Euler Angles, Quaternions, and Transformation Matrices

Working Relationships

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EULER ANGLES, QUATERNIONS, AND TRANSFORMATION MATRICES.

WORKING RELATIONSHIPS

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EULER ANGLES, QUATERNIONS, AND TRANSFORMATION MATRICES - 
WORKING RELATIONSHIPS 
By D. M. Henderson 
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1.0 INTRODUCTION 

Due to the extensive use of the quaternion in the onboard Space Shuttle Computer System, considerable analysis is being performed using relationships between the quaternion, the transformation matrix, and the Euler angles. This Internal Note offers a brief mathematical development of the relationships between the Euler angles and the transformation matrix, the quaternion and the transformation matrix, and the Euler angles and the quaternion. The analysis and equations presented here apply directly to current Space Shuttle problems. Appendix A presents the twelve three-axis Euler transformation matrices as functions of the Euler angles, the equations for the quaternion as a function of the Euler angles, and the Euler angles as a function of the transformation matrix elements. 

The equations of Appendix A are a valuable reference in Shuttle analysis work and this Internal Note is the only known document where each of the twelve Euler angle to quaternion relationships are given. Appendix B presents a group of utility subroutines to accomplish the Euler-matrix, quaternion-matrix, and Euler-quaternion relationships of Appendix A.
2.0 DISCUSSION

2.1 Euler Angle Transformation Matrices

The following analysis and utility subroutines are offered to simplify computer programs when working with coordinate transformation matrices and their relationships with the Euler Angles and the Quaternions. The coordinate transformation matrices discussed here are defined using the following figure.

Figure 1. Coordinate system and Euler angles.
The transformation matrix \( M \), is defined to transform vectors in the \( \vec{x} \)-system \((\vec{x}, \vec{y}, \vec{z})\) into the original \( x \)-system \((x, y, z)\) and is given by the equation,

\[
x = M \vec{x}
\]

where

\[
x = (x, y, z) \quad \text{and} \quad \vec{x} = (\vec{x}, \vec{y}, \vec{z}).
\]

Using the right-hand rule for positive rotations, the \( M \) matrix in (1) above is constructed by the following analysis. The first rotation in Figure 1 above is about the \( x \)-axis by the amount \( \theta_1 \). The single rotation about the \( x \)-axis results in the following transformation,

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
= \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_1 & -\sin \theta_1 \\
0 & \sin \theta_1 & \cos \theta_1
\end{pmatrix}
\begin{pmatrix}
\vec{x}' \\
\vec{y}' \\
\vec{z}'
\end{pmatrix}
\]

or \( x = \vec{x}' \) in matrix form. Rotation about the \( \vec{y}' \)-axis by the amount \( \theta_2 \) yields the intermediate transformation matrix:

\[
\begin{pmatrix}
\vec{x}' \\
\vec{y}' \\
\vec{z}'
\end{pmatrix}
= \begin{pmatrix}
\cos \theta_2 & 0 & \sin \theta_2 \\
0 & 1 & 0 \\
-\sin \theta_2 & 0 & \cos \theta_2
\end{pmatrix}
\begin{pmatrix}
\vec{x}'' \\
\vec{y}'' \\
\vec{z}''
\end{pmatrix}
\]

or \( \vec{x}'' = Y \vec{x}' \) in matrix form. Finally rotation about the \( \vec{z}'' \)-axis by the amount \( \theta_3 \) yields the intermediate transformation matrix,
\[
\begin{pmatrix}
\bar{x}'' \\
\bar{y}'' \\
\bar{z}''
\end{pmatrix} =
\begin{pmatrix}
\cos\theta_3 & -\sin\theta_3 & 0 \\
\sin\theta_3 & \cos\theta_3 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\bar{x}' \\
\bar{y}' \\
\bar{z}'
\end{pmatrix}
\] (4)

and in matrix form \( \bar{x}'' = Z\bar{x}' \). Now using the three equations,

\[
x = X\bar{x}'
\]

\[
\bar{x}' = Y\bar{x}''
\]

\[
\bar{x}'' = Z\bar{x}
\]

by substitution

\[
x = (X Y Z) \bar{x}.
\] (6)

Then from equation 1,

\[
M = (X Y Z)
\] (7)

Computation for the \( M \) matrix from the indicated matrix multiplication in equation (7) yields,

\[
M = \begin{pmatrix}
\cos\theta_2 \cos\theta_3 & -\cos\theta_2 \sin\theta_3 & \sin\theta_2 \\
\cos\theta_1 \sin\theta_3 + \sin\theta_1 \sin\theta_2 \cos\theta_3 & \cos\theta_1 \cos\theta_3 - \sin\theta_1 \sin\theta_2 \sin\theta_3 & -\sin\theta_1 \cos\theta_2 \\
\sin\theta_1 \sin\theta_3 - \cos\theta_1 \sin\theta_2 \cos\theta_3 & \sin\theta_1 \cos\theta_3 + \cos\theta_1 \sin\theta_2 \sin\theta_3 & \cos\theta_1 \cos\theta_2
\end{pmatrix}
\] (8)

The matrix \( M \) in equation (8) is a function of:

(1) The three Euler angles \( \theta_1, \theta_2, \) and \( \theta_3 \)

(2) The sequence of rotations used to generate the matrix.

By examination of equation (7) it is possible to show that there are twelve possible Euler rotational sequences. If the \( (X Y Z) \) notation in equation (7) represents a rotation about the \( X \) axis, then the \( Y \) axis and finally the \( Z \) axis, then the following per-
mutations of the rotational order represents the twelve possible Euler angle sets using three rotations,

\[
\begin{array}{ccc}
  X & Y & Z \\
  Y & X & Z \\
  Z & Y & X \\
  X & Y & X \\
  Y & X & Y \\
  Z & Z & Y \\
\end{array}
\]

(9)

Any repeated axis rotation such as XXY does not represent a three axis rotation but reduces to the two axis rotation XY. Hence the rotations described in (9) above represent all twelve possible sets of Euler angle defining sequences. Conversely then, for a given transformation matrix there are twelve Euler angle sets which can be extracted from the matrix. The Euler matrices corresponding to all possible rotational sequences of (9) above are presented in Appendix A.

