td>
</tr>
</tbody>
</table>
B. Analysis of $b_2$ and $\delta_2$ coefficient values for $\sigma_2(z)$

Fitted $b_2$-coefficient values from Table 4 for $\sigma_2(z)$ for the near normal (squares) and oblique (circles) shock waves are plotted as a function of stagnation pressure in Fig. 10. The $b_2$-coefficients represent maximum bow shock wave amplitude unsteadiness expressed as standard deviations at the density inflection point or density first derivative or $z = 0$ in Fig. 9. Data at $P_0 = 1454$ psi ($t = 18$ sec) are not included in the fits for model motion reasons discussed previously [25]. Slopes and correlation coefficients obtained using a linear least squares fit are shown in Fig. 10. Each fit was obtained assuming an intercept of zero since turbulence is defined as zero at zero pressure and hence zero flow.

Since $b_2$-coefficients are deconvolved from total unsteadiness, $b_2$-coefficients are not affected by instrument unsteadiness. Since no Schlieren images are available for all conditions, $b_2$-coefficient values are uncorrected for shock angle variation with increasing stagnation pressure and different shock locations probed due to model motion.

Both near normal and oblique shock deconvolved b-coefficient amplitudes and therefore peak $\sigma_2(z=0)$ amplitudes are linearly dependent on stagnation pressure and hence freestream density. Assuming linear wave theory [17], the frequency and amplitude of shock oscillations should correspond to the frequency and amplitude of the incident flow disturbances. It is concluded that each of these amplitudes is a direct measure of and scaling linearly with time-averaged facility freestream turbulence. Also, freestream turbulence must be increasing linearly with stagnation pressure and freestream density.

The ratio of slopes ($b_2$ normal / $b_2$ oblique) is 1.39. This is consistent with previous observations that shock unsteadiness amplitude is shock angle dependent being maximum for the near normal shock and reduced for the oblique shock [16]. Since the Schlieren image in fig. 2 and Rayleigh data were not acquired simultaneously, there is uncertainty regarding the location of the laser beam relative to the bow shock. The result is uncertainty determining the near normal shock angle. Figure 1 contains no information on oblique shock angle. Therefore, this ratio from measured shock angles cannot be obtained to confirm Fig. 10 slope ratio. Future planar LRS density measurements are recommended to acquire this information.

Increasing stagnation pressure will modestly flatten the near normal shock. Correlation coefficients in Table 4 indicates this effect on b-coefficients is negligible for the near normal shock data. Although fit to a straight line, visual inspection of and correlation coefficient for the oblique shock data suggests data may be curved. If true, this curvature is attributed to significant stagnation-pressure-induced bow-shock-angle variation. If Schlieren images were available to correct b-coefficients for shock angle, the author believes all $b_2$-coefficients for both the near normal and oblique shock at each stagnation pressure would be identical.
All data in table 4 are acquired off the model centerline. Maximum unsteadiness is expected for a normal shock at the sting / model centerline. Therefore, table 4 represent lower limits to both facility freestream turbulence and bow shock oscillation amplitudes.

In equation 5, the $\delta_2$ coefficient represents unsteadiness width associated with $\sigma_2(z)$. Table 4 shows computed unsteadiness width ($\delta_2$ coefficient) as a function of stagnation pressure for both the normal and oblique shocks. Based on results spanning 650-1254 psi, $\delta_2$ (normal shock) = 4.08 ± 0.21 (±1σ), $\delta_2$ (oblique shock) = 3.44 ± 0.84 (±1σ), and $\delta_2$ (normal) / $\delta_2$ (oblique) = 1.19 for $\sigma_2(z)$. To first order, unsteadiness widths are independent of stagnation pressure and shock angle within the errors of this experiment. To second order, inspection of $\delta_2$ coefficients in Table 4 shows a weak linear decrease with increasing stagnation pressure for both the near normal and oblique shock. LRS is observing weak stagnation-pressure-dependent changes in the shock shape which compress the oblique relative to the normal shock $\delta_2$ coefficients along the z-axis.

