Euler Angles, Quaternions, and Transformation Matrices

(NASA-TM-74839) SHUTTLE PROGRAM. EULER ANGLES, QUATERNIONS, AND TRANSFORMATION MATRICES WORKING RELATIONSHIPS (NASA) 42 p HC A03/MF A01 CSCL 22A N77-31234 Unclas G3/16 37332

Working Relationships

Mission Planning and Analysis Division

July 1977

NASA
National Aeronautics and Space Administration
Lyndon B. Johnson Space Center
Houston, Texas
SHUTTLE PROGRAM

EULER ANGLES, QUATERNIONS, AND TRANSFORMATION MATRICES - WORKING RELATIONSHIPS

By D. M. Henderson
McDonnell Douglas Technical Services Co., Inc.

JSC Task Monitor: B. F. Cockrell

Approved:  
Emil R. Schiesser, Chief  
Mathematical Physics Branch

Approved:  
Ronald L. Berry, Chief  
Mission Planning and Analysis Division

National Aeronautics and Space Administration
Lyndon B. Johnson Space Center
Mission Planning and Analysis Division
Houston, Texas

July 1977
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.0</td>
<td>DISCUSSION</td>
<td>2</td>
</tr>
<tr>
<td>2.1</td>
<td>Euler Angle Transformation Matrices</td>
<td>2</td>
</tr>
<tr>
<td>2.2</td>
<td>Transformation Matrices Using the Hamilton Quaternion</td>
<td>7</td>
</tr>
<tr>
<td>2.3</td>
<td>Euler Angle and Quaternion Relationships</td>
<td>9</td>
</tr>
<tr>
<td>3.0</td>
<td>REFERENCES</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>APPENDIX A - RELATIONSHIPS FOR THE THREE-AXIS EULER ANGLE ROTATION SEQUENCES</td>
<td>A-1</td>
</tr>
<tr>
<td></td>
<td>APPENDIX B - COMPUTER SUBROUTINES FOR THE RELATIONSHIPS</td>
<td>B-1</td>
</tr>
</tbody>
</table>
EULER ANGLES, QUATERNIONS, AND TRANSFORMATION MATRICES -
WORKING RELATIONSHIPS

By D. M. Henderson
McDonnell Douglas Technical Services Co., Inc.

1.0 INTRODUCTION

Due to the extensive use of the quaternion in the onboard Space Shuttle Computer System, considerable analysis is being performed using relationships between the quaternion, the transformation matrix, and the Euler angles. This Internal Note offers a brief mathematical development of the relationships between the Euler angles and the transformation matrix, the quaternion and the transformation matrix, and the Euler angles and the quaternion. The analysis and equations presented here apply directly to current Space Shuttle problems. Appendix A presents the twelve three-axis Euler transformation matrices as functions of the Euler angles, the equations for the quaternion as a function of the Euler angles, and the Euler angles as a function of the transformation matrix elements.

The equations of Appendix A are a valuable reference in Shuttle analysis work and this Internal Note is the only known document where each of the twelve Euler angle to quaternion relationships are given. Appendix B presents a group of utility subroutines to accomplish the Euler-matrix, quaternion-matrix, and Euler-quaternion relationships of Appendix A.
2.0 DISCUSSION

2.1 Euler Angle Transformation Matrices

The following analysis and utility subroutines are offered to simplify computer programs when working with coordinate transformation matrices and their relationships with the Euler Angles and the Quaternions. The coordinate transformation matrices discussed here are defined using the following figure.

Figure 1.- Coordinate system and Euler angles.
The transformation matrix $M$, is defined to transform vectors in the $\bar{x}$-system ($\bar{x}, \bar{y}, \bar{z}$) into the original $x$-system ($x, y, z$) and is given by the equation,

$$x = M\bar{x}$$

where

$$x = (x, y, z) \text{ and } \bar{x} = (\bar{x}, \bar{y}, \bar{z}).$$

Using the right-hand rule for positive rotations, the $M$ matrix in (1) above is constructed by the following analysis. The first rotation in Figure 1 above is about the $x$-axis by the amount $\theta_1$. The single rotation about the $x$-axis results in the following transformation,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \bar{x}' \\ \bar{y}' \\ \bar{z}' \end{pmatrix}$$

