Exploration of Failure and Potential Damage Markers in Ti-6Al-4V

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Summary

To improve prognostics for predicting component life, coupon tests were conducted on Ti-6Al-4V to investigate the development of damage. Various test types were employed (tension, creep, stress relaxation, and fatigue) under a wide range of temperatures and loading rates to engage various amounts of time-dependent and damage behavior. Both stiffness degradation and Poisson’s ratio were monitored to see if they were useful in indicating the initiation and accumulation of damage and subsequently the onset of failure. Stiffness exhibited large decreases with increasing strain when calculated using engineering stress and strain. However, minimal stiffness change was observed when using the true-stress and true-strain values. Detailed microscopy and image analysis were performed to document the actual physical damage that occurred within the various specimens. Two types of damage were observed: shallow surface cracks and internal pores. The severity of damage was greater in the region of localized necking and diminished further away from the neck. The area fraction of pores was higher in the necked region, yet even there it did not exceed more than a few percent and, in most cases, was significantly lower. It was observed that conditions that induced more time-dependent deformation (e.g., slower strain rate testing) resulted in a larger number of pores and larger pore sizes. The pores were always found to occur in the softer β-phase. The severity of observed damage was inconsistent with the large drop in engineering stiffness, but was more aligned with stiffness changes from the true-stress and -strain values.

Poisson’s ratio was monitored on every test and showed promise as a damage indicator. It was found to be more sensitive than either the ultimate tensile strength or the inception of tertiary creep. However, calculation of Poisson’s ratio was found to be more involved than expected because of anisotropy and the presence of strain gradients along the specimen at large applied strains. First, the material had a moderate texture and resulted in anisotropy of the strains. Second, the evolution of Poisson’s ratio as a function of axial strain showed strange and unexpected behaviors, which necessitated more extensive analyses of its development. This led to an investigation on the use of small versus large strain approaches and the importance of making consistent measurements; that is, axial and transverse strain measurements over the same volume of material. Additionally, finite element analysis of a tensile sample indicated that non-uniform strain occurred in the sample prior to attaining the maximum strength. This, along with the location at which transverse strain was measured, had a significant influence on the resulting values of Poisson’s ratio. Based on this, lessons learned are offered for future testing.

Introduction

Predicting component life is critical to aircraft safety. Life prediction schemes account for both deformation and damage progression during the service life of the part and dictate both inspection intervals and part replacement. Damage is often thought of as occurring when deformation reaches a critical stage, resulting in the formation of voids and cracks. The damage could be isolated to a singular location, perhaps as a result of a geometric, microstructural, or load singularity, or it could be more uniform and widely distributed throughout the part. As the amount of damage increases, a point can occur where the remaining material ligaments within the part are no longer able to withstand the applied load.
and failure occurs through either a reduction in its designed operation or catastrophic failure of the part. A diagnostic system could monitor component conditions, and coupled with a good prognostic’s scheme, predict the criticality of the condition-of-state and provide the correct course of action to maximize part usage and ensure safe system operation. As part of NASA’s IVHM (Integrated Vehicle and Health Management) program a prognostic project is advancing a viscoelastoplastic model with damage and being characterized for the titanium alloy Ti-6Al-4V (Ti-6-4).

Ti-6-4 is widely used in the aeronautics industry and experiences time-dependent deformation at elevated temperatures. Previous work by Lerch and Arnold (2014 and 2016) has thoroughly investigated the deformation behavior of this alloy over the temperature range 20 to 538 °C. It was discovered that Ti-6-4 experienced both viscoelastic and viscoplastic deformation. At low stresses deformation was viscoelastic (time dependent, nonlinear, and completely reversible, given sufficient time). This behavior occurred below a uniaxial threshold stress (\(Y\)) that delineated the boundary between reversible and irreversible deformation. This threshold was noted to be below, but in the range of, the proportional limit of the material for very slow rate loadings (e.g., \(1 \times 10^{-6} \text{ s}^{-1}\) total strain rate). For stresses above the threshold the material deformed by both viscoelastic and viscoplastic mechanisms. The majority of the deformation, particularly at larger strains, was of the irreversible (permanent), time-dependent nature. However, there was still a smaller amount of reversible, or recoverable, deformation that occurred. In this viscoplastic regime deformation was found to be highly dependent on the test temperature, the loading rate, and the prior deformation history. A large and detailed database was generated to allow characterization of deformation constitutive models. In particular, the advanced model GVIPS (generalized viscoplasticity with potential structure) developed by Saleeb and Arnold (2001 and 2004) handles both viscoelastic and viscoplastic deformation and has been extended to include material damage (Saleeb and Wilt, 2005). Two primary types of damage were introduced: stiffness reduction and strength reduction. Stiffness reduction mechanisms (e.g., microvoids and/or microcracking on the microstructural and mesostructural scales) result in softening due to a loss of load-carrying capability (i.e., reflected in a change in modulus) of the material. Strength reduction mechanisms (e.g., precipitate coarsening and intergranular defects) similarly produce a softening that is typically reflected in an increase in inelastic strain accumulation (e.g., tertiary creep) without necessarily a commensurate change in elastic modulus. Either type of damage can act independently or in combination within the model. Experimentally, stiffness degradation is relatively easy to observe, as would be strength degradation without stiffness degradation. However, both stiffness and strength degradations most likely interact before reaching the final failure of real materials, thus making it very difficult to differentiate these mechanisms experimentally. The GVIPS model contains a sufficient number of possible mechanisms (both deformation and damage) and can address those that occur or are active over a wide range of temperatures and loading conditions.

There have been many suggestions over the years proposing the tracking of stiffness as a measure of damage (see, for example, Lamaitre and Desmorat, 2005). Sample stiffness typically drops with increasing damage as the amount of load-carrying cross-sectional area decreases. Although stiffness is reasonably easy to measure on a test sample, it could also be determined in an actual component through mainly the change in its deflection. Therefore, this becomes a reasonable parameter to investigate as a damage indicator. In fatigue tests stiffness is easily tracked by monitoring the modulus for each cycle, wherein the modulus for both the loading and unloading portion of the cycle can be monitored. Differences between the two could appear to be due to the excessive displacement occurring while the crack is opening in the loading portion. Fatigue also is suitable for tracking stiffness reduction since such a test tends to fail through the minute growth of cracks over a long cyclic lifetime. This is in contrast to, for instance, a tensile test in which long and deep cracks may not form until much after the ultimate tensile strength (UTS) is achieved, closer to the end of the test, and result in abrupt failure.
Similarly, strength reduction (sample softening due to an increase in inelastic deformation) has also been recommended as a measure of damage (Saleeb and Wilt, 2005) if softening occurs in the absence of stiffness changes. Under strain- or displacement-controlled tests the stress response has been observed to both increase and/or decrease over the life of the test. An increase in stress (hardening) is usually associated with work hardening of the material, whereas a decrease (softening) is associated with damage as a result of a microstructure rearrangement. Note, however, that softening can also be caused by microstructural changes such as shearing of precipitates within a slip band, leading to a continued decrease in resistance to continued plastic flow. Such behavior is easily confounded with damage.

It has also been suggested that Poisson’s ratio may be a good parameter for monitoring damage due to directionality of damage, since it involves both the transverse ($e_t$) and axial ($e_a$) strains. Rahka and Laird (1983) have shown that Poisson’s ratio decreases with cycles for a Cr-Mo-V steel tested in strain-controlled fatigue. This was defined by them as a “drop in ductility,” connected to a rate of damage accumulation. They associated this drop with a change in the bulk specimen behavior (dislocation rearrangement rather than cracking). Cracking was observed near the end of life and was denoted by a more rapid dropoff in Poisson’s ratio. In a follow-on study Wang, Laird, and Rahka (1988) observed the formation of grain boundary cavitation that led to the decrease in Poisson’s ratio in the middle- to end-of-life regimes. However, the results presented do not appear to define a strong relationship between a change in Poisson’s ratio and damage. In fact, much discussion was introduced on this topic by other researchers commenting on this paper. Studies in concrete by Mazzotti and Savoia (2002) and Zhaoxia (1994) showed that Poisson’s ratio increased because of compressive loading and compressive creep as a result of cracking in the concrete. Whereas this was observed at high stress levels, there was minimal change at low and medium stresses.

Given the many types of potential physical damage that can occur in materials and the wide range of service conditions experienced by Ti-6-4 components, a variety of different tests were conducted in the current study and include tensile, creep, stress relaxation, and fatigue. These were performed over a range of temperatures and loading rates since different mechanisms may turn on and off under different loading regimes. Both axial and transverse strain as well as load (stress) were measured during all tests in an attempt to characterize damage. Stiffness was calculated at all loading and unloading excursions. During many of the tests, these loading and unloading excursions were inserted during the test to provide continual monitoring of the material stiffness. Since both real-time stress and strain are affected by reduction in area due to loading, data are presented in both engineering and true-stress and true-strain values. True values are typically considered in tensile testing, but are used to a much lesser extent in other types of tests. Consequently, both are examined with the hope of determining if one or the other type could better aid in determining the damage state. For the reader’s convenience the data is presented in various ways (e.g., stress-strain or strain-time space). Additionally, results from this study should provide insight to enable future modeling and experimentation, particularly in the area of integrated computational materials engineering.

This study is the final one in a large investigation of time-dependent deformation and damage of Ti-6-4. Three reports present the detailed experimental data. The first report (Lerch and Arnold, 2014) describes Ti-6-4’s viscoelastic behavior. The second report (Lerch and Arnold, 2016) documents its viscoplastic behavior, and now this last report provides more detailed information on the response domain associated with material damage and experimental issues associated with it. A number of conference presentations have been provided (Arnold, Lerch, and Sellers, 2013; Arnold et al., 2013 and 2014) regarding the characterization of the GVIPS model, and a number of future deformation and damage reports detailing these modeling activities will be forthcoming.

Definitions of symbols and acronyms used in this report are found in Appendix A.
Experimental Procedure

Coupon tests were performed on samples machined from one 0.625-in.-thick, mill-annealed Ti-6-4 plate purchased to AMS specification 4911. A detailed description of the plate’s pedigree was documented by Lerch and Arnold (2014). Test samples were excised from the plate with the longitudinal axis (load axis) of the samples being parallel to the final rolling direction. The microstructure can be viewed in Figure 1(a) and (b). The samples were machined into cylindrical dogbones 150 mm in length and having a 6.43-mm-gage diameter. The gauge section had a final hand polish parallel to the sample’s axis. Note that the other two dimensions of the plate were not tracked in the samples. As described in the previous two reports (Lerch and Arnold, 2014 and 2016), the plate contained a transverse texture (Figure 1(c)), which influenced anisotropic deformation, mostly manifesting in the radial directions, causing the cross-sectional area to become elliptical as strains increased. The cross-sectional shape of the samples was recorded and documented for most samples by measuring posttest diameters at several locations along the gage and in particular at the location of failure or at a localized neck if the sample did not fail. Three locations around the sample’s circumference were measured, and an ellipse fitted to the points. The major (a) and minor (b) axes of the ellipse were determined and an eccentricity $e$ calculated by

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

The angle $\theta$ between the right-hand side of the test rig and the major axis was also calculated (see Figure 2), and it varies randomly from test to test since the other two directions of the plate were not tracked.

In the previous report on viscoplasticity (Lerch and Arnold, 2016) a detailed discussion of the diameters and transverse strains was provided. In the present study, the results will be discussed in view of the developing sample shape and damage.

All samples were tested in the same hydraulically actuated load frame (Figure 2(a)). Heating was performed using direct induction with a group of induction coils at either end of the sample. This provided more unobstructed area in the gage of the sample. The temperature was monitored by a type-K thermocouple spot welded at the transition between the grip end and the radius of the sample. This thermocouple was calibrated against four thermocouples welded onto the gauge section of a dummy sample. The temperature gradient within the gauge section was within $\pm 1$ percent of the test temperature and remained constant throughout the test as a result of the coil configuration and their location. This was purposely designed to ensure a constant temperature during excessive sample movement in large strain tests. A 12.7-mm-gage, high-temperature, water-cooled extensometer was used to measure the axial strains. All tests also included measurement of transverse strains using an optical micrometer whose beam height is approximately 3 mm. The optical micrometer was originally positioned at the top of the sample, above the axial extensometer. This was done to have a clearer path to the gage surface since the measurements require line-of-site to the gauge. After all of the viscoelastoplastic tests for the earlier reports and a few of the damage tests were run, it was decided to move the optical micrometer to the center of the gage length in an attempt to have the transverse strain measured in the same location as the axial strain per the discussion in Appendix B. The experimental setup is shown in Figure 2(a), and additional details on the experimental setup can be found in Lerch and Arnold (2014 and 2016). Again, when inserting specimens for testing no attempt to align the specimen’s transverse directions was made, resulting in a random orientation of the transverse planes relative to the direction of transverse strain measurement indicated by the red arrow in Figure 2(b). In hindsight, this was an unfortunate oversight.
Figure 1.—Three-dimensional microstructure of Ti-6Al-4V plate. (a) Low magnification. (b) High magnification. (c) (00.2) pole figure.
Figure 2.—Tensile test of Ti-6Al-4V. (a) Test setup with axial extensometer measuring from front and optical micrometer measuring from right. (b) Graph indicating cross-sectional area of sample gauge and its relationship to test rig and strain measuring devices. $\theta$ is degree of rotation between right side of test rig and major axis of cross section.

Tests were conducted in air, under either load or strain control, depending on the specific test type: tension, creep, stress relaxation, or fatigue. The discrete temperatures examined in this study were 316, 427, and 538 °C, with one of the relaxation tests conducted at 20 °C and one of the creep tests at 482 °C. The rate of load application covered a range of strain rates from $10^{-3}$ to $10^{-3}$ s$^{-1}$, with $10^{-3}$ s$^{-1}$ being the most prevalent. When conducting tests under load control, an elastically equivalent load (stress) rate was used (i.e., $\sigma = E(\dot{\varepsilon})\dot{\varepsilon}$, where $E(\dot{\varepsilon})$ is the rate-dependent initial modulus and $\dot{\varepsilon}$ is the total loading strain rate). Load-controlled tests were primarily used for creep tests. All creep tests were performed under constant load, not stress. Several of the tests were unloaded to zero load at the end of the test, if failure had not occurred, and allowed to viscoelastically recover. Some tests were intermittently unloaded to determine the modulus after discrete deformation limits. In most tests, the unloading rates were equivalent to the loading rates. A complete test matrix description for the damage portion of the study is given in Table 1 and represents a subset of the total tests investigated during the full characterization study of Ti-6-4. Contrary to testing in the previous two reports, the tests in this study were loaded to high strains either to failure or near failure in an attempt to track damage evolution and ultimate failure.

**Results**

Data are collected, plotted, and discussed from the individual experiments: tensile, creep, stress relaxation, and fatigue tests. Test conditions are given in Table 1. The available room and elevated temperature tensile properties (e.g., modulus, proportional limit, yield stress, etc.) are also listed in Table 1. Key features (specimen shape factors, measurement location, and severity of necking) pertinent to the interpretation of test results are provided in Table 2.
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<th>Comments</th>
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<th>Proportional limit, MPa</th>
<th>Yield point, MPa</th>
<th>Final strain</th>
<th>Elasitic $\nu_{12}$ at temperature*</th>
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$\nu_{12}$ is Poisson’s ratio.
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<tr>
<td></td>
<td>Creep</td>
<td></td>
<td>538</td>
<td>0.001</td>
<td>206</td>
<td>Creep to 5% then unloaded and recovered</td>
<td>87</td>
<td>65</td>
<td></td>
<td>0.053</td>
</tr>
<tr>
<td>1</td>
<td>Modulus</td>
<td>20</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Creep</td>
<td></td>
<td>538</td>
<td>0.001</td>
<td>226</td>
<td>Creep to 8.5% then unloaded and recovered</td>
<td>85</td>
<td>115</td>
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<td>0.086</td>
</tr>
<tr>
<td>3</td>
<td>Modulus</td>
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</tr>
<tr>
<td></td>
<td>Creep</td>
<td></td>
<td>538</td>
<td>0.001</td>
<td>241</td>
<td>Creep to 11%</td>
<td>86</td>
<td>77</td>
<td></td>
<td>0.115</td>
</tr>
<tr>
<td>34</td>
<td>Modulus</td>
<td>20</td>
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<tr>
<td></td>
<td>Creep</td>
<td></td>
<td>538</td>
<td>0.001</td>
<td>241</td>
<td>Creep to 12% with unloads every 2%</td>
<td>87</td>
<td>74</td>
<td></td>
<td>0.120</td>
</tr>
<tr>
<td>85</td>
<td>Modulus</td>
<td>20</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>Creep</td>
<td></td>
<td>538</td>
<td>0.001</td>
<td>239</td>
<td>Creep to 9.7%</td>
<td>80</td>
<td>80</td>
<td>218</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Modulus</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Creep</td>
<td></td>
<td>538</td>
<td>0.001</td>
<td>243</td>
<td>Creep to 16% with unloads and thermal recovery</td>
<td>83</td>
<td>85</td>
<td>231</td>
<td></td>
</tr>
<tr>
<td>88</td>
<td>Modulus</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Creep</td>
<td></td>
<td>538</td>
<td>0.001</td>
<td>376</td>
<td>Creep to 13%</td>
<td>83</td>
<td>98</td>
<td>246</td>
<td></td>
</tr>
</tbody>
</table>

⁵ν₁₂ is Poisson’s ratio.
| Specimen no. | Test type   | Temperature, °C | Strain rate, s⁻¹ | Stress level, MPa | Comments                                      | Modulus, GPa | Proportional limit, MPa | Yield point, MPa | Final strain | Elastic ν₁₂ at temperature ¹ | ²
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>89B</td>
<td>Relaxation</td>
<td>20</td>
<td>0.001</td>
<td></td>
<td>Step relaxation 1.8%, 4%, 6%, 8%, 10%, and 12% Interrupted</td>
<td>112</td>
<td>749</td>
<td>801</td>
<td>0.120</td>
<td>0.29</td>
</tr>
<tr>
<td>27</td>
<td>Modulus</td>
<td>20</td>
<td></td>
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<td>112</td>
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</tr>
<tr>
<td>Relaxation</td>
<td>427</td>
<td>0.001</td>
<td></td>
<td></td>
<td>Relaxed at 1.2% and 1.8%</td>
<td>88</td>
<td>365</td>
<td>439</td>
<td>517</td>
<td>0.26</td>
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<tr>
<td>Relaxation</td>
<td>427</td>
<td>0.001</td>
<td></td>
<td></td>
<td>Step relaxation at 3.2%, 4.4%, and 6.6%</td>
<td>82</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Tension</td>
<td>427</td>
<td>0.001</td>
<td></td>
<td></td>
<td>Limit tripped at 11%, no relaxation</td>
<td>79</td>
<td></td>
<td></td>
<td></td>
<td>0.141</td>
</tr>
<tr>
<td>Relaxation</td>
<td>427</td>
<td>0.001</td>
<td></td>
<td></td>
<td>Relaxation at 12.6%</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Failed on subsequent loadup at 14.1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>77</td>
<td>Modulus</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td>116</td>
<td></td>
<td></td>
<td></td>
<td>0.22</td>
</tr>
<tr>
<td>Relaxation</td>
<td>538</td>
<td>0.001</td>
<td></td>
<td></td>
<td>Relaxed previously at 1.8% for 8 h</td>
<td>79</td>
<td>97</td>
<td>197</td>
<td>401</td>
<td>0.30</td>
</tr>
<tr>
<td>Relaxation</td>
<td>538</td>
<td>0.001</td>
<td></td>
<td></td>
<td>Step relaxation 4%, 8%, 12%, and 16% Failed on next loadup at 18%</td>
<td>79</td>
<td></td>
<td></td>
<td></td>
<td>0.183</td>
</tr>
</tbody>
</table>

| Specimen no. | Test type   | Temperature, °C | Strain rate, s⁻¹ | Stress level, MPa | Comments                                      | Modulus, GPa | Proportional limit, MPa | Yield point, MPa | Final strain | Elastic ν₁₂ at temperature ¹ | ²
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>57</td>
<td>Modulus</td>
<td>20</td>
<td>0.001</td>
<td>535</td>
<td>0 to 535 MPa fatigue to runout at (N = 580,948)</td>
<td>112</td>
<td>400</td>
<td>457</td>
<td>530</td>
<td>0.015</td>
</tr>
<tr>
<td>LCF</td>
<td>427</td>
<td>0.001</td>
<td>535</td>
<td></td>
<td>0 to 535 MPa fatigue to runout at (N = 580,948)</td>
<td>9</td>
<td>400</td>
<td>457</td>
<td>530</td>
<td>0.015</td>
</tr>
<tr>
<td>98</td>
<td>LCF</td>
<td>427</td>
<td>0.001</td>
<td>549</td>
<td>0 to 549 MPa fatigue to failure (N_f = 103,683)</td>
<td>92</td>
<td>436</td>
<td>464</td>
<td>533</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>Modulus</td>
<td>20</td>
<td>0.001</td>
<td>581</td>
<td>0 to 581 MPa fatigue to failure (N_f = 22,134)</td>
<td>93</td>
<td>446</td>
<td>484</td>
<td>550</td>
<td>0.024</td>
</tr>
<tr>
<td>LCF</td>
<td>427</td>
<td>0.01</td>
<td>581</td>
<td></td>
<td>0 to 581 MPa fatigue to failure (N_f = 22,134)</td>
<td>93</td>
<td>446</td>
<td>484</td>
<td>550</td>
<td>0.024</td>
</tr>
<tr>
<td>59</td>
<td>Modulus</td>
<td>20</td>
<td>0.001</td>
<td>597</td>
<td>0 to 597 MPa fatigue to failure (N_f = 9,735)</td>
<td>92</td>
<td>399</td>
<td>445</td>
<td>524</td>
<td>0.110</td>
</tr>
<tr>
<td>LCF</td>
<td>427</td>
<td>0.001</td>
<td>597</td>
<td></td>
<td>0 to 597 MPa fatigue to failure (N_f = 9,735)</td>
<td>92</td>
<td>399</td>
<td>445</td>
<td>524</td>
<td>0.110</td>
</tr>
<tr>
<td>58</td>
<td>Modulus</td>
<td>20</td>
<td>0.001</td>
<td>642</td>
<td>0 to 642 MPa fatigue to failure (N_f = 115)</td>
<td>91</td>
<td>430</td>
<td>464</td>
<td>530</td>
<td>0.133</td>
</tr>
<tr>
<td>LCF</td>
<td>427</td>
<td>0.001</td>
<td>642</td>
<td></td>
<td>0 to 642 MPa fatigue to failure (N_f = 115)</td>
<td>91</td>
<td>430</td>
<td>464</td>
<td>530</td>
<td>0.133</td>
</tr>
</tbody>
</table>

¹ν₁₂ is Poisson's ratio.
²LCF is low-cycle fatigue, \(N\) is number of cycles, \(N_f\) is number of cycles to failure, and \(R\) is load or strain ratio.
TABLE 1.—Ti-6Al-4V TEST SAMPLE MATRIX WITH TENSILE PROPERTIES (Concluded)

<table>
<thead>
<tr>
<th>Specimen no.</th>
<th>Test type</th>
<th>Temperature, °C</th>
<th>Strain rate, s⁻¹</th>
<th>Stress level, MPa</th>
<th>Comments</th>
<th>Modulus, GPa</th>
<th>Proportional limit, MPa</th>
<th>Yield point, MPa</th>
<th>Final strain</th>
<th>Elastic ν₁₂ at temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>74</td>
<td>Modulus</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td>115</td>
<td></td>
<td>81</td>
<td>246</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>LCF</td>
<td>538</td>
<td>0.001</td>
<td></td>
<td>Strain control</td>
<td></td>
<td>Strain control</td>
<td>N₀ = 900, R = -1, strain range = ±1%</td>
<td>66</td>
<td>91</td>
</tr>
<tr>
<td>73</td>
<td>Modulus</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td>114</td>
<td></td>
<td>81</td>
<td>246</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>LCF</td>
<td>538</td>
<td>0.00001</td>
<td></td>
<td>Strain control</td>
<td></td>
<td>Strain control</td>
<td>N = 176, R = 0, interrupted</td>
<td>66</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>Tensile</td>
<td>538</td>
<td>0.00001</td>
<td></td>
<td>Tension to failure</td>
<td></td>
<td>Tension to failure</td>
<td>76</td>
<td>0.142</td>
<td>0.34</td>
</tr>
</tbody>
</table>

aν₁₂ is Poisson’s ratio.

bLCF is low-cycle fatigue, N is number of cycles, N₀ is number of cycles to failure, and R is load or strain ratio.
TABLE 2.—Ti-6Al-4V TEST MATRIX WITH INFORMATION ON CROSS-SECTIONAL AREA, AND VALUES AT EITHER ULTIMATE TENSILE STRENGTH (UTS) OR INITIATION OF TERTIARY CREEP

(a) Tension

<table>
<thead>
<tr>
<th>Specimen no.</th>
<th>Eccentricity, (e)</th>
<th>Angle of sample notation, (\theta)</th>
<th>Micrometer location</th>
<th>Neck location</th>
<th>Severity of neck</th>
<th>Stress (MPa)</th>
<th>Rate, (b) (10^{-6}) s(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Engineering</td>
<td>True</td>
</tr>
<tr>
<td>79</td>
<td>0.56</td>
<td>132</td>
<td>Midgage</td>
<td>Midgage</td>
<td>Large</td>
<td>709 [0.112]</td>
<td>&gt;816 [0.160]</td>
</tr>
<tr>
<td>42</td>
<td>0.54</td>
<td>151</td>
<td>Midgage</td>
<td>Midgage</td>
<td>Large</td>
<td>745 [0.143]</td>
<td>&gt;871 [0.133]</td>
</tr>
<tr>
<td>95</td>
<td>0.52</td>
<td>142</td>
<td>Top</td>
<td>Midgage</td>
<td>Large</td>
<td>636 [0.080]</td>
<td>717 [0.162]</td>
</tr>
<tr>
<td>23</td>
<td>0.51</td>
<td>125</td>
<td>Top</td>
<td>Midgage</td>
<td>Large</td>
<td>655 [0.105]</td>
<td>747 [0.150]</td>
</tr>
<tr>
<td>69</td>
<td>0.36</td>
<td>10</td>
<td>Midgage</td>
<td>Midgage</td>
<td>Medium</td>
<td>632 [0.105]</td>
<td>708 [0.124]</td>
</tr>
<tr>
<td>39</td>
<td>0.65</td>
<td>170</td>
<td>Top</td>
<td>Top</td>
<td>Large</td>
<td>475 [0.055]</td>
<td>505 [0.062]</td>
</tr>
<tr>
<td>89T</td>
<td>0.54</td>
<td>149</td>
<td>Top</td>
<td>Top</td>
<td>Large</td>
<td>479 [0.046]</td>
<td>513 [0.094]</td>
</tr>
<tr>
<td>21</td>
<td>0.66</td>
<td>58</td>
<td>Midgage</td>
<td>Top</td>
<td>Large</td>
<td>519 [0.063]</td>
<td>565 [0.102]</td>
</tr>
<tr>
<td>37</td>
<td>0.54</td>
<td>10</td>
<td>Top</td>
<td>Top</td>
<td>Large</td>
<td>465 [0.049]</td>
<td>493 [0.080]</td>
</tr>
<tr>
<td>70</td>
<td>0.66</td>
<td>70</td>
<td>Top</td>
<td>Top</td>
<td>Large</td>
<td>530 [0.070]</td>
<td>579 [0.110]</td>
</tr>
<tr>
<td>97</td>
<td>0.65</td>
<td>54</td>
<td>Top</td>
<td>Top</td>
<td>Large</td>
<td>339 [0.018]</td>
<td>347 [0.024]</td>
</tr>
<tr>
<td>65</td>
<td>0.55</td>
<td>165</td>
<td>Top</td>
<td>Bottom</td>
<td>Large</td>
<td>270 [0.013]</td>
<td>273 [0.017]</td>
</tr>
<tr>
<td>15</td>
<td>0.60</td>
<td>167</td>
<td>Midgage</td>
<td>Top</td>
<td>Large</td>
<td>342 [0.023]</td>
<td>349 [0.023]</td>
</tr>
</tbody>
</table>

\(^a\)Values at UTS for tensile and relaxation tests. Values at initiation of tertiary creep.