Previous results obtained with physical probes measuring unsteadiness behind a bow shock in a gas cap conclude freestream unsteadiness (due presumably to TKE amplification) exponentially decreases with increasing stagnation pressure [36,37]. Offbody molecular-based measurements in figure 10 based on peak bow shock oscillation measurements (z=0) shows freestream unsteadiness linearly increases with increasing stagnation pressure. A similar albeit weak linear trend is observed for the deconvolved unsteadiness amplification coefficients for $\sigma_1(z)$ in Table 3 in the gas cap with increasing stagnation pressure. Further study is required to resolve these discrepancies.
Fig. 10. Bow shock peak unsteadiness versus stagnation pressure
3. Total Unsteadiness – Mechanisms 1 + 2

A. Data and Fits

Based on results in this manuscript, the time-averaged spatial unsteadiness profiles in Fig. 4 spanning the freestream, bow shock wave, and gas cap consists of two mechanisms. Computed fits for each mechanism (total \( \sigma_1(z) \) from Table 2 and \( \sigma_2(z) \) for Table 4 at \( P_0 = 1254 \text{ psi and } t = 18 \text{ sec} \)) are shown in Fig 11A. Total unsteadiness \( \sigma_{\text{total}}(z) \) is written as a linear combination of two independent and simultaneous unsteadiness mechanisms given by equation 6.

\[
\sigma_{\text{total}}(z) = \sigma_1(z) + \sigma_2(z) = a_1 + b_1 \cdot \tanh \left[ \frac{z}{\delta_1} \right] + b_2 \cdot \text{sech}^2 \left[ \frac{z}{\delta_2} \right]
\]

Experimental data points and computed \( \sigma_{\text{total}}(z) \) line from fit parameters in Tables 2 and 4 are shown in Fig. 11B. \( R^2 \) value is 0.99. Similar \( R^2 \) fit results are obtained for all data in Fig 4. This indicates good agreement between experimental data and the assumptions stated to analyze this data. Procedures for combining standard deviations were detailed in section III.D.1.

Figure 11A shows the primary stronger unsteadiness \( \sigma_2(z) \) is bow shock wave oscillation. This dominates across the bow shock from \( z = -3 \) to \( +2.5 \text{ mm} \) where density derivatives are maximum. The fractional contribution of \( \sigma_2(z) \) to the freestream \( (z < -10 \text{ mm}) \) and gas cap \( (z = +10 \text{ to } +15 \text{ mm}) \) unsteadiness is essentially zero; density gradients are negligible at these locations. Secondary weaker unsteadiness \( \sigma_1(z) \) is the result of freestream unsteadiness being amplified as it transitions from the freestream, though the near normal shock and into the gas cap. Its effect scales in direct proportion to local density. Each mechanism is spatially independent of the other and occur simultaneously.

For comparison with data, Fig. 11a uses total \( \sigma_1(z) \) whose freestream is dominated by instrument unsteadiness. Deconvolved \( \sigma_1(z) \) reduces total \( \sigma_1(z) \) in Fig. 11A by nearly a factor of 2. When compared to deconvolved \( \sigma_1(z) \), \( \sigma_2(z) \) unsteadiness will dominate across nearly the entire bow shock wave in Fig. 11B and all data in Fig. 4 from \( z=-5 \) to \( z=+5 \text{ mm} \).

Figure 11A displays significant symmetry. \( \sigma_2(z) \) is rotationally symmetric around \( z=0 \). \( \tanh(z) \) which forms \( \sigma_1(z) \) has a symmetry about \(+1 \) and \(-1 \). Since \( \sigma_2(z) \) is dominant, total bow shock unsteadiness has significant symmetry.

Tables 2 and 4 allow comparison of fitted bow shock unsteadiness widths (\( \delta \) coefficient values) for each mechanism for the near normal and oblique shock. For the normal shock \( \delta (\sigma_1(z)) = 3.60 \pm 0.18 \) and \( \delta (\sigma_2(z)) = 4.08 \pm 0.21 \). For the oblique shock \( \delta (\sigma_1(z)) = 2.50 \pm 0.14 \) and \( \delta (\sigma_2(z)) = 3.44 \pm 0.84 \). Errors represent \( (\pm 1 \sigma) \). Comparing \( \delta (\sigma_1(z)) \) to \( \delta (\sigma_2(z)) \) at each shock angle, these values are identical within errors of \( \pm 2\sigma \). Therefore, with the listed assumptions, bow shock unsteadiness width is independent of stagnation pressure and unsteadiness mechanism at the locations probed within the measurement uncertainties.
Fig. 11. A) Unsteadiness mechanisms fits
B) Total unsteadiness data and tanh(z) + sech^2(z) fit
Freestream unsteadiness at hypersonic speeds impacts the bow shock wave and forces it toward the model. The incompressible and subsonic gas cap relieves this local and essentially instantaneous pressure increase by moving fluid in all directions by collisional transfer but mostly away from the model. The observed time-averaged shape and symmetry of mechanism 2 is the result of these two forces. In contrast, turbulent kinetic energy transfer to the density field by mechanism 1 near the bow shock density inflection point (z=0) is much weaker. Mechanism 1 cannot force fluid upstream. Mechanism 2 contributes a small fraction of the total unsteadiness observed in the gas cap which decays exponentially with the extent of penetration.