or $x = \bar{x}'$ in matrix form. Rotation about the $\bar{y}'$-axis by the amount $\theta_2$ yields the intermediate transformation matrix:

$$\begin{pmatrix} \bar{x}' \\ \bar{y}' \\ \bar{z}' \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \bar{x}'' \\ \bar{y}'' \\ \bar{z}'' \end{pmatrix}$$

or $\bar{x}' = Y\bar{x}$ in matrix form. Finally rotation about the $\bar{z}''$-axis by the amount $\theta_3$ yields the intermediate transformation matrix,
\[
\begin{pmatrix}
\bar{x}^n \\
\bar{y}^n \\
\bar{z}^n
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_3 & -\sin \theta_3 & 0 \\
\sin \theta_3 & \cos \theta_3 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\bar{x} \\
\bar{y} \\
\bar{z}
\end{pmatrix}
\] (4)

and in matrix form \( \bar{x}^n = Z\bar{x}_1 \). Now using the three equations,

\[
\begin{align*}
x &= \bar{x}^n \\
\bar{x}^n &= Y\bar{x}^m \\
\bar{x}^m &= Z\bar{x}
\end{align*}
\] (5)

by substitution

\[
x = (X Y Z) \bar{x}.
\] (6)

Then from equation 1,

\[
M = (X Y Z)
\] (7)

Computation for the \( M \) matrix from the indicated matrix multiplication in equation (7) yields,

\[
M = \begin{pmatrix}
\cos \theta_2 \cos \theta_3 \\
\cos \theta_1 \sin \theta_3 + \sin \theta_1 \sin \theta_2 \cos \theta_3 \\
\sin \theta_1 \sin \theta_3 - \cos \theta_1 \sin \theta_2 \cos \theta_3
\end{pmatrix}
\begin{pmatrix}
\cos \theta_2 \sin \theta_3 \\
\cos \theta_1 \cos \theta_3 - \sin \theta_1 \sin \theta_2 \sin \theta_3 \\
\sin \theta_1 \cos \theta_3 + \cos \theta_1 \sin \theta_2 \sin \theta_3
\end{pmatrix}
\] (8)

The matrix \( M \) in equation (8) is a function of;

(1) The three Euler angles \( \theta_1, \theta_2, \) and \( \theta_3 \)
(2) The sequence of rotations used to generate the matrix.

By examination of equation (7) it is possible to show that there are twelve possible Euler rotational sequences. If the \((X Y Z)\) notation in equation (7) represents a rotation about the \( X \) axis, then the \( Y \) axis and finally the \( Z \) axis, then the following per-
mutations of the rotational order represents the twelve possible Euler angle sets using three rotations,

\begin{align*}
X Y Z & \quad Y X Z & \quad Z X Y \\
X Z Y & \quad Y Z X & \quad Z Y X \\
X Y X & \quad Y X Y & \quad Z X Z \\
X Z X & \quad Y Z Y & \quad Z Y Z
\end{align*}

(9)

Any repeated axis rotation such as XXY does not represent a three axis rotation but reduces to the two axis rotation XY. Hence the rotations described in (9) above represent all twelve possible sets of Euler angle defining sequences. Conversely then, for a given transformation matrix there are twelve Euler angle sets which can be extracted from the matrix.

The Euler matrices corresponding to all possible rotational sequences of (9) above are presented in Appendix A.

The utility subroutines "EULMAT" generates the transformation matrix from a given Euler sequence and the Euler angles. The utility subroutine "MATEUL" extracts the Euler angles from a given Euler rotational sequence. The convention is established that the Euler angles occur in the same sequences as the axis rotations. Using this concept and functional notation, equation (7) could be expressed as

\[ M = X Y Z = M(\theta_x, \theta_y, \theta_z) \]

(10)

and from (9)
\[
M = X Z X = M(\theta_x, \theta_z, \theta_x') \text{ etc.} \tag{11}
\]

This convention is assumed in both subroutines and also used throughout this design note when Euler angles are used. A brief explanation of the use of these two utility subroutines is given in Appendix B.