\(^b\)Creep rate at beginning of tertiary creep.
### TABLE 2.—Ti-6Al-4V TEST MATRIX WITH INFORMATION ON CROSS-SECTIONAL AREA, AND VALUES AT EITHER ULTIMATE TENSILE STRENGTH (UTS) OR INITIATION OF TERTIARY CREEP (Continued)

<table>
<thead>
<tr>
<th>Specimen no.</th>
<th>Eccentricity, ( e )</th>
<th>Angle of sample rotation, ( \theta )</th>
<th>Micrometer location</th>
<th>Neck location</th>
<th>Severity of neck</th>
<th>Stress (MPa)</th>
<th>Rate, ( 10^{-6} ) s(^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td>Engineering</td>
<td>True</td>
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<tr>
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<td>value</td>
</tr>
<tr>
<td>45</td>
<td>0.31</td>
<td>0</td>
<td>Midgage</td>
<td>Top</td>
<td>Small</td>
<td>591 [0.099]</td>
<td>649 [0.094]</td>
</tr>
<tr>
<td>35</td>
<td>0.32</td>
<td>90</td>
<td>Midgage</td>
<td>Top</td>
<td>Small</td>
<td>586 [0.099]</td>
<td>644 [0.095]</td>
</tr>
<tr>
<td>87</td>
<td>0.36</td>
<td>25</td>
<td>Midgage</td>
<td>Top</td>
<td>Medium</td>
<td>596 [0.120]</td>
<td>667 [0.113]</td>
</tr>
<tr>
<td>81</td>
<td>0.68</td>
<td>42</td>
<td>Top</td>
<td>Top</td>
<td>Large</td>
<td>376 [0.458]</td>
<td>393 [0.045]</td>
</tr>
<tr>
<td>94</td>
<td>0.25</td>
<td>150</td>
<td>Midgage</td>
<td>Top</td>
<td>Small</td>
<td>206 [0.019]</td>
<td>210 [0.018]</td>
</tr>
<tr>
<td>1</td>
<td>0.22</td>
<td>125</td>
<td>Midgage</td>
<td>Top</td>
<td>Small</td>
<td>226 [0.022]</td>
<td>231 [0.022]</td>
</tr>
<tr>
<td>3</td>
<td>0.26</td>
<td>18</td>
<td>Midgage</td>
<td>Top</td>
<td>Small</td>
<td>241 [0.025]</td>
<td>247 [0.025]</td>
</tr>
<tr>
<td>34</td>
<td>0.37</td>
<td>176</td>
<td>Midgage</td>
<td>Top</td>
<td>Medium</td>
<td>241 [0.025]</td>
<td>247 [0.025]</td>
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<tr>
<td>85</td>
<td>0.40</td>
<td>128</td>
<td>Top</td>
<td>Top</td>
<td>Medium</td>
<td>239 [0.026]</td>
<td>246 [0.025]</td>
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<tr>
<td>89</td>
<td>0.46</td>
<td>142</td>
<td>Midgage</td>
<td>Top</td>
<td>Medium</td>
<td>243 [0.018]</td>
<td>247 [0.018]</td>
</tr>
<tr>
<td>88</td>
<td>0.74</td>
<td>156</td>
<td>Top</td>
<td>Top</td>
<td>Large</td>
<td>376 [0.033]</td>
<td>389 [0.032]</td>
</tr>
</tbody>
</table>

\( ^a \)Values at UTS for tensile and relaxation tests. Values at initiation of tertiary creep.

\( ^b \)Creep rate at beginning of tertiary creep.
### TABLE 2.—Ti-6Al-4V TEST MATRIX WITH INFORMATION ON CROSS-SECTIONAL AREA, AND VALUES AT EITHER ULTIMATE TENSILE STRENGTH (UTS) OR INITIATION OF TERTIARY CREEP (Concluded)

#### (c) Relaxation

<table>
<thead>
<tr>
<th>Specimen no.</th>
<th>Eccentricity, $e$</th>
<th>Angle of sample rotation, $\theta$</th>
<th>Micrometer location</th>
<th>Neck location</th>
<th>Severity of neck</th>
<th>Stress (MPa) [strain]$^a$</th>
<th>Rate,$^b$ $10^{-6}$ s$^{-1}$</th>
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#### (d) Fatigue

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$^a$Values at UTS for tensile and relaxation tests. Values at initiation of tertiary creep.

$^b$Creep rate at beginning of tertiary creep.
Tensile

The 13 tensile test results for Ti-6-4 at various temperatures and strain rates are given in Figure 3 to Figure 15. The tension tests were conducted under strain control. A sample (79) tested at 316 °C (Figure 3) was strained at a rate of $10^{-3}$ s$^{-1}$. This rate was the highest rate used in this study and represented a rate at which most of the viscoelastic deformation would be locked in during loading (Lerch and Arnold, 2014).

In Figure 3(a) the final cross-sectional area of sample 79 is plotted and oriented with respect to the test rig and both extensometers, as was described in the report on viscoplasticity (Lerch and Arnold, 2016). In such figures, the final elliptical geometry of the specimen minimum cross section can be observed and compared to the circular, pretest area. The length of both the major (a) and minor (b) axes are given along with the eccentricity $e$ of the ellipse. The angle $\theta$ between the major axis and the plane perpendicular to the right side of the test rig is also displayed (see Figure 3(a)). The major and minor axes are presumed to represent the two untracked surfaces (short- and long-transverse planes) of the rolled plate. We considered transforming the measured transverse strain onto the major and minor axis, but we did not deem it worth the effort, considering the assumptions that would have to have been made. Sample 79 exhibits a reasonably large eccentricity ($e = 0.56$) as shown in Figure 3(a). Its major axis was rotated by $132^\circ$ from the right side of the test rig, indicating that the transverse extensometer measured a sample diameter that averages the length of the major and minor axes being oriented halfway between the two. The transverse extensometer is located on the right side of the rig and measures the diameter perpendicular (oriented front to back) to the optical beam direction. This diameter is indicated in the associated schematics, labeled as “transverse measurement.” The axial extensometer is located in the front of the rig pointing front to back. The location of the extensometer probes are indicated on the specimen geometry shown on the right of Figure 3(a). The vertical distance between these two probes is 12.7 mm. More details on the measurement of the sample diameters can be found in Appendix C, “Transverse Strain Behavior” in Lerch and Arnold (2016). Elements of the transverse strain are shown in Table 2, including the location along the gage where the strain was measured and where the localized neck formed. This is also indicated by the sample schematic in Figure 3(a), which depicts the axial location of the transverse optical measurement and the location of the minimum diameter (localized neck).

The stress-strain curves for sample 79 are shown in Figure 3(b) for the true stress $\sigma_{\text{true}} = S(1+e)$, where $S$ is engineering stress and $e$ is engineering strain. Note that this expression is not completely correct after the UTS has been reached, as will be explained later in the report. In Figure 3(b) the true stress versus true strain, true stress versus engineering strain, and engineering stress versus engineering strain curves are shown for clarity and background information. Henceforth, only two stress-strain curves (true and engineering stress) will be shown in which the strains for both curves are plotted in terms of the engineering strains, since this was the controlled test variable. Using the actual cross-sectional area from the transverse extensometer measurements to calculate the true stress gave similar results at least up to the UTS. Both engineering and true-stress values for all tests are presented to document the difference between the two stress measures, to indicate that the true stress is larger (often much larger) than the engineering stress and to ascertain which is more appropriate for monitoring damage. In the case of stiffness, the moduli were called either engineering ($E_{\text{eng}}$) or true ($E_{\text{true}}$), based on which stress and strain values they were calculated from. The engineering curve for sample 79 (Figure 3(b) and (c)) displays hardening until saturation is reached at the UTS. This occurs at an engineering value of 709 MPa and

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1See Equation (D.21) in Appendix D, where Poisson’s ratio $\nu = 0.5$ (constant volume) is assumed.
2The true modulus was calculated using the true axial strain values (assumed herein to be logarithmic strain), not the engineering strains.
Figure 3.—Tensile test result of Ti-6-4 sample 79 at 316 °C and strain rate of 10^{-3} s^{-1}. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity \( e \). Dashed circle is original sample circumference. (b) Stress-strain curves (engineering and true), showing ultimate tensile stress (UTS) and loading and unloading moduli \( E_{\text{eng}} = 97.22 \text{ GPa} \) and \( E_{\text{true}} = 97.37 \text{ GPa} \). \( \Delta F_{\text{eng}} = 37.2 \text{ percent} \), \( \Delta F_{\text{true}} = 4.4 \text{ percent} \). (c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio \( \nu_{\text{davg}} \). Final Poisson’s ratio = 0.19 for nonnecked areas.
an engineering strain of 0.112. The sample softens from that point onward until it reaches a strain of nearly 0.2, which was the maximum range of the extensometer. This limit strain corresponds to an engineering stress of 678 MPa, or a (Δσ) 4.4-percent decrease from the UTS. Upon attaining the limit strain, the sample was unloaded to zero stress and held at zero stress and allowed to recover viscoelastically for a small amount of time. This recovery was performed on all samples that reached the upper strain limit before failing, to gain additional insight into the recovery process. The recovery data are presented in Lerch and Arnold (2014) and are not discussed here since they were not related to the damage process. The true-stress curve shows continued hardening until a saturation of stress of 813 MPa was attained prior to unloading. The actual maximum stress at failure (or in this case, unloading) is shown at a stress of 866 MPa. This value was calculated using the measured cross-sectional area in the neck after the test.

Both the loading and unloading moduli are given in Figure 3(b), calculated from both the engineering and true stress-strain values. Both initial loading moduli (i.e., $E_{L,\text{eng}} = 97.22$ and $E_{L,\text{true}} = 97.37$ GPa) are nearly identical, with the true modulus being slightly larger. After a strain of 0.2, the engineering unloading modulus, compared to the initial loading modulus, has dropped by 37 percent ($ΔE_{\text{eng}}$) to a value of $E_{U,\text{eng}} = 61.04$ GPa, whereas the true modulus only decreased by 10 percent to $E_{U,\text{true}} = 87.91$ GPa.

Figure 3(c) duplicates the engineering stress-strain curve and compares it to the engineering stress-transverse strain response. The maximum transverse strain for this test was 0.124, significantly less than the final maximum axial strain of 0.2, as expected. Values for the instantaneous Poisson’s ratio were also calculated and plotted versus the engineering axial strain. The instantaneous Poisson’s ratio throughout the test was calculated using the expression

$$\nu = -\frac{\varepsilon_t}{\varepsilon}$$

(2)

where $\varepsilon_t$ is the transverse engineering strain and $\varepsilon$ is the axial engineering strain at a given moment in time or loading. Note henceforth this will be merely referred to as “Poisson’s ratio.” Isotropic behavior was assumed at the outset of the program, thus only a single, random oriented, transverse strain measurement was taken and only a single Poisson’s ratio calculated. Although not theoretically correct because of the material texture and corresponding anisotropy, this was the same approach taken in the viscoplastic paper (Lerch and Arnold, 2016). Poisson’s ratio is plotted in Figure 3(c) (green curve). It remains relatively constant throughout the elastic zone, then begins to increase rapidly during yielding, and reaches a plateau value of approximately 0.45 between the axial strains of 0.04 and 0.09. With additional straining, Poisson’s ratio again begins to increase, continuing to a value of 0.62 where the sample was finally unloaded. The black curve shows the true Poisson’s ratio plotted versus engineering axial strain. Both the true and the engineering Poisson’s ratios are similar until an axial strain of approximately 3 percent, at which point the true Poisson’s ratio exceeds the engineering values. Henceforth, only the engineering Poisson’s ratio will be shown. Both the values of Poisson’s ratio and its behavior shown in this figure are unexpected. Hence various responses for Poisson’s ratio were calculated and are discussed with relationship to the strain

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3The transverse strains are usually presented as positive values throughout this report for easier comparison.
4Poisson’s ratio is always plotted versus engineering axial strain in this report.
5This is called contraction ratio when the material is nonlinear (Goodno and Gere, 2009).
6This plate was specifically ordered to be cross rolled to minimize texturing. The existence and severity of the texture was not realized until later in the study.
7True Poisson’s ratio was calculated using the true transverse strain, $\varepsilon_{t,\text{true}}$ and dividing by the true axial strain, $\varepsilon_{\text{true}}$, yielding $\nu = -\varepsilon_{t,\text{true}}/\varepsilon_{\text{true}}$. 

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measurement techniques employed in this test program in Appendix B and Appendix D. A detailed finite element analysis of the actual test specimen and a representative tensile test was performed and is presented in Appendix B, with results illustrating how one might obtain various response profiles depending upon the combination of longitudinal and transverse measurements.

The final values for Poisson’s ratio after the tests are also given in the figure captions for the nonnecked area; in Figure 3(c) this value is 0.14. This value indicates the difference between Poisson’s ratios measured with the transverse micrometer (intended to measure the localized neck) compared to remote locations where the deformation was more homogeneous and represented more global conditions (i.e., more representative of pre-UTS strain states). Note that the UTS occurs at an axial strain of 0.112, which is higher than the beginning of the final increase in Poisson’s ratio that starts at a strain of 0.09. This implies that Poisson’s ratio may be more sensitive than the UTS for indicating the inception of damage. The stress-strain values and Poisson’s ratio associated with the UTS for each test are given in columns 7 to 10 of Table 2.

Also shown in the figures of monotonic tests such as Figure 3(c) is a point labeled “ν_{davg},” which represents the diameter-averaged Poisson’s ratio. This value was calculated by averaging the major and minor diameters at the minimum cross section after the test and using this value to calculate the transverse strain and subsequently Poisson’s ratio. This was done to reduce the effects of the random transverse orientation of the sample and reduce the amount of anisotropy contributing to the measurement of the Poisson’s ratio shown in the figure.

Another sample (42) was tested at 316 °C, but at a slower strain rate of $1 \times 10^{-5} \text{ s}^{-1}$. This sample had an eccentricity and major axis rotation (Figure 4(a)) similar to the previous sample, and therefore comparisons between the two tests response curves should illustrate primarily strain rate effects. Unfortunately, the angle of rotation was different enough to affect the transverse strain values. This test exhibited slightly higher moduli and UTS (Figure 4(b)) than the previous sample, which was contrary to expectation because sample 42 was conducted at a slower strain rate and resulted in lower yield points (given in Table 1) particularly evidenced by the proportional limit. It was shown in the viscoplastic paper (Lerch and Arnold, 2016) that it is within this temperature regime that dynamic strain aging occurred, which can give an inverse relationship between stress and loading rate (Lin et al., 2011). The tensile curves given in Figure 4(b) ($1 \times 10^{-5} \text{ s}^{-1}$ strain rate) also show some waviness between strains of 0.04 and 0.10, which is again consistent with dynamic strain aging. As was seen previously for sample 79 in Figure 3(b) ($1 \times 10^{-3} \text{ s}^{-1}$ strain rate), the true moduli for sample 42 in Figure 4(b) ($1 \times 10^{-5} \text{ s}^{-1}$ strain rate) are higher than the engineering moduli for both the loading and unloading cases. The unloading moduli are less than the original loading moduli by approximately the same amount observed in the fast-rate test. The UTS value for the engineering curve is 745 MPa, and it occurs at a strain of 0.143. For this test, the true-stress curve does not appear to saturate and reach an ultimate stress although its stress at unload is 871 MPa. The transverse strain is displayed in Figure 4(c); it reached slightly higher values (0.163) than for the previous, fast-rate sample (79). This was because the measured diameter was more aligned (by 9°) with the minor axis and exhibited a larger decrease in diameter per given strain than elsewhere around the circumference of the gage. The response for Poisson’s ratio is similar to that shown for sample 79 with the exception that the absolute values are slightly higher for the slow-strain-rate test: 0.40 compared to 0.28, even in the elastic regime. This is again due to the particular diameter that was measured by the transverse extensometer. Poisson’s ratio, in spite of the oscillations observed, again appears to contain a plateau of approximately 0.65 occurring at midstrains for the slow-rate test and 0.45 for the fast-rate test. The final value of Poisson’s ratio before unloading is 0.85 for the slow-rate test, and the value outside of the neck is 0.29. Note that the $\nu_{davg}$ value is significantly lower than what was measured with the transverse extensometer. This is again because the transverse measurement was more aligned with the minor axis.
Figure 4.—Tensile test of Ti-6-4 sample 42 at 316 °C and strain rate of 10⁻⁵ s⁻¹. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity e. Dashed circle is original sample circumference. (b) Stress-strain curves (engineering and true), showing ultimate tensile stress (UTS) and loading and unloading moduli $E_{\text{true}} = 90.38$ GPa and $E_{\text{true}} = 63.58$ GPa. $F_{\text{true}} = 99.23$ GPa $E_{\text{true}} = 99.41$ GPa. (c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{\text{davg}}$. Final Poisson’s ratio $= 0.20$ for nonnecked areas.
The next series of results are from tensile tests at 427 °C. Figure 5(a) indicates the final area shape and orientation for sample 23 tested at a strain rate of $10^{-3}$ s$^{-1}$; it is consistent with the fast-rate sample (79) tested at 316 °C. This sample was not unloaded but stopped at an engineering strain of 0.196. The sample contained a localized neck just below the top extensometer probe. The location of the transverse strain measurement was not in the center of the sample, but had been moved to a location between the top extensometer probe and the upper sample radius. Hence the measured transverse strain and the localized neck do not coincide as in the last two tests. Because of the increase in temperature, the strength of this sample (Figure 5(b)) is slightly less than the two samples tested at 316 °C. The UTS attained a value of 655 MPa at an axial strain of 0.105 and a transverse strain of 0.043. The loading moduli are also lower, having values near 91 GPa. For this sample the true-stress curve reaches a stress plateau of 744 MPa at a strain of 0.16. The transverse strain, shown in Figure 5(c), is very low compared with the associated longitudinal strain, having only a maximum of 0.063 achieved by the end of the test. This is in sharp contrast to those transverse strains observed for the tests conducted at 316 °C, which are well above 0.1.

This is significant since the final axial strains and the sample orientations of all three samples are nominally equivalent. However, in the tests at 316 °C the transverse extensometer measured the diameters at midgage, which is where the samples necked. For the sample tested at 427 °C, the transverse extensometer measured the diameter at the top of the gage, whereas necking occurred closer to midgage. Hence the transverse strain for this sample was not following the largest deformation during the necking process and therefore less total strain is exhibited. Similarly, in this test the specimen’s major axis was rotated by only 125° from the right side of the test rig, indicating that the transverse extensometer measured an average sample diameter that was more aligned with the major axis, and therefore one would expect a stiffer response (less strain). The behavior of the Poisson’s ratio also appears much different than for the previous two tests as a consequence of these same experimental artifacts. First, the value increased from an elastic-region value of 0.36 to a plateau value of 0.43 at an axial strain of 0.048. This maximum value occurs at a strain smaller than the axial strain at UTS (0.105). After maintaining a plateau, the value of Poisson’s ratio drops until reaching a value of 0.32 at the end of the test. Hence the values for Poisson’s ratio remain low (<0.5) throughout this test. The values of Poisson’s ratio at the end of the test are 0.22 in the nonnecked (bottom of gauge) areas. However the $\nu_{davg}$ value taken at the neck was 0.56 and is similar to the final value shown in Figure 3(c) for the test at 316 °C. The green curve shown in Figure 5(c) was obviously taken from a gauge location that felt less influence from necking. Moreover this suggests caution when interpreting the values of both transverse strain and Poisson’s ratio since they are influenced heavily by the location that they are measuring, as discussed in Appendix B.

Figure 6 shows results from a sample (69) tested at a slower strain rate of $1 \times 10^{-5}$ s$^{-1}$, but still at 427 °C. This sample exhibits a smaller change in diameter, as evidenced by the low eccentricity value of 0.36 (Figure 6(a)), in spite of attaining the same maximum strain (0.2). Moreover the major axis was only rotated 10° from the right side of the test frame. Hence the micrometer measured a diameter change nearly coincident with the minor axis. Measurement at the minor axis leads to the largest change in the diameter around the gage section circumference and therefore yields the largest values for transverse strain and Poisson’s ratio. The stress-strain curves are shown in Figure 6(b) and exhibit slightly lower stress values than for the previous, fast-rate sample, consistent with expected strain rate effects. The loading moduli are also slightly lower. This sample was unloaded upon reaching an engineering axial strain of 0.2, and the moduli values for unloading are $E_{u,eng} = 59.63$ and $E_{u,\text{true}} = 85.50$ GPa. Both of these are smaller than the values observed for unloading at 316 °C for the same maximum strain. The transverse strain is plotted in Figure 6(c) and reaches a value of 0.096 before unloading. For this sample both the measured diameter and the neck are in proximity to midgage; yet Poisson’s ratio displays a behavior similar to the fast-rate sample (23), but with the final transverse strain value larger than that in
Figure 5.—Tensile test of Ti-6Al-4V sample 23 at 427 °C and strain rate of 10⁻³ s⁻¹. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity ε. (b) Stress-strain curves (engineering and true) showing ultimate tensile stress (UTS) and loading moduli $E_L$ and $E_U$. $\Delta \sigma = 5.0$ percent. (c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{davg}$. Final Poisson’s ratio = 0.28 for nonnecked areas.
Figure 6.—Tensile test of Ti-6Al-4V sample 69 at 427 °C and strain rate of $10^{-5}$ s$^{-1}$. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true), showing ultimate tensile stress (UTS) and loading and unloading moduli $E_L$ and $E_U$. $\Delta E_{\text{eng}} = 34.5$ percent, $\Delta \sigma = 9.5$ percent. (c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{\text{davg}}$. Final Poisson’s ratio = 0.26 for nonnecked areas.
the fast-rate test at 427 °C (0.063), and is more consistent with the maximum values (0.124 and 0.163) from the tests at 316 °C. The UTS for this test occurs at an axial strain of 0.105 and is located on the decreasing portion of the Poisson’s ratio curve. Similar to the previous test at 427 °C, Poisson’s ratio decreases to the end of test after attaining a plateau at smaller strains. The $v_{davg}$ value of Poisson’s ratio at the end of the test is 0.74 compared to a measured value of 0.48.

To investigate the presence of damage and whether it manifests itself as stiffness degradation, the progression of the modulus as a function of strain was measured during an interrupted tensile test; the sample (95) was tested at 427 °C and a strain rate of $10^{-3}$ s$^{-1}$ (Figure 7). Periodic unloads (also at a strain rate of $10^{-3}$ s$^{-1}$) were conducted at 0.01 strain intervals (typically). This continued until the test was stopped at 0.2 strain. A final unload was not performed. The cross section diagram is shown in Figure 7(a) for sample 95 with an eccentricity of 0.52 and a rotation of the major axis of 142°. This was similar to most of the previous samples. The stress-strain curves are shown in Figure 7(b) and have a shape similar to the monotonically loaded sample 23 (Figure 5(b)) under the same test conditions, thus indicating that the multiple load-unload cycles did not greatly affect its tensile behavior. Note that reverse yielding did not occur during unloading. Sample 95 has a UTS value of 636 MPa, which occurs at an axial strain of 0.08 and a transverse strain of 0.04. For this sample the true-stress curve reaches a stress plateau of 717 MPa at a strain of 0.17. The loading and unloading moduli were calculated from the engineering and true stress-strain curves, listed in the Table 3, and plotted normalized against the initial modulus as a function of step in Figure 7(c), where step $L$ is the initial loadup. For the loading modulus, the first loadup yields the highest moduli of 89.41 and 90.36 GPa for the engineering and true values, respectively. The engineering modulus shows a continuous drop until reaching the last value of 59.07 GPa: a 34 percent drop. The true modulus shows only a slight decrease in values (8 percent) with slightly more scatter in the data. The final true modulus value is 83.45 GPa. The unloading moduli shows similar trends and are only slightly higher than the loading moduli at any given strain increment. Figure 7(d) shows the transverse strain behavior, which terminates at a value of 0.074 and is nearly the same as that observed for the other fast-rate test, sample 23 (Figure 5(c)). As in that sample, the diameter for sample 95 was measured at the top of the gage and the neck occurred midgage. The Poisson’s ratio also shows a similar behavior to that observed for sample 23, although sample 95 has slightly higher values due to its greater alignment with the minor axis; that is, rotation of 142° compared to 125° for sample 23. The maximum value of Poisson’s ratio (excluding unloads) of 0.51 occurs at a strain of 0.058, which is again before the UTS, and the $v_{davg}$ value of Poisson’s ratio is 0.63. Note in Figure 7(d) that the values for Poisson’s ratio actually increase for each unloading step (decrease in strain) as has been shown in the previous two papers (Lerch and Arnold, 2014 and 2016). It is interesting to note that the magnitude of the Poisson’s ratio increase during unloading decreases as the overall strain accumulation increases, suggesting some possible connection with the accumulation of damage.

The next series of results are from samples tensile tested at 538 °C, a temperature at which there is a great deal of time dependency. Sample 21 was tested at the fast strain rate of $10^{-3}$ s$^{-1}$ and the results are shown in Figure 8. The area cross section and its orientation are shown in Figure 8(a). The eccentricity (0.66) as well as the total amount of necking is higher than what has previously been observed at the two lower temperatures. The major axis of the ellipse from this sample was rotated 58° from the right of the test frame. Hence the diameter being measured with the optical micrometer is nearly coincident with the major axis, thus yielding transverse strains and Poisson’s ratios lower than what would have been given by the measurement of the minor axis. The sample schematic in Figure 8(a) indicates that the measured diameter (transverse strain) location is at midgage, whereas this sample necked at the top of the gage section.
Figure 7.—Tensile test of Ti-6Al-4V sample 95 at 427 °C and strain rate of $10^{-3}$ s$^{-1}$.
(a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true) showing ultimate tensile stress (UTS) and loading and unloading moduli $E_L$ and $E_U$. $\Delta E_{\text{eng}} = 34.0$ percent, $\Delta \sigma = 6.1$ percent. (c) Loading and unloading moduli $E_L$ and $E_U$ as function of loading step. (d) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{\text{davg}}$. Final Poisson’s ratio = 0.18 for nonnecked areas.
Figure 7.—Concluded.
TABLE 3.—TENSILE TEST RESULTS FOR Ti-6Al-4V
SAMPLE 95: STIFFNESS DEGRADATION
[At 427 °C and strain rate of 10⁻³ s⁻¹.]

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The tensile curves are shown in Figure 8(b). The sample (21) was not unloaded since it broke near the top radius at an axial strain of 0.133. Note that samples at the two lower temperatures all ran to strains of at least 0.2 yet displayed less of a reduction in area. Apparently the test temperature also plays a role in the degree of deformation. The loading moduli for sample 21 are \(E_{L,\text{eng}} = 85.13\) and \(E_{L,\text{true}} = 85.38\) GPa, which are much lower than those measured at the lower temperatures. The UTS occurs at engineering values of 519 MPa stress and 0.063 (axial) and 0.025 (transverse) strains. The true stress-strain curve reaches its UTS at a strain of 0.127 (near failure). Note that the maximum stress in the neck at failure (calculated using the posttest minimum diameters) is 963 MPa, nearly twice the stress in remote areas of the gauge, again indicating the intense strain in the neck at this temperature. The transverse strain is shown in Figure 8(c) and attains a maximum strain value of 0.051 at failure. The value for Poisson’s ratio begins at a value of 0.32 in the elastic regime and increases only slightly to a maximum of 0.41 at an axial strain of 0.034. However, the \(\nu_{\text{davg}}\) value measured at the neck is 2.1, significantly higher than any value observed thus far. The measured Poisson’s ratio shows less change past the UTS for this sample than all previously described samples.
Figure 8.—Tensile test of Ti-6Al-4V sample 21 at 538 °C and strain rate of 10^{-3} \text{s}^{-1}.
(a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity e. (b) Stress-strain curves (engineering and true), showing ultimate tensile stress (UTS) and loading moduli.
(c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{davg}$. Final Poisson’s ratio = 0.30 for nonnecked areas.
A second sample (39), also tested at 538 °C and the fast rate (10^{-3} s^{-1}), is shown in Figure 9. The major difference between samples 39 and 21 was their axis orientation, \( \theta \). Sample 39 had its major axis rotated 170° from the right side of the load frame (Figure 9(a)), resulting in transverse strain being nearly coincident with the minor axis. This time both the measured diameter (transverse strain) and the location of necking were at the top of the gage. Therefore, the expected values for transverse strain and Poisson’s ratio in this sample should be much larger than for the previous sample (21) tested under identical conditions, but whose transverse strain was measured at the center of the gage.

Figure 9(b) documents the tensile curves and indicates that the sample failed at a very low axial strain of 0.069. The UTS values occur at engineering values of 475 MPa stress and 0.055 (axial) and 0.075 (transverse) strains. The true-stress curve reaches its UTS at a strain of 0.062. The loading moduli for sample 39 are \( E_{\text{L,eng}} = 81.38 \) and \( E_{\text{L,true}} = 81.53 \) GPa. Note that the maximum stress in the neck at failure is 868 MPa, compared with 963 MPa for sample 21. Sample 21 appears to be stiffer and stronger, and has twice the ductility of sample 39. Sample 21 also necked more in spite of having nearly the same eccentricity as sample 39. The transverse strain and Poisson’s ratio are shown in Figure 9(c) and exhibit unique but understandable behavior given the analysis performed in Appendix B. The transverse strain at failure was 0.23, which is significantly larger than the axial failure strain (0.069). This yielded values for Poisson’s ratio that are extremely high and well over a value of 3.0 at failure. One reason for this is the fact that the minor axis was being measured and hence tracked the largest change in the sample diameter around the circumference. Another reason is that the transverse measurement coincided with the location of necking, thus yielding a larger decrease in diameter. Photogrammetry was also used on this sample to measure full-field strains and revealed similar results. For more detailed information on this test the reader is referred to Appendix C in Lerch and Arnold (2016).

