THE EFFECT OF YIELD STRENGTH
AND DUCTILITY TO FATIGUE DAMAGE

by
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ABSTRACT

The cumulative damage of aluminium alloys with different yield strength and various ductility due to seismic loads has been studied. The responses of an idealized beam with a centered mass at one end and fixed at the other end to El Centro's and Taft's earthquakes are computed by assuming that the alloys are perfectly elastoplastic materials and by using numerical technique. Consequently, the corresponding residual plastic strain can be obtained from the stress-strain relationship. The revised Palmgren-Miner cumulative damage theorem is utilized to calculate the fatigue damage. The numerical results show that in certain cases, the high ductility materials are more resistant to seismic loads than the high yield strength materials. The results also show that if a structure collapse during the earthquake, the collapse always occurs in the very early stage.
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INTRODUCTION

There are two different approaches to structural design. One is based on the concept of allowable stress and elastic behavior of materials and the other on the concept of ultimate load and inelastic behavior. Since the structure designed by using plastic theory is more economic than those by applying elastic theory, plastic design concept has been gradually accepted by most engineers and has been widely applied in constructions.

If the external forces are large enough, the plastic-design oriented structures, undoubtedly, will undergo plastic deformation and create residual plastic strain. The residual plastic strain will gradually accumulate if the structure is subjected to dynamic loads such as wind force, seismic loads, etc. If the cumulative plastic strain reaches the critical values, the structures will collapse. This process is usually termed low cycle fatigue failure. Hence the study of low cycle fatigue failure is of importance to the structures designed by using the concept of plastic theory.

There are many papers dealing with inelastic behavior of materials (2, 9, 10, 12, 13) and the fatigue damage of structures under the action of seismic loads. (4, 5, 6, 10). Few of them have stressed the importance of the comparison of different fatigue life among the same materials with different yield strength and ductility.

*The numerals in parenthese refer to the list of references.*
Since low cycle fatigue damage can be expressed in terms of plastic strain, the fatigue life of structural systems can always be improved by choosing the materials with high ductility. However, high ductile materials generally have low yield strength which will cause the larger deformation than the materials with high yield strength. It becomes a complicated problem to choose material for structural members to resist dynamic loading. Based on the given random behavior of plastic deformation, a method for evaluation of alternative materials to be used in structures subjected to random White Noise excitation was presented (15). Since there is not any white noise excitation in reality, it is desirable to study the fatigue damage of materials subjected to earthquake load in more details.

The purpose of this investigation is to study the fatigue damage of materials with different yield strength and various ductility due to seismic loads. The revised Palmgren-Miner cumulative damage theorem expressed in terms of plastic strain is used to compute the fatigue damage. As an example, aluminium alloys (5052 and 3030 groups) are considered on an ideal beam, subjected to 1948 El Centro's and 1951 Taft's earthquakes.
LOW CYCLE FATIGUE DAMAGE MODEL

In computing the fatigue damage, the Palmgren-Miner criterion (8), because of its simplicity, has been most widely used during the past decades. This theorem can be mathematically expressed as follows:

$$\sum_{i=1}^{N} d_i = D$$

Where $d_i$ is the fatigue damage during its cycle, and $D$ is the total cumulative damage under $N$ cycles. The theorem states that the fatigue failure will occur if the total damage $D$ reaches some critical level, which depends on material properties.

At present, this study is concentrated on the low cycle fatigue life which usually is referred to the case with life less than $10^5$ cycles. If a structure is supposed to collapse within such a limited cycle, the magnitude of cyclic loading may be large enough to cause the plastic strain in the materials. Hence the Palmgren-Miner criterion for low cycle fatigue life was developed to associate with plastic strain. Manson and Gross et. al. (3, 7) proposed the following low cycle fatigue life criterion by introducing plastic strain for predicting the total number of cycles of reversed-strain to cause fatigue failure:

$$N^m (\Delta e_t) = C$$

Where $\Delta e_t$ is the plastic tensil strain, $m$ is a constant depending on material properties. $N$ is the fatigue life and $C$ is
some constant. Later Yao and Munse (14) suggested that, for uniaxially loaded metal, the cumulative damage can be expressed in terms of material ductility and plastic strains as follows:

\[ \sum_{i=1}^{N} \frac{\sum (\Delta e_t)}{e_f} \frac{1}{m} = 1 \]  \hspace{1cm} <3>