It is interesting to note that a negative rotation in the single axis rotation matrices of equations (2), (3) and (4) will result in the formation of the transpose of the matrix. However the transpose of \( M \) in equation (7) is formed from reversing the order of multiplication and transposing the individual axis rotation equations, i.e.

\[
M^T = (X \ Y \ Z)^T = (Y \ Z)X^T = Z^T Y^T X^T. \tag{12}
\]

Hence the transposes of the matrices of (9) are easily formed by reversing the order of multiplication and transposing each single axis rotation equation. Using the notation in equations (10) and (11) above equation (12) could be written,

\[
M^T(\theta_x', \theta_y', \theta_z') = M(-\theta_z, -\theta_y, -\theta_x). \tag{13}
\]

It is recommended to avoid confusion that the forward transformation be computed and simply transposed to yield the reverse transformation matrix. All matrices of Appendix A are in the forward form, i.e. \( X = Mx \) and formed from (9).
2.2 Transformation Matrices Using the Hamilton Quaternion

The transformation matrix of equation (1) can be written as a function of the Hamilton Quaternion;

\[ q_1 = \cos \frac{\omega}{2} \]
\[ q_2 = \cos \alpha \sin \frac{\omega}{2} \]
\[ q_3 = \cos \beta \sin \frac{\omega}{2} \]
\[ q_4 = \cos \gamma \sin \frac{\omega}{2} \]

(14)

where \( \omega \) is the rotation angle about the rotation axis with \( \alpha, \beta, \) and \( \gamma \) direction angles with the \( x, y, \) and \( z \) axes respectively. Notice also that \( q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \), since \( \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \). The rotation angle, \( \omega \), is assumed positive according to the right-hand rule of axis rotation.

The matrix \( M \) becomes

\[
M = \begin{pmatrix}
(q_1^2 + q_2^2 - q_3^2 - q_4^2) & 2(q_2q_3 - q_1q_4) & 2(q_2q_4 + q_1q_3) \\
2(q_2q_3 + q_1q_4) & (q_1^2 - q_2^2 + q_3^2 - q_4^2) & 2(q_3q_4 - q_1q_2) \\
2(q_2q_4 - q_1q_3) & 2(q_3q_4 + q_1q_2) & (q_1^2 - q_2^2 - q_3^2 + q_4^2)
\end{pmatrix}
\]

(15)

For a more detailed discussion of the derivation of equation (15), see Reference 1. Using functional notation, equation (15) can be written

\[ M = M(q_1, q_2, q_3, q_4). \]

(16)

Unlike the Euler angle rotational sequences to describe the transformation matrix of equation (1), only two quaternions can be found from equation (15). The two quaternions are:
q₁   -q₁
q₂   -q₂
q₃   -q₃
q₄   -q₄

These two quaternions represent a positive rotation about
the rotation axis pointing in one direction and a positive
rotation about the same line of rotation pointing in the
opposite direction. Both quaternions of (17) satisfy equation
(15).

The utility subroutine "QMAT" generates the transformation
matrix from a given quaternion. The "QMAT" algorithm generates
the matrix as given in equation (15) without duplicating any
arithmetic operations. The subroutine "MATQ" extracts the positive
quaternion, i.e., q₁ > 0, from the transformation matrix and
normalizes the results to guarantee an orthogonal matrix. In
order to avoid any discontinuity in extracting the quaternion
from the transformation matrix, the procedure as described in
Reference 2 is used.

Early works by Hamilton (Reference 3) presented the quaternion
as having a scalar and a vector part, i.e.,

\[ q₁ = S \quad \vec{v} = (q₂, q₃, q₄) \]

(18)

and equation (16) could be expressed as,

\[ M = M(q₁, q₂, q₃, q₄) = M(S, \vec{v}) \]

(19)
For a given quaternion the following relationship is true (from (17) above),

\[ M(S, V) = M(-S, -V). \]  

(20)

The transpose of the transformation matrix is given by,

\[ \begin{align*} 
\mathbf{M}^\top(S, V) &= M(-S, -V) = M(S, -V). 
\end{align*} \]  

(21)

2.3 Euler Angle and Quaternion Relationships

By examination of equations (10) and (16) the equality,

\[ M(\theta_1, \theta_2, \theta_3) = M(\theta_1, \theta_2, \theta_3) = M(q_1, q_2, q_3, q_4) \]  