A third sample (89T), was step tested at a strain rate of 10^{-3} s^{-1} and a temperature of 538 °C and is shown in Figure 10. Its ellipse is shown in Figure 10(a) with its major axis rotated 149° from the right side of the test frame. Diameter measurements close to those of the minor axis were recorded and were coincident with those of the neck location. During this test the sample was loaded and unloaded every 0.01 strain until the test reached the extensometer limit of 0.2. As shown in Figure 10(b) the stress-strain behavior is quite similar to that of sample 39 (Figure 9(b)), which was tested at an identical strain rate, but without the load-unload excursions. Note that sample 89T continued to a strain of 0.2 and, although necked, did not break. Alternatively, sample 39 failed at a strain of 0.069. The values for the loading moduli for both engineering and true values are shown in Table 4 and plotted as normalized modulus versus step increment (or percent strain) in Figure 10(c). The engineering values show that the modulus for the first loadup was 82.26 GPa, which generally decrease until test termination at a value of 56.27 GPa: a 31.6 percent drop in stiffness. In contrast, the true values start at 82.35 GPa, rise to 86.68 GPa at a 0.05 strain,\(^8\) and then remain reasonably constant throughout the test. Both unloading moduli exhibit similar behavior to their loading counterparts, albeit slightly stiffer. The transverse strain plotted in Figure 10(d) was only captured through a transverse strain value of 0.05 (through the eighth loadup). At the same point, the axial strain has a value of 0.08. Consequently, the instantaneous Poisson’s ratio shows that during this initial portion of the test, it increases from an initial value of 0.41 up to a final value of 0.68 and is clearly showing a similar rapid rise after the UTS due to the localized necking, as sample 39 had exhibited.

\(^8\) It is not known if this increase in modulus at a strain of 0.05 is real or just due to scatter.
Figure 9.—Tensile test of Ti-6Al-4V sample 39 at 538 °C and strain rate of 10⁻³ s⁻¹.
(a) Sample cross section and rotation, indicating orientation of major (a) and minor
(b) axes with respect to right side of test frame and giving eccentricity ε. (b) Stress-
strain curves (engineering and true), showing ultimate tensile stress (UTS) and
loading moduli $E_{\text{eng}} = 81.38$ GPa and $E_{\text{true}} = 81.53$ GPa.
(c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{\text{davg}}$. Final Poisson’s ratio = 0.42 for nonnecked areas.
Figure 10.—Tensile test of Ti-6Al-4V sample 89T at 538 °C and strain rate of 10^{-3} \text{s}^{-1}.
(a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity \( e \).
(b) Stress-strain curves (engineering and true), showing ultimate tensile stress (UTS) and loading and unloading moduli \( E_L \) and \( E_U \). \( \Delta F_{\text{eng}} = 31.6 \text{ percent, } \Delta \sigma = 17.1 \text{ percent.} \)
(c) Loading and unloading moduli \( E \) as function of loading step. (d) Axial and transverse strain and Poisson's ratio, showing the diameter-averaged Poisson's ratio \( \nu_{\text{davg}} \). Final Poisson's ratio = 0.36 for nonnecked areas.
Figure 10.—Concluded.
TABLE 4.—TENSILE TEST RESULTS FOR Ti-6Al-4V SAMPLE 89T
[At 538 °C and strain rate of $10^{-3}$ s$^{-1}$]

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Two additional tests were conducted at 538 °C at a slightly slower strain rate of $5 \times 10^{-4}$ s$^{-1}$. The first, sample 37, is reported in Figure 11. The elliptical orientation is given in Figure 11(a), and it shows a major axis rotation of only 10°. Thus, the minor axis was aligned with the measured transverse strain component, similar to the earlier sample (39). Similarly, the transverse diameter measurement and the location of necking were both at the top of the gage section, once again suggesting that large values for the transverse strain and Poisson’s ratio should be expected.

For this test the sample was strained until a limit was tripped at 0.014 axial strain. The sample was then reloaded to failure at a strain of 0.102 (Figure 11(b)). Two sets of loading moduli were acquired. The initial loading moduli values are $E_{L, eng} = 82.34$ and $E_{L, true} = 82.07$ GPa. The reload values are $E_{L, eng} = 78.09$ and $E_{L, true} = 79.23$ GPa, which indicate a substantial drop (5 percent) in stiffness after minimally surpassing the sample’s 0.2% yield point (408 MPa). The UTS values occur (Figure 11(c)) at an engineering stress value of 465 MPa and associated strains of 0.049 (axial) and 0.040 (transverse). The true-stress curve reaches its UTS at a strain of 0.083. Because of the lower strain rate in this test, the UTS is slightly lower than that observed previously for sample 89T. However, the initial load modulus is slightly higher, which was not expected, but is certainly within specimen-to-specimen scatter. Similarly the area-adjusted (because of necking) maximum stress at failure is 876 MPa compared to the 868 MPa of sample 39. The transverse strain, shown in Figure 11(c), exhibits nearly a one-to-one correspondence with the axial strain, therefore...
yielding a Poisson’s ratio of slightly less than 1 throughout the test. The high values for both transverse strain and Poisson’s ratio are consistent with the fact that the minor axis was measured during the test, and the diameter measurement was coincidental with the location of necking. This agreed with the results from test sample 39, although the amount of necking and eccentricity is significantly reduced in sample 37.

The second sample tested under these conditions was sample 70, documented in Figure 12. Two major differences were observed compared to the previous test. First, the sample was strained to 0.018 and relaxed for 18 h and subsequently stopped with no unload. Thereafter it was reloaded to a failure strain of 0.177. Second, the orientation of the sample ellipse (Figure 12(a)) showed that the major axis was rotated by 70°, thus indicating that the transverse strain measurements tracked more the change in the major axis, which was opposite from the previous two samples. Similar to the previous two tests, the location of necking and the diameter (transverse strain) measurement both occurred at the top of the gage. Two sets of loading moduli were again calculated. The initial loadup yielded values of $E_{L,eng} = 84.11$ and $E_{L,true} = 84.33$ GPa. The reload values were $E_{L,eng} = 82.95$ and $E_{L,true} = 85.18$ GPa. Although this showed a mild drop (1.4 percent) in stiffness for the engineering value, the true modulus actually showed an increase. The UTS values occurred at engineering values of 530 MPa at an axial strain of 0.070, whereas those associated with the true-stress curve reached its UTS at a true axial strain of 0.116. The associated stress strain tensile curves are shown in Figure 12(b). The transverse strain was not measured during this test, so unfortunately no values for Poisson’s ratio could be calculated.

The next sample (97) was tested at 538 °C and a still slower strain rate of $1 \times 10^{-5}$ s$^{-1}$ until the test was interrupted at a strain of 0.11 when a significant neck was observed near the upper radius of the gauge. This was also the location measured by the transverse extensometer. The rotation of the elliptical area is displayed in Figure 13(a) and shows a 54° rotation between the right side of the load frame and the major axis of the final elliptical cross section. Thus, the transverse strain will primarily be measuring an average reduction in the sample’s “diameter.” In Figure 13(b) the stress-strain curves showed that the sample hit its ultimate strength directly after yielding (given a criterion of 0.2% offset) and then softened to near failure of the sample. This was also true, although to a lesser extent, for the true-stress curve. Reaching the UTS at low values of axial strain was characteristic of all tests at $10^{-5}$ s$^{-1}$ and 538 °C. The UTS values occurred at an engineering stress value of 339 MPa and engineering strains of 0.018 (axial) and 0.006 (transverse). The true-stress curve reached its UTS at an engineering strain of 0.025. The initial loadup moduli values of $E_{L,eng} = 76.66$ and $E_{L,true} = 76.78$ GPa were significantly lower than those for the faster rate tests, indicative of significant viscoelastic behavior at this temperature (see Lerch and Arnold, 2014). Transverse strain exhibited a similar behavior (Figure 13(c)), in which the UTS occurred shortly after yielding was observed. The final value for the transverse strain was 0.039. Consequently, Poisson’s ratio began at an upper value in the elastic region of 0.43 and started decreasing, long before the UTS, finally reaching a value of 0.30. This near-constant value over the life of the test was unusual, particularly given the specimen’s significantly reduced final elliptical cross section.

Results for two additional tests conducted under the same loading conditions, at 538 °C and a strain rate of $1 \times 10^{-5}$ s$^{-1}$ are shown in Figure 14 (sample 15) and Figure 15 (sample 65). Sample 65 was a repeat of sample 15 since it had terminated prematurely. Both tests consisted of intermittent unloads at 0.02 strain intervals. However, in these tests the unloads were performed at the faster rate of $10^{-3}$ s$^{-1}$. This was done since at this temperature, time dependency was more prevalent and thus the stiffness behavior for the slower rate tests was more nonlinear, and fitting a modulus to the curve led to a poorer fit. Thus, a more accurate stiffness was anticipated by using the faster unloading rate.
Figure 11.—Tensile test of Ti-6Al-4V sample 37 at 538 °C and strain rate of $5 \times 10^{-4}$ s$^{-1}$.
(a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity e. (b) Stress-strain curves (engineering and true) showing ultimate tensile stress (UTS), loading and reloading moduli $E_{\text{eng}}$ and $E_{\text{true}}$, $\Delta \sigma = 4.7$ percent. (c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{\text{davg}}$. Final Poisson’s ratio = 0.36 for nonnecked areas.
Figure 12.—Tensile test of Ti-6Al-4V sample 70 at 538 °C and strain rate of $5 \times 10^{-4}$ s$^{-1}$. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true), showing ultimate tensile stress (UTS) and loading moduli $E_L$. $\Delta \sigma = 9.2$ percent.
Figure 13.—Tensile test of Ti-6Al-4V sample 97 at 538 °C and strain rate of $10^{-5}$ s$^{-1}$.
(a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true) showing ultimate tensile stress (UTS) and loading moduli $E_{\text{true}}$ = 76.66 GPa and $E_{\text{true}}$ = 76.78 GPa. (c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{\text{davg}}$. Final Poisson’s ratio = 0.30 for nonnecked areas.
Figure 14(a) shows the rotation of the elliptical area for sample 15, which indicates a 167° rotation between the right side of the load frame and the major axis of the deformed specimen. Thus, the optical micrometer primarily measured the reduction in the minor axis. Note for this sample the diameter (i.e., transverse strain) was measured at midgage, and necking occurred at the top of the gage. The stress-strain curves are shown in Figure 14(b). The test was stopped at an axial strain of 0.078 because of experimental problems. However, a substantial localized neck was already observed near the top radius. The UTS values occur at engineering values of 342 MPa stress and strains of 0.023 (axial) and 0.012 (transverse). The true-stress curve reaches its UTS also at an engineering strain of 0.023. The strain rates and moduli associated with each load-unload increment are given in Table 5 and indicates that the loading moduli are significantly smaller than the unloading moduli, as one would expect because of the difference in applied strain rate. The engineering moduli exhibits a decrease with increasing deformation (Figure 14(c)), as does the true loading modulus (but to a lesser extent) with a significant decrease in the last step. The unloading modulus at the fast rate indicates little change with continued straining for both the engineering and true moduli. These results suggest that slow rates of loading and unloading may be better indicators for tracking damage. However, irrespective of loading rate, the unloading portions of the test should be extended to lower loads to provide a larger linear region for calculating the modulus, providing that the lower loads do not produce reversed yielding. Figure 14(d) displays the engineering stress-strain curves (both axial and transverse) and shows the final transverse strain to be 0.042. Poisson’s ratio, also shown in Figure 14(d), increases from an elastic value of 0.44 to a maximum of 0.53 at approximately the UTS and remains constant through the end of the test.

The duplicate test, sample 65, fortuitously had the nearly identical sample orientation to the former sample (15), with a rotation between the major axis and the right side of the test frame of 165° (Figure 15(a)). However, this time the diameter was measured at the top of the gage section, and the sample necked at the bottom of the gage section. The stress-strain curves (Figure 15(b)) show similar behavior to sample 15: the stress peaks immediately after yielding and slowly decreases until the test terminates at a strain of 0.17. The sample displayed a localized neck at the bottom radius of the specimen. The UTS occurs at engineering values of 270 MPa stress and strain values of 0.013 (axial) and 0.007 (transverse). The maximum strength for this sample is 21 percent lower than for the previous experiment under identical conditions. However, the initial moduli are only 5.4 percent lower. Loading and unloading moduli are given in Table 6. The initial loading step is designated “L.” The first unload occurs at a strain of 0.002 and is in the nominally elastic region and designated as step “1.” The subsequent unloads occur at strain intervals of 0.02. All moduli are plotted normalized as a function of loading step in Figure 15(c). The loading moduli (both engineering and true) and the engineering unloading modulus all decrease from the beginning of loading, with the largest decreases displayed by both loading (slow rate) moduli. The loading moduli show a total decrease of approximately 85 percent over the life of the test, although the loading modulus on the last step (8) was only calculated over a 5-MPa range because of the early yield values, bringing into question the accuracy of this value. Nonetheless, the penultimate loading still exhibits a large, approximately 45 percent, decrease in modulus compared with the initial values. The true unloading modulus remains constant throughout the test, similar to that of the previous sample. This again suggests that slower loading rates may be better indicators for tracking damage. Figure 15(d) shows both axial and transverse strain behavior; however, the transverse strain is only captured up to a transverse value of 0.03. Poisson’s ratio, shown in Figure 15(d), behaves similarly to other tests performed at this temperature, but here it continually increases from an initial value of 0.42 up to 0.69 at an axial strain of 0.05, when the transverse measurement is lost. Note that Poisson’s ratio continues to increase past the UTS, which occurs at an axial strain of 0.013.
Figure 14.—Tensile test of Ti-6Al-4V sample 15 at 538 °C and strain rate of $10^{-5}$ s$^{-1}$.
(a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true), showing ultimate tensile stress (UTS) and loading and unloading moduli $E$. Strain rate is $10^{-5}$ s$^{-1}$ for loading and $10^{-3}$ s$^{-1}$ for unloading. $\Delta E_{\text{eng}} = 48.2$ percent, $\Delta \sigma = 10.2$ percent. (c) Loading and unloading moduli $E_L$ and $E_U$ as function of loading step. (d) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{\text{avg}}$. Final Poisson’s ratio = 0.52 for nonnecked areas.
TABLE 5.—TENSILE TEST RESULTS FOR Ti-6Al-4V SAMPLE 15
[At 538 °C, loading strain rate of $10^{-3}$ s$^{-1}$, and unloading strain rate of $10^{-3}$ s$^{-1}$.]  

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Figure 15.—Tensile test of Ti-6Al-4V sample 65 at 538 °C and strain rate of $10^{-5}$ s$^{-1}$.
(a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true) showing ultimate tensile stress (UTS) and loading and unloading moduli $E$. Unloads are at faster strain rate. $\Delta E_{\text{eng}} = 37.5$ percent, $\Delta \sigma = 89.5$ percent. (c) Loading and unloading moduli $E_L$ and $E_U$ as function of loading step. Loading is at $10^{-5}$ s$^{-1}$ strain rate, and unloading is at $10^{-3}$ s$^{-1}$. (d) Axial and transverse strain and Poisson's ratio, showing the diameter-averaged Poisson's ratio $\nu_{\text{davg}}$. Final Poisson's ratio = 0.38 for nonnecked areas.
Figure 15.—Concluded.
TABLE 6.—TENSILE TEST RESULTS FOR Ti-6Al-4V SAMPLE 65
[At 538 °C, loading strain rate of 10⁻⁵ s⁻¹, and unloading strain rate of 10⁻³ s⁻¹.]

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Creep

Creep tests were conducted in load control at 427 and 538 °C, with one sample tested at a temperature of 482 °C. The loading and unloading rate was always the elastic stress equivalent of 10⁻³ s⁻¹. The creep hold time was 24 h if failure did not first occur. At the end of 24 h, the sample was unloaded to zero load and held for 24 h to allow recovery. The viscoelastic recovery has been presented in Lerch and Arnold (2014), and the creep curves (strain versus time) in Lerch and Arnold (2016). The strain-versus-time curves are repeated in this report for convenience. The strain, instantaneous Poisson’s ratio, and creep rate at which apparent tertiary creep commenced were determined from the creep curves and are shown in Table 2. Some of the creep tests also had periodic unloads inserted to monitor the change in modulus to enable identification of a possible stiffness degradation mechanism under creep.

Sample 45 (see Figure 16) was creep tested at 427 °C and unloaded after a 24-h hold time. The posttested cross-sectional area geometry is shown in Figure 16(a) and exhibits a low eccentricity of 0.31 with no rotation of the major axis with respect to the right side of the test frame. Thus the minor axis was directly measured for the transverse strain. The transverse strain was measured midgage, but sample necking occurred at the top of the gage. The applied engineering creep stress is 591 MPa, which is above the sample’s 0.2% yield strength of 537 MPa and significantly above the 427 °C average viscoelastic threshold stress of 170 MPa (Lerch and Arnold, 2014). The stress-strain curves are shown in Figure 16(b). The sample reaches a total engineering strain of 0.124 in 24 h and is subsequently unloaded. This sample is in apparent tertiary creep at this point, which initiates at a strain of approximately 0.099. Tertiary creep is defined as the point where the creep rate starts to increase after a minimum. The loading moduli are $E_{L,\text{eng}} = 93.50$ and $E_{L,\text{true}} = 93.57$ GPa and are consistent with other tests at this temperature and loading rate. The unloading moduli decrease to $E_{U,\text{eng}} = 72.19$ and $E_{U,\text{true}} = 90.81$ GPa, which is a significant drop of 23 percent for the engineering value. Note that while the engineering stress is constant throughout the creep period, the true stress-strain curve displays a continually increasing stress value from an initial

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₉The word “apparent” is used here since the actual test conducted was under constant load rather than constant stress (a criterion for truly identifying tertiary creep). Therefore, the increasing creep rate (i.e., apparent tertiary creep) could merely be an artifact of an increase in inelastic flow due to an increase in the actual stress being applied. For convenience, the term “tertiary creep” will, henceforth, typically only be used.
creep stress of 600 to a final creep value of 664 MPa, with the true stress being approximated\textsuperscript{10} by \(\sigma_{\text{true}} = S(1 + e)\). The maximum true stress at the end of creep (i.e., 687 MPa) was calculated from the cross-sectional area at the neck location. It is also noteworthy that the loading and unloading moduli for true values are nearly identical (3 percent reduction).

Total strain is plotted as a function of test time in Figure 16(c). Both axial and transverse strains exhibit the typical three-stage creep curve with tertiary creep occurring at 57,700 s. After 24 h (86,466 s) of creep the sample was unloaded to zero load and recovered. Both strains decrease rapidly during unloading and reach saturation during the recovery section. The evolution of Poisson’s ratio during the test can also be observed in Figure 16(c). Both axial and transverse engineering stress-strain curves are given in Figure 16(d), with the final transverse strain at unloading being 0.077. Instantaneous Poisson’s ratio, also shown in Figure 16(d), rapidly increases from an elastic value of 0.41 to a plateau value of 0.63 at a total strain of 0.04. There is a slight decrease in Poisson’s ratio starting at an approximate axial strain of 0.09 and continues to a final value of 0.62 at unload. This drop in Poisson’s ratio, at 0.09, coincides with the beginning of tertiary creep. There is a slight increase in Poisson’s ratio during unloading, which has been documented earlier in Lerch and Arnold (2016) and is observed during unloaded tensile tests. The \(\nu_{\text{avg}}\) value of Poisson’s ratio is plotted and has a value of 0.42, whereas the final value in the neck (not measured by the transverse extensometer) is 0.78.

Figure 17 shows the creep results at 427 °C for sample 35 tested at an applied engineering stress of 586 MPa and an elastic equivalent loading rate of \(10^{-3} \text{ s}^{-1}\). This test was intended to repeat the previous test but now includes periodic unloads at every 0.02 strain to enable monitoring of the change in modulus (assessment of stiffness degradation). Figure 17(a) shows the final eccentricity of this sample to be similar to sample 45 (i.e., 0.32). The axis of rotation, however, was 90° such that the transverse strain was measured along the major axis, instead of the minor axis as in the previous sample, and at midgage. A neck once again formed at the top of the gage. The creep stress of 586 MPa for this test is also above its 0.2% yield strength of 536 MPa and viscoelastic threshold of 385 MPa. Figure 17(b) displays the stress-strain results indicating that the 24-h creep period ended at a total engineering strain of 0.14, and the sample was subsequently unloaded to zero load. The initial loading moduli are \(E_{L,\text{eng}} = 92.71\) and \(E_{L,\text{true}} = 92.83\) GPa, and they decrease to values of \(E_{L,\text{eng}} = 71.64\) and \(E_{L,\text{true}} = 90.87\) GPa during the final load increment at 0.12. The intermittent moduli are shown in Table 7, normalized against the modulus at the first step, and plotted versus step in Figure 17(c). Although there is a significant drop in the engineering moduli over the life of the test, once again there is little observable change in the true moduli. The transverse stress-strain curve is shown in Figure 17(d) and indicates the end of creep to occur at a transverse strain value of 0.028. Poisson’s ratio was found to gradually drop (decrease) from an elastic value of 0.33 to 0.22 at the onset of unloading. The beginning of tertiary creep is indicated on each plot and occurred at a total axial strain of 0.10 or a modulus step of “5”. Note that there was no descriptive change in any of the curves indicating the initiation of tertiary creep. The total strain-time plot is shown in Figure 17(c) and displays an inverse relationship to the previous sample (Figure 16(c) and (d)). Note the transverse strain in Figure 17(d) is approximately half that of the previous test (see Figure 16(d)), even though the axial strain accumulated is almost the same in both tests. This explains why sample 35 Poisson’s ratio is significantly lower than sample 45. This difference in transverse strain, and consequently the Poisson’s ratio behavior, is consistent with the known anisotropy of the material; reflected in the fact that in the present case the transverse strain measurement is associated with the major axis, whereas in the previous sample (45) it was associated with the minor axis. Lastly, the onset of apparent tertiary creep can be observed in the axial strain curve to occur at 72,800 s.

\textsuperscript{10}Note this equation has an inherent assumption of Poisson’s ratio being equal to 0.5.
Figure 16.—Creep test of Ti-6Al-4V sample 45 at 591 MPa, 427 °C, and strain rate of $10^{-3}$ s$^{-1}$. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$.

(b) Stress-strain curves (engineering and true), showing inception of tertiary creep; loading and unloading moduli $E_L$ and $E_U$ are given, with $\Delta E_{eng} = 22.8$ percent; and calculated maximum true stress = 687 MPa. (c) Strain-time response (axial and transverse) and Poisson’s ratio, showing inception of tertiary creep. (d) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{davg}$. Final Poisson’s ratio = 0.39 for nonnecked area.
Figure 16.—Concluded.
Figure 17.—Creep test of Ti-6Al-4V sample 35 at 586 MPa, 427 °C, and strain rate of $10^{-3}$ s$^{-1}$. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true) showing inception of tertiary creep; loading and unloading moduli $E_L$ and $E_U$ are listed, with $\Delta E_{eq} = 22.7$ percent and calculated maximum true stress = 758 MPa. (c) Normalized loading and unloading moduli $E_L$ and $E_U$ as function of loading step. (d) Axial and transverse strain and Poisson's ratio, showing the diameter-averaged Poisson's ratio $\nu_{avg}$. Final Poisson's ratio = 0.34 for nonnecked area. (e) Strain-time response (axial and transverse) and Poisson's ratio, showing inception of tertiary creep.
Figure 17.—Concluded.
TABLE 7.—CREEP TEST RESULTS FOR Ti-6Al-4V SAMPLE 35
[At 427 °C and applied engineering stress = 586 MPa.]

<table>
<thead>
<tr>
<th>Step</th>
<th>Loading, $E_{L, eng}$</th>
<th>Unloading, $E_{U, eng}$</th>
<th>Loading, $E_{L, true}$</th>
<th>Unloading, $E_{U, true}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>92.71</td>
<td>-----</td>
<td>92.83</td>
<td>-----</td>
</tr>
<tr>
<td>1</td>
<td>85.49</td>
<td>85.17</td>
<td>89.86</td>
<td>90.00</td>
</tr>
<tr>
<td>2</td>
<td>83.02</td>
<td>83.96</td>
<td>90.14</td>
<td>91.90</td>
</tr>
<tr>
<td>3</td>
<td>79.97</td>
<td>80.12</td>
<td>90.85</td>
<td>89.46</td>
</tr>
<tr>
<td>4</td>
<td>78.53</td>
<td>76.56</td>
<td>90.64</td>
<td>89.17</td>
</tr>
<tr>
<td>5</td>
<td>73.90</td>
<td>73.45</td>
<td>90.46</td>
<td>89.17</td>
</tr>
<tr>
<td>6</td>
<td>71.64</td>
<td>71.71</td>
<td>90.87</td>
<td>90.44</td>
</tr>
<tr>
<td>7UL</td>
<td>-----</td>
<td>69.15</td>
<td>-----</td>
<td>89.96</td>
</tr>
</tbody>
</table>

Figure 18 reveals the results at 427 °C for sample 87 tested at an applied engineering creep stress of 596 MPa and an elastic equivalent loading rate of $10^{-3}$ s$^{-1}$. Posttest analysis indicates that sample 87 was oriented such that the major axis was rotated 25° from the right side of the test machine and had a final eccentricity of 0.36 (see Figure 18(a)). Note that tests on samples 45, 35, and 87 (Figure 16 to Figure 18) were crept under similar conditions (and thus can all be viewed as repeats) and developed similar eccentricities. The transverse strain for sample 87 was again measured at midgage, with necking occurring fortuitously once again at the top of the gage section. The stress-strain curves are shown in Figure 18(b). At an axial strain of 0.120 a limit was tripped, causing the test to terminate and the sample to cool to room temperature. Tertiary creep had already begun at this strain. The test was restarted and the sample crept, once again at an applied engineering creep stress of 596 MPa, to a strain of 0.134 at which point there is an unload-load cycle. The sample continues creeping until it fractures at a strain of 0.21. The moduli are displayed in Table 8 and plotted normalized versus loading step in Figure 18(c), which clearly shows a continual decrease in the engineering moduli and no change (or actually an increase) in the true moduli. The engineering stress-strain curve, with both axial and transverse strains, is shown in Figure 18(d) and indicates that fracture occurs at a transverse strain of 0.106. Poisson’s ratio increases from an elastic value of 0.34 to a maximum value of 0.64 at an axial strain of 0.033. It begins to decrease at a total axial strain of 0.078, which is earlier than the observation of the beginning of tertiary creep (0.120). Except for the period during unloading, Poisson’s ratio continues to decrease until fracture occurs at a value of 0.51. The value for $v_{davg}$ is shown with the asterisk and is equal to 0.39. The value of Poisson’s ratio for the necked area is 1.22. Strain-time histories for axial, transverse and Poisson’s ratio are shown in Figure 18(e), where the onset of tertiary creep appears to take place at 11,100 s. Note that the transverse strain measurement, for sample 87, is associated with neither the minor (sample 45) nor the major (sample 35) axes—as was the case for the prior two tests. It does, however, lean toward the minor axis and so it is not surprising that its behavior (strain and Poisson’s ratio) reflects that of sample 45 both in magnitude and history.
Figure 18.—Creep test of Ti-6Al-4V sample 87 at 596 MPa, 427 °C, and strain rate of $10^{-3}$ s$^{-1}$. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true) showing inception of tertiary creep; loading and unloading moduli $E_L$ and $E_U$ are listed, with $\Delta E_{\text{eng}} = 15.2$ percent and calculated maximum stress $= 1,222$ MPa. (c) Normalized loading and unloading moduli $E_L$ and $E_U$ as function of loading step. (d) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{\text{avg}}$. Final Poisson’s ratio $= 0.36$ for nonnecked area. (e) Strain-time response (axial and transverse) and Poisson’s ratio, showing inception of tertiary creep.
Figure 18.—Concluded.
TABLE 8.—CREEP TEST RESULTS FOR Ti-6Al-4V SAMPLE 87
[At 427 °C and applied engineering stress of 596 MPa.]