Where \( e_f \) is the ultimate plastic strain and is constant for a given material, \( \Delta e_t \) is the plastic strain at tensile cycle and \( m \) is material constant dependent on ambient and loading rate. Usually \( m \) is defined as function of plastic strain in tensile and compressive cycles, denoted by \( e_c \) and \( e_t \), respectively.

\[ \frac{1}{m} = 1 - 0.086 \left( \frac{\Delta e_t}{\Delta e_c} \right) \]  \hspace{1cm} <4>

The cyclic history of plastic strains in tensile and compressive cycles is illustrated in Figure 1. Equation <3> and <4> will be considered as the fatigue damage criterion through this investigation.

Figure 1. Cyclic History of Plastic Strain
Consider a beam with a centered mass at one end and fixed at the other end subjected to a fluctuating excitation as shown in Figure 2.

![Figure 2. Mechanical Model](image)

The equation of motion of this spring-dash-pot system is well known.

\[ M \ddot{x} + C \dot{x} + G(x) = M \ddot{y} \]  

Where \( M \) is the concentrated mass of the beam.

\( C \) is the damping of the mechanical system.

\( x \) is the deformation of the beam.

\( \ddot{y} \) is the accelerogram of seismic loads.

\( G(x) \) is the forced-deformation function dependent on the stress-strain curve.

The force-deformation function, \( G(x) \), can be derived from the stress-strain relationship of a given material. For simplifying the computation, the aluminium alloys considered here are assumed to be elasto-plastic materials. The stress-strain diagram of such a material is shown in Figure 3.
Let $A$ be the cross-sectional area of the beam.

$E$ be the modulus of elasticity.

$L$ be the length of beam.

$e_r$ be the residual strain.

$e_y$ be the yielding strain.

$f_{yt}$ be the yielding tensile stress.

$f_{yc}$ be the compressive yielding stress.

If $f_{yt} = f_{yc} = f_y$, the force-deformation function can be formulated as follows:

$$G(x) = EA \left( \frac{x}{L} - e_r \right) \quad \text{if} \quad \left| \frac{x}{L} - e_r \right| < |e_y|$$

$$G(x) = A_f y \quad \text{if} \quad \left| \frac{x}{L} - e_r \right| > |e_y|$$  \hspace{1cm} <6>$$

Since the analytical solution of equation (5) is not available, Runge-Kutta Numerical integration was used to obtain the deformation, $x$. From the known deformation and stress-strain curve, the cyclic history of plastic strain can be found, then the fatigue damage can be calculated from equation <3>.
NUMERICAL EXAMPLES

In order to illustrate the low cycle fatigue damage of structures subjected to dynamic loads, several aluminium alloys are studied. The mechanical properties of these materials are tabulated in Table (1).

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Yield strength ksi</th>
<th>Ductility %</th>
<th>Modulus of Elasticity $10^3$ ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alclad 3003 - H12</td>
<td>18</td>
<td>20</td>
<td>10.0</td>
</tr>
<tr>
<td>Alclad 3003 - H14</td>
<td>21</td>
<td>16</td>
<td>10.0</td>
</tr>
<tr>
<td>Alclad 3003 - H16</td>
<td>25</td>
<td>14</td>
<td>10.0</td>
</tr>
<tr>
<td>Alclad 3003 - H18</td>
<td>27</td>
<td>10</td>
<td>10.0</td>
</tr>
<tr>
<td>5052 - H32</td>
<td>28</td>
<td>18</td>
<td>10.0</td>
</tr>
<tr>
<td>5052 - H34</td>
<td>31</td>
<td>14</td>
<td>10.0</td>
</tr>
<tr>
<td>5052 - H36</td>
<td>35</td>
<td>10</td>
<td>10.0</td>
</tr>
<tr>
<td>5052 - H38</td>
<td>37</td>
<td>8</td>
<td>10.0</td>
</tr>
</tbody>
</table>

Table 1. Mechanical Properties of Aluminium Alloys

Since the natural frequency can not be well defined if the structural systems allow to undergo plastic deformation, the terminology of "artificial frequency" is adopted. Let $p_a$ be
the artificial frequency which is defined as \( \sqrt{\frac{AE}{ML}} \). If both sides of equation (5) are divided by the mass \( M \), it becomes

\[
\ddot{x} + 2c p_a \dot{x} + g(x) = \ddot{y}
\]

Where \( c \) is the damping ratio, \( p_a \) is the artificial frequency and \( g(x) \) is \( \frac{G(x)}{M} \). It is to be noted that \( g(x) \) can be reformulated in terms of artificial frequency.