(22)

can be written. Based on an equality for each element of the matrix the following nine equations must be true;

\[ \begin{align*}
\cos \theta_2 \cos \theta_3 &= q_1^2 + q_2^2 - q_3^2 - q_4^2 \\
-\cos \theta_2 \sin \theta_3 &= 2(q_2q_3 - q_1q_4) \\
\sin \theta_2 &= 2(q_2q_4 + q_1q_3) \\
\cos \theta_1 \sin \theta_3 + \sin \theta_1 \sin \theta_2 \cos \theta_3 &= 2(q_2q_3 + q_1q_4) \\
\cos \theta_1 \cos \theta_3 - \sin \theta_1 \sin \theta_2 \sin \theta_3 &= q_1^2 - q_2^2 + q_3^2 - q_4^2 \\
-\sin \theta_1 \cos \theta_2 &= 2(q_3q_4 - q_1q_2) \\
\sin \theta_1 \sin \theta_3 - \cos \theta_1 \sin \theta_2 \cos \theta_3 &= 2(q_3q_4 - q_1q_3) \\
\sin \theta_1 \cos \theta_3 + \cos \theta_1 \sin \theta_2 \sin \theta_3 &= 2(q_3q_4 + q_1q_2) \\
\cos \theta_1 \cos \theta_2 &= q_1^2 - q_2^2 - q_3^2 + q_4^2.
\end{align*} \]  

(23)

It is possible to solve for the values of the quaternion using the trigonometric half angle identities. For this Euler sequence, i.e. \( X(\theta_1) \ Y(\theta_2) \ Z(\theta_3) \), the following quaternion results;
\begin{align*}
q_1 &= -\sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2} \sin \frac{\theta_3}{2} + \cos \frac{\theta_1}{2} \cos \frac{\theta_2}{2} \cos \frac{\theta_3}{2} \\
q_2 &= +\sin \frac{\theta_1}{2} \cos \frac{\theta_2}{2} \cos \frac{\theta_3}{2} + \sin \frac{\theta_2}{2} \sin \frac{\theta_3}{2} \cos \frac{\theta_1}{2} \\
q_3 &= -\sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2} \cos \frac{\theta_3}{2} + \sin \frac{\theta_2}{2} \cos \frac{\theta_1}{2} \cos \frac{\theta_3}{2} \\
q_4 &= +\sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2} \cos \frac{\theta_3}{2} + \sin \frac{\theta_3}{2} \cos \frac{\theta_1}{2} \cos \frac{\theta_2}{2}
\end{align*}

(24)

Appendix A gives the quaternion as a function of the Euler angles for each of the twelve Euler rotational sequence presented in Section 2.1. The special equations for the quaternion as functions of the Euler angles, like equations (24) above, are sometimes cumbersome to use, especially when multiple Euler sequences are utilized. Less coding, but perhaps more computer operations, are required by using the more general method of first generating the matrix M from the given Euler angle set and then simply extracting the quaternion from the matrix. This method is effected by first a call to "EULMAT" and then a call to "MATQ".
3.0 REFERENCES


APPENDIX A

RELATIONSHIPS FOR THE THREE-AXIS EULER ANGLE ROTATION SEQUENCES

The twelve Euler matrices for each rotation sequence are given based on the single axis rotation equations (2), (3) and (4). The Euler matrices transform vectors from the system that has been rotated into vectors in the stationary system. Also presented here are the equations for the quaternion as a function of the Euler angles and the Euler angles as a function of the matrix elements for each rotation sequence.
(1) $M = M(X(\theta_1), Y(\theta_2), Z(\theta_3)) = XYZ$

Axis Rotation Sequence: 1, 2, 3

$$M = \begin{bmatrix}
\cos \theta_2 \cos \theta_3 & -\cos \theta_2 \sin \theta_3 & \sin \theta_2 \\
\sin \theta_1 \sin \theta_2 \cos \theta_3 & -\sin \theta_1 \sin \theta_2 \sin \theta_3 & -\sin \theta_1 \cos \theta_2 \\
+\cos \theta_1 \sin \theta_3 & +\cos \theta_1 \cos \theta_3 & +\sin \theta_1 \cos \theta_3 \\
-\cos \theta_1 \sin \theta_2 \cos \theta_3 & \cos \theta_1 \sin \theta_2 \sin \theta_3 & \cos \theta_1 \cos \theta_2 \\
+\sin \theta_1 \sin \theta_3 & +\sin \theta_1 \cos \theta_3 & 0
\end{bmatrix}$$