<table>
<thead>
<tr>
<th>Step</th>
<th>Modulus, GPa</th>
<th>Engineering stress</th>
<th>True stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loading, $E_{L,\text{eng}}$</td>
<td>Unloading, $E_{U,\text{eng}}$</td>
<td>Loading, $E_{L,\text{true}}$</td>
</tr>
<tr>
<td>L1</td>
<td>89.98</td>
<td>-----</td>
<td>90.29</td>
</tr>
<tr>
<td>L2</td>
<td>78.13</td>
<td>-----</td>
<td>96.84</td>
</tr>
<tr>
<td>3</td>
<td>76.26</td>
<td>74.03</td>
<td>97.14</td>
</tr>
</tbody>
</table>

Only one creep test, sample 81, was conducted at a test temperature of 482 °C, and its results are given in Figure 19. The sample was loaded at an equivalent elastic strain rate of $10^{-3}$ s$^{-1}$ to a creep stress of 376 MPa and then crept to failure, which occurred at an axial strain of 0.104. This sample had a fairly high posttest eccentricity (0.68) and was rotated 42° with respect to its major axis (Figure 19(a)). The transverse strain measurement coincides with the neck location at the top of the gage section. The stress-strain curves (Figure 19(b)) indicate that the loadup is nominally linear. This is in contrast to all creep tests performed at 427 °C that experienced large-scale plasticity during loading. The creep stress for sample 81 is between its proportional limit and its estimated 0.02% yield stress of 403 MPa (Lerch and Arnold, 2016). Tertiary creep begins at an axial strain of 0.046. The transverse strain is shown in stress-strain space in Figure 19(c) and strain-time space in Figure 19(d) and indicates fracture occurs at a transverse strain of 0.165, which is significantly larger than the axial strain at failure, 0.104. Poisson’s ratio begins at an elastic value of 0.31 and increases to failure, yielding a final value of 1.59. Values for an assumed $\nu_{\text{davg}}$ are given in Figure 19(c) for both the neck ($\nu = 2.87$) and nonnecked region ($\nu = 0.46$). There appears to be an inflection point at the beginning of tertiary creep, but it would be hard to identify it as the beginning of tertiary creep by only examining the curve for Poisson’s ratio given in Figure 19(c) in real time. Thus, using this value as an in-service monitoring parameter would be difficult.

The continually increasing Poisson’s ratio is a result of actually measuring the transverse strain at the localized necking location. The strain-time curves are given in Figure 19(d) and show the onset of tertiary creep occurring at 15,600 s; this is easily observed in all three curves (axial, transverse, and Poisson’s ratio) on this plot by their inflection points.

A number of creep tests were performed at the highest test temperature of 538 °C. At the lowest stress level of 206 MPa, sample 94 (see Figure 20) was crept to an axial strain of 0.053 and then unloaded to zero load. This sample was oriented 150° to its major axis and exhibits a low eccentricity of 0.25 (Figure 20(a)). The transverse strain represents predominantly the deformation (dimensions) along the minor axis of the specimen. The stress-strain curves are shown in Figure 20(b). Note that the engineering unloading modulus is much smaller (a 14.4-percent drop) during the final unloading compared to the initial loading modulus. This is even exhibited by the true values, yet to a lesser extent (4.7-percent drop). The initiation of the tertiary regime occurs at an axial strain of 0.019. The axial and transverse strain are shown in Figure 20(c) and (d) and denote that unloading occurs at strains of 0.030 (transverse) and 0.053 (axial). Poisson’s ratio begins at a value of 0.50 in the elastic regime and shows a small increase, maximizing at a value of 0.57, and remains constant over an axial strain of 0.01 to 0.02. A very small decrease follows thereafter, ending at a value of 0.53. The $\nu_{\text{davg}}$ takes on a value of 0.34, and the value in the neck is 0.69. Strain-time curves are presented in Figure 20(d) and include the final unload and recovery period. Tertiary creep begins at 21,200 s.
Figure 19.—Creep test of Ti-6Al-4V sample 81 at 376 MPa, 482 °C, and strain rate of $10^{-3}$ s$^{-1}$. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true) showing loading moduli $E_L$ and inception of tertiary creep. (c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{davg}$. Final Poisson’s ratio = 0.43 for nonnecked area. (d) Strain-time response (axial and transverse) and Poisson’s ratio showing inception of tertiary creep.
Figure 19.—Concluded.
Figure 20.—Creep test of Ti-6Al-4V sample 94 at 206 MPa, 538 °C, and strain rate of $10^{-3}$ s$^{-1}$. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true) showing loading and unloading moduli $E_E$ and $E_U$ and inception of tertiary creep. $\Delta F_{e,n} = 13.9$ percent. (c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{davg}$. Final Poisson’s ratio $= 0.46$ for nonnecked area. (d) Strain-time response (axial and transverse) and Poisson’s ratio showing inception of tertiary creep.
Figure 20.—Concluded.
Sample 1 test conditions are similar to those of sample 94, except the target stress value was increased roughly 10 percent beyond sample 94. Sample 1 had a 125° rotation of its major axis and an eccentricity of 0.22 (Figure 21(a)). The stress-strain curves (Figure 21(b)) show that creep occurs at a stress of 226 MPa, which is slightly below the sample’s estimated 0.02% yield strength of 230 MPa (Lerch and Arnold, 2016). The sample crept until reaching a total strain of 0.086, at which point the sample was unloaded to 0 load and allowed to recover. The unloading moduli are lower than the initial loading moduli, thus suggesting damage of the stiffness-degradation type. Apparent tertiary creep begins at a total axial strain of 0.022. The transverse strain (Figure 21(c)) hits a maximum of 0.031 before the sample is unloaded. Poisson’s ratio begins from an elastic value of 0.24 and increases during the initial creep stage, finally maximizing at a value of 0.4 at an axial strain of approximately 0.01, which is a smaller strain than where tertiary creep is observed. There is a very slight decrease in Poisson’s ratio thereafter, which reaches a final value of 0.36 before unload. The $\nu_{dav}$ is 0.45, and in the necked area it is 0.94. Strain-time curves are presented in Figure 21(d) and include the final unload and recovery period. Tertiary creep begins at 27,600 s and corresponds with the beginning of a slight drop in Poisson’s ratio. Note the Poisson’s ratio curvature (as a function of time) is very similar (although different in magnitude) to that of samples tested at 427 °C: compare samples 35 and 1 (transverse along major axis) and then samples 45 and 94 (transverse along minor axis).

Sample 3 was crept at a slightly higher stress of 241 MPa, which is still below the average 0.02% yield strength at this temperature of 538 °C (Lerch and Arnold, 2016). The sample was oriented 18° to the major axis (which means the transverse strain was associated with the minor axis) and has a low eccentricity of 0.26 (Figure 22(a)). This test ran in creep until fracture (Figure 22(b)) at a total strain of 0.115. The transverse strain at fracture (Figure 22(c)) is 0.069. Poisson’s ratio begins at an elastic value of 0.61, increases very slightly to 0.63 near the beginning of the apparent tertiary regime (0.025), and decreases very slightly until attaining a value of 0.60 at fracture. The $\nu_{dav}$ is 0.45, and in the necked area it is 2.89. Strain-time curves are presented in Figure 22(d) and indicate that tertiary creep begins at 17,800 s and roughly corresponds with the peak in Poisson’s ratio.

Figure 23 displays results for a repeat test, sample 34, crept at 538 °C and 241 MPa, again below its average 0.2% yield strength, but this time intermediate unload/reloads were planned to assess damage. The sample orientation was 176° from the right side of the test rig (Figure 23(a)), which means that the transverse measurement is associated almost entirely with the minor axis. Eccentricity is a bit higher for this sample at 0.37. Figure 23(b) shows the stress-strain curves with unload-load cycles every 0.02 strain increments. The final fracture of the sample occurs at a total axial strain of 0.120. The moduli at each strain increment are given in Table 9 and plotted normalized versus loading step in Figure 23(c). As typically observed in previous creep tests, the moduli taken from the engineering values display a continuous drop in moduli over the length of the test, whereas the moduli from the true values remain largely unchanged. There is a slight dropoff of the final unloading true modulus, which could just be attributed to scatter. Strain-time curves are presented in Figure 23(d), and stress-strain curves in Figure 23(e). The transverse strain reaches a value of 0.071 at failure. Poisson’s ratio increases from an elastic value of 0.40 to a maximum of 0.6 near the tertiary point (0.025) (see Figure 23(d)). It remains nearly constant through the end of the test. Note that the final value for Poisson’s ratio in the neck is 2.62, but the $\nu_{dav}$ value in the nonnecked area is 0.47. Tertiary creep begins at 14,700 s, which does not correspond with any perceptible behavioral change in Poisson’s ratio.
Figure 21.—Creep test of Ti-6Al-4V sample 1 at 226 MPa, 538 °C, and strain rate of $10^{-3}$ s$^{-1}$. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$.

(b) Stress-strain curves (engineering and true) showing loading and unloading moduli $E_L$ and $E_T$ and inception of tertiary creep. $\Delta E_{\text{eng}} = 18.9$ percent. (c) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{\text{davg}}$. Final Poisson’s ratio = 0.45 for nonnecked area. (d) Strain-time response (axial and transverse) and Poisson’s ratio showing inception of tertiary creep.
Figure 21.—Concluded.
Figure 22.—Creep test of Ti-6Al-4V sample 3 at 241 MPa, 538 °C, and strain rate of \(10^{-3} \text{ s}^{-1}\). (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity \(e\). (b) Stress-strain curves (engineering and true) showing loading moduli \(E_L\) and inception of tertiary creep. (c) Axial and transverse strain and Poisson's ratio, showing diameter-averaged Poisson's ratio \(\nu_{\text{davg}}\). Final Poisson's ratio = 0.39 for nonnecked area. (d) Strain-time response (axial and transverse) and Poisson's ratio showing the inception of tertiary creep.
Figure 22.—Concluded.

(c) Tertiary  Fracture

(d) Axial  Transverse  Poisson’s ratio

$\nu_{avg} = 2.91$
Figure 23.—Creep test of Ti-6Al-4V sample 34 at 241 MPa, 538 °C, and strain rate of 10⁻³ s⁻¹. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity e. (b) Stress-strain curves (engineering and true) showing loading and unloading moduli $E_L$ and $E_U$, and inception of tertiary creep. $\Delta E_{eng} = 19.5$ percent. (c) Normalized loading and unloading moduli $E_L$ and $E_U$ as function of loading step. (d) Strain-time response (axial and transverse) and Poisson’s ratio showing inception of tertiary creep. (e) Axial and transverse strain and Poisson’s ratio, showing the diameter-averaged Poisson’s ratio $\nu_{davg}$. Final Poisson’s ratio = 0.46 for nonnecked area.
Figure 23.—Concluded.
Sample 85 was tested at an applied engineering creep stress of 239 MPa, which is above its 0.02% yield strength of 218 MPa. The sample was rotated 128° from the right of the test rig and its eccentricity is 0.40 (Figure 24(a)). The stress-strain curves in Figure 24(b) indicate failure at an axial strain of 0.097. The final transverse strain shown in Figure 24(c) is 0.062. Poisson’s ratio begins at 0.27, increasing continuously to the end of the test to a value of 0.65. This behavior differs from prior tests at this temperature, but it must be remembered that in this test necking and transverse strain measurement coincided, thus explaining the increase in transverse strain and consequently Poisson’s ratio beyond the onset of tertiary creep. Also, the transverse strain increase is not as large since it is associated more with the major axis than the minor. Tertiary creep begins at an axial strain of 0.026 and at a time of 3,480 s as shown in Figure 24(d).

Figure 25 shows the results from a thermal recovery test, sample 89. A thermal recovery test is one in which after prior inelastic flow (e.g., creep) a specimen is unloaded to a lower but nonzero stress and held at temperature for some time to enable thermal recovery of internal stress (back stress), through such mechanisms as thermal annihilation of dislocations (see Lerch and Arnold, 2016), and subsequently reloaded. The sample was rotated 142° from the right of the test rig, and its deformed cross section eccentricity was determined to be 0.46 (Figure 25(a)). This sample was crept at a stress of 243 MPa with an unload-reload cycle at an axial strain of 0.033 (Figure 25(b)). The sample is again unloaded to a stress of 74 MPa, at an axial strain of 0.075, and held at this stress for 8 h to allow recovery, and then reloaded to 243 MPa, where it crept until a limit is tripped at a total strain of 0.160. It was, however, near failure because a localized neck had formed near the top radius. The moduli are given in Table 10 and plotted normalized as a function of loading step in Figure 25(c). The first unload displays a 4-percent change in modulus for the engineering value (Figure 25(b)). The reloading modulus decreases by 5 percent for the engineering value and actually increases by 2.6 percent for the true value. Once again the moduli taken from the engineering values depict a continuous decrease, whereas the true moduli exhibit only a slight decrease, small enough that it could be attributed to data scatter as opposed to damage. The transverse strain, shown in Figure 25(d), reaches a value of 0.065 when the limit tripped. Poisson’s ratio begins at 0.27, increasing to a value of 0.45 slightly before the onset of the tertiary region (axial strain = 0.018), followed by a decrease until reaching 0.40 at the end of the test. Poisson’s ratio in the neck is 0.85 (taken from posttest diameter measurements). The $v_{davg}$ value in the nonnecked area is 0.43. Stress-versus-time curves for axial and transverse stresses and Poisson’s ratio are shown in Figure 25(e). Note that tertiary creep begins early in the

![Image](image-url)
test at 3,660 s, which represents only 7 percent of the total test time. Hence both unloads and the thermal recovery occurred after significant inelastic straining. During the second unload, both axial and transverse strain exhibit a small decrease in values (Figure 25(c)). Both strains increased slightly during the thermal recovery period, indicating that the recovery stress of 74 MPa was still high enough to cause forward-going creep. At the end of recovery and upon reloading to the originally applied stress, a slight primary creep regime followed by secondary creep was reinitiated before strains continued at an increasing rate (apparent tertiary creep). It is interesting to note that the Poisson’s ratio remains reasonably constant during and after the thermal recovery period with only a slight decrease occurring after reload.

Figure 24.—Creep test of Ti-6Al-4V sample 85 at 239 MPa, 538 °C, and strain rate of 10⁻³ s⁻¹. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity e. (b) Stress-strain curves (engineering and true) showing loading moduli \( E_{\text{eng}} \) and inception of tertiary creep. (c) Axial and transverse strain and Poisson’s ratio showing the diameter-averaged Poisson’s ratio \( \nu_{\text{davg}} \). Final Poisson’s ratio = 0.46 for nonnecked area. (d) Strain-time response (axial and transverse) and Poisson’s ratio showing inception of tertiary creep.
Figure 24.—Concluded.
Figure 25.—Creep test of Ti-6Al-4V sample 89 at 243 MPa, 538 °C, and strain rate of $10^{-3}$ s$^{-1}$. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true) showing loading and unloading moduli $E_L$ and $E_U$ and inception of tertiary creep. $\Delta E_{eng} = 13.5$ percent. (c) Loading and unloading moduli $E_L$ and $E_U$ as function of loading step. (d) Axial and transverse strain and Poisson’s ratio. Final Poisson’s ratio = 0.33 for nonnecked area. (e) Strain-time response (axial and transverse) and Poisson’s ratio showing inception of tertiary creep.
Figure 25.—Concluded.
TABLE 10.—CREEP TEST RESULTS FOR Ti-6Al-4V SAMPLE 89
[At 538 °C and applied engineering stress of 243 MPa.]

<table>
<thead>
<tr>
<th>Step</th>
<th>Modulus, GPa</th>
<th>Engineering stress</th>
<th>True stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loading, $E_{L,\text{eng}}$</td>
<td>Unloading, $E_{U,\text{eng}}$</td>
<td>Loading, $E_{L,\text{true}}$</td>
</tr>
<tr>
<td>L</td>
<td>82.67</td>
<td>-----</td>
<td>81.94</td>
</tr>
<tr>
<td>1</td>
<td>79.77</td>
<td>72.52</td>
<td>84.06</td>
</tr>
<tr>
<td>2</td>
<td>71.51</td>
<td>65.77</td>
<td>81.81</td>
</tr>
</tbody>
</table>

The final creep test at 538 °C, sample 88, was conducted at the highest stress of 376 MPa, which is between the sample’s 0.02 and 0.2 percent estimated yield. The specimen’s deformed elliptical cross section was rotated 156° from the right side of the test machine and exhibited the highest eccentricity in this study of 0.74 (Figure 26(a)). The test crept until fracture at an axial strain of 0.132 (Figure 26(b)) and a transverse strain of 0.165 (Figure 26(c)). Note that the transverse strain is larger in magnitude than the axial strain since the measurement location coincided with both the neck and the minor axis. This led to very large Poisson’s ratios, which begin at an elastic value of 0.28 and increase during the creep test to a final measured value of 1.53 as well as to a value of 2.20 using the minimum neck dimensions. The nonnecked $\nu_{\text{avg}}$ value is 0.21, about the same as the initial elastic value. The tertiary creep phase initiates at 0.033, which is the first inflection point in the Poisson’s ratio curve with the second (0.1) being associated with onset of significant necking (see Figure 26(d)). Strains as a function of time are plotted in Figure 26(d) and depict the typical three-stage creep behavior. It is observed that the primary creep regime is very short, which is typical at high homologous temperatures. The majority of the creep time is spent in the apparent tertiary regime (i.e., the zone in which the true stress is constantly increasing, and thus so too is the inelastic strain rate). Finally, the average failure strain of Ti-6-4 under creep at 538 °C appears to be 12.5 percent strain.

**Stress Relaxation**

Three step-relaxation tests were conducted to examine modulus change as a function of deformation history. Three test temperatures were examined: 20, 427, and 538 °C. The loading rate for all tests was $10^{-3}$ s$^{-1}$. The relaxation period was designed to be 24 h.

The first test (sample 89B) occurred at room temperature (Figure 27). The sample was oriented with the major axis being 90° from the right side of the test rig (Figure 27(a)). Consequently, the measured transverse strain was fully aligned with the major axis. The final eccentricity was very low at 0.15. Note that necking occurred midgage and transverse strain was measured at the top of the gage section for this specimen. Figure 27(b) shows the stress-strain curves and indicates multiple relaxation periods, each occurring after a 2 percent strain increment. This first load cycle was performed as an initially stand-alone relaxation test at a slow loading rate of $6 \times 10^{-7}$ s$^{-1}$, with the relaxation occurring at 0.018 axial strain. There is also a loading curve plotted (see Figure 27(b)) for a second sample (10) loaded at a strain rate of $10^{-3}$ s$^{-1}$ for comparison. Note that even at room temperature, Ti-6-4 exhibits rate-dependent yielding, with the sample loaded at the faster rate showing higher stresses, which is more comparable with the remaining load steps for sample

---

$\nu_{\text{avg}}$ is defined as $T/T_m$, where $T$ is the applied temperature and $T_m$ is the melting temperature of the material (on absolute scale). Note “high” is typically considered to be $T/T_m > 0.25$. 

11Homologous temperature is defined as $T/T_m$, where $T$ is the applied temperature and $T_m$ is the melting temperature of the material (on absolute scale). Note “high” is typically considered to be $T/T_m > 0.25$. 

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Step relaxation was also performed at 427 °C, as shown in Figure 28. The sample (27) was oriented at a 90° angle between its major axis and the right side of the load frame (Figure 28(a)), which is a similar orientation to the previous sample, 89B. The eccentricity is extremely high having a value of 0.71, which is perhaps associated with the higher test temperature. Also, for this specimen necking and transverse strain measurement coincided at the top of the gage section.

The test started with a two-step relaxation occurring at axial strains of 0.012 and 0.018 for 24 h each, followed by an unload to zero load and a 19-h recovery. The sample was later exposed to another multistep relaxation history at strains of 0.03, 0.044, and 0.066. The relaxation period at 0.066 strain was only 7 h in length because of a hydraulic shutdown. The sample was reloaded to 0.115 strain at which point a limit was tripped. The sample was once again reloaded for a relaxation at 0.126 for 24 h and thereafter loaded to fracture at a final axial strain of 0.141. The complete sample stress-axial strain loading history is shown in Figure 28(b). Also shown in this figure are the engineering and true stress-strain curves for a second sample (23) that was monotonically loaded in tension to failure under the same conditions to indicate that the step relaxation process did not largely effect the peak stresses. Any differences could be easily attributed to specimen-to-specimen scatter. A similar observation was made for Ti-6-4 by Evans (1987) where he stated that “relaxation did not significantly alter the deformation characteristics of the alloy.” The loading moduli are listed in Table 12 and also plotted normalized versus step in Figure 28(c). Once again, the engineering values show a decrease over the life of the test, exhibiting an initial to final difference of 14 percent. The true values were constant until step six (0.115), at which point they jumped by 9 percent.
Figure 26.—Creep test of Ti-6-4 sample 88 at 376 MPa, 538 °C, and strain rate of $10^{-5}$ s$^{-1}$. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity $e$. (b) Stress-strain curves (engineering and true) showing loading moduli $E_L$ and inception of tertiary creep. Calculated maximum true stress = 776 MPa. (c) Axial and transverse strain and Poisson’s ratio. Final Poisson’s ratio = 0.22 for nonnecked area. (d) Strain-time response (axial and transverse) and Poisson’s ratio showing inception of tertiary creep.
Figure 26.—Concluded.
Figure 27.—Stress relaxation test of Ti-6-4 sample 89B at 20 °C and strain rate of 10^{-3} \text{s}^{-1}. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame, and giving eccentricity \( e \). (b) Stress-strain curves (engineering and true) showing ultimate tensile stress (UTS) and loading moduli, \( E_L \). First step was loaded at strain rate of 6\times10^{-7} \text{s}^{-1}. Change in engineering modulus \( \Delta E_{\text{eng}} = 17.5 \text{ percent} \). (c) Engineering and true loading moduli \( E_L \) as function of loading step. (d) Axial and transverse strain and Poisson’s ratio. Nonnecked Poisson’s ratio = 0.02.
TABLE 11.—STRESS RELAXATION TEST RESULTS FOR Ti-6Al-4V SAMPLE 89B
[At 20 °C and strain rate of 10^{-3} \text{s}^{-1}.]

<table>
<thead>
<tr>
<th>Step</th>
<th>Engineering stress</th>
<th>True stress</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Loading, $E_{L,\text{eng}}$</td>
<td>Unloading, $E_{U,\text{eng}}$</td>
</tr>
<tr>
<td>L (slow)</td>
<td>112.01</td>
<td>103.05</td>
</tr>
<tr>
<td>L (fast)</td>
<td>110.33</td>
<td>Relax</td>
</tr>
<tr>
<td>1</td>
<td>102.66</td>
<td>Relax</td>
</tr>
<tr>
<td>2</td>
<td>98.36</td>
<td>Relax</td>
</tr>
<tr>
<td>3</td>
<td>95.07</td>
<td>Relax</td>
</tr>
<tr>
<td>4</td>
<td>90.95</td>
<td>Relax</td>
</tr>
</tbody>
</table>
Figure 28.—Stress relaxation test of Ti-6-4 sample 27 at 427 °C and strain rate of 10^−3 s^{-1}. (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to the right side of test frame, and giving eccentricity e. (b) Stress-strain curves (engineering and true), showing ultimate tensile stress (UTS) and loading moduli, \( E_{L} \). Tensile sample 23 plotted for comparison. Change in engineering modulus \( \Delta E_{\text{eng}} = 14.3 \) percent, and stress, \( \Delta \sigma = 8.0 \) percent. (c) Loading moduli \( E_{L,\text{eng}} \) and \( E_{L,\text{true}} \) as function of loading step. (d) Axial and transverse strain and Poisson’s ratio, showing diameter-averaged Poisson’s ratio \( \nu_{\text{davg}} \). Nonnecked Poisson’s ratio = 0.05.
Figure 28.—Concluded.

TABLE 12.—STRESS RELAXATION TEST RESULTS FOR Ti-6Al-4V SAMPLE 27
[At 427 °C and strain rate of 10^{-3} s^{-1}.]

<table>
<thead>
<tr>
<th>Step</th>
<th>Modulus, GPa</th>
<th>Engineering stress</th>
<th>True stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loading, $E_{L,\text{eng}}$</td>
<td>Unloading, $E_{U,\text{eng}}$</td>
<td>Loading, $E_{L,\text{true}}$</td>
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<tr>
<td>L</td>
<td>88.86</td>
<td>-----</td>
<td>89.18</td>
</tr>
<tr>
<td>1</td>
<td>88.02</td>
<td>Relax</td>
<td>90.50</td>
</tr>
<tr>
<td>2</td>
<td>87.92</td>
<td>87.21</td>
<td>90.30</td>
</tr>
<tr>
<td>3</td>
<td>85.65</td>
<td>Relax</td>
<td>90.63</td>
</tr>
<tr>
<td>4</td>
<td>83.91</td>
<td>Relax</td>
<td>90.81</td>
</tr>
<tr>
<td>5</td>
<td>81.95</td>
<td>-----</td>
<td>89.87</td>
</tr>
<tr>
<td>6</td>
<td>79.08</td>
<td>Relax</td>
<td>97.05</td>
</tr>
<tr>
<td>7</td>
<td>76.11</td>
<td>Relax</td>
<td>97.78</td>
</tr>
</tbody>
</table>
The stress-transverse strain history is shown in Figure 28(d) and depicts the end of the transverse strain at sample failure at a value of 0.060. Note that the transverse strain continues to increase during the relaxation dwell as was pointed out earlier by Lerch and Arnold (2016). This is because the axial strain was controlled during the test, but the transverse strain was not held constant. This can be best observed in the relaxation for the sixth step near 0.05 transverse strain where the relaxation line has a slight negative slope rather than a pure vertical slope if the transverse strain had been constant. Poisson’s ratio is also plotted in Figure 28(d) and begins at an elastic value of 0.26. It remains relatively constant until the first relaxation (0.012 strain) at which point it increases. In fact, it increases every time there is either a relaxation or unloading. The final value of Poisson’s ratio at fracture is 0.42. The value of Poisson’s ratio in the necked area is 1.79.

The final relaxation test was performed at a temperature of 538 °C and is displayed in Figure 29. The sample was oriented such that its major axis coincided with the transverse strain’s optical beam, $\theta = 0$ (Figure 29(a)), which is a 90° difference compared to those of the last two samples. Consequently, the measured transverse strain was fully aligned with the minor axis. The eccentricity is intermediate to the previous two relaxation tests and has a value of 0.56. Figure 29(b) depicts the loading history of this sample (77). Sample 77 was first loaded to a strain of 0.018 and relaxed for 8 h, ending with an unplanned shutdown. The test was restarted and relaxed for 24 h at strains of 0.04, 0.079, and 0.119. A final relaxation occurred at a strain of 0.158, but stopped after 16 h of relaxation. The test was subsequently restarted and failed upon loading at an axial strain of 0.183. Engineering values from two additional tensile samples (21 and 39) tested under similar conditions are also plotted in this figure to again indicate that the maximum stresses are similar (given specimen variability) to those observed in a monotonically loaded test. The change in the loading modulus is similar to that shown for the other two relaxation tests where there is a decrease in the engineering modulus and no change in the true modulus (see Table 13 and Figure 29(c)). There is again a jump in the final loadup for both moduli and this may be due to the influence of a large crack forming outside of the gage section as in the previous sample; hence, this may be an indicator of damage. The transverse strain response is shown in Figure 29(d). Failure takes place at a transverse strain of 0.172. This is nearly the same strain as that obtained in the axial direction. Hence, this yields a Poisson’s ratio of almost 1 at the end of the test. This is understandable since necking and transverse strain measurement coincide, thus producing transverse strain magnitudes similar to those occurring at the location of axial strain measurement. Poisson’s ratio behaves similarly to the previous sample, with the exception that the values are larger, which is consistent with the fact that the transverse strain represents the minor axis and experiences the most reduction in diameter around the circumference. The maximum value of Poisson’s ratio in the necked area is 1.53, which indicates that the transverse extensometer was not quite measuring the nadir of the neck, but a slightly larger diameter. The final Poisson’s ratio in the nonnecked area is 0.22.

**Fatigue**

Five samples underwent cyclic loading to produce a stress-life (S-N) curve (Figure 30). These tests were conducted in load control at a load ratio of zero (i.e., $R_\sigma = \sigma_{\text{min}}/\sigma_{\text{max}} = 0$). Tests were run at 427 °C and at an elastic stress-equivalent rate of $10^{-3}$ s$^{-1}$. An inverted S-shaped curve was fitted to the data shown in the semilogarithmic plot in Figure 30. The upper most point was taken from the value for a tensile test (sample 23), using its UTS and a life of one-half cycle. The upper and lower asymptotes on this curve are given by the UTS of 655 MPa and the estimated fatigue endurance limit of 515 MPa that is based on two standard deviations from the average 0.2% yield stress definition (see Table II in Lerch and Arnold, 2016). The fatigue data are plotted on the traditional double logarithmic plot of total strain range versus life in Figure 31. The strain range was calculated at half-life by dividing the maximum stress by the
modulus. At half-life the stress-strain behavior was predominantly linear elastic. The fatigue life equation is also given in Figure 31 and shows the exponent to be \(-0.06\). This is typical for most metallic materials (Manson and Halford, 2006).

Figure 29.—Stress relaxation test of Ti-6-4 sample 77 at 538 °C and strain rate of \(10^{-3}\) s\(^{-1}\). (a) Sample cross section and rotation, indicating orientation of major (a) and minor (b) axes with respect to right side of test frame and giving eccentricity \(e\). (b) Stress-strain curves (engineering and true) showing ultimate tensile stress (UTS) and loading moduli, \(E_L\). Tensile samples 21 and 39 are plotted for comparison. Change in engineering modulus \(\Delta E_{\text{eng}} = 10.5\) percent, and stress, \(\Delta \sigma = 6.2\) percent. (c) Loading moduli \(E_L\) as function of loading step. (d) Axial and transverse strain and Poisson’s ratio, showing diameter-averaged Poisson’s ratio \(\nu_{\text{davg}}\). Nonnecked Poisson’s ratio = 0.22.
Figure 29.—Concluded.

