Computation of fiber orientation in X-ray micro-tomography reconstructions

Presented by Federico Semeraro
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Technical Session #1:
Micro-tomography based analysis
MOTIVATION & OBJECTIVES
Modeling Thermal Protection Systems (TPS)

Macroscale Modeling
Full scale material response solvers, using volume-averaged techniques to solve conservation equations for ablation

Microscale Modeling
Used to inform material properties and material response parameters used in macro-scale modeling

Simulation of surface temperature for MSL heatshield*1

Lachaud and Mansour, JTHT 2013

Porous Microstructure Analysis (PuMA)


CT Reconstruction of FiberForm
Challenges in Micro-scale modeling

As NASA moves towards woven TPS materials, our modeling must adapt.
Objectives

Computation of Effective Material Properties

Fiber Orientation Estimation*¹

Physical and Numerical Model*²


FIBER ORIENTATION METHODS
Overview

• Ray Casting (novel)
• Artificial Flux*¹
• Structure Tensor*²


Ray Casting

\[ \theta \in [0, 180^\circ) \quad \phi \in [0, 360^\circ) \]

\[ N = \left( \frac{180^\circ}{d\psi} - 1 \right) \left( \frac{360^\circ}{d\psi} \right) + 2 \]
Artificial Flux

\[ T_{i+1/2} = \frac{k_{i+1}}{k_i + k_{i+1}} T_{i+1} + \frac{k_i}{k_i + k_{i+1}} T_i \]
Structure Tensor

4 Steps:

1. \[ \nabla I_{\sigma}(x) = \nabla(\sigma \ast I(x)) \]

2. \[ \nabla I_{\sigma} \nabla I_{\sigma}^T = \begin{pmatrix} I_x^2 & I_x I_y & I_x I_z \\ I_x I_y & I_y^2 & I_y I_z \\ I_x I_z & I_y I_z & I_z^2 \end{pmatrix} \]

3. \[ J_\rho(x) = \rho \ast (\nabla I_{\sigma} \nabla I_{\sigma}^T) \]

4. Local orientation vector \( v \) is the eigenvector related to the smallest eigenvalue of \( J_\rho(x) \)

\[ (I(x + v)I(x))(x))^2 \approx 0 \]
Conductivity Tensor Rotation

\[
v = v_x i + v_y j + v_z k
\]

\[
k'' = \begin{bmatrix}
  k^{\text{Long.}} & 0 & 0 \\
  0 & k^{\text{Trans.}} & 0 \\
  0 & 0 & k^{\text{Trans.}}
\end{bmatrix}
\]

\[
\theta = \arcsin v_z \quad \phi = \arctan \frac{v_y}{v_x}
\]

\[
q = \left( R^{-1} k'' R \right) \nabla T
\]

\[
R = \begin{bmatrix}
  \cos \theta & 0 & -\sin \theta \\
  0 & 1 & 0 \\
  \sin \theta & 0 & \cos \theta
\end{bmatrix} \begin{bmatrix}
  \cos \phi & \sin \phi & 0 \\
  -\sin \phi & \cos \phi & 0 \\
  0 & 0 & 1
\end{bmatrix}
\]
APPLICATION TO MATERIALS
Parametric Study

Artificial Straight Fibers  2D Weave  3D Weave

Methods’ Inputs

Ray Casting (RC)
  Ray angle separation $d\psi$

Artificial Flux (AF)
  Solver Tolerance

Structure tensor (ST)
  Kernel window sizes:
  1. $\sigma$
  2. $\rho$

Methods’ Performance

Mean Angular Error

$$\mu_E = \sum_{n=1}^{N_{solid}} \frac{\alpha_n(x)}{N_{solid}}$$
Results on Artificial Fibrous Samples

<table>
<thead>
<tr>
<th>Resolution (vox)</th>
<th>$\mu_{E,ST}(^\circ)$</th>
<th>$\mu_{E,AF}(^\circ)$</th>
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</thead>
<tbody>
<tr>
<td>$200^3$</td>
<td>20</td>
<td>17</td>
</tr>
<tr>
<td>$400^3$</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>$600^3$</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>$800^3$</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

Increasing fiber radius

Structure Tensor (ST)  Artificial Flux (AF)  Ray Casting (RC)
Results on Real Fibrous Samples

FiberForm\textsuperscript{TM} 800\textsuperscript{3} voxels
\[ \mu_\alpha = 21.6^\circ \]

Morgan Felt 800\textsuperscript{3} voxels
\[ \mu_\alpha = 8.5^\circ \]
Results on Artificial Woven Samples

<table>
<thead>
<tr>
<th>Design</th>
<th>Resolution (vox)</th>
<th>$\mu_{E,ST}$ (°)</th>
<th>$\mu_{E,AF}$ (°)</th>
<th>$\mu_{E,RC}$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
<td>200$^2 \times 50$</td>
<td>4.9</td>
<td>14.0</td>
<td>16.5</td>
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<tr>
<td>Tows</td>
<td>400$^2 \times 100$</td>
<td>4.6</td>
<td>12.5</td>
<td>17.6</td>
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<tr>
<td>Circular</td>
<td>600$^2 \times 150$</td>
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<td>11.8</td>
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<td>Tows</td>
<td>800$^2 \times 200$</td>
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<tr>
<td>Elliptical</td>
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<tr>
<td>Tows</td>
<td>400 $\times 800 \times 160$</td>
<td>25.9</td>
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<tr>
<td>Elliptical</td>
<td>600 $\times 1200 \times 240$</td>
<td>25.9</td>
<td>15.6</td>
<td></td>
</tr>
<tr>
<td>Tows</td>
<td>800 $\times 1600 \times 320$</td>
<td>26.5</td>
<td></td>
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</tbody>
</table>

Tow Curvature and Contact

2D Weave

3D Weave

Structure

Artificial Flux (AF)

Ray Casting (RC)
Improved Workflow for Weave Orientation

Individual tow segmentation

Mean Filtering

Artificial Flux (AF) $\mu_{E,AF} = 2.3^\circ$

Structure Tensor (ST) $\mu_{E,ST} = 1.7^\circ$
Real Woven Sample*1

Summary

1. Ray Casting:
   - Performs well on artificial straight fibers ($\mu \sim 3 - 5^\circ$) and similar to other methods on binarized woven structures ($\mu \sim 10 - 15^\circ$). Slight improvement in new workflow ($\mu \sim 9^\circ$)
   - **Limitation:** affected by large fiber curvatures and computational expensive

2. Artificial Flux:
   - Easy to use because independent on inputs. Performs similar to other methods on binarized weaves ($\mu \sim 15^\circ$). Very accurate when using new workflow for woven materials ($\mu \sim 1 - 3^\circ$)
   - **Limitation:** Performs poorly on artificial straight fibers due to regions not being in the path of heat flux through the material ($\mu \sim 15 - 20^\circ$)

3. Structure Tensor:
   - Performs effectively on artificial straight fibers ($\mu \sim 1 - 5^\circ$) and similar to other methods on binarized weaves ($\mu \sim 20^\circ$). Very accurate when using new workflow for woven materials ($\mu \sim 1 - 2^\circ$)
   - **Limitation:** hard to define optimal window a priori. For high resolutions, window must be sufficiently large, which can be very expensive