$q_1 = -\sin \theta_1 \sin \theta_2 \sin \theta_3 + \cos \theta_1 \cos \theta_2 \cos \theta_3$
$q_2 = \sin \theta_1 \cos \theta_2 \cos \theta_3 + \sin \theta_2 \sin \theta_3 \cos \theta_1$
$q_3 = -\sin \theta_1 \sin \theta_2 \cos \theta_3 + \sin \theta_2 \cos \theta_1 \cos \theta_3$
$q_4 = \sin \theta_1 \sin \theta_2 \cos \theta_3 + \sin \theta_2 \cos \theta_1 \cos \theta_3$

$$\theta_1 = \tan^{-1}\left(\frac{m_{23}}{m_{33}}\right)$$
$$\theta_2 = \tan^{-1}\left(\frac{m_{12}}{\sqrt{1-m_{13}^2}}\right)$$
$$\theta_3 = \tan^{-1}\left(\frac{-m_{12}}{m_{11}}\right)$$
(2) \( M = M(X(\theta_1), Z(\theta_2), Y(\theta_3)) = XZY \)

Axis Rotation Sequence: 1, 3, 2

\[
M = \begin{bmatrix}
\cos\theta_2\cos\theta_3 & -\sin\theta_2 & \cos\theta_2\sin\theta_3 \\
\cos\theta_1\sin\theta_2\cos\theta_3 & \cos\theta_1\cos\theta_2 & \cos\theta_1\sin\theta_2\sin\theta_3 \\
+\sin\theta_1\sin\theta_3 & \cos\theta_1\sin\theta_3 & -\sin\theta_1\cos\theta_3 \\
\sin\theta_1\sin\theta_2\cos\theta_3 & \sin\theta_1\cos\theta_2 & \sin\theta_1\sin\theta_2\sin\theta_3 \\
-\cos\theta_1\sin\theta_3 & \sin\theta_1\cos\theta_3 & +\cos\theta_1\cos\theta_3
\end{bmatrix}
\]

\( q_1 = +\sin\theta_1\sin\theta_2\sin\theta_3 + \cos\theta_1\cos\theta_2\cos\theta_3 \)

\( q_2 = +\sin\theta_1\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3\cos\theta_1 \)

\( q_3 = -\sin\theta_1\sin\theta_2\cos\theta_3 + \sin\theta_3\cos\theta_1\cos\theta_2 \)

\( q_4 = +\sin\theta_1\sin\theta_2\cos\theta_3 + \sin\theta_2\cos\theta_1\cos\theta_3 \)

\( \theta_1 = \tan^{-1}\left(\frac{m_{32}}{m_{22}}\right) \)

\( \theta_2 = \tan^{-1}\left(\frac{-m_{12}}{\sqrt{1-m_{12}^2}}\right) \)

\( \theta_3 = \tan^{-1}\left(\frac{m_{13}}{m_{11}}\right) \)
Axis Rotation Sequence: 1, 2, 1

\[
M = \begin{bmatrix}
\cos \theta_2 & \sin \theta_2 \sin \theta_3 & \sin \theta_2 \cos \theta_3 \\
\sin \theta_1 \sin \theta_2 & \cos \theta_1 \cos \theta_3 & -\cos \theta_1 \sin \theta_3 \\
-\cos \theta_1 \sin \theta_2 & \sin \theta_1 \cos \theta_3 & \cos \theta_1 \cos \theta_2 \cos \theta_3 \\
\end{bmatrix}
\]

\[
q_1 = \cos \theta_2 \cos \left(\frac{1}{2}(\theta_1 + \theta_3)\right)
\]

\[
q_2 = \cos \theta_2 \sin \left(\frac{1}{2}(\theta_1 + \theta_3)\right)
\]

\[
q_3 = \sin \theta_2 \cos \left(\frac{1}{2}(\theta_1 - \theta_3)\right)
\]

\[
q_4 = \sin \theta_2 \sin \left(\frac{1}{2}(\theta_1 - \theta_3)\right)
\]

\[
\theta_1 = \tan^{-1}\left(\frac{m_{21}}{-m_{31}}\right)
\]

\[
\theta_2 = \tan^{-1}\left(\frac{\sqrt{1 - m_{11}^2}}{m_{11}}\right)
\]

\[
\theta_3 = \tan^{-1}\left(\frac{m_{12}}{m_{13}}\right)
\]
\[ M = M(x(\theta_1), z(\theta_2), x(\theta_3)) = XZX \]