TABLE 13.—STRESS RELAXATION TEST RESULTS FOR Ti-6Al-4V SAMPLE 77
[At 538 °C and strain rate of 10⁻³ s⁻¹.]

<table>
<thead>
<tr>
<th>Step</th>
<th>Engineering stress Modulus, GPa</th>
<th>True stress Modulus, GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loading, ( E_{L,eng} )</td>
<td>Unloading, ( E_{U,eng} )</td>
</tr>
<tr>
<td>L</td>
<td>78.69</td>
<td>-----</td>
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<tr>
<td>1</td>
<td>78.68</td>
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<td>71.47</td>
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<td>4</td>
<td>67.89</td>
<td>Relax</td>
</tr>
<tr>
<td>5</td>
<td>70.46</td>
<td>Relax</td>
</tr>
</tbody>
</table>
Figure 30.—Fatigue tests of Ti-6-4 at 427 °C and stress-equivalent rate of $10^{-3}$ s$^{-1}$.

Figure 31.—Fatigue tests of Ti-6-4 at 427 °C and stress-equivalent rate of $10^{-3}$ s$^{-1}$, plotted as function of total strain range at half-life.
Since these tests were conducted at a high temperature and with a positive mean stress, the strain ratcheted with cycles, showing an initial rapid increase in strain increment due to the large amounts of deformation, followed by a continually decreasing rate of strain increase. Figure 32 illustrates this behavior, given a semilogarithmic plot, as the strain increases in an upward fashion. As expected, the maximum strain is higher for higher stressed samples. The lower stressed samples maintain more of a constant strain over a longer period of time before increasing at a rapid rate. The curves in this figure are ordered with respect to their maximum applied stress and indicate more time dependency (ratcheting) as the stress increases. Sample 56 falls in line with a lower stressed sample (98). This is due to the order-of-magnitude higher loading rate \(10^{-2} \text{ s}^{-1}\) for sample 56, which reduces the amount of its time-dependent deformation. A similar trend is seen for sample 98, which employed an order-of-magnitude rate increase \(10^{-2} \text{ s}^{-1}\) for the cycles after \(N = 80607\). The rate increase was applied to accelerate testing and reach fracture in a reasonable time frame. This accounts for the slope change to a more horizontal strain curve after the rate change. Although implying continually increasing damage, these curves show there was no singular event that would indicate imminent final fracture. A similar trend using true-strain values is not shown mainly because the overall strains were too small and resulted in true-strain values that are nearly identical to the engineering values. An illustration of the stress-strain curves for these stress-controlled tests is shown in Figure 57 of Lerch and Arnold (2016) for sample 58, the highest loaded test.

In an attempt to track damage, the reduction in modulus as a function of cycles is plotted in Figure 33. Loading moduli were calculated from both the engineering and true values. As shown in several prior instances, the engineering moduli (solid symbols) exhibit substantial decreases in their values over the life of the test with some of the largest drops occurring between the first and second cycle. The true moduli (open symbols) display minimal change between the beginning and the near-final cycles. There is certainly no evidence of significant degradation observed in the true modulus, except for perhaps sample 58, which was tested at the highest stress level and was observed to fail mostly by tensile overload. However, even for this specimen the drop in true modulus is difficult to separate from scatter. Similar to the previous figure, there are no singular events in any of the modulus curves that would suggest an impending failure.

In addition to the stress-controlled fatigue tests, two strain-controlled cyclic tests were also conducted but at 538 °C and strain rates of \(10^{-3}\) and \(10^{-5} \text{ s}^{-1}\). Three stress-strain curves at different cycles for these two types of strain-controlled tests are shown in Figure 55 for sample 74 and Figure 56 for sample 73 of Lerch and Arnold (2016). Both the maximum and minimum (negative) engineering stresses are plotted as a function of cycles in Figure 34 and generally show a gradual decrease in both values. The circular symbols represent sample 74, which was cycled between strain limits of ±0.01 (strain ratio \(R_e = e_{\text{min}}/e_{\text{max}} = -1\)) at the fast loading rate. There is a consistent decrease in stress with increasing cycles and a large decrease near the end of the life (800 < \(N < 900\)), which could be a precursor to the upcoming failure. The second sample (73, square symbols) was cycled at the slower rate and between strain limits of 0 and 0.018 (\(R_e = 0\)). After the first cycle, the maximum and minimum stresses are nearly fully reversed for both tests. The data in Figure 34 show no definitive trends in the peak stresses for sample 73. Although, the test was interrupted at cycle 176, there is no indication of failure based on its stress response (Figure 34) or change in modulus (Figure 35).

The loading and unloading moduli are shown in Figure 35 for both of these strain-controlled cyclic tests (samples 73 and 74). For the fully reversed test, sample 74, there is minimal change in the engineering loading modulus over the life of the test. There is a much larger change in the unloading modulus with an identifiable decrease beginning at cycle 400 and a large dropoff occurring between 800 and failure at 900 cycles. For sample 73, which has a positive mean strain, there is an initial decrease in both the engineering loading and unloading moduli up to cycle 13 where it saturates and is constant throughout the remainder of the fatigue test. This may imply that there is no damage at this point. For both samples true loading moduli at a few selected cycles were also calculated and are shown in Figure 35. There is no noticeable change in these values throughout the test for either sample.
Figure 32.—Fatigue tests of Ti-6-4 at 427 °C and stress-equivalent rate of $10^{-3} \text{s}^{-1}$ for maximum strain as function of cycle number $N$.

Figure 33.—Fatigue tests of Ti-6-4 at 427 °C and stress-equivalent rate of $10^{-3} \text{s}^{-1}$ for loading modulus $E_L$ as function of cycle number $N$. 
Figure 34.—Strain-controlled fatigue tests of Ti-6-4 at 538 °C for maximum and minimum stresses as function of cycle number $N$.

Figure 35.—Strain-controlled fatigue tests of Ti-6-4 at 538 °C for loading and unloading moduli $E_L$ and $E_U$ as function of cycle number $N$. 
Microscopy

Given the large discrepancy between engineering and true modulus degradation evolution it was decided that a metallography study needed to be done to ascertain whether or not physical damage was actually present in the various specimens. Consequently, scanning electron microscopy (SEM) examination of the gauge surfaces was performed on the samples, and damage was documented from the fracture surface along the gauge length. The samples were then mounted to view the area underneath the extensometer probes and were polished to approximately mid-diameter. Both optical and SEM images were taken of the metallographic specimens to document internal damage.

Surface Cracks

All of the tensile samples had some degree of surface cracking on the gauge surfaces. At the lower temperatures, the crack density was much less than in the samples tested at 538 °C. Hence the cracking will be described only for samples tested at 538 °C, although similar observations were made for the samples tested at lower temperatures. The density of cracks was higher near the localized neck and decreased further away from the neck. An example of this behavior is observed in Figure 36 for sample 538-39 tested to failure at 6.9 percent strain and at a strain rate of $10^{-3}$ s$^{-1}$. The highest crack density is observed near the fracture surface on the right side of the image. Images of the cracking are shown at much higher magnification in Figure 37 for three different distances from the fracture surface. Near the fracture surface (Figure 37(a)) the deformation and damage state is very high and individual cracks are more difficult to resolve because of their coalescence. Moving away from the fracture surface (Figure 37(b) and (c)), individual cracks can be clearly observed. The crack lengths in these remote areas measure approximately 10 µm and have spacings of approximately 5 µm.

Sample 538-15 tested at a strain rate of $10^{-5}$ s$^{-1}$ was interrupted at a strain of 8 percent. This sample was unloaded every 2 percent strain to monitor the modulus. By a strain of 8 percent the sample already exhibited a large neck near the top radius. Cracking at the minimum diameter (neck) is shown in Figure 38, and the crack density again drops upon moving to either side of the minimum diameter. The crack densities are shown at higher magnification in Figure 39. The surface cracks have a length of 20 to 30 µm and a spacing of 5 to 10 µm, with the crack spacing being smaller near the minimum diameter. It can also be observed that the surface cracks on this specimen tend to be wider than those in the previous, faster rate sample. This suggests that more time-dependent deformation occurs for the slower rate sample.

These cracks are shown in cross-sectional images in Figure 40. One optical micrograph (part (a)) and two SEM images (parts (b) and (c)) display several surface cracks. The wider cracks measure a few microns in width. It is significant to note that the depth of these cracks is less than 0.5 µm so that although the crack damage shown on the gauge surfaces in the previous figures appears to be severe, it is extremely shallow, and as observed in Figure 40(c), does not even traverse much of the surface grains. The backscattered electron image (Figure 40(c)) indicates that most of the surface cracks are transgranular in the α grains (darker, hexagonal close packed (hcp) phase), although there are a lesser number of intergranular cracks in the β phase (lighter, body-centered cubic (bcc) phase).

Surface cracks likewise form in the creep samples. Figure 41 shows the cracking at two different areas on sample 427-45 tested at a creep stress of 591 MPa. This test was crept to 12 percent strain and then unloaded. A small neck had formed near the top radius of the sample. At a distance of 4 mm from the neck (Figure 41(a)) few cracks can be observed. Although the cracks have a wider opening in the neck (Figure 41(b)), they still are small and infrequent compared to those in the tensile specimens. Their length is in the 5 to 20 µm range and have a spacing of ~10 µm. These are again shallow cracks (Figure 42), having a depth <1 µm.
Figure 36.—Scanning electron microscopy (SEM) image of crack distribution on gauge surface of Ti-6Al-4V tensile sample 538-39 tested at strain rate of $10^{-3}$ s$^{-1}$. Load axis is horizontal.

Figure 37.—Scanning electron microscopy (SEM) images of Ti-6Al-4V tensile sample 538-39 tested at strain rate of $10^{-3}$ s$^{-1}$ showing crack morphology on gauge surface at three different distances from fracture. Load axis is horizontal. (a) 1 mm. (b) 5 mm. (c) 9 mm (midgage).
Figure 38.—Scanning electron microscopy (SEM) image of crack distribution on gauge surface of Ti-6Al-4V tensile sample 538-15 tested at strain rate of $10^{-5}$ s$^{-1}$. Load axis is horizontal.

Figure 39.—Scanning electron microscopy (SEM) images of crack morphology on gauge surface at three different distances from fracture of Ti-6Al-4V tensile sample 538-15 tested at strain rate of $10^{-5}$ s$^{-1}$. Load axis is horizontal. (a) 1 mm. (b) 5 mm. (c) 9 mm (midgage).
Figure 40.—Depth of surface cracks near fracture surface of Ti-6Al-4V tensile sample 538-15 tested at strain rate of $10^{-5}$ s$^{-1}$. Load axis is horizontal. (a) Optical image. (b) Secondary electron scanning electron microscopy (SEM) image. (c) Backscattered electron SEM image.

Figure 41.—Scanning electron microscopy (SEM) Images of crack morphology on gauge surface at two different distances from fracture in Ti-6Al-4V creep sample 427-45 tested at creep stress of 591 MPa. Load axis is horizontal and fracture surface is to right. (a) 4 mm. (b) At neck.
At a test temperature of 538 °C several samples were creep tested, and surface cracks are observed to be related to the applied creep loads. At a stress of 101 MPa, sample 538-76 (see Lerch and Arnold, 2016) displayed no surface cracks. This sample crept to only 1 percent strain and was still in the secondary creep regime. At a higher stress of 206 MPa sample 538-94 was crept to 5 percent strain and then unloaded. This sample was into the tertiary regime and developed a small neck near the upper radius. The extent of surface cracking near the neck is shown in Figure 43(a). Cracks are up to 75 μm in length, substantially longer than any of the previously discussed samples. The crack spacing is on the order of 25 to 50 μm. The cracks are again very shallow (Figure 43(b)), being less than 2 μm.

At a stress level of 241 MPa sample 538-3 was crept to fracture at 12 percent strain. At a distance of 4 mm from the fracture surface, there are many more surface cracks than there were for the previous sample tested at a lower stress. The cracks’ lengths appear to be a bit shorter, <50 μm, with a crack spacing of 25 μm (Figure 44(a)). Note that the extent of cracking at 4 mm is at least as severe as for the previous sample in the necked region. For sample 538-3 the cracking in the neck (Figure 44(b)) exhibits very wide cracks.

Fatigue samples tested at 427 °C and with a load ratio of zero exhibit a somewhat different display of surface cracks. At the highest stress level (642 MPa) the cracks appear as a ±45° cross pattern with smaller transverse cracks interspersed (Figure 45). The fracture surface of sample 427-58 reveals only typical tensile fracture morphology. At a lower maximum stress (sample 427-59), the cracking pattern appears more like tensile sample 538-39 (Figure 37). Cracks are nearly all transverse and very wide (Figure 46(a)) in the neck. At 3 mm from the fracture surface the crack lengths are less than 35 μm (Figure 46(b)).

At a still lower maximum stress of 581 MPa sample 427-56 (Figure 47) again shows the cross pattern of cracks similar to those in Figure 45. However, this sample was tested at a strain rate of 0.01 s⁻¹, an order of magnitude higher than the other fatigue samples in this group. Therefore, less time-dependent deformation would be expected in this sample.

Sample 427-98 (Figure 48) was tested with a maximum stress of 549 MPa and shows only a transverse cracking pattern. The cracks in the necked area, however, are quite long: as long as 80 μm and having a crack spacing of 10 μm. At a distance of 3 mm from the fracture surface (Figure 48(b)) the surface cracks have the same approximate length but become thinner, with slightly larger spacing.
Figure 43.—Crack morphology and depth of surface cracks near neck of Ti-6Al-4V creep sample 538-94 tested at stress level of 206 MPa. Load axis is horizontal and neck is to right. (a) Gauge surface, secondary electron scanning electron microscopy (SEM) image. (b) Cross section, backscattered electron SEM image.

Figure 44.—Secondary electron SEM images of crack morphology of Ti-6Al-4V creep sample 538-3 tested at stress level of 241 MPa. Load axis is horizontal and neck is to right. (a) 4 mm from fracture surface. (b) At fracture surface.
Figure 45.—SEM images of crack morphology of Ti-6Al-4V fatigue sample 427-58 at maximum stress of 642 MPa on gauge surface at two different distances from fracture. Load axis is horizontal. (a) At fracture surface. (b) 4 mm from fracture, lower resolution. (c) 4 mm from fracture, higher resolution.

Figure 46.—SEM images of crack morphology of Ti-6Al-4V fatigue sample 427-59 at maximum stress of 597 MPa on gauge surface at two different distances from fracture. Load axis is horizontal. (a) At fracture surface. (b) 3 mm from fracture.
At the lowest stress level of 535 MPa, no surface cracks were observed on this sample (427-57) and the test was stopped (runout) at 580,948 cycles.

Optical images of transverse sections of the cracks are shown in Figures 49(a) and (b) for fatigue samples tested with low and high maximum stresses, respectively. The cracks in the low stressed sample (427-98) contain some deep cracks on the order of 10 µm. Additionally, the very small surface cracks discussed previously are also observed. In fact, the two larger cracks shown in Figure 49(a) initiated at these shallow, but wider cracks. Figure 49(b) shows surface cracking for a highly stressed fatigue sample (427-59) and only the smaller surface cracks appear. They can be as deep as 6 µm and can be very wide, particularly near the fracture surface.

For the fatigue samples tested at 538 °C, the sample tested at a strain ratio of zero (538-73) and stopped after only 176 cycles exhibit no cracking or other forms of damage. Sample 538-74 was tested in a fully reversed (±1 percent), strain-controlled manner and failed after 900 cycles. The surface cracks are shown in Figure 50 at both the fracture surface and 4 mm from the fracture. Both areas display very long cracks (several millimeters). The cracks are much more wide open near the fracture surface and become very tight in more remote regions. The cracks in this sample can be very deep on the order of hundreds of microns (Figure 51).
Figure 49.—Optical images of crack depth near fracture for Ti-6Al-4V fatigue samples. Load axis is horizontal. (a) Sample 427-98 tested at maximum stress of 549 MPa. (b) Sample 427-59 tested at maximum stress of 597 MPa.

Figure 50.—SEM images of crack morphology of Ti-6Al-4V fatigue sample 538-74 on gauge surface at two different distances from fracture tested at ±1 percent strain. Load axis is horizontal. (a) At fracture surface. (b) 4 mm from fracture.

Figure 51.—Optical images of crack depth for Ti-6Al-4V fatigue sample 538-74 tested at ±1 percent strain. Load axis is horizontal. (a) At fracture surface. (b) In nonnecked area.
Internal Pores

Longitudinal cross sections of the tested samples revealed some amount of pore formation in all of the specimens. Similar to the surface cracks, the severity of pore formation was greater near the fracture surface in the necked area. The pores tended to diminish away from the neck. Since the samples were from rolled plate, pores were not observed in the initial material. Image analysis was performed on the samples, and the details are reported in Appendix C. Only the highlights will be repeated here.

Pores were found in each sample and had equivalent circular diameters of a few microns. In reality, the pores were elliptical as evidenced by an average circularity of 0.85. The aspect ratio of these pores was consistent among the samples examined and averages 1.5. The major axis of the pores was aligned at various random angles to the loading axis with the grand average over all tests being 69°. There appeared to be a slight increase in the pore angle with increasing temperature, but more tests would be needed to confirm this.

The area fraction of pores is very low. For tensile tests at 316 °C the average volume fraction is the same for both samples and is 0.015 percent. At 427 °C the average is between 0.015 and 0.02 percent. At 538 °C the average volume fraction is higher and dependent on strain rate. For the three fast-rate tests the average area fraction is approximately 0.05 percent. Of these three tests, sample 538-89 has the lowest area fraction of pores and this sample was subjected to unloads every 1 percent strain to monitor any change in modulus as a function of damage.

For the tensile samples tested at a lower strain rate of $5 \times 10^{-4}$ s$^{-1}$, the pore fraction was significantly higher. Sample 538-37 had an area fraction of 0.15 percent, but 538-70 had a fraction of 0.3. The latter sample (538-70) had a relaxation at the first 1.8 percent strain followed by tension to failure. It is possible that the relaxation portion affected the amount of pores.

Finally, the three samples tested at the lowest strain rate of $10^{-5}$ s$^{-1}$ had still a larger area fraction of pores. Sample 538-97 was a simple tension test to failure at 11 percent strain and had an area fraction of 0.72 percent. Sample 538-65 had an average area fraction of 0.42 percent, but this test had unloads every 2 percent. The anomaly of the group was sample 538-15, which had a similar, slow loading rate but whose unloading rate was 0.001 s$^{-1}$, 2 orders of magnitude faster than its loading rate. This sample had a very low area fraction of pores of 0.08, which is more like those of the fast-rate tensile tests at 538 °C. The low area fraction of pores was not expected because slow/fast rate tests in fatigue tend to accentuate creep mechanisms, enhance pore formation, and lead to lower fatigue lives (Baik and Raj, 1982; Rao, Schuster, and Halford, 1996). However, these observations are based on tests that included unloading into the compressive regime, whereas the current tests do not, nor do they exhibit reverse yielding during the unload.

For the creep samples tested at 427 °C, the pore area fraction was again very low and observed to be a function of creep stress. At a stress level of 375 MPa (427-57; Lerch and Arnold, 2016), the area fraction was only 0.006 percent. After the 24-h creep time, this sample had only attained the very low strain value of 0.0065. Sample 427-45 was crept at a higher stress of 591 MPa and attained 12 percent strain before unloading. This was sufficient to place the sample into tertiary creep and form a small neck. The area fraction of pores equaled 0.012 percent, twice as large as the previous, lower stress level test. The third test, sample 427-87, was tested to failure at 21 percent strain at a similar stress level (596 MPa). This sample had an area fraction of pores of 0.004 percent, lower than the other two samples. However, this sample also had one unload at 11 percent strain and another at 13 percent before continuing on to failure.
Four creep samples tested at 538 °C were examined. Sample 538-76 (Lerch and Arnold, 2106) was crept at a low stress level of 101 MPa. This sample only attained a strain of 0.015 and was still in the secondary regime of creep. The area fraction of pores was only 0.015. At a higher stress level of 206 MPa in the current study, sample 538-94 crept until reaching a strain of 5 percent and was then unloaded. The sample was already in the tertiary regime and contained a small neck. The area fraction was actually lower than the previous sample and equaled 0.009 percent. It is unknown why the area fraction for this sample is lower.

Sample 538-3 was crept at a stress level of 241 MPa and continued until fracture at 11 percent strain. This sample has a high pore area fraction of 0.53 percent. Sample 538-34 was crept at the same stress level of 241 MPa, but was subjected to unloads every 2 percent before failing at 12 percent strain. It has a very low area fraction of 0.03 percent.

Size of the pores was also measured and the area of the average pore was only a few square microns. For tensile tests at 316 and 427 °C the average area is 1.8 μm². It is interesting to note that the average pore size for sample 427-95 corresponds to the smallest pore area (1.32 μm²) of the samples tested at 427 °C. This sample had been subjected to unloading every 1 percent strain.

The area of the average pore for samples tensile tested at 538 °C and a fast strain rate (10^{-3} s^{-1}) is at least twice as large as those at the lower temperatures, with a value of 3.7 μm². Note that the smallest average pore area of the three samples tested at the fast strain rate again belongs to a sample (427-89) that experienced periodic unloads. When tested at slower strain rates, the pore area greatly increased. Both samples 538-70 and 538-37 were tested at strain rates of 5×10^{-4} s^{-1}, and pore areas of 10 μm² were produced. The samples tested at the slowest rate of 10^{-5} s^{-1} had similar pore areas to this. The smallest of the group was 538-15, which was the slow/fast test and only had a pore area of 6.1 μm².

For creep samples tested at a temperature of 427 °C it was observed that the average pore size was dependent on stress level. For the lowest stress level of 375 MPa, sample 427-57 had an average pore area of 1.4 μm². At the higher stress level of 591 MPa, sample 427-45 had a substantially larger average pore area of 5.1 μm². When tested at a similar stress level (596 MPa) however, but with unloads at 11 and 13 percent strain, the pore area for sample 427-87 dropped to 1.6 μm².

At creep temperatures of 538 °C the samples again exhibit a relationship between pore size and stress level. With increasing stress levels of 101, 206, and 241 MPa, samples 538-76, 538-94, and 538-3 had pore areas of 4.2, 6.6, and 7.4 μm², respectively. The pore size again dropped for the additional sample 538-34, which was tested at 241 MPa (same as 538-3), but experienced unloads every 2 percent strain. This sample had an average pore size of 3.9 μm².

Backscattered electron images (Figure 52 and Figure 53) revealed for both creep and tension tests that the pores formed either in the softer, β phase (lighter phase in the figures) or at the α/β interface. The resolution was not sufficient to determine which. However, it can definitely be concluded that pores did not form within the α grains. It was also determined that the location of pore formation was not altered by temperature, test type, strain rate, or creep stress level.
Discussion

Life prediction is an important task for aircraft safety. Conservativeness is desired to avoid catastrophic failure. Alternatively, if the prognostic scheme is too conservative, parts are removed long before any damage has begun, resulting in reduced financial gains. Hence the goal to be accurate is highly valued. Ideally one would like to track the damage from the point of local ductility exhaustion and the start of the first microvoid or crack through its continued progression to the end of life. Thus, this work aimed to follow various trends during experiments on coupons of mill-annealed Ti-6-4 and to examine the data for possible connections to damage growth and ultimate failure. The program scope did not allow for the systematic study of fractional life damage to determine via microscopic examination specifically when physical damage initiated. The experimental test matrix was designed to provide results over a large range of conditions that can also be used to characterize any damage model. Of particular interest is the GVIPS model developed by Saleeb and Arnold (2001, 2004). Important in this and all models designed to simulate multiaxial behavior, is at the minimum the characterization and validation of transverse strain under uniaxial loading; thus, great pains were taken to measure this parameter on every sample.
throughout each test. To accomplish this, an optical micrometer was employed since this device was (1) fairly easy to implement, (2) was noncontacting (would not initiate cracks), (3) was reasonably unobtrusive, (4) had sufficient resolution, and (5) would not be affected by high test temperatures. The micrometer measured the diameter change at one plane along the axis of the sample’s gage section. Samples were monitored throughout the test program at either the top of the gage near the radius or at midgage. Only one location per test was monitored. This method had two minor drawbacks. First, the micrometer only measured the diameter at one location on the gage, and it was fortuitous if this location happened to coincide with the localized neck that formed at large strains. Even a lower probability existed of having it centered about the nadir of the localized neck. Moreover, it was very difficult to locate the precise position along the gage length that was being measured and therefore it was only generally noted. This is because the micrometer did not actually measure an exact planar section at the given location, but was more of a thin volume element.12 Second, the micrometer’s reading could also be affected by nearby objects such as extensometer probes, induction coils, and other portions of the experimental setup. This occasionally led to erroneous readings during various parts of the test. For example, the values for Poisson’s ratio given in Figure 22(c) were high even in the elastic regime and suspected to be a result of these complications. However, the trend in Poisson’s ratio was believed to be real even if the absolute value was somewhat off. This interference in the transverse strain signal was also what caused the transverse strain to stop recording during portions of the test as shown at the beginning of the test in Figure 27(d).

Unlike many of the tests in the previous two works on Ti-6-4, most of the tests in this study were performed to high strain values because the majority of the tests were either taken to failure or near failure. Therefore, the expectation was that samples would have incurred large amounts of damage (e.g., cracks, pores, phase coarsening, etc.). Most of the unbroken samples showed localized necks, some of which were severe as indicated by their eccentricity value in Table 2. Examination of the gage surface of these samples generally showed large-scale deformation as indicated by cracks and voids, surface roughness, and flow lines. A number of samples were examined metallographically and did show both gage surface (e.g., Figure 36 and Figure 37) and internal damage (e.g., Figure 53). In all cases it was apparent that microcracking, void nucleation, and so forth was present, thereby indicating the presence of both diffuse damage and localization and coalescing of damage into mesocracks near the location of final failure. Longitudinal cross-sectional slices were taken and indicated gradients of damage (see Appendix C) within the failed specimens, thus confirming again the presence of load-induced damage (cracking, void nucleation, etc.). However, it must be emphasized that the volume fraction of the material impacted by the presence of damage was extremely low. Even when the pores were quantified directly at the minimum diameter of the localized neck (sample 538-65), they were found to be only slightly more severe than the average over the 3-mm distance (see Appendix C). This shows that either (1) Damage (i.e., pores) does not grow significantly, nor does it link up to cause final failure (which implies that cracks between the pores develop to cause failure), or (2) The growth and link-up of pores occurs at the final few microstrains before failure—an amount nearly impossible to capture by an interrupted test.

Ideally interrupted tests should be performed to strains experiencing lower amounts of damage to better investigate the inception of damage. This was not within the scope of this study. However, given this definitive evidence of the presence of damage (albeit small) at end of life, the question before us is “what measurements and properties, if any, can be used as effective indicators of this damage initiation and accumulation as well as a precursor to failure itself?” To answer this, loading and unloading moduli and instantaneous Poisson’s ratio were examined.

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12This device uses a light curtain, not a laser.
Young’s Modulus

Stiffness (modulus) change, often associated with loss of cross-sectional load-carrying area (e.g., cracking or void growth), is a desired parameter for input into a stiffness reduction model. During this study the modulus was calculated from both the engineering and true stress-strain curves (in all of the figures the true stress is plotted against engineering strain). For ease of reference both types of curves were given in this report. The data show, as expected, that the true stress is always larger than the engineering stress for any given strain and the true strain is always smaller than its engineering counterpart at any given moment in the test. This behavior is consistent with the definition of engineering and true stress and true strain (see Appendix D). It was also observed that the stiffness calculated from the true stress-strain curve is larger than its engineering counterpart. When possible, unloading modulus was calculated in addition to the loading modulus. These two values could conceivably be different, particularly if damage was involved, since cracks would have opened in tension and closed in compression and may have led to a discernable difference in stiffness. In general there is very little difference found between the loading and unloading moduli. Even though the unloading modulus is slightly lower than the loading modulus regardless of whether they are calculated from the engineering or true values, they exhibited similar trends.