In order to make the comparison, two seismic loads are utilized in this study. One is 1948 El Centro's earthquake NS component with maximum peak in the order of 0.312g, the other is 1951 Taft's earthquake S69E component with a peak value in the order of 0.157g, where \( g \) denotes the gravitational force.

Because of the high peak value of El Centro's earthquake, the artificial frequency of the mechanical system varies from 50 rad/sec to 70 rad/sec, and for the case of Taft's earthquake, the frequency varies from 35 rad/sec to 45 rad/sec. With the length of beam, \( L \), being 10 inches and the information shown in Table 1, the fatigue damages of aluminium alloys subjected to El Centro's and Taft's earthquakes are computed by the aid of 360/65 IBM computer. The computer program is listed in Appendix. The results are tabulated in Tables 2, 3, and 4, and also plotted in Figures 4, 5, and 6.
<table>
<thead>
<tr>
<th>Cumulative damage</th>
<th>5052 - H32</th>
<th>5052 - H34</th>
<th>5052 - H36</th>
<th>5052 - H38</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>2.9920149</td>
<td>3.1137094</td>
<td>2.5031681</td>
<td>3.9078865</td>
</tr>
<tr>
<td>52.5</td>
<td>1.5593367</td>
<td>1.5319510</td>
<td>1.7257175</td>
<td>1.8590279</td>
</tr>
<tr>
<td>55</td>
<td>0.97939056</td>
<td>1.1105976</td>
<td>1.4277601</td>
<td>1.2899484</td>
</tr>
<tr>
<td>57.5</td>
<td>0.85163534</td>
<td>0.96612877</td>
<td>1.1536770</td>
<td>0.82214361</td>
</tr>
<tr>
<td>60</td>
<td>0.62915808</td>
<td>0.66661555</td>
<td>0.44669420</td>
<td>0.58149391</td>
</tr>
<tr>
<td>62.5</td>
<td>0.32020706</td>
<td>0.32498884</td>
<td>0.4687804</td>
<td>0.31088388</td>
</tr>
<tr>
<td>65</td>
<td>0.32253897</td>
<td>0.32985848</td>
<td>0.14389163</td>
<td>0.1470437</td>
</tr>
<tr>
<td>67.5</td>
<td>0.25728476</td>
<td>0.17950523</td>
<td>0.13773221</td>
<td>0.13794059</td>
</tr>
<tr>
<td>70</td>
<td>0.12210464</td>
<td>0.090874553</td>
<td>0.06572252</td>
<td>0.06312008</td>
</tr>
</tbody>
</table>

Table 2 Fatigue Damages of 5052 Group Alloys Subjected to El Centro's Earthquake
<table>
<thead>
<tr>
<th>Cumulative damage Frequency</th>
<th>Alclad 3003 - H12</th>
<th>Alclad 3003 - H14</th>
<th>Alclad 3003 - H16</th>
<th>Alclad 3003 - H18</th>
</tr>
</thead>
<tbody>
<tr>
<td>52.5</td>
<td>3.9831467</td>
<td>4.1246414</td>
<td>4.5256672</td>
<td>4.8264341</td>
</tr>
<tr>
<td>55</td>
<td>2.9735933</td>
<td>3.6988564</td>
<td>1.9604654</td>
<td>2.4367723</td>
</tr>
<tr>
<td>57.5</td>
<td>2.9497690</td>
<td>2.8602915</td>
<td>1.5088053</td>
<td>2.1295109</td>
</tr>
<tr>
<td>60</td>
<td>2.5558205</td>
<td>1.7559586</td>
<td>1.2991552</td>
<td>1.6246128</td>
</tr>
<tr>
<td>62.5</td>
<td>1.9535789</td>
<td>1.6068087</td>
<td>1.0175724</td>
<td>0.6890013</td>
</tr>
<tr>
<td>65</td>
<td>1.8724232</td>
<td>0.9148501</td>
<td>0.67170912</td>
<td>0.51194036</td>
</tr>
<tr>
<td>67.5</td>
<td>1.2662811</td>
<td>0.7284845</td>
<td>0.52865228</td>
<td>0.48792696</td>
</tr>
</tbody>
</table>