The utility subroutines "EULMAT" generates the transformation matrix from a given Euler sequence and the Euler angles. The utility subroutine "MATEUL" extracts the Euler angles from a given Euler rotational sequence. The convention is established that the Euler angles occur in the same sequences as the axis rotations. Using this concept and functional notation, equation (7) could be expressed as

\[ M = X \ Y \ Z = M(\theta_x, \theta_y, \theta_z) \]

(10)

and from (9)
\[ M = X Z X = M(\vartheta_x, \vartheta_z, \vartheta_x') \text{ etc.} \quad (11) \]

This convention is assumed in both subroutines and also used throughout this design note when Euler angles are used. A brief explanation of the use of these two utility subroutines is given in Appendix B.

It is interesting to note that a negative rotation in the single axis rotation matrices of equations (2), (3) and (4) will result in the formation of the transpose of the matrix. However the transpose of \( M \) in equation (7) is formed from reversing the order of multiplication and transposing the individual axis rotation equations, i.e.

\[ M^T = (X Y Z)^T = (Y Z)^T X^T = Z^T Y^T X^T. \quad (12) \]

Hence the transposes of the matrices of (9) are easily formed by reversing the order of multiplication and transposing each single axis rotation equation. Using the notation in equations (10) and (11) above equation (12) could be written,

\[ M^T(\vartheta_x, \vartheta_y, \vartheta_z) = M(-\vartheta_z, -\vartheta_y, -\vartheta_x). \quad (13) \]

It is recommended to avoid confusion that the forward transformation be computed and simply transposed to yield the reverse transformation matrix. All matrices of Appendix A are in the forward form, i.e. \( X = MX \) and formed from (9).
2.2 Transformation Matrices Using the Hamilton Quaternion

The transformation matrix of equation (1) can be written as a function of the Hamilton Quaternion;

\[
\begin{align*}
q_1 &= \cos \omega/2 \\
q_2 &= \cos \alpha \sin \omega/2 \\
q_3 &= \cos \beta \sin \omega/2 \\
q_4 &= \cos \gamma \sin \omega/2 ,
\end{align*}
\]

(14)

where \( \omega \) is the rotation angle about the rotation axis with \( \alpha, \beta, \) and \( \gamma \) direction angles with the \( x, y \) and \( z \) axes respectively. Notice also that \( q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1, \) since \( \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1. \) The rotation angle, \( \omega, \) is assumed positive according to the right-hand rule of axis rotation.

The matrix \( M \) becomes

\[
M = \begin{pmatrix}
(q_1^2 + q_2^2 - q_3^2 - q_4^2) & 2(q_2 q_3 - q_1 q_4) & 2(q_2 q_4 + q_1 q_3) \\
2(q_2 q_3 + q_1 q_4) & (q_1^2 - q_2^2 + q_3^2 - q_4^2) & 2(q_3 q_4 - q_1 q_2) \\
2(q_2 q_4 - q_1 q_3) & 2(q_3 q_4 + q_1 q_2) & (q_1^2 - q_2^2 - q_3^2 + q_4^2)
\end{pmatrix}
\]

(15)

For a more detailed discussion of the derivation of equation (15), see Reference 1. Using functional notation, equation (15) can be written:

\[
M = M(q_1, q_2, q_3, q_4).
\]

(16)

Unlike the Euler angle rotational sequences to describe the transformation matrix of equation (1), only two quaternions can be found from equation (15). The two quaternions are:
\[ \begin{align*}
q_1 &\quad -q_1 \\
q_2 &\quad -q_2 \\
q_3 &\quad -q_3 \\
q_4 &\quad -q_4
\end{align*} \tag{17} \]

These two quaternions represent a positive rotation about the rotation axis pointing in one direction and a positive rotation about the same line of rotation pointing in the opposite direction. Both quaternions of (17) satisfy equation (15).

The utility subroutine "QMAT" generates the transformation matrix from a given quaternion. The "QMAT" algorithm generates the matrix as given in equation (15) without duplicating any arithmetic operations. The subroutine "MATQ" extracts the positive quaternion, i.e., \( q_1 > 0 \), from the transformation matrix and normalizes the results to guarantee an orthogonal matrix. In order to avoid any discontinuity in extracting the quaternion from the transformation matrix, the procedure as described in Reference 2 is used.

Early works by Hamilton (Reference 3) presented the quaternion as having a scalar and a vector part, i.e.,

\[ q_1 = S, \quad \vec{V} = (q_2, q_3, q_4) \tag{18} \]

and equation (16) could be expressed as,

\[ M = M(q_1, q_2, q_3, q_4) = M(S, \vec{V}). \tag{19} \]
For a given quaternion the following relationship is true (from (17) above),
\[ M(S, V) = M(-S, -V). \] (20)

The transpose of the transformation matrix is given by,
\[ M^T(S, V) = M(-S, V) = M(S, -V). \] (21)

2.3 Euler Angle and Quaternion Relationships

By examination of equations (10) and (16) the equality,
\[ M(X(\theta_1), Y(\theta_2), Z(\theta_3)) = M(\theta_1, \theta_2, \theta_3) = M(q_1, q_2, q_3, q_4) \] (22)
can be written. Based on an equality for each element of the matrix the following nine equations must be true;

\[
\begin{align*}
\cos \theta_2 \cos \theta_3 &= q_1^2 + q_2^2 - q_3^2 - q_4^2 \\
-\cos \theta_2 \sin \theta_3 &= 2(q_2q_3 - q_1q_4) \\
\sin \theta_2 &= 2(q_2q_4 + q_1q_3) \\
\cos \theta_1 \sin \theta_3 + \sin \theta_1 \sin \theta_2 \cos \theta_3 &= 2(q_2q_3 + q_1q_4) \\
\cos \theta_1 \cos \theta_3 - \sin \theta_1 \sin \theta_2 \sin \theta_3 &= q_1^2 - q_2^2 + q_3^2 - q_4^2 \\
-\sin \theta_1 \cos \theta_2 &= 2(q_3q_4 - q_1q_2) \\
\sin \theta_1 \sin \theta_3 - \cos \theta_1 \sin \theta_2 \cos \theta_3 &= 2(q_3q_4 - q_1q_3) \\
\sin \theta_1 \cos \theta_3 + \cos \theta_1 \sin \theta_2 \sin \theta_3 &= 2(q_3q_4 + q_1q_2) \\
\cos \theta_1 \cos \theta_2 &= q_1^2 - q_2^2 - q_3^2 + q_4^2 .
\end{align*}
\] (23)