The largest difference in stiffness behavior is seen when comparing engineering ($E_{\text{eng}}$) and true ($E_{\text{true}}$) modulus values. In all cases $E_{\text{eng}}$ decreases with increasing strain (assumed related to loss of load-carrying cross-sectional area due to damage accumulation), whereas $E_{\text{true}}$ exhibits very little change during the test. For tensile tests strained to 0.2 and unloaded, the reduction in $E_{\text{eng}}$ is approximately 35 percent, whereas the reduction in $E_{\text{true}}$ is only 7 percent. There is a slight dependence of modulus reduction as a function of temperature, whereby samples tested at the higher temperature exhibit slightly less reduction for both $E_{\text{eng}}$ and $E_{\text{true}}$. The one outlier under tension loading is sample 65, which was tested at 538 °C and a slow strain rate of $10^{-5}$ s$^{-1}$. This sample exhibits a 90 percent reduction in $E_{\text{eng}}$ and an 83 percent reduction in $E_{\text{true}}$. This suggests that at low strain rates it is damage mechanisms, which truly affect modulus (e.g., void nucleation, etc.), that may be more active, thus making modulus a better indicator of damage accumulation. In fact, image analysis (Appendix C) shows that sample 538-65 did indeed have the highest area volume fraction and largest size of pores in the entire study.

The creep tests indicate a similar trend: the $E_{\text{eng}}$ values exhibit a greater reduction (20 percent) compared to a reduction of only 5 percent for the $E_{\text{true}}$ values. The amount of reduction does not readily appear to be a function of the final creep strain, although there are not enough data to state this definitively. There may have been a dependence on creep stress level since the three samples tested at 538 °C (94, 1, and 34), which have increasing stress levels (206, 226, and 241 MPa), display increasing reductions in the engineering modulus (14, 19, and 20 percent, respectively). Again more data would have been required to confirm this. It is interesting to note that similar changes in $E_{\text{eng}}$ as a function of total strain applied do occur whether the loading is tensile or creep: Compare Table 3 and Table 7 for examples suggesting that strain is more important than type of test (e.g., tension, creep, or fatigue).

The fatigue tests display very little reduction in $E_{\text{eng}}$ for most tests: a few percent range for tests at the higher lives. Only the two samples (59 and 58) tested at the highest stress values (short lives) had a significant reduction and were on the order of 20 percent. The evolution of modulus ($E_{\text{eng}}$) degradation was very distinctive; however, see Figure 33. Sample 58 failed mostly through a tensile overload mode and also exhibits a very high cross-sectional eccentricity. However, sample 59, which had the second highest load but still failed via cyclic mechanisms, displays a smaller eccentricity. Moreover, it is not
understood why the $E_{\text{true}}$ values changed as they did for the fatigue samples. For half of the samples the final modulus is larger than the initial value, although by amounts less than 1.5 percent.

The type of physical damage was identified by posttest microscopy. For all samples, damage appears as two primary mechanisms: internal voids and surface cracks. Both of these types of physical damage appear to be very small. In fact, no sample shows metallographic evidence of damage sufficient to cause such reductions in stiffness as suggested by the change in engineering modulus. There were very high densities of surface cracks on the samples; however, the cracks were very shallow (see Figure 40) and did not even span a significant portion of each surface grain. If this damaged area were eliminated in calculation of the cross-sectional area, assuming that it would carry no load, it would only result in a reduction of cross-sectional area of less than 0.06 percent. It is hard to believe that this little damage could reduce the modulus as much as is observed. Similarly, the amount of actual pores observed within the samples was incredibly small. The average area fraction of pores ranged from 0 to 1 percent with the majority of the samples being below the sample average of 0.1 percent. The exception to this was creep sample 538-3, which had the highest pore area fraction at the fracture surface of 3 percent. Cases such as tensile sample 316-42 had an average pore area fraction of only 0.01 percent, yet display a modulus drop after 20 percent axial strain of 36 and 9 percent for the engineering and true moduli, respectively. It is hard to rationalize either of these reductions based on the small amount of damage observed, even recognizing that both surface cracks and internal pores occur at the same time. Similarly tension sample 427-95 exhibits continually decreasing moduli (both engineering and true) from unload cycle 1, and yet the observed sample average area pore fraction is only 0.02 percent. Moreover, the moduli dropped 15 and 3 percent (engineering and true, respectively) by the UTS, and it is assumed that there is no damage before this point.

Periodic unloads were employed in some of the tests to monitor any change in modulus. Although this seemed to have no effect on the stress-strain curve, it did appear to reduce the amount of damage via area fraction of pores and to a lesser extent the pore size. Hence future testing employing unloading has to be done with the knowledge that the damage state within the material will be different than for tests that do not involve unloading.

To assess the effect of surface damage, an axisymmetric finite element analysis of the test specimen was conducted, similar to that in Appendix B. In this analysis, a line of shallow surface cracks (idealized by reducing the element stiffness of the outermost ring of elements (0.0025 mm in size) to almost zero) was simulated to assess the impact, if any, such surface cracking might have on the strain that a surface mounted virtual extensometer would record. The analysis showed that although the axial strain measure was increased slightly, its effect on the effective stiffness was less than 0.5 percent.

Consequently, it appears that the engineering modulus provides an erroneous measure of stiffness reduction because of the illegitimate use of engineering stress and strain measures when in fact significant changes in geometry have occurred at large magnitudes of strain (i.e., larger than 5 percent). Thus, this renders the engineering modulus inappropriate as an indicator of damage. These results reinforce the need for consistency (between experimental measurements and mathematical theory) when characterizing a given material constitutive model. For example, if one utilizes engineering stress and strain the resulting model parameters will take on values that would be significantly different than those obtained if one used true-stress and true-strain curves to calibrate these parameters. Also, one may resort to erroneously introducing a damage mechanism (e.g., stiffness reduction) into a given model in order to “fit” the data when in actuality little, if any, damage is actually present in the material. Furthermore, given merely uniaxial data, one may feel confident in an isotropic model’s ability to reproduce the response curves for a given material (with slight or strong anisotropic behavior) when its multiaxial simulation capability would be sorely lacking.
Another possible method for tracking damage is through determining Poisson’s ratio, which necessitates the measurement of transverse strain. This was done on nearly every sample, yet since the location of this measurement and transverse rotation of specimen was not always ideal or adequately tracked, the extraction of a consistent and meaningful Poisson’s ratio from these measurements is problematic whether engineering or true strain based. Yet in some cases the usefulness of Poisson’s ratio as a damage indicator is encouraging. In all cases the measurement of transverse strains was experimentally accurate (even though the interpretation of Poisson’s ratios is questionable, see Appendix B) and instructive for understanding the multiaxial behavior of the material (e.g., anisotropy) and verifying or validating the multiaxial capability of a given constitutive model. The multiaxial deformation produced even under uniaxial loading is clearly impacted by the presence of internal defects, material anisotropy, load gradients (whether thermal or mechanical), and changes in local fields throughout the specimen, particularly toward the end of life (e.g., during necking).

In the case of tension and a temperature of 316 °C, Poisson’s ratio was found to increase continually until the end of the tests. Only two samples (79 and 42) were tested at this temperature, but both displayed the same behavior. It can be observed in both sets of data (Figure 3(c) and Figure 4(c)) that Poisson’s ratio transitions from a lower initial elastic value, increases significantly and plateaus, then proceeds to increase again, as one might envision. The actual magnitude of Poisson’s ratio at the plateau suggests the material is anisotropic. The occurrence of the UTS does not correspond with a characteristic increase in the value of Poisson’s ratio; however, a change in the rate of increase in Poisson’s ratio prior to attaining the UTS (this occurs at approximately an axial strain of 0.09 in Figure 3(c)) does take place, which may have some significance. Perhaps Poisson’s ratio is a more sensitive indicator of the beginning of necking, or perhaps it indicates the initiation of damage. Again, interrupted testing to look for damage would be required. Poisson’s ratio for these two experiments, although calculated inconsistently according to results detailed in Appendix B, was measured at midgage and coincided with the location of axial strain measurement. Fortuitously, necking occurred at midgage as well, thus allowing the capture of the rapid increase in transverse deformation (strain) commensurate with the necking of the cross-sectional area. It is unfortunate that no loading and unloading modulus checks were conducted during either of these tests.

Tensile tests at 427 °C exhibit a different behavior for Poisson’s ratio. Poisson’s ratio increases to a maximum value and then decreases until failure (Figure 5(c)). Its maximum value occurs at approximately half the axial strain at which the maximum stress (UTS) is attained. Similar to the tests at 316 °C, the strain at which the UTS occurs does not reflect any conspicuous change in the trend of Poisson’s ratio. At first this decrease in Poisson’s ratio was thought to be attributable to the fact that necking once again occurs at midgage, yet in this case the transverse strain was measured at the top of gage area while the axial strain was measured via extensometer over the gage length, thus causing an increase in axial strain without a commensurate increase in transverse. However, this behavior is persistent even when both axial and transverse measurements were taken at midgage (see sample 69, Figure 6(c)) and significantly less necking occurred. Further, in this test (427-69) the sample axis rotation was such that the transverse strain measurement was almost directly in line with the minor axis. Appendix D illustrates how the anisotropy of Ti-6-4 plays a significant role in matching the observed behavior in Poisson’s ratio. Furthermore, the difference between small strain and large strain assumptions is illustrated, as is the assumption of compressibility during damage versus that of incompressibility in specimen dilatation; the latter is important to accurately simulate the Poisson’s ratio history measured in Figure 6(c) (see Figure B.7). Note however, that the damage simulated in Figure B.7 corresponds to that associated with an engineering modulus reduction. At 538 °C Poisson’s ratio tends to increase continually to failure (Figure 9(c)). However, at low
strain rates it tends to reach a plateau and remain approximately constant throughout the test (see, for example, Figure 14(d)).

During creep tests Poisson’s ratio was more dependent on both sample orientation and whether or not the neck and the transverse measurement location coincided. Only three creep tests were conducted at 427 °C and they were all tested under the same conditions. The location of transverse strain measurement and the neck failed to coincide in all three tests. The only difference was that the transverse strain for sample 35 followed the minor axis and in the other two samples (45 and 87) it tracked the major axis. For sample 35 (Figure 17(d)) the instantaneous Poisson’s ratio decreased from the beginning of the test. For the other two samples (45 and 87, Figure 16(d) and Figure 18(d) respectively), the instantaneous Poisson’s ratio increased to a maximum value of 0.62 at an axial strain of approximately 0.04, and then decreased to failure. In none of the three tests did the beginning of tertiary creep manifest itself by significantly altering the curve of instantaneous Poisson’s ratio versus axial strain. At 482 °C only one creep test was conducted (sample 81). The localized neck formed at the top of the gage in the same location as the transverse strain was measured. Hence the instantaneous Poisson’s ratio not only increased continuously, but attained very high values, reaching a measured value of 1.6 at failure. It was observed in the Poisson’s ratio versus axial strain curve in Figure 19(c) that the inflection point laid in the general vicinity of the inception of tertiary creep. This implied that the inflection point may indicate the initiation of damage. For creep tests at 538 °C the behavior of Poisson’s ratio again depended on the location of the neck and the transverse strain measurement. If they did not coincide with one another, then Poisson’s ratio tended to increase slightly and then decreased slightly (Figure 20(c), Figure 21(c), Figure 22(c), Figure 23(d), and Figure 25(e)). If they did coincide, then Poisson’s ratio increased until failure (Figure 24(c) and Figure 26(c)), attaining some high values (>0.5). For these cases the inflection point in the curve was again similar to the inception point of tertiary creep.

Only a few relaxation tests were run and all were step tests. The test at 20 °C indicated that Poisson’s ratio increased in the beginning of the test and decreased thereafter (Figure 27(d)), although there were data missing prior to a strain of 0.01. The sample necked midgage, but the transverse strain was measured above that. The Poisson’s ratio behavior was similar to the creep tests at 538 °C. At the two higher temperatures of 427 and 538 °C the instantaneous Poisson’s ratio generally increased throughout the test (Figure 28(d) and Figure 29(d)). For both of the samples, the neck coincided with the location of strain measurement. Since in the test at 538 °C the minor axis was measured, the values for Poisson’s ratio were larger than for the test at 427 °C where the major axis was measured. Nonetheless, the general trends were similar. All three of these tests indicated that no discerning change in Poisson’s ratio behavior was observed near the UTS. Moreover there was nothing in the characteristics of Poisson’s ratio at the end of the tests to indicate that failure was imminent.

Although transverse strain was also measured during the fatigue tests, there was little change in the diameter during the experiments due to the small applied strains in any given cycle. Therefore there was little change in the transverse strain and subsequently no discernable change in the values of Poisson’s ratio. At least for the types of cyclic tests performed in this report, Poisson’s ratio did not seem to be a suitable indicator for end of life. However, cyclic data were only collected on a logarithmic count; to clarify, data were not taken for every cycle. It was possible that major changes happened in the last 1 percent or so of life. In order to detect this, every cycle would have to have been recorded and the ending cycles examined. This would have greatly increased the data file sizes and made manipulation of these files unwieldy. Moreover, it was something that would have to be planned in advance of testing.

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13 Remember that all creep tests are constant load tests and not constant stress tests, therefore this increase in creep strain could be attributed to increases in localized stress and not necessarily the onset of damage.
Comments and Recommendations

In this study two predominate parameters—loading and unloading modulus and instantaneous Poisson’s ratio—were investigated to see if they were useful in indicating both the initiation and accumulation of damage and, more importantly, the onset of failure. It was hoped that one or both of these parameters would exhibit significant changes, paralleling the accumulation of damage, with a discernable, dramatic indication of final failure before its occurrence. This was expected since actual physical damage in the form of surface cracks and pores were observed through metallography in many—if not most—of the specimens tested.\(^{14}\) In general, this was not the case when examining either of these parameters or any other features (e.g., yield stress, transverse strain, UTS, and strain to failure) from this study. The loading and unloading modulus as determined from the engineering stress-strain curves do display large changes. However, these changes appeared to be more closely connected to geometric factors (such as reduction in cross-sectional area and/or the distortion of cross-sectional area from circular to elliptical during the tests) rather than damage itself because when this change in cross-sectional area was accounted for by utilizing true-stress and true-strain measures, very little change in stiffness was observed. Perhaps a specific study to solely investigate the true modulus in more detail would be more fruitful. Moreover, the large changes in engineering modulus are not consistent with the small amount of physical damage observed in the specimens with respect to volume fraction. However, the smaller changes in true modulus are consistent with the amount of damage observed.

In Appendix B and Appendix D, many issues are discussed that question the meaning and interpretations of the experimental measures after the sample has reached its UTS. Values between the UTS and a sample’s failure appear highly dependent on measurement location, localized necking, multiaxial stress and strain states, strain gradients, and damage. Reporting a value of failure strain in the presence of localizing events (e.g., necking) is highly suspect, since necking outside the gage section can cause significantly lower axial strains to be recorded (see Appendix B, Figure B.6). Yet this parameter (failure strain)\(^{15}\) is widely reported without reference to failure location and used to compare materials; it is also used as a parameter in many failure models. A more reliable parameter may be the value of axial strain at UTS, since it is less susceptible to variation from localizing events, although this value has significant conservatism built into it and thus ductility could be significantly underrepresented, depending upon the test temperature. If one examines the strains at the UTS given in Table 2, it can be observed that at given test conditions these values have minimal scatter. Appendix B indicates that any errors in the calculated Poisson’s ratio are minimal up to about 10 percent strain, the strain at UTS at 427 °C for Ti-6-4. Moreover, Appendix D indicates that the error between engineering and true strain, as well as differences between large and small strains, are also relatively negligible at this point. A similar discussion can apply to the rupture strain under creep conditions.

Although specimen-to-specimen and test-to-test inconsistencies are observed in Poisson’s ratio and inconsistencies exist in its interpretation, Poisson’s ratio still shows promise as an indicator of damage. Changes in its evolution rate as a function of axial strain were noted well before attainment of the UTS as well as before the initiation of tertiary creep. Hence Poisson’s ratio may be a parameter that, if monitored, could indicate damage accumulation. As discussed in both the above text and in Appendix B and Appendix D, Poisson’s ratio and its trends are influenced by the material volume, location of transverse strain measurement, anisotropy, and large strain versus small strain assumptions. Consequently, a more

\(^{14}\)After careful analysis, however, the actual volume fraction of this physical damage was found to be relatively small.

\(^{15}\)Reduction in area at the necking location is frequently reported for a tensile test and is often used as a measure of ductility.
detailed and well planned study will need to be done before a definitive statement can be made. To that end, the following is a list of lessons learned regarding the specification and execution of such a test program:

1. FEA is needed up front to give insight into the planned experimental measurements, to understand the importance and sensitivity of parameters to be measured. The accuracy, however, of such an analysis is dependent upon the accuracy of the characterization of the material constitutive model, which in turn is dependent upon the availability of accurate experimental results, suggesting that
   a. A model material (one that is already reasonably well known) be used for any damage study
   b. Consistency and continuous updating of model and test matrix be required throughout the test program
2. Avoid the use of anisotropic materials, as it immensely complicates the study. Isotropy should be verified by metallographically examining the microstructure and conducting a few simple tensile tests up front before finalizing the material choice. This may require several material purchases until an isotropic material is found.
3. Use samples taken from only one lot of material to reduce scatter. Lot-to-lot variability should be taken into account after a model is created and the damage is understood.
4. If material anisotropy is unavoidable, then accurate tracking of specimen orientation relative to material anisotropy (e.g., rolling direction, etc.) must be maintained for all directions during material testing.
5. The number of testing variables should be limited and more repeats performed to better account for the scatter in both material properties and damage distribution.
6. Full-field displacement measurements should be used if possible, but require line of sight to the gage section and extensive data reduction.
7. The measurement of the transverse and axial strains should occur over the same volume of material to be consistent.
8. Interrupted tests should be conducted to track the initiation and earlier stages of physical damage and to ascertain if visible damage occurs with modulus changes at low strain values.
9. Although unloading is required to assess stiffness reduction, it does appear to affect the physical damage state. This should be considered if image analysis of damage will be performed.
10. Image analysis must be performed to identify the extent of physical damage. This is a resource-intensive analysis made somewhat easier by today’s available software. Nonetheless, the results still remain difficult to explain because of large spatial inhomogeneities and gradients in the damage.
11. If compression is not of interest (i.e., buckling is not a concern), a larger sample size can be used. This would be beneficial, as it would make observations easier by providing more volume to allow additional instrumentation to be employed on and around the sample. However, this must be balanced by the disadvantages of the increased difficulty in maintaining uniform temperature in the gage section, and potentially higher surface area to volume ratio, resulting in larger scatter in measured properties.
12. Damage and failure typically occur beyond 5 percent strain; therefore, small strain approximations are not valid and should be avoided.
13. Calculations of strains and Poisson’s ratio must be performed using either small- or large-strain approaches exclusively, not both. For examination of damage it makes the most sense to use large-strain assumptions and present results in context of true stress and true strain.
14. The classical proportional relationship between the transverse and longitudinal strains only applies to true-strain values (i.e., \(e_{t,\text{true}} = -\nu e_{\text{true}}\), see Appendix D). Consequently, if one wishes to utilize
engineering strain (typically restricted to small strains) then the expression \((1 + \varepsilon) = (1 + e)^v\) for Poisson’s ratio must be used when the magnitude of engineering strains become large (i.e., \(>5\) percent).

15. A consistently calculated instantaneous Poisson’s ratio will saturate at 0.5 for isotropic materials or at some value less than 1 for anisotropic materials, if no damage is present.

16. To capture the rapid rise in apparent Poisson’s ratio after the elastic region, anisotropy may have to be incorporated, depending upon the magnitude of the rise.

17. Damage induces material compressibility and thus influences the dilatation, resulting in a decrease in the instantaneous Poisson’s ratio. This implies that a consistent measurement of Poisson’s ratio could be used to track damage.

18. Such a study will be resource intensive. Therefore, if one truly wants to impact or enable integrated computational materials engineering, one must be willing to invest significant resources in both modeling and experimentation to provide sufficient fidelity to draw definitive conclusions.

Conclusions

This report represents the final experimental paper in a trilogy of articles investigating deformation and damage development in annealed Ti-6-4. Herein the accumulation of damage and its eventual culmination to failure were documented. Various test types and conditions were employed to try to activate a wide range of damage mechanisms. Sample stiffness and strength were monitored along with axial and transverse strain (implicitly instantaneous Poisson’s ratio) to determine if any of these attributes could be used to track or prognosticate damage and indicate the impending failure. However, the primary goal of this work is to provide material characterization and validation data for the generalized viscoplasticity with potential structure (GVIPS) constitutive model. The major conclusions from this study are

1. Actual physical damage in the form of shallow surface cracks and internal pores was observed in the tested specimens. The actual volume fraction of such damage relative to overall material volume was quite small.
2. True-stress and true-strain measures should be employed when interpreting data for a damage study to enable correct conclusions to be drawn.
   a. The modulus calculated from the engineering stress and strain curves shows large decreases (approx. 35 to 40 percent) with increasing deformation and damage. However, this was believed to be fictitious because of the continually decreasing cross-sectional area of the sample. Moreover, it was not consistent with the small amounts of physical damage observed in the samples.
   b. The true modulus, calculated from the true-stress and true-strain curves, showed minimal (approx. 5 percent) changes with increasing deformation and damage. This smaller stiffness reduction appears to be much more consistent with the small amounts of physical damage observed within the various specimens and certainly more realistic than observed changes in the engineering modulus. Unfortunately, considering scatter in the data, it appears difficult to use this parameter to track damage as a function of deformation.
   c. Modulus taken at slow strain rates displays a larger decrease in true modulus with increasing damage, thus suggesting that slow strain rate testing may be more useful for characterizing damage evolution.
3. Consistent Poisson’s ratio appears promising as a damage indicator and can provide insight into the onset of necking rather than simple examination of the ultimate tensile strength (UTS) in the stress-strain curves. It may also be useful in identifying the beginning of apparent tertiary creep by determining the inflection point in the curve of Poisson’s ratio versus axial strain.
a. Transverse strain and thus, implicitly, Poisson’s ratio are difficult values to measure. In future studies, the measurement of transverse strain and axial strain needs to be done consistently (i.e., measurement of axial and transverse strain should be over the same volume of material) to make an accurate interpretation of instantaneous Poisson’s ratio possible. Furthermore, transverse strain output should be maximized, probably through sample design, to minimizing the effects of signal noise.

b. Monitoring transverse strain and thus Poisson’s ratio during the test provides immediate validation on the multiaxial capability of a given model.

4. Because of material texture and the resulting anisotropy, the transverse strain and Poisson’s ratio are highly dependent on the rotation of the sample coordinates with respect to the specific diameter being measured by the transverse micrometer. Their values are also dependent on the axial position within the sample gauge that is being measured and its relation to the localized necking. Consequently, it is extremely important to document these relative directions and locations to ensure traceability and thus pedigree of the measurements. This is particularly important if the component design includes texture and anisotropy.

5. There were minimal fatigue tests conducted so few concluding statements can be made regarding the effect of fatigue. It appeared as if none of the parameters investigated tracked very well with damage. However, damage accumulation during fatigue probably occurred near the very end of life, and these particular tests were not designed to capture enough data in that regime.

6. One must remember that strain measurements are average measurements over a specified volume of material, and as such one needs to understand and report the implied length scale over which measurements are being taken when dealing with strain magnitudes that exceed the UTS (i.e., where material volume elements contain gradients due to local instabilities) of a given material.

7. For small-strain studies, wherein deformation model characterization takes place, a number of experimental and analytical complexities are eliminated. For instance, consistency within the transverse and axial strain measurements is not necessary, the strain gradients in the sample can be neglected, and small strain approximation can be used. However, damage studies, by definition, require large strains, and thus incorporation of consistent true-stress and true-strain measures.
### Appendix A.—Nomenclature

- **$A$** instantaneous gage cross-sectional area
- **$A_o$** initial gage cross-sectional area
- **$a$** major axis of gage cross section ellipse
- **$B$** scaling factor for damage accumulation
- **$b$** minor axis of gage cross section ellipse
- **bcc** body-centered cubic
- **$C_{ijkl}$** elastic stiffness tensor
- **$D$** damage variable
- **$d$** sample diameter
- **$\Delta d$** change in sample diameter
- **$d_o$** original sample diameter
- **$E$** elastic (Young’s) modulus
- **$E(\dot{e})$** rate-dependent initial modulus
- **$E_{\text{eng}}$** elastic modulus based upon engineering stress and engineering strain
- **$E_L$** loading modulus
- **$E_{\text{true}}$** elastic modulus based upon true stress and true strain
- **$E_U$** unloading modulus
- **$E_{x,y,z}$** elastic modulus in material directions x, y, and z
- **$e$** eccentricity
- **$e$** axial engineering strain
- **$\dot{e}$** loading engineering strain rate
- **$e_D$** engineering strain due to damage
- **$e_E$** elastic engineering strain at proportional limit
- **$e_I$** inelastic engineering strain
- **$e_{\text{onset}}$** engineering strain at which damage begins accumulating
- **$e_{ij}$** total engineering strain tensor
- **$e_{\text{max}}$** maximum engineering strain
- **$e_{\text{min}}$** minimum engineering strain
- **$e_p$** plastic engineering strain
- **$e_t$** transverse engineering strain
- **$e_{t,\text{true}}$** true transverse strain
- **$e_{\text{true}}$** true axial strain
- **$e_x$** engineering strain in x-direction (in loading direction)
- **$e_y$** engineering strain in y-direction
- **$e_z$** engineering strain in z-direction
- **$F$** applied load
- **GVIPS** generalized viscoplasticity with potential structure
hcp hexagonal close packed

$L$ load step

$l$ instantaneous gage length

$l_o$ original gage length

$N$ number of load cycles

$N_f$ fatigue life, failure

$PL$ proportional limit

$R_e$ strain ratio

$R_\sigma$ load ratio

$S$ engineering stress

SEM scanning electron microscopy

$T$ applied temperature

$T_m$ melting temperature

UTS ultimate tensile stress/strength

$Y$ uniaxial threshold stress

$\alpha$ hexagonal close packed (hcp) phase in titanium

$\beta$ body-centered cubic (bcc) phase in titanium

$\delta$ dilatation

$\varepsilon^{D}_{ij}$ damage strain tensor

$\varepsilon^{E}_{ij}$ reversible elastic (viscoelastic) strain tensor

$\varepsilon^{I}_{ij}$ irreversible inelastic (viscoplastic) strain tensor

$\varepsilon^{T}_{ij}$ total strain tensor

$\varepsilon^{th}_{ij}$ reversible thermal strain tensor

$\Delta \varepsilon_i$ total strain range

$\theta$ angle of rotation between right side of test rig and major axis of sample cross section

$\nu$ instantaneous Poisson’s ratio

$\nu^e$ elastic Poisson’s ratio

$\nu^p$ plastic Poisson’s ratio

$\nu_{davg}$ diameter-averaged Poisson’s ratio

$\sigma$ stress

$\dot{\sigma}$ stress (load) rate

$\Delta \sigma$ stress range

$\sigma_{ij}$ stress tensor

$\sigma_{max}$ maximum stress

$\sigma_{min}$ minimum stress

$\sigma_{true}$ true stress
\( \sigma_{0.2\%} \) 0.2\% yield point

\( \psi_{ij} \) anisotropic Poisson’s ratio in elastic regime

\( \psi_{yx} \) anisotropic Poisson’s ratio \((-e_y/e_x)\)

\( \psi_{zx} \) anisotropic Poisson’s ratio \((-e_y/e_x)\)
Appendix B.—Finite Element Analysis of Dogbone Cylindrical Specimen

The cylindrical specimen was modeled using finite element analysis assuming an elastoplastic J2 constitutive model with kinematic hardening to represent the Ti-6-4 material at 427 °C. Classic small-strain theory was assumed as well. The tensile behavior for sample 23 was used to characterize the model parameters (modulus $E = 91$ GPa and Poisson’s ratio $\nu = 0.36$; stress and associated plastic strain values are shown in Table B.1). Two Abaqus (Dassault Systemes) finite element meshes, one with 10,710 C3D8 elements and a finer mesh with 114,728 elements, were used to confirm that results had converged. The coarser 10,710-element mesh was used to produce all the results shown here, where reduced integration was used for all elements in the larger grip area. The dimensions of the actual specimen design used in all these experiments and analyzed herein are given in Figure 1 of Lerch and Arnold (2014). Per agreement with the experiments, the top of the specimen was fixed while the bottom portion was subjected to a constant displacement rate, as indicated in Figure B.1. The corresponding axial and transverse strain components are also shown. This is a typical test specimen geometry and is specifically designed to provide a gage volume that has uniform stress and strain. However, it will be shown that the total strain experiences a gradient.