Table 3 Fatigue Damages of 3003 Group Alloys Subjected to El Centro's Earthquake
<table>
<thead>
<tr>
<th>Cumulative damage</th>
<th>5052-H32</th>
<th>5052-H34</th>
<th>5052-H36</th>
<th>5052-H38</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>18</td>
<td>14</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>35</td>
<td>1.9330864</td>
<td>2.2903147</td>
<td>2.4820929</td>
<td>2.7911043</td>
</tr>
<tr>
<td>37.5</td>
<td>1.6505632</td>
<td>1.3604631</td>
<td>1.720997</td>
<td>1.5659914</td>
</tr>
<tr>
<td>40</td>
<td>1.0455103</td>
<td>0.99359804</td>
<td>0.7776531</td>
<td>0.68719399</td>
</tr>
<tr>
<td>42.5</td>
<td>0.63108575</td>
<td>0.43125600</td>
<td>0.32834977</td>
<td>0.31635483</td>
</tr>
<tr>
<td>45</td>
<td>0.31469911</td>
<td>0.18018502</td>
<td>0.14462858</td>
<td>0.14523304</td>
</tr>
</tbody>
</table>

*Table 4 Fatigue Damages of 5052 Group Alloys Subjected to Taft's Earthquake*
Figure 4. Fatigue Damages of 5052 group alloys due to El Centro's Earthquake
Figure 5. Fatigue Damage of 3003 Group Alloys due to El Centro's Earthquake
Figure 6. Fatigue Damage of 5052 Group Alloys due to Taff's Earthquake.
DISCUSSION AND CONCLUSION

For Table 2, 3, and 4 or Figures 4, 5, 6, they show that in general, the fatigue damage will increase as the stiffness of the beam decreases. However, in some cases, the fatigue damages of higher stiffness member are smaller than that of lower stiffness member as it can be seen in frequencies 62.5 rad/sec and 65.0 rad/sec for alloys 5052-H32, and 5052-H36. This phenomenon is due to the fact that the frequency of the beam coincide with the frequency of seismic load.

Since a structural system is usually defined as "collapse" if the cumulative fatigue damage reaches one, the discussion here will concentrate on those materials whose cumulative damage smaller than one. Under Taft's earthquake, the cumulative damages increase as the yield strength of alloys 5052 group decreases regardless the ductility of materials. But it is not the case for the same materials subjected to El Centro's earthquake. The results show that in certain frequency domain, the material with high ductility is stronger to resist seismic load than the material with high yield strength. For easy comparison, the cumulative damage historigrams are plotted versus time as shown in Figure 7 and 8 for alloys 5052 group in the frequencies of 60.0 rad/sec and 62.5 rad/sec.

The damage historigrams are also plotted for alloys 5052 subjected
to Taft's earthquake in some frequency domain. It is very interesting to note that the cumulative damages pile up very fast in the early stage of earthquake action as shown in Figures 7, 8, 9, and 10. In other words, fatigue damage is always created during the first 5-second period for the 30-second duration El Centro's earthquake, and the first 12-second period for 60-second duration Taft's earthquake. It can be concluded that if a structure can survive during the first quarter period of earthquake action, it surely will not collapse and under this earthquake. On the contrary, it can be said that if a structure collapses during the earthquake, the failure always occurs in the very early stage.
Figure 7. Fatigue Damage Histogram for 5052 Group Alloy's in the Frequency of 60 rad/sec Due to El Centro's Earthquake.
Figure 9. Fatigue Damage Histogram for 5052 - H38 Alloy Due to Taft's Earthquake.
Figure 10. Fatigue Damage Histogram For 5052 Group Alloys in the Frequency of 42.5 rad/sec due to Taft's Earthquake.
ACKNOWLEDGEMENT