It is possible to solve for the values of the quaternion using the trigonometric half angle identities. For this Euler sequence, i.e. \( X(\theta_1) Y(\theta_2) Z(\theta_3) \), the following quaternion results;
\[
q_1 = \sin^2\alpha_1 \sin^2\beta_2 \sin^2\beta_3 + \cos^2\alpha_1 \cos^2\beta_2 \cos^2\beta_3
\]
\[
q_2 = +\sin^2\alpha_1 \cos^2\beta_2 \cos^2\beta_3 + \sin^2\beta_2 \sin^2\beta_3 \cos^2\alpha_1
\]
\[
q_3 = -\sin^2\alpha_1 \sin^2\beta_3 \cos^2\beta_2 + \sin^2\beta_2 \cos^2\beta_1 \cos^2\beta_3
\]
\[
q_4 = +\sin^2\alpha_1 \sin^2\beta_2 \cos^2\beta_3 + \sin^2\beta_3 \cos^2\beta_1 \cos^2\beta_2
\]

(24)

Appendix A gives the quaternion as a function of the Euler angles for each of the twelve Euler rotational sequence presented in Section 2.1. The special equations for the quaternion as functions of the Euler angles, like equations (24) above, are sometimes cumbersome to use, especially when multiple Euler sequences are utilized. Less coding, but perhaps more computer operations, are required by using the more general method of first generating the matrix \( M \) from the given Euler angle set and then simply extracting the quaternion from the matrix. This method is effected by first a call to "EULMAT" and then a call to "MATQ".
3.0 REFERENCES


APPENDIX A
RELATIONSHIPS FOR THE THREE-AXIS EULER ANGLE ROTATION SEQUENCES

The twelve Euler matrices for each rotation sequence are given based on the single axis rotation equations (2), (3) and (4). The Euler matrices transform vectors from the system that has been rotated into vectors in the stationary system. Also presented here are the equations for the quaternion as a function of the Euler angles and the Euler angles as a function of the matrix elements for each rotation sequence.
(1) $M = M(X(\theta_1), Y(\theta_2), Z(\theta_3)) = XYZ$

Axis Rotation Sequence: 1, 2, 3

$$M = \begin{bmatrix}
\cos\theta_2\cos\theta_3 & -\cos\theta_2\sin\theta_3 & \sin\theta_2 \\
\sin\theta_1\sin\theta_2\cos\theta_3 & -\sin\theta_1\sin\theta_2\sin\theta_3 & -\sin\theta_1\cos\theta_2 \\
+\cos\theta_1\sin\theta_3 & +\cos\theta_1\cos\theta_3 & \cos\theta_1\cos\theta_2 \\
\end{bmatrix}$$

$$q_1 = -\sin\theta_1\sin\theta_2\sin\theta_3 + \cos\theta_1\cos\theta_2\cos\theta_3$$

$$q_2 = \sin\theta_1\cos\theta_2\cos\theta_3 + \sin\theta_2\sin\theta_3\cos\theta_1$$

$$q_3 = -\sin\theta_1\sin\theta_2\cos\theta_3 + \sin\theta_2\cos\theta_1\cos\theta_3$$

$$q_4 = \sin\theta_1\sin\theta_2\cos\theta_3 + \sin\theta_3\cos\theta_1\cos\theta_2$$

$$\theta_1 = \tan^{-1}\left(\frac{-m_{23}}{m_{33}}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{m_{13}}{\sqrt{1-m_{13}^2}}\right)$$

$$\theta_3 = \tan^{-1}\left(\frac{-m_{12}}{m_{11}}\right)$$
(2) $M = M(X(\theta_1), Z(\theta_2), Y(\theta_3)) = XYZ$

Axis Rotation Sequence: 1, 3, 2

$$M = \begin{bmatrix}
\cos\theta_2\cos\theta_3 & -\sin\theta_2 & \cos\theta_2\sin\theta_3 \\
\cos\theta_1\sin\theta_2\cos\theta_3 & \cos\theta_1\cos\theta_2 & \cos\theta_1\sin\theta_2\sin\theta_3 \\
+\sin\theta_1\sin\theta_3 & \sin\theta_1\cos\theta_2 & \sin\theta_1\sin\theta_2\sin\theta_3 \\
-\cos\theta_1\sin\theta_3 & \sin\theta_1\cos\theta_2 & \cos\theta_1\sin\theta_2\sin\theta_3
\end{bmatrix}$$

$q_1 = +\sin\theta_1\sin^2\theta_2\sin\theta_3 + \cos\theta_1\cos^2\theta_2\cos\theta_3$
$q_2 = +\sin\theta_1\cos^2\theta_2\cos\theta_3 - \sin\theta_2\sin^2\theta_3\cos\theta_1$
$q_3 = -\sin\theta_1\sin\theta_2\cos\theta_3 + \sin^2\theta_3\cos\theta_1\cos\theta_2$
$q_4 = +\sin\theta_1\sin^2\theta_3\cos\theta_2 + \sin^2\theta_2\cos\theta_1\cos\theta_3$

$$\theta_1 = \tan^{-1}\left(\frac{m_{32}}{m_{22}}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{-m_{12}}{\sqrt{1-m_{12}^2}}\right)$$

$$\theta_3 = \tan^{-1}\left(\frac{m_{13}}{m_{11}}\right)$$
Axis Rotation Sequence: 1, 2, 1

\[
M = \begin{bmatrix}
\cos \theta_2 & \sin \theta_2 \sin \theta_3 & \sin \theta_2 \cos \theta_3 \\
\sin \theta_1 \sin \theta_2 & \cos \theta_1 \cos \theta_3 & -\cos \theta_1 \sin \theta_3 \\
-\cos \theta_1 \sin \theta_2 & +\sin \theta_1 \cos \theta_3 & -\sin \theta_1 \cos \theta_2 \cos \theta_3 
\end{bmatrix}
\]

\[
q_1 = \cos \frac{\theta_2}{2} \cos \left(\frac{\theta_1 + \theta_3}{2}\right)
\]

\[
q_2 = \cos \frac{\theta_2}{2} \sin \left(\frac{\theta_1 + \theta_3}{2}\right)
\]

\[
q_3 = \sin \frac{\theta_2}{2} \cos \left(\frac{\theta_1 - \theta_3}{2}\right)
\]

\[
q_4 = \sin \frac{\theta_2}{2} \sin \left(\frac{\theta_1 - \theta_3}{2}\right)
\]

\[
\theta_1 = \tan^{-1} \left( \frac{m_{21}}{-m_{31}} \right)
\]

\[
\theta_2 = \tan^{-1} \left( \frac{\sqrt{1 - m_{11}^2}}{m_{11}} \right)
\]

\[
\theta_3 = \tan^{-1} \left( \frac{m_{12}}{m_{13}} \right)
\]
(4) \[ M = M(x(\theta_1), z(\theta_2), x(\theta_3)) = XZX \]