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B.1 Virtual Experiment Assuming No Necking

The experimental stress-strain curve as well as predicted axial and transverse strain component distributions at six specific axial strain locations are shown in Figure B.2. Clearly after 5 percent axial strain (position 2), strain gradients (although slight) begin to appear within the gage region (defined by the extensometer zone), with significant gradients appearing after the ultimate tensile strength, UTS (position 3). By position 5 significant localization of plasticity occurs, thus implying that this localization proceeds the onset of ultimate failure. In Figure B.3 multiple axial and transverse strain “measurement” interpretations are shown as a function of time. These measurements are associated with three locations: (1) over the entire extensometer gage length (see area denoted as extensometer in Figure B.1), (2) at midgage (see slice (ii) in Figure B.1), and (3) at the top of gage (see slice (i) in Figure B.1). Two basic ways of calculating (virtually measuring) strains are conducted: (1) averaging the nodal displacements along two given planes (be they horizontal or vertical) to obtain a change in length and then dividing by the original length (this corresponds to the experimental measurement procedure) or (2) averaging the given strain component (axial or transverse) over the entire specified area or volume. In Figure B.3 it is clear that the measurements at midgage provide the largest responses (black curves), those at the top provide the smallest responses (purple curves), and those averaged over the entire extensometer section are in between (blue curves). The significance of how one calculates the longitudinal and transverse strain is more evident when they are subsequently used to infer (or calculate) other properties such as the instantaneous Poisson’s ratio response as a function of applied loading. Figure B.4 shows the importance of being consistent: only when measurements obtained from the same area or volume of material are combined is one able to retrieve the known Poisson’s ratio behavior for an isotropic elastic/plastic
material (assuming small strain, see Eq. (D.7) in Appendix D) wherein the inelastic strain contribution is incompressible (i.e., $\nu = 0.5$). Note that all three measures (top, midway, and overall) provide the same instantaneous response that saturates to a value of 0.5 when extensive plastic strain is present; however, when transverse measurements obtained from material regions inconsistent with those of the longitudinal strain measurement are combined, one obtains inaccurate (yet understandable) results for Poisson’s ratio (see the black and purple curves in Figure B.4). For instance, in the case when transverse strains are measured at midgage (the largest, see Figure B.3) and divided by the axial strain obtained from the extensometer (blue line, Figure B.3), one would expect a commensurate increase in Poisson’s ratio (beyond 0.5) as shown in Figure B.4. Similarly, when transverse strain is measured at the top of the gage area (solid purple line, Figure B.3) and divided by the larger extensometer axial strain (blue line, Figure B.3) one would expect a decrease in instantaneous Poisson’s ratio, as shown in Figure B.4. The interesting aspect regarding this particular Poisson’s ratio case is that it actually does not reach the theoretical limit of 0.5 as it should in this virtual experimental case, clearly indicating a problem of inconsistency.

Similarly, given that strain gradients are present within the specimen and that the magnitude of transverse strain is dependent upon the vertical location of measurement, Figure B.4 illustrates another important point: that the actual shape of the corresponding Poisson’s ratio curve is highly dependent upon the location of the transverse measurement. In Figure B.4, if the measurement was taken in between (i) and (ii) or (ii) and (iii) instead of at the middle or top of the gage section (as before), the resulting response curve can change significantly (see the corresponding curves labeled “Location (i) to (ii)” or “Location (ii) to (iii)”). This result is particularly important in view of the fact that the actual vertical transverse strain measurement location is fixed to the load frame during the test, yet during deformation the specimen is vertically moving during the test. Therefore, the magnitude of the transverse measurement, irrespective of consistency aspects, can change (as the “viewing window” is moving into a region of lower transverse strain); and thus potentially the instantaneous Poisson’s ratio is also changing—particularly at larger axial strain levels, or once past the UTS of the specimen.

Figure B.2.—Longitudinal stress-strain response of Ti-6Al-4V with axial and transverse strain profiles.
Figure B.3.—Simulated strain calculations based on location of strain measurement.
(a) Axial strain. (b) Transverse strain.
An illustration of this is shown in Figure B.5, where virtual strain gages are introduced to replicate two types of Poisson’s ratio measurements. The first type is that obtained when strains are measured over a consistent volume of material; that is, when the height of the virtual axial gage is equal to the length of the transverse strain’s optical shadow. If both longitudinal and transverse “strain gages” are fixed to the specimen at the middle of the gage area, the solid orange line results. If the transverse strain measurement is fixed to the test frame so that the specimen is moving through the observation window as the specimen deforms, the dashed orange line is the result. Clearly the observation window produces a slightly decreasing instantaneous Poisson’s ratio measurement since the transverse strain measurement decreases as one “moves” toward the top of the gage section once the axial strain becomes large (e.g., >10 percent). Note that even when a consistent volume of material is maintained for both longitudinal and transverse strains and both measurements are not fixed to the specimen, then a measurement inconsistent with theory is produced. The second example corresponds to that actually obtained in the laboratory—axial strain measured over the entire gage length via an extensometer that is attached to the specimen while the transverse strain is only measured over a thin section in the middle of the gage section. The solid black curve in Figure B.5 corresponds to the measurement when both axial and transverse “gages” are fixed to the specimen, whereas the dashed black line corresponds once again to the fixed observation window for the transverse strain measurement, as in the experiment. Once again, the observation window (that corresponding to how the measurement is actually obtained) gives a reduced instantaneous Poisson’s ratio measurement.
B.2 Virtual Experiments With Necking

Figure B.6 illustrates the stress-strain response curves for virtual tests when necking of the specimen is present. Necking is simulated by reducing the yield stress to 95 percent in two rows of elements across the entire specimen centered about the dashed lines in Figure B.1 where (i) necking is at the top, (ii) necking is at midgage, and (iii) necking is at the bottom of the gage section. All three locations of necking were observed in the various experiments presented in the body of this document (although only one, sample 538-65, necked at the bottom). Also plotted in Figure B.6 for reference purposes is the “No necking” curve, which is identical to that shown in Figure B.2. Strain contours are also shown in the insert of Figure B.6 at positions 3 (UTS) and 5 (severe necking near failure). As similarly shown in Figure B.2, significant strain gradients are present through the thickness and along the length of the gage section once the loading has exceeded the UTS. Also the location of plasticity localization shifts from top, to middle, to bottom, consistent with the imposed location of necking, as expected. Note that both stress-strain curves associated with top and bottom necking coincide and develop a smaller axial strain by the time the simulation is ended compared with the case of no necking or midgage necking. The axial strain measured and plotted in Figure B.6 is associated with that obtained from the virtual extensometer (which provides an average measure of strain over the extensometer gage length), and thus one would expect that the midgage necking case would have the largest recorded axial strain. Note that the stress-strain and final axial strain measured vary—post UTS—since significant gradients within the gage length arise. This variation in response (which is clearly dependent upon the given volume of material over which the measurement is taken) is a very important fact when reporting and/or using such values as strain to failure.

Figure B.7 illustrates the corresponding instantaneous Poisson’s ratio virtual measurements for the case of necking at midgage; as expected the response is similar to the case with no necking except for the occurrence when the midgage transverse measurement is combined with the axial extensometer measurement (see black curve, Figure B.7). In this case the instantaneous Poisson’s ratio of 0.5 is reached at 5 percent axial strain instead of 10 percent, and it achieves an overall larger value of approximately 0.9 at the end of the simulation.
compared with 0.8 in Figure B.4. Also note that significant deviation already occurs prior to reaching the UTS (10 percent strain) in Figure B.7. This is because of the significantly larger transverse strain accumulated at midgage due to the presence of necking. Note that if a consistent axial strain measurement (one associated with the same volume of material as that of the transverse strain) is used, one can recover the theoretical instantaneous Poisson’s ratio response (see orange curve Figure B.7). This once again strongly suggests the need for consistency when interpreting experimental measurements, particularly when combining them to obtain some other property response like Poisson’s ratio.

The other two cases of necking at the top and bottom of the gage section are shown in Figure B.8 and Figure B.9, respectively. In the case of necking at the top of the gage section, the inconsistent response curves switch: the Poisson’s ratio obtained from the transverse strain measured at the top of the gage now increases above that of the theoretical 0.5 as plasticity and necking proceeds (see the purple curve in Figure B.8), whereas that obtained from the transverse strain measured at midgage (see the black curve in Figure B.8) now decreases. Note that both deviate significantly from saturation after the UTS is reached (see Figure B.8) and at approximately the same axial strain. In the case of necking at the bottom of the gage section (Figure B.9), both inconsistent Poisson’s ratio measurements remain below the theoretical limit and drop off significantly. Note that the Poisson’s ratio obtained from the top measurement (purple curve) begins its dropoff well before that obtained from the midgage measurement (black curve) and obtains a significantly lower final value than that produced from the midgage measurement. It is interesting to note that in both cases for necking outside the extensometer gage section, the measured axial strain (from the virtual extensometer) is 20 percent lower than the maximum axial strain obtained when there is either necking at midgage or no necking. This is a consequence of the localized plasticity region occurring outside the extensometer section.

It is important to note that in all cases, with and without necking, the initial Poisson’s ratio as well as the instantaneous Poisson’s ratio response obtained remains the same up until 1 percent strain, regardless of the measurement technique.

Figure B.6.—Stress-strain curves and strain distribution plots for simulations with necking at bottom, middle, and top of the gage area.
Figure B.7.—Instantaneous Poisson’s ratio response, assuming necking location is in middle of gage section.

Figure B.8.—Instantaneous Poisson’s ratio response, assuming necking location is at top of gage section.
Figure B.9.—Instantaneous Poisson’s ratio response, assuming necking location is at bottom of gage section.

B.3 Full-Field Measurement

To confirm the veracity of the above results, a room-temperature tensile test was conducted on a rectangular, flat, dogbone-shaped specimen (gage length = 26 mm, gage width = 10 mm, and thickness = 3.1 mm) composed of Ti-6-4 machined from a different plate than all other tests in this report. During this test axial extensometer, 0/90 strain rosette, and full-field strain measurements were obtained to enable verification of the strain measurements and the trends observed in the elevated-temperature finite element analysis conducted previously. For this sample the fundamental tensile properties are $E = 107$ GPa, $\nu = 0.37$, proportional limit $PL = 897$ MPa, 0.2% yield point $\sigma_{0.2\%} = 950.4$ MPa, and UTS = 1,043.2 MPa. At the end of the test a total reduction in area of 40.3 percent was observed. Note that the extensometer slipped shortly after attaining the UTS, and therefore the strains from the extensometer do not continue to failure.

The tensile response of this specimen is provided in Figure B.10, where it is shown that all axial strain measures—irrespective of technique—provide similar (if not identical) curves up to 2 percent axial strain. Comparing the current specimen’s tensile response represented by the red dashed line in Figure B.10 to a prior 20°C test (see Sample 10, Figure 3 from Lerch and Arnold, 2016), it is clear that the two responses are very similar, considering that the samples were taken from two different plates. Further, from the ratio of the axial and transverse strains from the 0/90 rosette, at each stage in the loading history, one can obtain the instantaneous Poisson’s ratio (green curve in Figure B.10) that is consistent with what has been observed before. Subsequent to the near-saturated elastic zone, a zone of rapid increase is observed, followed by another apparent saturation zone up to the UTS. Unfortunately, the strain gages fell off upon reaching 2 percent strain so an exact effective inelastic Poisson’s ratio value could not be attained. However, upon extrapolation when projecting out it appears that this value would be approximately 0.57.

Figure B.11 illustrates the flat specimen as well as the location of the axial extensometer, and each colored line identifies the location of four virtual extensometers, via the full-field strain measurement technique, by which both axial (red line, corresponding to long axial gage; and orange line, corresponding to short axial gage) and transverse (green line, upper gage; and blue line, midgage) strain measurements were obtained. Figure B.12 and Figure B.13 provide the corresponding measured axial and transverse
Figure B.10.—Room-temperature stress-strain response from different strain measurements. Ultimate tensile stress (UTS) is indicated.

Figure B.11.—Test setup with colored lines indicating location of virtual extensometers utilized via full-field measurement technique.
stress-strain curves for each of these gage areas. The inserts in these two figures illustrate the full-field strain contours over the entire specimen very near to failure. Clearly these full-field as well as individual “virtual” gage measurements indicate that as loading increases and the strains exceed the UTS (e.g., approx. 7 percent), then significant strain gradients arise throughout the specimen. Consequently, as one might expect, the average axial strain measurement associated with the long gage (red curve) is smaller in magnitude than the average associated with the short axial gage (orange curve) at any given stress state, since the largest axial strain exists at the center of the specimen and decreases as one proceeds to move up or down the specimen (see Figure B.12). Similarly, the average transverse strain measurement associated with the top gage (green curve) is smaller in magnitude than the average transverse strain associated with the midgage (blue curve) at any given stress state, since the largest transverse strain exists at the center of the specimen (particularly since necking occurred there) and decreases as one proceeds to move up or down the specimen (see Figure B.13). Note that the discrepancy between the transverse strain at the top and midgage and that of the long and short gage axial strain both occur after UTS has been reached—thus suggesting that up to the UTS very little strain gradients exist within the test specimen. Furthermore, these results emphasize the importance of clearly understanding and reporting the implied length scale over which measurements are being taken when dealing with strains magnitudes that exceed the UTS (i.e., material volume elements that contain gradients) of a given material. This brings into question the validity of using typically reported strain-to-failure measurements in the literature within failure models.

Taking the ratio of these various strain measures provides four different instantaneous Poisson’s ratio measurements, which are shown in Figure B.14. All four of these Poisson’s ratios would be considered inconsistent, since each of the four strains (two axial and two transverse) were averaged over different areas or volumes of materials. The shapes of these curves agree very well with those obtained from the finite element analysis simulation, depicted in Figure B.7.

![Figure B.12.—Stress-strain curves corresponding to axial measurements over entire gage length (long gage axial strain) and over midgage length (short gage axial strain). Ultimate tensile stress (UTS) is indicated.](image-url)
Figure B.13.—Stress-strain curves corresponding to transverse measurements at midgage and at top of gage just above externally attached extensometer. Ultimate tensile stress (UTS) is indicated.

Figure B.14.—Four Instantaneous Poisson’s ratio curves obtained by mixing and matching transverse and axial strain measurements from Figure B.12 and Figure B.13.
To verify the simulation results obtained in Figure B.7 wherein the Poisson’s ratio rises rapidly and then saturates to a constant value (0.5 for isotropic material or a value less than 1.0 but greater than 0.5 for an anisotropic material), two specific material volumes, one at the center of the specimen in the vicinity of necking (denoted by the red rectangle in Figure B.15) and one toward the top of the gage area (denoted by the green rectangle in Figure B.15) were selected for examination. The volumes were purposely taken to be of different sizes. In each of these zones both the axial and transverse strains associated with each facet (within these zones) was averaged to obtain axial and transverse strains associated with the red and green zones, respectively. Next, the ratio of average transverse and average axial strains were obtained for both the red and green zones at each stage of loading. These Poisson’s ratios correspond to the solid red and green lines in Figure B.15. Note that the upper Poisson’s ratio (green line) starts out at a relatively constant elastic value of 0.33 and then rapidly raises to the constant value of 0.57 as inelastic strains accumulate and remains there until the test specimen fails. Similar behavior is observed for the midgage Poisson’s ratio (solid red line) except here the initial elastic value is 0.37, but the next plateau still occurs around 0.57 to 0.58. In contrast to the upper zone, the instantaneous Poisson’s ratio corresponding to the midzone, at around 20 percent axial strain, begins to drop gradually and significantly as necking and/or potentially damage takes place near end of life. Both of these Poisson’s ratio measurements are consistent and agree qualitatively with the earlier simulations done at 427 °C (see Figure B.7). Note the drop associated with the red line could also be associated with the difference between a fixed frame of reference and that of a moving one (see Figure B.5), with the current full-field measurements being associated with a moving frame. Most important are the results obtained when the averaged axial and transverse strain measurements from different volumes of material are mixed (see dashed curves in Figure B.15); as these two instantaneous Poisson’s measures are inconsistent. Note that the “Transverse, top; longitudinal, middle” (red dashed) line here corresponds to the “Longitudinal, extensometer; transverse, top” (purple) line in Figure B.7 and confirms experimentally the trends associated with the virtual experiments. These full-field results clearly demonstrate (1) the presence of strain gradients throughout the gage area of a test specimen past the UTS and (2) the importance of using the strain measurements over the same volume of material if one wants to have an instantaneous Poisson’s ratio that is consistent with continuum theory.

Figure B.16 provides a final illustration of the special variation of Poisson’s ratio throughout the specimen, as it shows in the inset a contour of Poisson’s ratio at a long axial strain of 0.169. Clearly, the magnitudes and distribution of Poisson’s ratios would change as a function of applied loading level (i.e., strain magnitude), with the first 0.067 extensometer strain providing approximately a uniform or constant value of Poisson’s ratio throughout the specimen. Note, the average values at midgage (red line) and upper gage (green line) at an extensometer strain of 0.12 correspond very closely to the pointwise values in the inset; again, this illustrates the need to associate a volume element length scale with a given measurement, particularly when dealing with averages in the presence of severe gradients and moderately large volume elements.
Figure B.15.—Consistent and inconsistent instantaneous Poisson’s ratio responses.

Figure B.16.—Poisson’s ratio as function of load level at two locations, upper gage and midgage, and contour of Poisson’s ratio for each facet measured over length of specimen (see inset). Contour shown corresponds to long axial strain of 0.169, which is very near end of test.
Appendix C.—Image Analysis of Damage

This appendix documents pore damage observed in the tested samples. A combination of optical and scanning electron microscopy (SEM) images were used to document the damage in each sample, and a commercial image analysis program was used to provide quantification of the pores. The purpose of this appendix is to document the morphology of the pores, present what we believe are important characteristics of the damage, and address the problems associated with documenting damage. This will hopefully be of use in future studies relating to damage mechanics as well as integrated computational materials engineering (ICME; Wikipedia contributors, 2018).

Most of the samples tested in this study were sectioned and polished halfway through the thickness for metallographic examination. Samples were oriented such that material removal occurred from the 270° orientation toward the back of the rig; that is, parallel to the axial extensometer probes. Images were referenced with respect to the distance from the fracture surface, or minimum diameter if there was not complete failure, with this being the zero location. Generally, two images were captured at the fracture surface (0 mm) and then each millimeter up to 6 mm, with the remaining two images taken at 10 mm from the fracture. From the complete set of images, the pertinent values are given in Table C.1 for the tensile, creep, and relaxation samples. Image analysis was not performed on the fatigue samples, although limited basic metallography was performed.

For the image analysis, optical images were taken at a magnification of 10× using the Nikon Eclipse Ma200 microscope. The pixel resolution for each image was the same at 2560 × 1920 pixels (0.5 μm/pixel). The ImageJ FIJI package was used to identify and measure pores. A threshold contrast was selected for the pores, and they were then automatically identified. Human intervention only occurred when an obvious stain or debris appeared, and those “spots” were then manually excluded. Partial pores occurring on the edges were included.

In Table C.1 samples are grouped by test type (tension followed by creep then relaxation) and temperature. The first column is the sample number, and its corresponding strain rate for the tensile tests and stress level for creep is in column two. The creep samples were loaded at only one strain rate of 0.001 s⁻¹. Column three gives the maximum strain attained, whether due to failure or subsequent unloading. The average number of pores counted is shown in columns 4 and 5 for distances <3 and ≥3 mm from the fracture surface, respectively. These pores can be observed in Figure C.1 for tensile sample 538-65, which was near failure at a strain of 17 percent. Note the higher density of pores is at the point of minimum diameter, and they continue to be high until approximately 3 mm on either side of the minimum diameter, after which the pore concentration is much less. Hence, the separation in the number of pores is set to 3 mm.

Columns 6 and 7 represent the average area fraction of pores, again separated by the 3 mm division. The eighth column is the average pore area, calculated from pore sizes over the entire sample. A circle was fit to each pore and the average circular diameter is given in column nine. The average circularity is shown in column 10 and represents how close the average pore is to a circle. However, as shown by the circularity being a value less than one, the pores are better represented as ellipsoids whose aspect ratio is listed in column 11. The last column, ellipsoid angle, gives the average angle between the major axis of the pore ellipsoid and the horizontal (load) axis.
TABLE C.1.—TI-6AL-4V IMAGE ANALYSIS DATA SHOWING AVERAGE VALUES

(a) Tensile tests

<table>
<thead>
<tr>
<th>Sample</th>
<th>Strain rate, s⁻¹</th>
<th>Max. strain, percent</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Pores &lt;3 mm</td>
<td>Pores ≥3 mm</td>
</tr>
<tr>
<td>316 °C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>79</td>
<td>0.001</td>
<td>20</td>
<td>162</td>
</tr>
<tr>
<td>42</td>
<td>0.00001</td>
<td>20</td>
<td>93</td>
</tr>
<tr>
<td>427 °C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>0.001</td>
<td>20</td>
<td>152</td>
</tr>
<tr>
<td>23</td>
<td>0.001</td>
<td>20</td>
<td>159</td>
</tr>
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<td>69</td>
<td>0.00001</td>
<td>20</td>
<td>123</td>
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<tr>
<td>538 °C</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>0.001</td>
<td>6.9</td>
<td>288</td>
</tr>
<tr>
<td>21</td>
<td>0.001</td>
<td>13.3</td>
<td>206</td>
</tr>
<tr>
<td>89</td>
<td>0.001</td>
<td>20</td>
<td>245</td>
</tr>
<tr>
<td>70</td>
<td>0.0005</td>
<td>17.7</td>
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<td>37</td>
<td>0.0005</td>
<td>10</td>
<td>153</td>
</tr>
<tr>
<td>15</td>
<td>0.00001</td>
<td>8</td>
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<tr>
<td>97</td>
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<td>1,609</td>
</tr>
<tr>
<td>65(--)</td>
<td>0.00001</td>
<td>17</td>
<td>296</td>
</tr>
<tr>
<td>65(+)</td>
<td>------------</td>
<td>---</td>
<td>519</td>
</tr>
<tr>
<td>65(nadir)</td>
<td>------------</td>
<td>---</td>
<td>154</td>
</tr>
</tbody>
</table>

(b) Creep tests (10⁻³ s⁻¹)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Strain rate, s⁻¹</th>
<th>Max. strain, percent</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Pores &lt;3 mm</td>
<td>Pores ≥3 mm</td>
</tr>
<tr>
<td>427 °C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>375</td>
<td>1.1</td>
<td>76</td>
</tr>
<tr>
<td>45</td>
<td>591</td>
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<tr>
<td>87</td>
<td>596</td>
<td>21</td>
<td>35</td>
</tr>
<tr>
<td>538 °C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>76</td>
<td>101</td>
<td>1.3</td>
<td>54</td>
</tr>
<tr>
<td>94</td>
<td>206</td>
<td>5</td>
<td>22</td>
</tr>
<tr>
<td>85</td>
<td>239</td>
<td>9.7</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>241</td>
<td>11</td>
<td>1,006</td>
</tr>
<tr>
<td>34 (step)</td>
<td>241</td>
<td>12</td>
<td>61</td>
</tr>
<tr>
<td>88</td>
<td>376</td>
<td>13</td>
<td>548</td>
</tr>
</tbody>
</table>
TABLE C.1.—Ti-6Al-4V IMAGE ANALYSIS DATA SHOWING AVERAGE VALUES (Concluded)

(c) Stress relaxation tests (10⁻³ s⁻¹)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Strain rate, s⁻¹</th>
<th>Max. strain, percent</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pores &lt;3 mm</td>
</tr>
<tr>
<td>20-89b(−) Step relaxation</td>
<td>12</td>
<td>539</td>
<td>61</td>
</tr>
<tr>
<td>20-89b(+) Step relaxation</td>
<td>578</td>
<td>115</td>
<td>0.142</td>
</tr>
<tr>
<td>427-27   Step relaxation</td>
<td>15</td>
<td>65</td>
<td>7</td>
</tr>
<tr>
<td>538-77   Step relaxation</td>
<td>17</td>
<td>230</td>
<td>9</td>
</tr>
</tbody>
</table>

Figure C.1.—Ti-6Al-4V tensile specimen 538-65, indicating high concentration of pores within ±3 mm length of minimum diameter.

As indicated earlier, there are more pores in the localized necked area than in areas where there was a more uniform reduction in area. Comparison of the numbers in column 4 with those of column 5 show that the localized neck has approximately 2 to 6 times the average number of pores. The total number of pores per image (photo) is shown for the tension, creep, and relaxation samples in Figure C.2 as a function of distance from the fracture. For the samples tested in tension, there is a sum total number of pores over the combined imaged regions of approximately 1,000 to 2,000, and this appears to be independent of failure strain and strain rate. The exception to this is at 538 °C and at the slowest strain rates. For these samples, 538-97 and 538-65, the number of imaged pores totaled in the tens of thousands, which suggests a greater influence of time-dependent deformation mechanisms. There are two other noteworthy samples from the 538 °C tensile tests. Sample 538-70 had nearly 7,000 total imaged pores, a factor of 3 higher than what was observed for sample 37 (also tested at a strain rate of 5×10⁻⁴ s⁻¹) and for all samples tested at faster rates. Sample 538-70 was first strained to 1.8 percent, relaxed for 18 h, and subsequently pulled to failure in tension. It is speculated that this initial relaxation had a significant influence on the latter pore formation. The second oddity was 538-15, which had a low amount of total imaged pores (2,086) despite being tested at the slowest strain rate. However, this test employed a fast-rate (0.001 s⁻¹) unload, which may have dramatically decreased the pore formation.
Figure C.2.—Total number of observed pores per image of Ti-6Al-4V samples for each test type at given distance from fracture surface. (a) Tensile test. (b) Creep test. (c) Relaxation test.
Since the intensity of pores is higher at the localized neck, we wanted to see if there was a section in the localized neck, probably at the nadir, that had a very high area fraction of pores that would eventually lead to failure. We more closely examined sample 538-65 and performed pore analyses at the nadir, across the full diameter, and over a gage length (thickness of the slice) of 0.7 mm. The results are shown in the last row in Table C.1 part (a), the tensile test section. It is observed that the average pore area fraction is slightly higher (0.78 percent) than elsewhere in the sample, but not unusually high. The pores at the minimum diameter are also observed to be a bit more circular as shown by the slightly higher circularity and the lower aspect ratio. Moreover, the pores have a slightly larger diameter and pore area than all other samples, including at other locations in sample 65. It can therefore be concluded that the pores have not grown across a major portion of the diameter (cross-sectional area) at this point, even though the sample was strained to 17 percent and contained a significant localized neck (i.e., it is very close to failure).

The total number of imaged pores in the creep samples (Figure C.2(b)) ranged between 300 and 1,000, much lower than what was observed in the tensile samples. The exception to this was sample 538-3, which had an extremely high number of pores (7,800), and 538-88, which had a total of 3,700 imaged pores. At first glance, there does not appear to be a relationship between the maximum strain attained in the creep samples and the total number of imaged pores. There is a marginally higher pore count for samples tested at 538 °C compared with those tested at 427 °C.

The total number of imaged pores in the step relaxation samples are 3,900 (20 °C), 432 (427 °C), and 223 (538 °C). These are generally in the same range as those observed for the tension and creep samples. It is not known why the relaxation sample tested at 20 °C had a higher pore count than the other two sample types, but it could be due to the higher limit stress that the samples relaxed to at 20 °C.

The area fraction of pores trends with the total number of imaged pores for the three test types. Columns 6 and 7 in Table C.1 show the average area fraction of pores, again separated by the 3-mm demarcation line. The area fraction of pores near the fracture surface (minimum diameter) is higher than in further remote areas (i.e., beyond the localized neck) with the factor ranging between 0.5 and 40, and at lower temperature the difference is usually smaller than at higher temperatures. There is approximately a factor of 100 between the area fraction of pores at the fracture compared to those at remote areas for creep samples 34 and 88 and relaxation sample 77, all tested at 538 °C. The area fraction of pores per image is plotted as a function of distance from the fracture surface for the tensile samples in Figure C.3 (trend lines shown for easier viewing) and separated by test temperature. There is minimal difference in area fraction among the samples tested at 316 and 427 °C and very little difference between the area fractions near the fracture surface compared to those of more remote regions. However, at a test temperature of 538 °C, there is a greater difference between the area fractions of pores near the fracture surface than in more remote areas. Moreover, the area fraction at this temperature is approximately an order of magnitude higher than that observed at the two lower test temperatures. The tests at 538 °C and at slower strain rates tend to have higher area fractions of pores; this is consistent with the total number of imaged pores discussed earlier.

Figure C.4 shows the pore area fraction for the creep samples as a function of distance from the minimum diameter. There is a trend for the area fraction to be higher near the minimum diameter by a factor between 1 and 20. However, samples 538-34 and 538-88 have factors of 86 and 77, respectively. There is no difference in the average area fraction among the samples, with the exception of specimen 538-3, and to a lesser extent 538-88, which have significantly higher average area fractions than all of the other creep samples. This trend was previously displayed by the total number of imaged pores.
Figure C.3.—Area fraction of pores in Ti-6Al-4V tensile samples as function of distance from fracture surface at different temperatures. (a) 316 °C. (b) 427 °C. (c) 538 °C.
The step relaxation samples (Figure C.5) have a similar order of magnitude of pore area fraction as most of the other test samples. However, there is a large increase of pore fraction near the fracture surface of the relaxation specimens, moreso than observed in most of the other test samples.

The data indicate that in all test samples, the area fraction of pores is extremely small. The majority of the samples contained less than 0.1 percent in any given image. The highest area fraction in any image was 3 percent for samples 538-97 (slow-rate tensile) and 538-3 (creep), which had the anomalously high number of pores. The generally small area fraction of pores implies a very low state of damage.