Axis Rotation Sequence: 1, 3, 1

\[
M = \begin{bmatrix}
\cos \theta_2 & -\sin \theta_2 \cos \theta_3 & \sin \theta_2 \sin \theta_3 \\
\cos \theta_1 \sin \theta_2 & \cos \theta_1 \cos \theta_2 \cos \theta_3 & -\cos \theta_1 \cos \theta_2 \sin \theta_3 \\
\sin \theta_1 \sin \theta_2 & \sin \theta_1 \cos \theta_2 \cos \theta_3 & -\sin \theta_1 \cos \theta_2 \sin \theta_3 \\
\end{bmatrix}
\]

\[ q_1 = \cos \theta_2 \cos \left( \frac{1}{2}(\theta_1 + \theta_3) \right) \]
\[ q_2 = \cos \theta_2 \sin \left( \frac{1}{2}(\theta_1 + \theta_3) \right) \]
\[ q_3 = -\sin \theta_2 \sin \left( \frac{1}{2}(\theta_1 - \theta_3) \right) \]
\[ q_4 = \sin \theta_2 \cos \left( \frac{1}{2}(\theta_1 - \theta_3) \right) \]

\[ \theta_1 = \tan^{-1} \left( \frac{m_{31}}{m_{21}} \right) \]

\[ \theta_2 = \tan^{-1} \left( \frac{\sqrt{1 - m_{11}^2}}{m_{11}} \right) \]

\[ \theta_3 = \tan^{-1} \left( \frac{m_{13}}{-m_{12}} \right) \]
(5) \( M = M(Y(\theta_1), X(\theta_2), Z(\theta_3)) \) = YXZ

Axis Rotation Sequence: 2, 1, 3

\[
M = \begin{bmatrix}
\sin\theta_1\sin\theta_2\sin\theta_3 + \cos\theta_1\cos\theta_2 \\
\cos\theta_2\sin\theta_3 \\
\cos\theta_1\sin\theta_2\sin\theta_3 - \sin\theta_1\cos\theta_2 \\
-sin\theta_1\cos\theta_3 \\
\end{bmatrix}
\]

\[
q_1 = \sin^2\theta_1\sin^2\theta_2\sin^2\theta_3 + \cos^2\theta_1\cos^2\theta_2\cos^2\theta_3
\]

\[
q_2 = \sin^2\theta_1\sin^2\theta_3\cos^2\theta_2 + \sin^2\theta_2\cos^2\theta_1\sin^2\theta_3
\]

\[
q_3 = \sin^2\theta_1\cos^2\theta_2\cos^2\theta_3 - \sin^2\theta_2\sin^2\theta_3\cos^2\theta_1
\]

\[
q_4 = -\sin^2\theta_1\sin^2\theta_2\cos^2\theta_3 + \sin^2\theta_3\cos^2\theta_1\cos^2\theta_2
\]

\[
\theta_1 = \tan^{-1}\left(\frac{m_{31}}{m_{33}}\right)
\]

\[
\theta_2 = \tan^{-1}\left(\frac{-m_{23}}{\sqrt{1-m_{23}^2}}\right)
\]

\[
\theta_3 = \tan^{-1}\left(\frac{m_{21}}{m_{22}}\right)
\]
(6) \( M = M(\gamma(\theta_1), \ Z(\theta_2), \ X(\theta_3)) = YZX \)