Axis Rotation Sequence: 1, 3, 1

\[
M = \begin{bmatrix}
\cos\theta_2 & -\sin\theta_2\cos\theta_3 & \sin\theta_2\sin\theta_3 \\
\cos\theta_1\sin\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3 & -\cos\theta_1\cos\theta_2\sin\theta_3 \\
\sin\theta_1\sin\theta_2 & \sin\theta_1\cos\theta_2\cos\theta_3 & -\sin\theta_1\cos\theta_2\sin\theta_3 \\
+\cos\theta_1\sin\theta_3 & +\cos\theta_1\cos\theta_3 & +\cos\theta_1\cos\theta_3
\end{bmatrix}
\]

\[ q_1 = \cos\theta_2\cos(\frac{1}{2}(\theta_1 + \theta_3)) \]
\[ q_2 = \cos\theta_2\sin(\frac{1}{2}(\theta_1 + \theta_3)) \]
\[ q_3 = -\sin\theta_2\sin(\frac{1}{2}(\theta_1 - \theta_3)) \]
\[ q_4 = \sin\theta_2\cos(\frac{1}{2}(\theta_1 - \theta_3)) \]

\[ \theta_1 = \tan^{-1}\left(\frac{m_{31}}{m_{21}}\right) \]
\[ \theta_2 = \tan^{-1}\left(\frac{\sqrt{1 - m_{11}^2}}{m_{11}}\right) \]
\[ \theta_3 = \tan^{-1}\left(\frac{m_{13}}{-m_{12}}\right) \]
(5) \[ M = M(Y(\theta_1), X(\theta_2), Z(\theta_3)) = YXZ \]

Axis Rotation Sequence: 2, 1, 3

\[
M = \begin{bmatrix}
\sin \theta_1 \sin \theta_2 \sin \theta_3 & \sin \theta_1 \sin \theta_2 \cos \theta_3 & \sin \theta_1 \cos \theta_2 \\
+\cos \theta_1 \cos \theta_3 & -\cos \theta_1 \sin \theta_3 & \\
\cos \theta_2 \sin \theta_3 & & \\
\cos \theta_1 \sin \theta_2 \sin \theta_3 & \cos \theta_1 \sin \theta_2 \cos \theta_3 & \cos \theta_1 \cos \theta_2 \\
-\sin \theta_1 \cos \theta_3 & & +\sin \theta_1 \sin \theta_3
\end{bmatrix}
\]

\[ q_1 = \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \theta_3 + \cos^2 \theta_1 \cos^2 \theta_2 \cos^2 \theta_3 \]

\[ q_2 = \sin^2 \theta_1 \sin^2 \theta_3 \cos^2 \theta_2 + \sin^2 \theta_2 \cos^2 \theta_1 \cos^2 \theta_3 \]

\[ q_3 = \sin^2 \theta_1 \cos^2 \theta_2 \cos^2 \theta_3 - \sin^2 \theta_2 \sin^2 \theta_3 \cos^2 \theta_1 \]

\[ q_4 = -\sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \theta_3 + \sin^2 \theta_3 \cos^2 \theta_1 \cos^2 \theta_2 \]

\[ \theta_1 = \tan^{-1}\left(\frac{m_{31}}{m_{33}}\right) \]

\[ \theta_2 = \tan^{-1}\left(\frac{-m_{23}}{\sqrt{1-m_{23}^2}}\right) \]

\[ \theta_3 = \tan^{-1}\left(\frac{m_{21}}{m_{22}}\right) \]
\( M = M(Y(\theta_1), Z(\theta_2), X(\theta_3)) = YZX \)

Axis Rotation Sequence: 2, 3, 1

\[
M = \begin{bmatrix}
\cos \theta_1 \cos \theta_2 & -\cos \theta_1 \sin \theta_2 \cos \theta_3 & \cos \theta_1 \sin \theta_2 \sin \theta_3 \\
\sin \theta_2 & \cos\theta_2 \cos \theta_3 & -\cos \theta_2 \sin \theta_3 \\
-\sin \theta_1 \cos \theta_2 & \sin \theta_1 \sin \theta_2 \cos \theta_3 & \sin \theta_1 \sin \theta_2 \sin \theta_3 \\
\end{bmatrix}
\]

\( q_1 = -\sin \theta_1 \sin \theta_2 \sin \theta_3 + \cos \theta_1 \cos \theta_2 \cos \theta_3 \)

\( q_2 = +\sin \theta_1 \sin \theta_2 \cos \theta_3 + \sin \theta_3 \cos \theta_1 \cos \theta_2 \)

\( q_3 = +\sin \theta_1 \cos \theta_2 \cos \theta_3 + \sin \theta_2 \sin \theta_3 \cos \theta_1 \)

\( q_4 = -\sin \theta_1 \sin \theta_2 \cos \theta_3 + \sin \theta_2 \cos \theta_1 \cos \theta_3 \)

\( \theta_1 = \tan^{-1} \left( \frac{-m_{31}}{m_{11}} \right) \)

\( \theta_2 = \tan^{-1} \left( \frac{m_{21}}{\sqrt{1 - m_{21}^2}} \right) \)

\( \theta_3 = \tan^{-1} \left( \frac{-m_{23}}{m_{22}} \right) \)
(7) \[ M = M(Y(\theta_1), X(\theta_2), Y(\theta_3)) = YXY \]