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REFERENCES


DIMENSION PRMT(5), Y(2), DERY(2), AUX(8,2), YDIS1(3000)
  S(500), AC(500), ACCEL(3000), XT(3000), YDIS2(3000)
  OMEGA(2), ETA(2), DULTY(2), YEYLD(2), CON(2), Q(2), YDMIN(2), YDMAX(2)
  QMAX(2), QMIN(2), YMAX(2), YMIN(2), YEYLD(2), TL(2)
  DAMGC(2), DAMGV(2), K(2), KT1(3000), KT2(3000)
COMMON/VI/ACCEL, OMEGA, ETA
COMMON/VP/YMAX, YMIN, YEYLD, QMAX, QMIN, Q, SK1, SK2, QME
COMMON/VO/DULTY, TL, NL
COMMON/VO/YDIS1, YDIS2, YDMIN, YDMAX, NT, XT, DMU, TT, APH, BPM, CPM
EXTERNAL FCT, OUTP
READ(5,905), TEN, DEL
905 FORMAT(15,F10.5)
READ(5,906), (PRMT(I), I=1,5)
906 FORMAT(5F10.5)
READ(5,907), (I( ), AC(I ), I=1,15)
907 FORMAT(4(F6.2,F12.7))
J=2
IR=0
IFN1=IFN+1
S(IFN1)=PRMT(2)
AC(IFN1)=0.
DO 110 I=1,15
   DI=(S(I+1)-S(I))/DEL
   ID=DI
   IID=ID+0.5
   IF (IID-ID) 80,90,100
60 ID=ID+1
GO TO 90
100 WRITE(6,904)
904 FORMAT(2X,12HERROR IN AC,I3)
90 IR=IR+1
ACCEL(J-1)=AC(J-1)*0.3864
120 CONTINUE
ACCEL(J)=ACCEL(J-1)+(AC(J)+1)-AC(J))*0.3864/DI
110 DO 20 J=1,IR
20 CONTINUE
READ(5,907) APH, BPM, CPM
907 FORMAT(3F10.6)
READ(5,908) N
NDIM=N*2
900 FORMAT(15)
WRITE(6,999)
999 FORMAT(1H1)
0F 20

MAIN

DATE = 72215 17/10/44

31 READ(5,922) (TL(I),I=1,N)

READ(5,908) TLMU,LCHECK

908 FORMAT(F10.5,15)

DO 20 I = 1,N

READ(5,901) (OMGA(I),ETA(I),DULTY(I),EYELD(I),CON(I))

901 FORMAT(5F10.5)

YELD(I)=EYELD(I)*TL(I)

Y(I)=EYELD(I)*TL(I)

Q(I)=OMGA(I)**2*EYELD(I)

SK1(I)=OMGA(I)**2

SK2(I)=SK1(I)*CON(I)

QM(I)=OMGA(I)**2

QMIN(I)=-QM(I)

YMAX(I)=YELD(I)

YMIN(I)=-YELD(I)

WRITE(6,923) (OMGA(I),ETA(I),DULTY(I),EYELD(I),CON(I),TL(I))

923 FORMAT(2X,10H FREQUENCY F10.5,17H DAMPING RATIO F10.5,11H DUCTILITY F10.5,11H YIELDING F10.5,13H STIFFNESS 2 F10.5,3H L 10.5)

CONTINUE

20 CONTINUE

DIM=NDIM

DO 10 I = 1,NDIM

Y(I)=0.

DERY(I)=1./DIM

CONTINUE

10 CONTINUE

DMU=DEL*TLMU

TT=0.

NT=1

CALL RKGS(PRMT,Y,DERY,NDIM,HLF,FCT,OUTP,AUX)

NL=NT

CALL DAMAGE(YDIS1,YELD(I),DAMGC(1),DAMGV(1),K(1),KTI1,TL(1),

DULTY(I),NL,KT1,1).