Figure 11A shows unsteadiness mechanism 2 (σ2(z) - bow shock unsteadiness) created by freestream unsteadiness can contribute to time-averaged bow shock wave thickness. Since all bow shock thickness measurements in the literature were performed using time averaged methods, all bow shock thickness measurements should be considered upper limits to true bow shock thickness. Statistically, unsteadiness results in this study are identical within experimental errors. However, assuming δ(σ1(z)) = 3.60 (ie unsteadiness width from mechanism 1 was assumed equal to shock thickness from density curves) and δ(σ2(z)) = 4.08 (shock unsteadiness thickness from sech² fits) and near normal shock results represent the true values, results crudely suggest ~ 10-20% of time-averaged bow shock thickness is due to freestream unsteadiness in the 31M10 facility.
B. Relative Mechanism Coupling Efficiencies of Freestream Unsteadiness

Table 5 compares the coupling efficiency of freestream unsteadiness energy into each individual mechanism at the location of maximum unsteadiness for each individual mechanism. This occurs in the gas cap for $\sigma_1(\text{gas cap})$ and the bow shock number density inflection point at $z=0$ for $\sigma_2(z)$. Using equations 3 and 5, maximum unsteadiness is given as deconvolved $\sigma_1(\text{gas cap}) = a_1 + b_1$ and $\sigma_2(z=0) = b_2$. Data are taken from Tables 3 and 4.

Peak unsteadiness and relative unsteadiness ratios $\sigma_2(z=0) / \sigma_1(\text{gas cap})$ are listed in Table 5 for both near normal and oblique shocks. It is noted that all flow field state variables ($T, \rho, V, P$) are different at the point of maximum unsteadiness for each mechanism. Since $\text{sech}^2(z=0) = \text{tanh}(\text{gas cap}) = 1$, the amplitude ratio is independent of the mathematical functions associated with each mechanism.

<table>
<thead>
<tr>
<th>Po (PSI)</th>
<th>$\sigma_1(\text{gas cap})$</th>
<th>$\sigma_2(z=0)$</th>
<th>$\sigma_2 / \sigma_1$</th>
<th>$\sigma_2(z=0)$</th>
<th>$\sigma_2 / \sigma_1$</th>
</tr>
</thead>
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<tr>
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<td>1.07</td>
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<td>-</td>
<td>2.26</td>
<td>2.12</td>
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<tr>
<td>1454</td>
<td>1.23</td>
<td>4.67</td>
<td>3.78</td>
<td>3.98</td>
<td>3.23</td>
</tr>
</tbody>
</table>

Table 5. Unsteadiness Coefficient Peak Amplitudes and Lower-Limit Ratios

Since LRS data represent time averaged results, this analysis assumes time-averaged 3D isotropic freestream turbulence. Assuming LIA theory, computed unsteadiness amplitudes at any given point in the bow shock and gas cap should each be proportional to freestream turbulence. As concluded in section III.D.2.B above, freestream unsteadiness increases in direct proportion to freestream pressure. Therefore, unsteadiness amplitude at the location of maximum unsteadiness should increase linearly with stagnation pressure for both the near normal and oblique shock data.
Data in table 5 is not consistent with these expectations. It shows the ratio increasing with increasing Po. This is attributed to errors in $\sigma_1$ (gas cap) due to fit errors in the a coefficient which is heavily affected by instrument noise. Therefore, since unsteadiness is maximum at Po = 1454 psi and results are expected to have the least error at Po = 1454 psi, the only conclusion from table 5 is that the ratio at Po = 1454 psi represent lower limits to the ratio $\sigma_2(z=0) / \sigma_1$ (gas cap) at the locations stated for each shock location probed. It also supports the assumption that each mechanism is independent.