Axis Rotation Sequence: 2, 3, 1

\[
M = \begin{bmatrix}
cos\theta_1cos\theta_2 & -cos\theta_1sin\theta_2cos\theta_3 & cos\theta_1sin\theta_2sin\theta_3 \\
+sin\theta_1sin\theta_3 & +sin\theta_1cos\theta_3 & \\
-sin\theta_1cos\theta_2 & sin\theta_1sin\theta_2cos\theta_3 & -sin\theta_1sin\theta_2sin\theta_3 \\
+cos\theta_1sin\theta_3 & +cos\theta_1cos\theta_3 & \\
\end{bmatrix}
\]

\[
q_1 = -sin^2\theta_1sin^2\theta_2sin^2\theta_3 + cos^2\theta_1cos^2\theta_2cos^2\theta_3
\]

\[
q_2 = +sin^2\theta_1sin^2\theta_2cos^2\theta_3 + sin^2\theta_3cos^2\theta_1cos^2\theta_2
\]

\[
q_3 = +sin^2\theta_1cos^2\theta_2cos^2\theta_3 + sin^2\theta_2sin^2\theta_3cos^2\theta_1
\]

\[
q_4 = -sin^2\theta_1sin^2\theta_3cos^2\theta_2 + sin^2\theta_2cos^2\theta_1cos^2\theta_3
\]

\[
\theta_1 = tan^{-1}\left(-\frac{m_{31}}{m_{11}}\right)
\]

\[
\theta_2 = tan^{-1}\left(-\frac{m_{21}}{\sqrt{1-m_{21}^2}}\right)
\]

\[
\theta_3 = tan^{-1}\left(-\frac{m_{23}}{m_{22}}\right)
\]
(7) \( M = M(Y(\theta_1), X(\theta_2), Y(\theta_3)) = YXY \)

Axis Rotation Sequence: 2, 1, 2

\[
M = \begin{bmatrix}
-sin\theta_1 cos\theta_2 sin\theta_3 & sin\theta_1 sin\theta_2 & sin\theta_1 cos\theta_2 cos\theta_3 \\
+cos\theta_1 cos\theta_3 & cos\theta_2 & +cos\theta_1 sin\theta_3 \\
sin\theta_2 sin\theta_3 & cos\theta_2 & -sin\theta_2 cos\theta_3 \\
-cos\theta_1 cos\theta_2 sin\theta_3 & cos\theta_1 sin\theta_2 & cos\theta_1 cos\theta_2 cos\theta_3 \\
-sin\theta_1 cos\theta_3 & -sin\theta_1 sin\theta_3 & -sin\theta_1 sin\theta_3
\end{bmatrix}
\]

\( q_1 = +cos^2\theta_2 cos(\frac{1}{2}(\theta_1 + \theta_3)) \)

\( q_2 = +sin^2\theta_2 cos(\frac{1}{2}(\theta_1 - \theta_3)) \)

\( q_3 = +cos^2\theta_2 sin(\frac{1}{2}(\theta_1 + \theta_3)) \)

\( q_4 = -sin^2\theta_2 sin(\frac{1}{2}(\theta_1 - \theta_3)) \)

\( \theta_1 = \tan^{-1}\left(\frac{m_{12}}{m_{32}}\right) \)

\( \theta_2 = \tan^{-1}\left(\frac{\sqrt{1-m_{22}^2}}{m_{22}}\right) \)

\( \theta_3 = \tan^{-1}\left(\frac{m_{21}}{-m_{23}}\right) \)
(8) \[ M = M(Y(\theta_1), Z(\theta_2), Y(\theta_3)) = YZY \]

**Axis Rotation Sequence: 2, 3, 2**

\[
M = \begin{bmatrix}
\cos \theta_1 \cos \theta_2 \cos \theta_3 & -\cos \theta_1 \sin \theta_2 & \cos \theta_1 \cos \theta_2 \sin \theta_3 + \sin \theta_1 \cos \theta_3 \\
-\sin \theta_1 \sin \theta_3 & \cos \theta_1 & \sin \theta_1 \cos \theta_2 \sin \theta_3 \\
\sin \theta_2 \cos \theta_3 & \cos \theta_2 & \sin \theta_2 \sin \theta_3 \\
-\sin \theta_1 \cos \theta_2 \cos \theta_3 & \sin \theta_1 \sin \theta_2 & -\sin \theta_1 \cos \theta_2 \sin \theta_3 + \cos \theta_1 \cos \theta_3 \\
-\cos \theta_1 \sin \theta_3 & \sin \theta_1 & -\cos \theta_1 \cos \theta_2 \sin \theta_3 + \sin \theta_1 \cos \theta_3
\end{bmatrix}
\]