CALL DAMAGE(YDIS2,YELD(2),DAMGC(2),DAMGV(2),K(2),KTI2,TL(2),

DULTY(2),NL,KT2,2)

TEND1=XT(KTI1)

TEND2=XT(KTI2)

WRITE(6,800) TEND1,TEND2

800 FORMAT(2X,10H FREQUENCY F10.5,10X,10H DAMPING RATIO F10.5,11H DUCTILITY F10.5,11H YIELDING F10.5,13H STIFFNESS 2 F10.5,3H L 10.5)

DO 30 I = 1,N

WRITE(6,921) N, YMAX(I),YMIN(I)

30 WRITE(6,920) N,DAMGC(1),DAMGV(1),K(I)

920 FORMAT(10X,13,6H STORY,16H DAMAGE FACTOR 1P2E16.7,20H NUMBER OF

1 CYCLES 17//)

921 FORMAT(10X,13,6H STORY,19H MAX DISPLACEMENT 1PE16.7,19H MIN DISP

1 LACEMENT 1PE16.7//)

K1=K(1)
K2 = K2
DO 140 I = 1, K1
   J = K1(I)
   WRITE (6, 940) I, XT(I)
140      DO 150 I = 1, K2
   J = K2(I)
   WRITE (6, 950) I, XT(I)
940      FORMAT (2X, 12HNO OF CYCLES 15, 2X, 5HTIME =1PE16.7, 2X, 2H01)
950      FORMAT (2X, 12HNO OF CYCLES 15, 2X, 5HTIME =1PE16.7, 2X, 2H02)
IF (LCHECK) 31, 32, 31
32      CONTINUE
STOP
END

C
SIMPLETIF RKG(S(PHMT, Y, DFRY, NDM, IHLF, FCT, OUTP, AUX)
C
DIMENSION Y(7), DERY(2), AUX(8, 2), A(4), B(4), C(4), PRMT(5)
DO 1 I = 1, NDM
1   AUX(I, 1) = 0.6666667*DERY(I)
      Y = PRMT(1)
      XFNM = PRMT(2)
      H = PRMT(3)
      PRT(5) = 0.
      CALL FCT(X, Y, DFRY)
C
C   FOR TEST
   IF (H*(XEND-X)) 28, 27, 2
C
C   PREPARATIONS FOR RUNGE-KUTTA METHOD
2   A(1) = .5
      A(2) = 2928932
      A(3) = 1.707107
      A(4) = 1666667
      B(1) = .5
      B(2) = 1.
      B(3) = 1.
      B(4) = 2.
      C(1) = .5
      C(2) = 2928932
      C(3) = 1.707107
      C(4) = .5
C
C   PREPARATIONS OF FIRST RUNGE-KUTTA STEP
   DO 3 I = 1, NDM
      AUX(I, 11) = Y(I)
      AUX(2, 11) = DFRY(I)
      AUX(3, 11) = 0.
3   AUX(6, 11) = 0.
    TDFEC = 0
    H = H*H
    IHLF = 1
    ISTEP = 0
    IEND = 0
C
C   START OF A RUNGE-KUTTA STEP
4   IF((X+H-XEND)*H) 7, 6, 5
      H = XEND - X
5   FEND = 1
   C
C
RECORDING OF INITIAL VALUES OF THIS STEP

7 CALL OUT(X,Y,DERY,IRFC,NDIM,PRMT)
   IF(PRMT(5))40,8,40
9 ITEST=0
9 ISTEP=ISTEP+1

C

C START OF INNERMOST RUNGE-KUTTA LOOP

J=1
10 Aj=A(J)
   B=r(J)
   CJ=C(J)
   DO 11 J=1,NDIM
      H=H*DERY(J)
      R2=AJ*(R1-BJ*AUX(6,J))
      Y(J)=Y(J)+R2
      R2=R2+R2
11 AUX(6,J)=AUX(6,J)+R2-CJ*R1
   IF(J-4)12,15,15
12 J=J+1
   IF(J-3)13,14,13
13 X=X+.5*H
14 CALL FCT(X,Y,DERY)
   GOTO 10
C