Axis Rotation Sequence: 2, 1, 2

\[
M = \begin{bmatrix}
-sin\theta_1 \cos\theta_2 \sin\theta_3 & \sin\theta_1 \sin\theta_2 & \sin\theta_1 \cos\theta_2 \cos\theta_3 \\
+\cos\theta_1 \cos\theta_3 & \cos\theta_2 & +\cos\theta_1 \sin\theta_3 \\
-sin\theta_2 \sin\theta_3 & \cos\theta_2 & -\sin\theta_2 \cos\theta_3 \\
-cos\theta_1 \cos\theta_2 \sin\theta_3 & \cos\theta_1 \sin\theta_2 & \cos\theta_1 \cos\theta_2 \cos\theta_3 \\
-sin\theta_1 \cos\theta_3 & \cos\theta_1 \sin\theta_2 & -\sin\theta_1 \sin\theta_3 \\
\end{bmatrix}
\]

\[ q_1 = +\cos\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3)) \]
\[ q_2 = +\sin\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3)) \]
\[ q_3 = +\cos\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3)) \]
\[ q_4 = -\sin\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3)) \]

\[ \theta_1 = \tan^{-1} \left( \frac{m_{12}}{m_{32}} \right) \]

\[ \theta_2 = \tan^{-1} \left( \frac{\sqrt{1-m_{22}^2}}{m_{22}} \right) \]

\[ \theta_3 = \tan^{-1} \left( \frac{m_{21}}{-m_{23}} \right) \]
(8) \( M = M(Y(\theta_1), Z(\theta_2), Y(\theta_3)) = YZY \)

Axis Rotation Sequence: 2, 3, 2

\[
M = \begin{bmatrix}
\cos \theta_1 \cos \theta_2 \cos \theta_3 & -\cos \theta_1 \sin \theta_2 & \cos \theta_1 \cos \theta_2 \sin \theta_3 \\
-\sin \theta_1 \sin \theta_3 & \cos \theta_2 & \sin \theta_1 \cos \theta_2 \sin \theta_3 \\
\sin \theta_2 \cos \theta_3 & \sin \theta_1 \sin \theta_2 & -\sin \theta_1 \cos \theta_2 \sin \theta_3 \\
-\cos \theta_1 \sin \theta_3 & -\sin \theta_1 \sin \theta_2 & \cos \theta_1 \cos \theta_2 \sin \theta_3
\end{bmatrix}
\]

\( q_1 = +\cos \theta_2 \cos \left( \frac{1}{2}(\theta_1 + \theta_3) \right) \)

\( q_2 = +\sin \theta_2 \sin \left( \frac{1}{2}(\theta_1 - \theta_3) \right) \)

\( q_3 = +\cos \theta_2 \sin \left( \frac{1}{2}(\theta_1 + \theta_3) \right) \)

\( q_4 = +\sin \theta_2 \cos \left( \frac{1}{2}(\theta_1 - \theta_3) \right) \)

\( \theta_1 = \tan^{-1} \left( \frac{m_{32}}{-m_{12}} \right) \)

\( \theta_2 = \tan^{-1} \left( \frac{\sqrt{1 - m_{22}^2}}{m_{22}} \right) \)

\( \theta_3 = \tan^{-1} \left( \frac{m_{23}}{m_{21}} \right) \)
(9) \[ M = M(Z(\theta_1), X(\theta_2), Y(\theta_3)) = ZXY \]

Axis Rotation Sequence: 3, 1, 2

\[
M = \begin{bmatrix}
-sin\theta_1 sin\theta_2 sin\theta_3 & -sin\theta_1 cos\theta_2 & sin\theta_1 sin\theta_2 cos\theta_3 \\
+cos\theta_1 cos\theta_3 & cos\theta_1 cos\theta_2 & -cos\theta_1 sin\theta_2 cos\theta_3 \\
cos\theta_1 sin\theta_2 sin\theta_3 & cos\theta_1 sin\theta_2 & +sin\theta_1 sin\theta_3 \\
+sin\theta_1 cos\theta_3 & sin\theta_2 & cos\theta_2 cos\theta_3 \\
-cos\theta_2 sin\theta_3 & sin\theta_2 & cos\theta_2 \cos\theta_3 \\
\end{bmatrix}
\]

\[ q_1 = -sin^2\theta_1 sin^2\theta_2 sin^3\theta_3 + \cos^2\theta_1 cos^2\theta_2 cos^2\theta_3 \]

\[ q_2 = -sin^2\theta_1 sin^2\theta_3 cos^2\theta_2 + \sin^2\theta_2 cos^2\theta_1 cos^2\theta_3 \]

\[ q_3 = +sin^2\theta_1 sin^2\theta_2 cos^2\theta_3 + \sin^2\theta_3 cos^2\theta_1 cos^2\theta_2 \]

\[ q_4 = +sin^2\theta_1 cos^2\theta_2 cos^2\theta_3 + \sin^2\theta_2 sin^2\theta_3 \cos^2\theta_1 \]

\[ \theta_1 = \tan^{-1}\left( \frac{-m_{12}}{m_{22}} \right) \]

\[ \theta_2 = \tan^{-1}\left( \frac{m_{32}}{\sqrt{1-m_{32}^2}} \right) \]

\[ \theta_3 = \tan^{-1}\left( \frac{-m_{31}}{m_{33}} \right) \]
\[ M = M(Z(\theta_1), Y(\theta_2), X(\theta_3)) = ZYX \]

Axis Rotation Sequence: 3, 2, 1

\[
M = \begin{bmatrix}
\cos \theta_1 \cos \theta_2 & \cos \theta_1 \sin \theta_2 \sin \theta_3 & \cos \theta_1 \sin \theta_2 \cos \theta_3 \\
-\sin \theta_1 \cos \theta_3 & \sin \theta_1 \sin \theta_2 \sin \theta_3 & \sin \theta_1 \sin \theta_2 \cos \theta_3 \\
-\sin \theta_2 & \cos \theta_2 \sin \theta_3 & \cos \theta_2 \cos \theta_3
\end{bmatrix}
\]

\[
q_1 = +\sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \theta_3 + \cos^2 \theta_1 \cos^2 \theta_2 \cos^2 \theta_3
\]

\[
q_2 = -\sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \theta_3 + \sin^2 \theta_2 \cos^2 \theta_1 \cos^2 \theta_3
\]

\[
q_3 = +\sin^2 \theta_1 \sin^2 \theta_3 \cos^2 \theta_2 + \sin^2 \theta_2 \cos^2 \theta_1 \cos^2 \theta_3
\]

\[
q_4 = +\sin^2 \theta_1 \cos^2 \theta_2 \cos^2 \theta_3 - \sin^2 \theta_2 \sin^2 \theta_3 \cos^2 \theta_1
\]

\[
\theta_1 = \tan^{-1}\left(\frac{m_{21}}{m_{11}}\right)
\]

\[
\theta_2 = \tan^{-1}\left(\frac{-m_{31}}{\sqrt{1-m_{31}^2}}\right)
\]

\[
\theta_3 = \tan^{-1}\left(\frac{m_{32}}{m_{33}}\right)
\]
(11) \[ M = M(Z(\theta_1), X(\theta_2), Z(\theta_3)) = ZXZ \]