To further understand these independent unsteadiness amplification mechanisms, consider the unsteadiness for each mechanism at z=0. At a single point in space, all flow-field state parameters are common to both mechanisms. This includes all flow field state variables (T, $\rho$, V, P) and freestream unsteadiness of a given value. Since $\tanh(z=0) = 0$, $\sigma_1(z=0) = a$. Computed values for a which represent $\sigma_1(z=0)$ are taken from Table 3. Essentially, this value is 0.9 independent of stagnation pressure. Hence, ratios in Table 5 are increased by ~1.1/0.9 or ~20% at z=0. As above, this is due to a large instrument error for freestream unsteadiness. Assuming the TKE amplification ratio of 2.9 discussed above is correct, freestream unsteadiness would be 0.38 at 1254 psi, $\sigma_1(z=0) = 0.74$ and $\sigma_2(z) / \sigma_1(z) = 5.7$. This ratio is only ~50% greater than the results in Table 5. Overall, this suggests momentum transfer from freestream turbulence to the bow shock (bow shock oscillation) at the location of max density gradient produces ~5 ± 1 times greater unsteadiness than TKE amplification at the z=0 spatial location.
4. Explanation of Relative and Product Standard Deviation Spatial Profile Correlations

A previous manuscript [14] has shown that peak values of relative standard deviation \(\sigma_{\text{total}}(z) / \rho(z)\) and product standard deviation \(\sigma_{\text{total}}(z) \cdot \rho(z)\) correlate with local maximum and local minimum values of the second derivative profile of \(\rho(z)\). The second derivative of \(\rho(z)\) is given by equation 7.

\[
d^2 \rho(z) / dz^2 = a^* \tanh (z / \delta) \cdot \text{sech} (z / \delta) \tag{7}
\]

Fig. 11 shows that \(\sigma_{\text{total}}(z)\) is dominated by \(\sigma_2(z)\) and hence \(\sigma_{\text{total}}(z) \sim \text{sech}^2(z)\). For brevity in this section, \(\sigma_{\text{total}}(z)\) will be written \(\sigma(z)\). Figure 12 plots computed bow shock wave density second derivative (tanh(z) * sech(z), equation 7, black data) versus \(\sigma(z) / \rho(z) \sim \text{sech}^2(z) / \rho(z)\) (green data) and \(\sigma(z) \cdot \rho(z) \sim \text{sech}^2(z) \cdot \rho(z)\) (red data). Sech^2(z) is computed from parameters in table 3 for Po = 1454 psi (t=0 dataset). \(\rho(z)\) is the corresponding experimental data. Figure 11 shows that multiplying computed unsteadiness (\(\sigma(z) \sim \text{sech}^2(z)\)) by spatial gas cap density \(\rho(z)\) shifts the sech^2(z) function so its maximum corresponds to the local maximum of \(d^2 \rho(z) / dz^2\) near \(z = +14\) while sech^2(z) \(\cdot \rho(z)\) shifts the sech^2(z) function so its maximum corresponds to the local minimum of \(d^2 \rho(z) / dz^2\) near \(z = +16\) mm. Peak correlations in Fig. 11 cannot be derived rigorously from mathematics. Instead, it appears that multiplication and division of \(\sigma(z)\) by \(\rho(z)\) fortuitously shifts peak \(\sigma(z)\) to correspond approximately to peak second derivative spatial locations of \(\rho(z)\).

![Fig. 12. Peak location comparisons - d2 ρ(z) / dz2, σ(z) * ρ(z), and σ(z) / ρ(z).](image)
IV. Conclusions

This study demonstrates the first application of laser Rayleigh scattering to observe, quantify, and provide mechanisms along with mathematics to explain and model tunnel-noise-induced bow shock and gas cap oscillations at Mach 10. Time-averaged spatially-resolved density and density unsteadiness is observed along a line. This approach samples flow in the freestream, through the bow shock, and in the gas cap ahead of a blunt body model. Both near normal shock and oblique shock data are acquired simultaneously. Line data are acquired vertically along the z-axis. Fits of the z-axis spatial density profile to a tanh(z) function demonstrate the density profile along the z-axis is stretched version of the standard streamwise or x-axis density profile. Z-axis imaging effectively expands the bow shock wave and allows observation of relevant features within the spatial resolution of typical intensified camera instrumentation.

Unsteadiness is expressed as density standard deviation. Amplitude not frequency results are presented. Two different unsteadiness mechanisms are observed as flow passes from the freestream, through the bow shock and into the gas cap. Both observed unsteadiness mechanisms are initiated by freestream turbulence. The first mechanism and lower amplitude effect is the well-known amplification of turbulent kinetic energy as flow transitions from the ~1500 m/s freestream to the stagnation region. It increases in direct linear proportion to gas density in the bow shock and gas cap. It is described by a tanh(z) function. The second mechanism and higher amplitude effect is physical oscillation of the bow shock. It scales with the spatial first derivative of density and is described by a sech^2(z) function. Both mechanisms are shock-angle dependent. Total unsteadiness (fit using tanh(z) + sech^2(z)) is expressed as number density standard deviation and modeled as a linear combination of the latter two independent, simultaneous, and nonlinear unsteadiness mechanisms.