The area (size) of an average pore is given in column 8 of Table C.1. In general the pore size is only a few square microns. The average area is consistent as a function of distance from the fracture for the two lower tensile temperatures as shown in Figure C.6. At 538 °C the pore size does increase closer to the fracture surface. Except for the samples tested at 538 °C and the slowest strain rates, all other tensile samples have similar pore sizes in the range of a few square microns. At slower strain rates, the pore area increases, being in the range of 10 to 20 μm² at 538 °C. It also appears that at 316 and 427 °C the slower strain rate samples may have slightly larger pore areas (see samples 316-42 and 427-69 in Table C.1), indicating a time dependency on pore size. Optical micrographs of typical pores are shown in Figure C.7 for
a sample tested in tension at 427 and another at 538 °C, both taken from 2 mm from the fracture surface and at a strain rate of 1×10⁻⁵ s⁻¹. The images show the size and distribution of the pores, which are much larger and more prevalent at the higher temperature.

Pore area in the creep samples is given in Figure C.8 as a function of distance from the fracture surface. Generally the pore size is consistent with those of the tensile specimens. At any given temperature, there does not appear to be a relationship between pore size and either creep stress level or maximum strain.

The average pore area is similarly only a few square micrometers for the relaxation samples as well. The pore size as a function of location to the fracture surface is given in Figure C.9 and shows that the pores increase in size by up to a factor of 10 near the fracture.

The pores were fit to a circle, and average diameters were calculated and are given in column 9 of Table C.1. The average circular diameter for the pores is approximately 1.5 μm for all tests, being higher and averaging 2.7 μm for temperatures of 538 °C and slower strain rates. There is a very slight dependence of pore diameter on strain rate at 316 and 427 °C test temperatures; they are larger with decreasing strain rate.

Average circularity of the pores is listed in column 10 of Table C.1, where the value of 1 indicates a perfect circle and a value near 0 indicates a highly elongated polygon. Among all the tests listed in the table, the value for circularity is constant at about 0.85, indicating that the pores are slight ellipsoids. The aspect ratio of the ellipsoids is given in column 11 and is consistent through the tests, having an average value of 1.5. The angle between the major axis of the pores and the horizontal axis of the image (coincides with the loading axis) is shown in the last column, 12. The average values in an image range between 20° and 90°, with the grand average of 69°. There is a slight increase in ellipsoid angle as a function of temperature, with higher temperatures leading to larger angles. Pore morphology and orientation is shown in the SEM image in Figure C.10. The dashed line in the figure denotes a 69° angle with respect to the horizontal. Most of the elliptical pores should be aligned about this axis.

![Figure C.5.—Area fraction of pores in Ti-6Al-4V relaxation samples as function of distance from fracture surface.](image)
Figure C.6.—Average area (size) of pores in Ti-6Al-4V tensile samples as function of distance from fracture surface at different temperatures. (a) 316 °C. (b) 427 °C. (c) 538 °C.
Figure C.7.—Optical images comparing pores in tension samples tested at strain rate of $1 \times 10^{-5}$ s$^{-1}$ at two different temperatures, 2 mm from fracture surface. (a) Sample 427-69. (b) Sample 538-97.

Figure C.8.—Average area (size) of pores in Ti-6Al-4V creep samples at two different temperatures as function of distance from fracture surface. (a) 427 °C. (b) 538 °C.
Figure C.9.—Average area (size) of pores in Ti-6Al-4V relaxation samples as function of distance from fracture surface.

Figure C.10.—Scanning electron micrograph (SEM) of section of Ti-6Al-4V tensile sample 538-39 showing elongated pores. Dashed line indicates 69° angle from horizontal (load) axis.
Appendix D.—Interpretation of Poisson’s Ratio Results

Given that the trend for Poisson’s ratio \( \nu \) as a function of strain varied from test to test and its behavior was often unexpected, a more detailed examination of Poisson’s ratio is required so that some proposed explanations can be made. The need for consistency when calculating Poisson’s ratio given experimental measurements at different locations is illustrated in Appendix B. Yet even when this inconsistency is virtually duplicated the experimental results are not fully explained, as the magnitude and slope changes of the measured instantaneous Poisson’s ratio and that simulated are not the same, although the general direction after saturation is, for the most part, in line with the inconsistency explanation. In this appendix, the influence of strain measure assumption with respect to large magnitudes of strain and the material symmetry assumption will be addressed to provide further insight into the observed behavior.

The most well known and widely used constitutive model is Hooke’s law:

\[
\sigma_{ij} = C_{ijkl} \epsilon^{E}_{kl}
\]

(D.1)

where \( \sigma_{ij} \) and \( \epsilon^{E}_{ij} \) are the elastic stress and strain components, respectively, and the \( C_{ijkl} \) are the elastic stiffness tensor components. In the case of an isotropic material, the \( C_{ijkl} \) contain only two model parameters: Young’s modulus \( E \) and Poisson’s ratio \( \nu \). Extension into the thermal and irreversible regimes has been accomplished by assuming an additive decomposition (applicable for small strain theory) of the total strain tensor \( \epsilon^{T}_{ij} \) into four components:

\[
\epsilon^{T}_{ij} = \epsilon^{E}_{ij} + \epsilon^{P}_{ij} + \epsilon^{D}_{ij} + \epsilon^{th}_{ij}
\]

(D.2)

where \( \epsilon^{E}_{ij} \) is the total applied strain; \( \epsilon^{P}_{ij} \), the reversible elastic or viscoelastic strain; \( \epsilon^{D}_{ij} \), the irreversible inelastic or viscoplastic strain; \( \epsilon^{th}_{ij} \), the damage strain; and \( \epsilon^{th}_{ij} \), the reversible thermal strain.

The term “damage” can be used very broadly to denote some degradation of performance that ultimately leads to failure of a given component or structure. However, one must realize that a given damage variable has some mechanistic origin within the material, such as nucleation and growth of voids, cavities, or microcracks and other microscopic or mesoscopic defects, which later coalesce into distinct fracture modes at the culmination of the failure process (e.g., propagating mesocracks and/or macrocracks or softening in localization zones). Therefore, damage will manifest phenomenologically in some form of softening behavior. The damage often manifests by decreasing the Young’s modulus (i.e., as a stiffness reduction mechanism); alternatively, it may result in a decrease in residual strength or increase in inelastic strain, thus being categorized as a strength reduction. Pure strength reduction would cause an increase in the inelastic flow \( \epsilon^{E}_{ij} \) without a corresponding change in stiffness (Young’s modulus, \( E \)). Figure D.1 illustrates the difference between a stiffness reduction damage and inelasticity on the unloading modulus. Three potential scenarios are shown: (1) all nonlinearity is due to inelasticity only (i.e., no change in unloading modulus or stiffness, \( \epsilon^{P}_{ij} = 0 \)), (2) all nonlinearity is due to damage (significant reduction in modulus such that all strain is recovered during unloading, i.e., \( \epsilon^{D}_{ij} = 0 \)), and (3) some combination of damage and inelasticity. Note that the inelasticity shown in Figure D.1 could potentially be enhanced by strength reduction damage. In reality, to discriminate between strength reduction damage and inelasticity, one would need additional testing beyond the tensile response shown in this figure.
Substitution of Equation (D.2) into Equation (D.1) yields a stress-strain relation (known as the Generalized Hooke’s Law) that incorporates both reversible and irreversible strains:

\[
\sigma_{ij} = C_{ijkl} \left( \varepsilon_{kl} - \varepsilon_{kl}^e - \varepsilon_{kl}^D - \varepsilon_{kl}^h \right)
\]  
(D.3)

Figure D.2 illustrates a typical uniaxial tensile response of a given metallic material in which elastic and inelastic strain, and possibly damage, is observed. The stress is typically plotted using engineering stress and strain quantities; i.e., engineering stress \( S \) is defined as load \( F \) divided by original area \( A_o \) \( (S = F/A_o) \), and axial engineering strain \( e \) is associated with the change in length over original length. The elastic, or Young’s, modulus \( E \) is then associated with the linear slope of the initial stress-strain curve, and Poisson’s ratio \( \nu \) is the ratio of the strain in the direction perpendicular to the load (i.e., transverse) \( e_t \) and that in the load direction (i.e., axial) \( e \):

\[
\nu = -\frac{e_t}{e}
\]  
(D.4)

If deformation involves strains that induce incompressible behavior (e.g., inelastic strains) then the Poisson’s ratio as given in Equation (D.4) will remain constant until the proportional limit is exceeded and then change throughout the remaining deformation history. Herein, this is called the instantaneous Poisson’s ratio, whereas others have defined it to be the contraction ratio or apparent Poisson ratio.
Two different expressions have been developed (by examining the dilatation of the material or change in volume per unit volume) to indicate how Poisson’s ratio will evolve with loading. The first comes from assuming small strains, isotropic material behavior, and summing the normal strain components to achieve the current dilatation $\delta$:

$$\delta = (1 - 2\nu)e$$  \hspace{1cm} (D.5)

Then by partitioning that dilatation into elastic and plastic dilatation (see Jones, 2009, and Nadia, 1963) one can obtain an expression for the instantaneous Poisson’s ratio $\nu$:

$$\nu = \nu^p - \frac{e^E}{e}(\nu^p - \nu^e)$$  \hspace{1cm} (D.6)

where $\nu^p = 0.5$ is the plastic Poisson’s ratio (i.e., implying incompressibility of inelastic strain), and $\nu^e$ and $e^E$ are elastic Poisson’s ratio and strain, which remain constant at $\sigma_y/E$ beyond the proportional limit (i.e., outside the elastic region).

Alternatively, the current unit volume ($\delta + 1$) can be written as

$$\delta + 1 = (1 + e_x)(1 + e_y)(1 + e_z)$$  \hspace{1cm} (D.7)

such that in the case of isotropic material (i.e., $e_y = e_z = -ve$ and $e_x = e$),

$$\delta + 1 = (1 + e)(1 - ve)^2$$  \hspace{1cm} (D.8)
Solving for Poisson’s ratio $\nu$, an expression can be obtained describing how the instantaneous Poisson’s ratio will evolve with applied total strain (see Stang, Greenspan, and Newmann, 1946):

$$\nu = \left[ 1 - \left( \frac{1 + \delta}{1 + e} \right)^{1/2} \right] \frac{1}{e}$$  \hspace{1cm} (D.9)

Note, because of the inclusion of higher order terms within Equation (D.9), it was thought that large magnitudes of engineering strain ($e$) were admissible, albeit inconsistent, with the assumption of engineering strain; that is, small strain theory. This inconsistency will be discussed subsequently. Equations (D.6) and (D.9) are equivalent when higher order terms (i.e., $\nu e^2 - 2\nu e^2 + \nu^2 e^3$) are ignored or when strains are small.

Figure D.3 illustrates the difference between the evolutions of the instantaneous Poisson’s ratio derived from these two expressions, assuming perfectly plastic material behavior (i.e., no work hardening). Clearly in the elastic regime Poisson’s ratio remains constant (at the assumed elastic value of 0.3) at all applied strains (i.e., $e = e^E$), resulting in dilatation. Upon yielding, inelastic strain $e^I$ (which does not correspond to a change in dilatation, as it is incompressible) is generated (i.e., $e = e^E + e^I$ for the small strain assumption and $e = e^E e^I$ for large strains), both of which cause a rapid increase in the Poisson’s ratio. In the case of the small-strain assumption (grey curve) the instantaneous Poisson’s ratio will saturate at 0.5 once the elastic strain is overwhelmed by the inelastic strain (i.e., at $e^I/e^E > 10$; here Poisson’s ratio is within 96 percent of the 0.5 limit\(^{16}\)). Whereas Equation (D.9) reaches a maximum value early (i.e., at $e^I/e^E = 5$) and thereafter decreases, the maximum value never reaches the theoretical incompressible limit value of 0.5. Clearly, the character of the two curves is quite different, with the higher order case (Eq. (D.9)) reflecting what has been seen experimentally (both in the current study and others; see Stang, Greenspan, and Newmann, 1946).

\(^{16}\)During years of uniaxial testing, we have never observed Poisson’s ratio to ever saturate for isotropic materials at exactly 0.5. It always falls slightly short of this limit.
Figure D.4 illustrates the importance of damage on the strain assumption. If damage is compressible and induces dilatation,

$$\delta + 1 = \left(1 + e^E + e^D\right) \left[1 - \nu \left(e^E + e^D\right)\right]^2$$  \hspace{1cm} (D.10)

where $e^D$ is the strain due to damage. If damage is incompressible and does not cause a change in dilatation,

$$\delta + 1 = \left(1 + e^E\right) \left(1 - \nu e^E\right)^2$$  \hspace{1cm} (D.11)

Clearly, when damage is assumed compressible (Eq. (D.10)), the instantaneous Poisson’s ratio Eq. (D.9)) will remain constant ($\nu = \nu^e$) over all strains; whereas if it is assumed incompressible (Eq. (D.11)), Poisson’s ratio (Eq. (D.9)) will evolve in the same way as if the nonlinearity were due to inelasticity, as expected (see orange line in Figure D.4). Experimental evidence suggests that damage (e.g., void nucleation, microcracking) does indeed invoke compressible strain contributions (albeit potentially directionally dependent) (Saanouni, 2012).

In Figure D.3 and Figure D.4 the definition of engineering strain, $e = \Delta l/l_o$ (which is predicated upon the assumption of small strains), was used in the calculation of dilatation. Here, $\Delta l$ is the change in gage length $l$, from the original gage length $l_o$. This raises the question of what constitutes small strains. Note typical experimental strain measurements also correspond to this definition of engineering strain. To answer this question of “smallness,” two normal strain measures will be examined: the first is the engineering strain as previously defined, and the second is the true strain (or logarithmic strain) $e_{true}$, which is defined as

$$e_{true} = \int_{l_o}^{l} \frac{dx}{x} = \ln\left(\frac{l}{l_o}\right)$$  \hspace{1cm} (D.12)

which can be shown to be related to the engineering strain, $e$:

$$e_{true} = \ln(1 + e)$$  \hspace{1cm} (D.13)

since $l = el_o + l_o$. Clearly, the true strain is always less than the engineering strain, and the difference between these two increases as the engineering strain increases. Dividing the error in strain\(^{17}\) (gold dashed line in Figure D.5) by the axial engineering strain, one can determine that the maximum error factor is 0.5 between true and engineering strain. Figure D.5 illustrates these two strain measures as a function of axial engineering strain. Consequently, depending upon the error one is willing to accept, the definition of “small strain” can be determined. For example, assuming 1 percent error is acceptable, a 2 percent axial strain would be the cutoff for small strain; or assuming an allowable 5 percent error, then the upper limit of axial engineering strain would be 10 percent.

\(^{17}\)Error = (true – engineering)/engineering.
Figure D.4.—Evolution of Poisson’s ratio, assuming isotropic elasticity and damage.

Figure D.5.—Measurement of engineering and true strains. (a) Measured strains. (b) Error (difference between engineering and true strains, divided by engineering strain).
Note that proportionality of the transverse strain to that of the longitudinal (axial) strain via Poisson’s ratio as described in Equation (D.4) is actually only valid for true strain (or engineering strain when strains are limited to 2 percent or less, assuming a 1-percent error is acceptable).

\[
\int_{y_o}^{y} \frac{dy}{y} = -\nu \int_{x_o}^{x} \frac{dx}{x}
\]

or

\[
\epsilon_{t,\text{true}} = -\nu \epsilon_{\text{true}}
\]

where \(\epsilon_{t,\text{true}}\) is the true transverse strain and \(\epsilon_{\text{true}}\) is the true axial (longitudinal) strain.

Consequently, if one desires to develop consistent relationships in terms of arbitrarily large engineering strain magnitudes, then the following relationship should be utilized instead of Equation (D.4) to relate transverse engineering strain to axial (longitudinal) engineering strain:

\[
(1 + \epsilon_{t}) = (1 + \epsilon)^{-\nu}
\]

The importance of utilizing this relationship can be seen in the derivation of the true stress (the conjugate measure to the true strain), \(\sigma_{\text{true}} = S(A_o/A)\). The ratio of original and current cross-sectional areas \((A_o/A)\) can be shown to be equal to the ratio of current length to original length \((l/l_o)\), which is equal to

\[
\frac{A_o}{A} = \frac{l}{l_o} = 1 + \epsilon
\]

assuming constant volume (incompressibility, \(\nu = 0.5\)) and small strain. If one desires a more general expression, one can assume a circular specimen cross section such that

\[
d = d_o (1 + \epsilon_{t}); \quad \text{where } \epsilon_{t} = \frac{\Delta d}{d_o}
\]

Then applying Equation (D.16) one can express the change in diameter as a function of axial strain, \(\epsilon\):

\[
d = d_o (1 + \epsilon)^{-\nu}
\]

so that the change in cross-sectional area becomes

\[
\frac{A_o}{A} = (1 + \epsilon)^{2\nu}
\]

Thus, the true stress \(\sigma_{\text{true}}\) expressed in terms of engineering stress \(S\) and engineering strain \(\epsilon\) with no limitation on the engineering strain magnitude or compressibility of the material is

\[
\sigma_{\text{true}} = S(1 + \epsilon)^{2\nu}
\]
This differs from the expression one would obtain if one used Equation (D.4) to relate the transverse strain to axial strain in the above, so that Equation (D.19) would become

\[ d = d_o (1 - ve) \]  \hspace{1cm} (D.22)

Equation (D.21) then becomes

\[ \sigma_{\text{true}} = \frac{S}{(1 - ve)^2} \]  \hspace{1cm} (D.23)

where \( S = F/A_o \). Equations (D.21) and (D.23) only agree in the case of small strains (<2 to 3 percent). Both Equation (D.21), simplified to incompressible material (\( v = 0.5 \)) (see Dowling, 1999), and Equation (D.23) (see Lamaitre and Chaboche, 1990), have been documented in the literature; yet Equation (D.21) is the only valid expression for all magnitudes of engineering strains.

Revisiting the derivation of instantaneous Poisson’s ratio (see Eq. (D.7)), given Equation (D.16) the current unit volume \((\delta + 1)\) can be written as

\[ \delta + 1 = (1 + e)^{-v} (1 + e)^{-v} (1 + e) \]  \hspace{1cm} (D.24a)

\[ \ln(\delta + 1) = \ln(1 + e)^{1-2v} \]  \hspace{1cm} (D.24b)

so that

\[ v = \frac{1}{2} \frac{\ln(1+\delta)}{2\ln(1+e)} \]  \hspace{1cm} (D.25)

thus demonstrating that the instantaneous Poisson’s ratio will saturate at 0.5, given an isotropic material, when incompressibility (inelastic strain) is active and large strain magnitudes are present. This is in sharp contrast to Equation (D.9) results, which are based solely on the small strain assumption that utilizes the assumption of proportionality to transverse and longitudinal engineering strains given in Equation (D.4). Figure D.6 illustrates these results up to an axial strain of 0.3. Once again, it is evident that strain magnitudes above 2 percent result in significant differences between instantaneous Poisson’s ratio calculations from Equations (D.6), (D.9), and (D.25). Note \( \ln(1+\delta) \) is a measure of dilatation since it equals the trace of the true-strain tensor; whereas unless strains remain “small” so that all higher order terms are eliminated, Equation (D.8) does not equal the trace of the engineering strain tensor. Also for the case of damage in which compressible behavior is induced it is clear that the instantaneous Poisson’s ratio will be less than 0.5.

Utilizing Equations (D.9) and (D.25), which were derived based on the assumption of isotropic material behavior, the resulting instantaneous Poisson’s ratio (given the measured strain \( e = e_i \)), see grey and orange curve, respectively in Figure D.7) significantly underpredicts the rapid rise of the measured Poisson’s ratio at 427 °C for sample 69 (see green curve in Figure D.7). Although in the case of Equation (D.9) the general shape of the curve is similar to that measured.
Figure D.6.—Influence of large strain magnitude on instantaneous Poisson’s ratio. Purple curve is the error between Equation (D.9), which incorporates higher order terms, and Equations (D.6) and (D.25).

Figure D.7.—Experimentally measured tensile response of Ti-6-4 at 427 °C, specimen 69; measured (exp) Poisson’s ratio $\nu_{yx}$ and computed instantaneous Poisson’s ratios assuming isotropic (iso) and anisotropic (aniso) behavior of the material, are shown. Ultimate tensile stress (UTS) is indicated.
Now assuming that the material is anisotropic and that proportionality factors $\psi$ are small (i.e., $\psi_{yx} = -e_y/e_x$, $\psi_{zx} = -e_z/e_x$, and $e = e_x$), the total dilatation of Equation (D.7) becomes

$$\delta + 1 = (1 + e)(1 - \psi_{yx} e)(1 - \psi_{zx} e)$$  \hspace{1cm} (D.26)

where small strain proportionality factors are assumed. Solving Equation (D.26) for the two instantaneous Poisson’s ratios yields the following two expressions:

$$\psi_{yx} = \left[ 1 - \frac{(1 + \delta)}{(1 - \psi_{yx} e)(1 + e)} \right] \frac{1}{e}$$  \hspace{1cm} (D.27)

and

$$\psi_{zx} = \left[ 1 - \frac{(1 + \delta)}{(1 - \psi_{zx} e)(1 + e)} \right] \frac{1}{e}$$  \hspace{1cm} (D.28)

where in the case of stress-controlled loading (e.g., creep test) the initial values of $\psi_{yx}$ and $\psi_{zx}$ correspond directly to Poisson’s values (i.e., $\psi_{yx} = \nu_{yx}$ and $\psi_{zx} = \nu_{zx}$); whereas in the case of strain-controlled loading (e.g., tension test) the initial values of $\psi_{yx}$ and $\psi_{zx}$ are significantly more complicated expressions that combine all three Poisson’s ratios and two of the three stiffnesses:

$$\psi_{yx} = \frac{E_x}{E_y} \left[ \frac{(1 - \nu_{yx}^2)(v_{xy} + v_{xz}v_{yz}) - (v_{xz} + v_{xy}v_{yz})(v_{yz} + v_{xy}v_{xz})}{(v_{yz} + v_{xy}v_{xz})^2 - (1 - \nu_{xy}^2)(1 - \nu_{xz}^2)} \right]$$  \hspace{1cm} (D.29)

and

$$\psi_{zx} = \frac{E_x}{E_z} \left[ \frac{(1 - \nu_{zx}^2)(v_{zx} + v_{xy}v_{yz}) - (v_{yz} + v_{xy}v_{xz})(v_{yz} + v_{xy}v_{xz})}{(v_{yz} + v_{xy}v_{xz})^2 - (1 - \nu_{xy}^2)(1 - \nu_{xz}^2)} \right]$$  \hspace{1cm} (D.30)

Similarly, using the large-strain Poisson’s ratio relationship (Eq. (D.16)), the anisotropic form of Equation (D.24b) becomes

$$1 + \delta = (1 + e)^{-\psi_{yx}} (1 + e)^{\psi_{zx}} (1 + e)$$  \hspace{1cm} (D.31)

where the expression for the instantaneous anisotropic Poisson’s ratios become

$$\psi_{yx} = 1 - \psi_{zx} - \frac{\ln(1 + \delta)}{\ln(1 + e)}$$  \hspace{1cm} (D.32)

$$\psi_{zx} = 1 - \psi_{yx} - \frac{\ln(1 + \delta)}{\ln(1 + e)}$$  \hspace{1cm} (D.33)
Employing Equation (D.27), the instantaneous Poisson’s ratio associated with the measured transverse Poisson’s ratio resulting from a strain-controlled tension test can simulate very accurately the rapid rise and subsequent decrease in the measured Poisson’s ratio up to the ultimate tensile strength (UTS) (see the solid black curve in Figure D.7). This curve was obtained by assuming that the initial Poisson’s ratios take on the following values, $\psi_{yx}^E = 0.375$ and $\psi_{xx}^E = 0.31$, and that the volume dilatation (Eq. (D.26)) only includes elastic strain up to yielding. Similarly, Equation (D.32), represented by the solid blue line in Figure D.7, accurately captures the rapid rise of the measured instantaneous Poisson’s ratio, but since it consistently incorporates large strains in its formulation it saturates to a fixed value of $1 - \psi_{xx}^E$ as the applied strain becomes large. In this case, the initial instantaneous Poisson’s ratios take on the following values, $\psi_{yx}^E = 0.375$ and $\psi_{xx}^E = 0.35$, with the elastic strain taken to be the same as before.

Although, these two sets of initial instantaneous Poisson’s ratio values have not been confirmed experimentally, Ti-6-4 was shown to possess texture variation in the three directions (rolling, short-transverse, and long-transverse directions) that are consistent with orthotropy; also, dynamic moduli measurements at various temperatures confirmed that at least transverse isotropy exists (see Lerch and Arnold, 2014). Consequently, it is believed that the observed initial rapid increase in instantaneous Poisson’s ratio to values greater than 0.5 is indicative of and attributable to material anisotropy. The next question is, “What is driving the more rapid decrease in the experimental Poisson’s ratio (green curve) relative to the simulated (black or blue curve) values in Figure D.7 after attaining the UTS (in the case of the black curve) or 5 percent strain in the case of Equation (D.32) (the blue curve).

To answer this, it is important to remember that the measured total strain $e$ in Figure D.7 includes contributions from the elastic, inelastic, and damage strains. Therefore, the strain used in Equations (D.27) and (D.32) contained damage while the dilatation, $\delta + 1$, contained only the elastic strain, implying that the damage strain was incompressible. As mentioned previously, damage strain typically will induce dilatation and thus should be considered compressible. If damage is not included in the dilatation equation (Eqs. (D.26) or (D.31)) it will cause the instantaneous Poisson’s ratio to be higher than it should be, as previously illustrated in Figure D.4. To include damage in the dilatation equation one clearly needs to know the amount of damage at any given point along the stress-strain curve. A measure of the damage strain $e^D$ that occurred during the test associated with stiffness degradation can be obtained by comparing the initial loading modulus and the unloading modulus obtained at the end of the test in Figure 6(b) of the report. The engineering unloading modulus was found to be approximately 66 percent that of the initial. Therefore, if the classical uniaxial stiffness reduction expression were used, $S = E(1 - D)e$ (see Lamaitre and Chaboche, 1990), it can be shown that $D = e^D/e = 0.34$, so that $e^D = 0.065$ at unload, occurring at total strain of 0.19. Next assuming a multiplicative expression (consistent with large strain formulations) for the strain that induces dilatation,

$$e = e^E e^D$$  \hspace{1cm} (D.34)

and assuming an exponential expression for the accumulation of damage strain (see Table D.1),

$$e^D = e^{B(e - e^{\text{onset}})}$$  \hspace{1cm} (D.35)

Note the maximum elastic strain $e^E$ used for both equations was 0.005. The resulting instantaneous Poisson’s ratio in Figure D.7 (depicted by the black and blue dashed lines), in which damage is now assumed to be compressible, appears to simulate the experimental response very well. Note Equation (D.32) is valid for all strain levels and thus clearly demonstrates the need to include the compressibility...
effect of the damage strains within the dilatation to correctly reproduce the observed Poisson’s ratio response. Note that to quantitatively match the experimental response in Figure D.7 the erroneous (since the actual physical damage observed does not agree with this amount of damage) stiffness degradation associated with the engineering modulus was inappropriately employed. Remember, however, that the accuracy of the experimental measurement (and thus quantitative magnitude of damage) is questionable because of inconsistencies in the current measurements as discussed in Appendix B. Finally, for completeness Figure D.8 illustrates the difference between engineering Poisson’s ratio (one in which the experimental engineering axial and transverse strain are ratioed; green curve) and the true Poisson’s ratio (one in which the true axial and true transverse experimental strains are ratioed; red curve). Once again, Equation (D.32) is calibrated to match the experimental data both without damage (solid blue curve) and with damage (dashed blue curve). Here \( \psi_{yx}^{e} = 0.37 \) and \( \psi_{zx}^{e} = 0.325 \), the damage parameters \( \epsilon^{onset} = 0.09 \) and \( B = 25 \), and the elastic strain is taken to be the same as before.

In conclusion, the above calculations demonstrate clearly the need to account for true stress, true strain, anisotropy, compressible elastic strain, incompressible inelastic strain, and compressible damage strain (beginning at some onset strain) to accurately reproduce the experimentally observed Poisson’s ratio behavior.

<table>
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<th>Equation</th>
<th>Beginning strain, ( \epsilon^{onset} )</th>
<th>Scaling factor, ( B )</th>
</tr>
</thead>
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<tr>
<td>(D.27)</td>
<td>0.11</td>
<td>27</td>
</tr>
<tr>
<td>(D.32)</td>
<td>0.05</td>
<td>21</td>
</tr>
</tbody>
</table>

Figure D.8.—Experimentally measured tensile response of Ti-6-4 at 427 °C, specimen 69; measured (exp) Poisson’s ratios \( \nu_{yx} \), both engineering and true, and computed instantaneous Poisson’s ratios assuming isotropic (iso) and anisotropic (aniso) behavior of the material. Ultimate tensile stress (UTS) is indicated.
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