Axis Rotation Sequence: 3, 1, 3

\[ M = \begin{bmatrix}
-sin\theta_1cos\theta_2sin\theta_3 & -sin\theta_1cos\theta_2cos\theta_3 & sin\theta_1sin\theta_2 \\
+cos\theta_1cos\theta_3 & -cos\theta_1cos\theta_3 & -cos\theta_1sin\theta_2 \\
cos\theta_1cos\theta_2sin\theta_3 & cos\theta_1cos\theta_2cos\theta_3 & -sin\theta_1sin\theta_3 \\
+sin\theta_1cos\theta_3 & -sin\theta_1sin\theta_3 & cos\theta_2 \\
sin\theta_2sin\theta_3 & sin\theta_2cos\theta_3 & cos\theta_2
\end{bmatrix} \]

\[ q_1 = +cos\theta_2cos(\frac{1}{2}(\theta_1 + \theta_3)) \]
\[ q_2 = +sin\theta_2cos(\frac{1}{2}(\theta_1 - \theta_3)) \]
\[ q_3 = +sin\theta_2sin(\frac{1}{2}(\theta_1 - \theta_3)) \]
\[ q_4 = +cos\theta_2sin(\frac{1}{2}(\theta_1 + \theta_3)) \]

\[ \theta_1 = \tan^{-1}\left(\frac{m_{13}}{-m_{23}}\right) \]
\[ \theta_2 = \tan^{-1}\left(\frac{\sqrt{1-m_{33}^2}}{m_{33}}\right) \]
\[ \theta_3 = \tan^{-1}\left(\frac{m_{31}}{m_{32}}\right) \]
12) \( M = M(Z(\theta_1), Y(\theta_2), Z(\theta_3)) = ZYZ \)

Axis Rotation Sequence: 3, 2, 3

\[
M = \begin{bmatrix}
\cos \theta_1 \cos \theta_2 \cos \theta_3 & -\cos \theta_1 \cos \theta_2 \sin \theta_3 & \cos \theta_1 \sin \theta_2 \\
-\sin \theta_1 \sin \theta_3 & -\sin \theta_1 \cos \theta_3 & \\
\sin \theta_1 \cos \theta_2 \cos \theta_3 & -\sin \theta_1 \cos \theta_2 \sin \theta_3 & \sin \theta_1 \sin \theta_2 \\
+\cos \theta_1 \sin \theta_3 & +\cos \theta_1 \cos \theta_3 \\
-\sin \theta_2 \cos \theta_3 & \sin \theta_2 \sin \theta_3 & \cos \theta_2 
\end{bmatrix}
\]

\[
q_1 = +\cos \frac{\theta_2}{2} \cos \left( \frac{\theta_1 + \theta_3}{2} \right)
\]
\[
q_2 = -\sin \frac{\theta_2}{2} \sin \left( \frac{\theta_1 - \theta_3}{2} \right)
\]
\[
q_3 = +\sin \frac{\theta_2}{2} \cos \left( \frac{\theta_1 - \theta_3}{2} \right)
\]
\[
q_4 = +\cos \frac{\theta_2}{2} \sin \left( \frac{\theta_1 + \theta_3}{2} \right)
\]

\[
\theta_1 = \tan^{-1} \left( \frac{m_{23}}{m_{13}} \right)
\]
\[
\theta_2 = \tan^{-1} \left( \frac{\sqrt{1-m_{33}^2}}{m_{33}} \right)
\]
\[
\theta_3 = \tan^{-1} \left( \frac{m_{32}}{-m_{31}} \right)
\]
APPENDIX B
COMPUTER SUBROUTINES FOR THE RELATIONSHIPS

The following subroutines with a brief description of their use are presented in this appendix.

(1) "EULMAT" - Generates the transformation matrix from a given set of Euler angles and an axis rotation sequence.
(2) "MATEUL" - Extracts the Euler angles from the given transformation matrix and an axis rotation sequence.
(3) "QMAT" - Generates the transformation matrix from a given quaternion.
(4) "MATQ" - Extracts the quaternion from a given transformation matrix.
(5) "YPRQ" - Generates the quaternion directly from the yaw-pitch-roll Euler angles.
(6) "POSNOR" - Computes the positive-normalized quaternion from the given quaternion.
NAME: EULMAT

PURPOSE: Generates a 3 x 3 transformation matrix from a given sequence and Euler angle set.

INPUT: ISEQ - Rotation Sequence (Integer Array (3); i.e., 1, 2, 3)
        EUL - Euler Angles in radians, in "ISEQ"
        Order; ARRAY..(3)

OUTPUT: A - The 3 x 3 transformation matrix

ALGORITHM REFERENCE: Appendix A; Euler Sequences (1) thru (12).
EULER ANGLES TO THE TRANSFORMATION MATRIX

FOR IS = FULMAT, EULMAT
FOR 52E3-02/19/77-06:42:23 1, 67

SUBROUTINE FULMAT ENTRY POINT EULMAT

STORAGE USED: CODE(I) G0325; DATA(I) F03154; BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)

     1032  SIN
     1035  COS

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

  7021  0.121, 1021  1.21  26, 13 1200  1081  107718
  7021  0.142, 1626  1.31  36, 144 1650  1151  11546
  7021  -1.147 3CL  1.32  36, -78  P  1171  11724
  7020 R 12,747 SINA  8007 P 80555 TEMP  8007 P 80555