C END OF INNERMOST RUNGE-KUTTA LOOP

C

C TEST OF ACCURACY

15 IF(ITEST)16,16,20
C

C IN CASE ITEST=0 THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY

16 DO 17 I=1,NDIM
17 AUX(4,I)=Y(I)
   ITEST=1
   ISTEP=ISTEP+ISTEP-2
18 XH=.5H
   X=X-H
   DO 19 I=1,NDIM
      Y(I)=AUX(I+1)
      DERY(I)=AUX(I+2)
19 AUX(4,I)=AUX(3,I)
   GOTO 9
C

C IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE

20 IMD=ISTEP/2
   IF(ISTEP-IMD-IMD)21,23,21
21 CALL FCT(X,Y,DERY)
DO 22 I=1,NDIM
AUX(5,1)=Y(I)
22 AUX(7,1)=DERY(I)
GOTO 9

C COMPUTATION OF TEST VALUE DELT
23 DELT=0.
DO 24 I=1,NDIM
24 DELT=DELT+AUX(P,I)*ABS(AUX(4,I)-Y(I))
IF(DELT-PRMT(4))28,28,25

C ERROR IS TOO GREAT
25 IF(IHLE=10)26,36,36
26 DO 27 I=1,NDIM
27 AUX(4,1)=AUX(5,1)
ISTEP=ISTEP+ISTEP-4
X=X-H
IFND=0
GOTO 18

C RESULT VALUES ARE GOOD
28 CALL FCT(X,Y,DERY)
DO 29 I=1,NDIM
AUX(1,1)=Y(I)
AUX(2,1)=DERY(I)
AUX(3,1)=AUX(6,1)
Y(I)=AUX(5,1)
29 DERY(I)=AUX(7,1)
CALL OUTP(X-H,Y,DERY,IHLE,NDIM,PRMT)
IF(PRMT(5))14,14,14
30 DO 31 I=1,NDIM
Y(I)=AUX(1,1)
31 DERY(I)=AUX(2,1)
IF(IFND)32,32,39
C INCREMENT GETS DOUBLED
32 IHLE=IHLE-1
ISTEP=ISTEP/2
H=H+H
IF(IHLE)4,33,33
33 IMOD=ISTEP/2
IF(ISTEP-IMOD-IMOD)14,34,4
34 IF(DELT-0.2*PRMT(4))135,35,4
35 IHLE=IHLE-1
ISTEP=ISTEP/2
H=H+H
GOTO 4
SUBROUTINE FCT(X,Y,DERY)

DIMENSION Y(4), DERY(4), ACCEL(3900),

1OMGA(2), ETA(2), YMAX(2), YMIN(2), YELD(2), QMAX(2), QMIN(2), Q(2), SK1(2)

CALL OUTF(X,Y,DERY, IHLF, NDIM, PRMT)

RETURN
END

C

RETURN TO CALLING PROGRAM.

CALL FCT(X,Y,DERY).

GOTO 39

CALL OUTF(X,Y,DERY, IHLF, NDIM, PRMT)

RETURN
END
SUBROUTINE DAMAGE(Y,YELD,DAMGC,DAMGV,K,KTIME,TL,DULTY,NL,KT,IDX)
DIMENSION Y(3000),DPP(3000),DPN(3000),KT(3000)
KTIME=0
DO 401 I=1,NL
  DPP(I)=0.
  DPN(I)=0.
  SK=1./TL
  YMAX=0.
  YMIN=0.
  EPN=0.
  DAMGV=0.
  DAMGC=0.
  N=1
  J=1
402 IF (ABS(Y(N))>YELD) 403,403,404
  N=N+1
  IF (N-NL) 402,402,420
403 IF (Y(N)) 405,405,406
  YYE=Y(N)-YELD
404 IF (Y(N)-YMAX) 410,410,406
405 EPP=YYE*SK
  YMAX=Y(N)
406 DPP(J)=EPP-EPN
  KT(J)=N
407 N=N+1
  IF (N-NL) 408,408,420
408 IF (Y(N)) 409,409,406
409 YYE=Y(N)+Y(N)-YMAX
  GO TO 406
410 YYN=Y(N)-(YMAX-2.*YELD)
411 IF (YYN) 411,407,407
412 EPN=YYN*SK
  YMIN=Y(N)
  IF (DPP(J)) 412,414,413
413 WRITE(6,999) 
999 FORMAT (10X,16H ERROR IN DPP(J)/)
  RETURN
414 DPN(J)=EPN-EPP
415 N=N+1
  IF (N-NL) 416,416,420
416 IF (Y(N)) 417,417,418
417 YYN=YYN+Y(N)-YMIN
  EPN=YYN*SK
  YMIN=Y(N)
  GO TO 414
418 YYE=Y(N)-YMIN-2.*YELD
IF (YYE) 415, 415, 406