Since both mechanisms are driven by common freestream turbulence, the ratio of energy coupling into each mechanism at any given spatial location is constant. Total amplitudes at any spatial location will increase with increasing freestream turbulence, freestream density, and stagnation pressure. Still, different results are observed as a function of spatial location. This indicates each mechanism is spatially independent of the other, occur simultaneously, are modelled using different mathematical functions, increase linearly with freestream turbulence, and are linearly additive. At the bow shock location where the spatial density derivative is maximum and hence mechanism 2 is maximum, the ratio of their standard deviation amplitudes (mechanism 2 / mechanism 1) is estimated to be 5 ± 1. Laser Rayleigh scattering permits quantification of spatially resolved unsteadiness, unsteadiness amplification across the shock, dual mechanisms, along with amplitude and width of unsteadiness associated with each mechanism.

This manuscript quantifies relationships between unsteadiness mechanism coefficients and various flow field and tunnel parameters. Bow shock
oscillation amplitude is linearly correlated with freestream turbulence, freestream
density and stagnation pressure. Assuming a linear amplification model of
transition and turbulence, time-averaged bow shock oscillation amplitude is a
new, nonintrusive, offbody, and direct measure of time-averaged freestream
turbulence. Instantaneous images show instantaneous turbulence levels.

Assuming time-averaged isotropic freestream unsteadiness, each
streamline impinging on a hypersonic bow shock has an unsteadiness spatial
profile described by the sum of a sech² and tanh function. Maximum oscillation
in this flow field is due to bow shock oscillation ie sech² function. The
unsteadiness spatial profile amplitude across the bow shock wave is driven by
the local shock angle which affects the density spatial derivative. Therefore, a
curved bow shock wave exhibits a unique localized unsteadiness amplitude
which varies spatially with shock angle along the entire shock front. For time-
averaged unsteadiness, the bow shock converts isotropic freestream
unsteadiness into anisotropic bow shock unsteadiness. At hypersonic speeds,
different parts of the bow shock cannot communicate via collisions; this maintains
the bow shock spatial anisotropic unsteadiness.

Logically, one would expect the nonuniform bow shock spatial
unsteadiness to move into the gas cap to create a time-averaged unsteadiness
profile which varies spatially with shock angle. Counterintuitively, time-averaged
gas cap unsteadiness within the errors in this study is nearly spatially uniform,
approximately independent of bow shock angle and local oscillation amplitude,
and increases linearly with local density. Hence, the gas cap to first order
recreates the time-averaged isotropic unsteadiness of the freestream. The
magnitude of gas cap unsteadiness is attributed primarily to amplification of
time-averaged freestream turbulent kinetic energy across the shock. The mathematics of the
sech² function show that the time-averaged bow-shock oscillation amplitude
decays exponentially with increasing gas-cap penetration. Therefore, time-
averaged bow shock oscillation is a spatially localized phenomena. Given a
sufficient distance behind the bow shock, time-averaged bow shock
unsteadiness has essentially no effect or at best a minimal secondary effect on
time-averaged gas cap unsteadiness. The effect decreases from the bow shock
maximum density derivative toward the model surface as a sech² function and
becomes zero at locations in the gas cap where the density derivative is zero.

Assuming time-dependent anisotropic unsteadiness, each streamline
impinging on the hypersonic bow shock will have a unique unsteadiness
character. Localized instantaneous strikes push the bow shock towards the
model surface and create instantaneous gas cap unsteadiness. In the stagnated
subsonic gas-cap flow, there is a damping mechanism whereby localized
unsteadiness is partially dispersed by gas collisional energy transfer. This
process will reduce localized anisotropic unsteadiness (density waves) induced
in the gas cap from bow shock oscillations but not eliminate time-dependent
unsteadiness.

Phenomenologically, this study shows freestream turbulence impacts the
bow shock and creates a physical movement. A simple momentum transfer
argument from the 1500 m/s freestream to the near zero velocity gas cap
supports the experiment result. Therefore, freestream turbulence impacts contribute to time-averaged bow shock thickness. All literature measurements of bow shock thickness are based on time-averaged measurements. Bow shock thickness measurements at a given Mach number in different facilities will be affected by the facility-dependent-level of freestream turbulence. Therefore, all current measurements of bow shock thickness are upper limits to true bow shock thickness. In future, all bow shock wave thickness measurements should be obtained in freestreams with minimal unsteadiness using high-spatial-resolution instantaneous molecular-based methods.

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