 SUBROUTINE FULMAT(ISL, EUL, A)
 DIMENSION IS0(3), EUL(1), A(3,3)
 DIMENSION X(3,3,3,3), P(3,3)
 DO 10 IC = 1, 3
 DO 10 J = 1, 3
 X(IC,J) = 0
 10 CONTINUE
 IF(1) SENO(K, K2) 60 TO 100
 SINA = SINA(1) EUL(1)
 COSA = COSA(1) EUL(1)
 IF trưng 60 TO 20
 IF(2) SENO(K, K2) 60 TO 30
 X(K, J) = 0
 X(K, J) = SINA
 X(K, J) = COSA
 20 X(K, J) = SINA
 X(K, J) = COSA
 X(K, J) = SINA
 X(K, J) = COSA
 30 X(K, J) = SINA
 X(K, J) = COSA
 X(K, J) = SINA
 X(K, J) = COSA
EULER ANGLES TO THE TRANSFORMATION MATRIX

(CONTINUED)

0.0163  0.0155  0.0162  0.0161  0.0164  0.0167  0.0172  0.0173  0.0175  0.0177  0.0201  0.0223
0.018  0.0187  0.0193  0.020  0.0249  0.023  0.0214  0.0215  0.0215  0.0217  0.0217
0.015  0.0158  0.0164  0.0167  0.017  0.0174  0.0175  0.0177  0.0201  0.0223  0.0223

END OF COMPILATION: NO DIAGNOSTICS.
NAME: MATEUL

PURPOSE: Extracts the Euler angles from the given transformation matrix and the required Euler rotational sequence.

INPUT: ISEQ - Rotation sequence, (Integer Array (3), i.e., 1,2,3.)
A - The 3 x 3 transformation

OUTPUT: EUL - The Euler angles, in "ISEQ" order ARRAY(3).

ALGORITHM REFERENCE: Appendix A; Euler angles as a function of the matrix elements, sequences (1) thru (12).
TRANSFORMATION MATRIX TO THE CULER ANGLES

SUBROUTINE MATEUL ENTRY POINT D72336

STORAGE USED: CODE (11) 202353; DATA (5), 20053; BLANK COMMON (2)

EXTERNAL REFERENCES (BLOCK, NAME)

103 SORT
1040 AIANC
1045 MPR3

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

141 1 300541C L...1 37266 13L 3701 7-77
141 1 20115 F L...1 37266 4L 3701 7-77
141 1 20117 F L...1 37266 6L 3701 7-77
141 1 P 5 - 1 E SIGN 20116 (C) 3703 R 12014 C3 3003 1 12016 1
141 1 P 3 T 30051 J 20116 (C) 3703 K 12012 JU 3003 1 12016 1

SUBROUTINE MATEUL (ISE, CUL)

DIMENSION A (21, 21), G (13, 13)

GO TO 7

IF (K .EQ. 1) GO TO 7

IF (K .NE. 2) GO TO 15

SIGN = -1

IF (K .NE. 2) GO TO 15

SIGN = 1

GO TO 3

IF (SIGN .GT. 0) L = 2

GO TO 3

IF (SIGN .LT. 0) L = 2

GO TO 3

IF (SIGN .GE. 0) L = 2

GO TO 3

SIGN = -1

GO TO 3

IF (SIGN .GE. 0) L = 2

GO TO 3

SIGN = 1

GO TO 3
TRANSFORMATION MATRIX TO THE EULER ANGLES

(CONTINUED)

30152 29# IF (I EQ. NE. LE. C) L = 1
30153 30# GO TO 30
30154 31# 25. CSIGN=-1.0
30155 32# IF (L EQ. NE. LE. C) L = 3
30156 33# GO. J = J-1. N = 13
30157 34# FNSIGN=1.0
30158 35# FDSIGN=1.0
30159 36# IF (N= EQ. 2) GO TO 70
30160 37# IF (N= EQ. 1) GO TO 50
30161 38# IF (I EQ. K NE. 0) GO TO 40
30162 39# FNSIGN=BSIGN
30163 40# JJ=1
30164 41# GO TO 45
30165 42# JJ=L
30166 43# IF (KSIGN. GT. J-1) FDSIGN=-1.0
30167 44# FNUM=FNSIGN*A(I+J)
30168 45# FDEN=FDSIGN*A(I+J)
30169 46# GO TO 90
30170 47# 50 IF (I EQ. K NE. 0) GO TO 55
30171 48# FNSIGN=BSIGN
30172 49# JJ=K
30173 50# GO TO 60
30174 51# 55 FDSIGN=BSIGN
30175 52# II=I
30176 53# JJ=J
30177 54# 60 FNUM=FNSIGN*A(J+I)
30178 55# FDEN=FDSIGN*A(I+J)
30179 56# GO TO 90
30180 57# 70 IF (I EQ. K NE. 0) GO TO 80
30181 58# FNUM=CSIGN*A(I+K)
30182 59# FDEN=SQR(T1-O-A(I+K)**2)
30183 60# GO TO 90
30184 61# 80 FNUM=SQR(T1-O-A(I+K)**2)
30185 62# FDEN=A(I+K)**2
30186 63# 90 CALL = ATAN2(FNUM,FDEN)
30187 64# 100 CONTINUE
30188 65# RETURN
30189 66# END

END OF COMPILED AND NO DIAGNOSTICS.

ORIGINAL PAGE IS OF POOR QUALITY
NAME: QMAT

PURPOSE: Generates the transformation matrix from the given quaternion.

INPUT: Q - The quaternion; ARRAY(4).

OUTPUT: A - The 3 x 3 transformation matrix

ALGORITHM REFERENCE: Equation (15) from Section 2.2.
QUATERNION TO THE TRANSFORMATION MATRIX

SUBROUTINE QMAT
ENTRY POINT QMAT

FOR 573-02/19/77-06:24:19 (19)

STORAGE USED: CODE (11), CODE (13); DATA (1) T0 (J) ; BLANK COMMON (2)

EXTERNAL REFERENCES (BLOCK, NAME)

... MARKS...

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

JACO R 007827 INJOU JACO R 007830 PZ JACO R 007802 TEMP

END OF COMPIILATION: NO DIAGNOSTICS.

B-9
NAME: MATQ

PURPOSE: Extracts the positive quaternion from the given transformation matrix.

INPUT: A - The 3 x 3 transformation matrix

OUTPUT: Q - The positive quaternion; ARRAY(4).

ALGORITHM REFERENCE: See Reference 2.
TRANSFORMATION MATRIX TO THE QUATERNION

FOR: MAT0-MAT0
FOR: SGE3-02/19/77-06:24:21 (C)

SUBROUTINE MAT0 ENTRY POINT 000093

STORAGE USED: CODE(10-500,20; DATAP) 2D USE; BLANK COMMON(1)

EXTERNAL REFERENCES (BLOCK, NAME)

SUBROUTINE MAT0

DIMENSION A(3,3),C(1,1,14)

TIC: 1.