419 YYN=Y(N)+YELD
GO TO 411

420 K=J
DO 422 J=1,K
IF (DPP(J)) 421,422,421
421 COES=1.0-0.86*DPP(J)/DPP(J)
DAMGV=DAMGV+(DPP(J)/DULTY)**CGES
IF (KTIME.NE.0) GO TO 423
IF (DAMGV.GE.1.) KTIME=K(J)
423 DAMGC=DAMGC+(DPP(J)/DULTY)**1.86
WRITE(6,998) DAMGC,Y(J),IDX
998 FORMAT(6,998) DAMGC=1PE16.7,2X,6DAMGV=1PE16.7,2X,12HNO. OF CYCLES
115,15,5HISTORY)
422 CONTINUE
RETURN
END

SUBROUTINE PLAS(QM,Y,YMAX,YMIN,YELD,QMAX,QMIN,Q,SK1,SK2,QME)

IF (Q) 212,201,208
201 IF (ABS(Y) 202,202,203
202 QM=SK1*Y
GO TO 216
203 IF (Y) 204,206,206
204 QM=QMIN+(Y-YMIN)*SK2
205 O=-1.
YMIN=Y
QMIN=QM
GO TO 216
206 QM=QMAX+(Y-YMAX)*SK2
207 O=1.
YMAX=Y
QMAX=QM
GO TO 216
208 IF (Y-YMAX) 209,206,206
209 YMA2=YMAX-Y-2.*YELD.
IF (YMA2) 210,211,211
210 QM=QMAX+(Y-YMAX)*SK1
GO TO 216
211 QM=QMAX-2.*QME-YMA2*SK2
GO TO 205
212 IF (Y-YMIN) 204,204,213
213 YM12=Y-YMIN-2.*YELD
IF (YM12) 214,214,215
214 QM=QMIN+(Y-YMIN)*SK1
GO TO 216
215 QM=QMIN+2.*QME+YM12*SK2
GO TO 207
216 RETURN
END
SUBROUTINE OUTP(X,Y,DERY,IHLF,NDIM,PRMT)

DIMENSION Y(4),DFRY(4),PRMT(5),YDIS1(3000),YDIS2(3000),XT(3000)
1,YDMAX(21),YDMIN(2)

COMMON/V0/YDIS1,YDIS2,YDMIN,YDMAX,NT,XT,DMU,TT,APM,BPM,CPM

N=NDIM/2
DO 212 I=1,N
J=2*I-1
IF (Y(J) 211,212,213
211 IF(YDMIN(I)-Y(J)) 212,212,214
214 YDMIN(I)=Y(J)
GO TO 212
213 IF (YDMAX(I)-Y(J)) 215,212,212
215 YDMAX(I)=Y(J)
212 CONTINUE
IF (IHLF-10) 216,216,217
217 WRITE(6,910).X,IHLF.
910 FORMAT(10X,2HX=,F10.5,10X,5MIHLF*»I3)/
RETURN
216 IF(X-XT) 227,228,226
226 IF(NT-3000) 225,226,226
225 CHEK=ABS(Y(1))
DO 219 I=2,NDIM
IF (CHEK-ABS(Y(I))) 219,219,218
218 CHEK=ABS(Y(I))
219 CONTINUE
220 IF(CHEK.EQ.0.0) RETURN
IF (CHEK-1.0E-03) 221,221,222
221 PRMT(4)=APM
RETURN
222 ICHEK=CHEK
IF (ICHEK) 223,224,223
224 CHEK=CHEK*10.
GO TO 222
223 CHEK=ICHEK
PRMT(4)=CHEK/CHEKI*BPM
RETURN
226 WRITE(6,911).X
911 FORMAT(10X,8HINT=3000,F10.5)
PRMT(5)=1.
RETURN