IF(I) EQ. 6) GO TO 15
IF(I) EQ. 3) GO TO 90
IF(I) EQ. 1) GO TO 20

1: TEMP = A(I,1) + A(I,2) + A(I,3) + A(I,4)

10: TEMP = A(I,1) + A(I,2) + A(I,3) + A(I,4)

15: TEMP = A(I,1) + A(I,2) + A(I,3) + A(I,4)

20: TEMP = A(I,1) + A(I,2) + A(I,3) + A(I,4)

30: IF(TEMP LT BIG) GO TO 40

40: IF(I) EQ. 6) GO TO 50

50: IF(I) EQ. 3) GO TO 90

60: IF(I) EQ. 1) GO TO 20

70: TEMP = A(I,3) + A(I,4)

80: IF(TEMP LT BIG) GO TO 40

90: CONTINUE

100: CONTINUE

110: CONTINUE

120: CONTINUE

130: CONTINUE

140: CONTINUE

150: CONTINUE

160: CONTINUE

170: CONTINUE

180: CONTINUE

190: CONTINUE

200: CONTINUE

210: CONTINUE

220: CONTINUE

230: CONTINUE

240: CONTINUE

250: CONTINUE

260: CONTINUE

270: CONTINUE

280: CONTINUE

290: CONTINUE

300: CONTINUE

310: CONTINUE

320: CONTINUE

330: CONTINUE

340: CONTINUE

350: CONTINUE

360: CONTINUE

370: CONTINUE

380: CONTINUE

390: CONTINUE

400: CONTINUE

410: CONTINUE

420: CONTINUE

430: CONTINUE

440: CONTINUE

450: CONTINUE

460: CONTINUE

470: CONTINUE

480: CONTINUE

490: CONTINUE

500: CONTINUE

510: CONTINUE

520: CONTINUE

530: CONTINUE

540: CONTINUE

550: CONTINUE

560: CONTINUE

570: CONTINUE

580: CONTINUE

590: CONTINUE

600: CONTINUE

610: CONTINUE

620: CONTINUE

630: CONTINUE

640: CONTINUE

650: CONTINUE

660: CONTINUE

670: CONTINUE

680: CONTINUE

690: CONTINUE

700: CONTINUE

710: CONTINUE

720: CONTINUE

730: CONTINUE

740: CONTINUE

750: CONTINUE

760: CONTINUE

770: CONTINUE

780: CONTINUE

790: CONTINUE

800: CONTINUE

810: CONTINUE

820: CONTINUE

830: CONTINUE

840: CONTINUE

850: CONTINUE

860: CONTINUE

870: CONTINUE

880: CONTINUE

890: CONTINUE

900: CONTINUE

910: CONTINUE

920: CONTINUE

930: CONTINUE

940: CONTINUE

950: CONTINUE

960: CONTINUE

970: CONTINUE

980: CONTINUE

990: CONTINUE

EOF

END

LIST OF COMPILATION: NO DIAGNOSTICS.
NAME: YPRQ

PURPOSE: Generates the quaternion directly from the yaw-pitch-roll Euler angles, i.e., a 3, 2, 1 Euler sequence.

INPUT: YPR - The yaw-pitch-roll Euler angles; ARRAY (3).

OUTPUT: QO - The positive quaternion, ARRAY (4).

ALGORITHM REFERENCE: Appendix A, the quaternion equations for Euler sequence (10), a 3, 2, 1 sequence.

NOTE: This subroutine calls "POSNOR" to take the normal of the positive quaternion.
Yaw-Pitch-Roll Euler Angles to the Quaternion

SUBROUTINE YPRQ ENTRY POINT D7C114

STORAGE USED: CODE(1) MOD1:1; DATA(D) MOD2:2; RANK COMMON(2) CU

EXTERNAL REFERENCES (BLOCK, NAME)

J003 POSNOR
J004 COS
J005 SIN
J006 NEPR31

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

J000 R 7 7.1: CP
J000 P 7 7.4 HY
J000 R 7 7.1: SY
J0101 1
J0102 2
J0103 3
J0104 4
J0105 5
J0106 6
J0107 7
J0108 8
J0109 9
J0110 10
J0111 11
J0112 12
J0113 13
J0114 14
J0115 15
J0116 16
J0117 17
J0118 18
J0119 19
J0120 20
J0121 21
J0122 22
J0123 23

END OF COMPILATION: NO DIAGNOSTICS.
NAME: POSNOR

PURPOSE: To output the positive and normalized quaternion from the given quaternion.

INPUT: Q - The quaternion; ARRAY (4).

OUTPUT: QO - The positive-normalized quaternion; ARRAY (4).

ALGORITHM REFERENCE:

1. If the sign of Q(I) is negative:
   Set QO(I) = -Q(I) for I = 1, 2, 3, 4.

2. Set QO(I) = QO(I)/TEMP
   where TEMP = \sqrt{QO_1^2 + QO_2^2 + QO_3^2 + QO_4^2}
SELECTIONS THE POSITIVE QUATERNION AND NORMALIZES

SUBROUTINE POSNOR ENTRY POINT 000.55

STORAGE USED: CODE(11) DOOLS; DATA(0) DOOLS; BLANK COMMON(2) C

EXTERNAL REFERENCES (BLOCK, NAME)
  0003   SRT1
  0004   NORMIS

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)
  3001   STACK 1110
  3002   DOOLS 1210
  3003   DOOLS 1220 TEMP

SUBROUTINE POSNOR(I, Q0)
DIMENSION Q0(I), Q0(I+1)

30103   2 I
30104   3 TEMP=50.0
30105   4 IF(Q1(1).LE.3.0) TEMP=1.0
30106   5 SUM=.
30107   6 DO 1 I=1,4
30110   7 Q0(I)=Q0(I+1)
30111   8 SUM=SUM+Q0(I)*Q0(I)
30112   9 Q0(I+1)=Q0(I)
30113   10 CONTINUE
30114   11 TEMP=1.0/SQRT(SUM)
30115   12 DO 1 I=1,4
30116   13 Q0(I)=TEMP*Q0(I)
30117   14 CONTINUE
30118   15 RETURN
  05123   16 END

END OF Compilation: NO DIAGNOSTICS.

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