\[ q_1 = \cos \theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3)) \]
\[ q_2 = \sin \theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3)) \]
\[ q_3 = \cos \theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3)) \]
\[ q_4 = \sin \theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3)) \]

\[ \theta_1 = \tan^{-1}\left(\frac{m_{32}}{-m_{12}}\right) \]

\[ \theta_2 = \tan^{-1}\left(\frac{\sqrt{1 - m_{22}^2}}{m_{22}}\right) \]

\[ \theta_3 = \tan^{-1}\left(\frac{m_{23}}{m_{21}}\right) \]
(9) \( M = M(Z(\theta_1), X(\theta_2), Y(\theta_3)) = ZXY \)

Axis Rotation Sequence: 3, 1, 2

\[
M = \begin{bmatrix}
-sin\theta_1 sin\theta_2 sin\theta_3 & -sin\theta_1 cos\theta_2 & sin\theta_1 sin\theta_2 cos\theta_3 \\
+cos\theta_1 cos\theta_3 & cos\theta_1 cos\theta_2 & -cos\theta_1 sin\theta_2 cos\theta_3 \\
+sin\theta_1 cos\theta_3 & cos\theta_1 sin\theta_2 & +sin\theta_1 sin\theta_3 \\
-cos\theta_2 sin\theta_3 & sin\theta_2 & cos\theta_2 cos\theta_3
\end{bmatrix}
\]

\[
q_1 = -sin^2\theta_1 sin^2\theta_2 sin^2\theta_3 + cos^2\theta_1 cos^2\theta_2 cos^2\theta_3
\]

\[
q_2 = -sin^2\theta_1 sin^2\theta_3 cos^2\theta_2 + sin^2\theta_2 cos^2\theta_1 cos^2\theta_3
\]

\[
q_3 = +sin^2\theta_1 sin^2\theta_2 cos^2\theta_3 + sin^2\theta_3 cos^2\theta_1 cos^2\theta_2
\]

\[
q_4 = +sin^2\theta_1 cos^2\theta_2 cos^2\theta_3 + sin^2\theta_2 sin^2\theta_3 cos^2\theta_1
\]

\[
\theta_1 = \tan^{-1} \left( \frac{-m_{12}}{m_{22}} \right)
\]

\[
\theta_2 = \tan^{-1} \left( \frac{m_{32}}{\sqrt{1-m_{32}^2}} \right)
\]

\[
\theta_3 = \tan^{-1} \left( \frac{-m_{31}}{m_{33}} \right)
\]
(10) \( M = M(Z(\theta_1), Y(\theta_2), X(\theta_3)) = ZYX \)

Axis Rotation Sequence: 3, 2, 1

\[
M = \begin{bmatrix}
\cos \theta_1 \cos \theta_2 & \cos \theta_1 \sin \theta_2 \sin \theta_3 & \cos \theta_1 \sin \theta_2 \cos \theta_3 \\
-sin \theta_1 \cos \theta_2 & \sin \theta_1 \sin \theta_2 \sin \theta_3 & \sin \theta_1 \sin \theta_2 \cos \theta_3 \\
-sin \theta_2 & \cos \theta_2 \sin \theta_3 & \cos \theta_2 \cos \theta_3
\end{bmatrix}
\]

q_1 = +\sin \frac{1}{2} \theta_1 \sin \frac{1}{2} \theta_2 \sin \frac{1}{2} \theta_3 + \cos \frac{1}{2} \theta_1 \cos \frac{1}{2} \theta_2 \cos \frac{1}{2} \theta_3

q_2 = -\sin \frac{1}{2} \theta_1 \sin \frac{1}{2} \theta_2 \cos \frac{1}{2} \theta_3 + \sin \frac{1}{2} \theta_2 \cos \frac{1}{2} \theta_1 \cos \frac{1}{2} \theta_2

q_3 = +\sin \frac{1}{2} \theta_1 \sin \frac{1}{2} \theta_2 \cos \frac{1}{2} \theta_3 + \sin \frac{1}{2} \theta_2 \cos \frac{1}{2} \theta_1 \cos \frac{1}{2} \theta_3

q_4 = +\sin \frac{1}{2} \theta_1 \cos \frac{1}{2} \theta_2 \cos \frac{1}{2} \theta_3 - \sin \frac{1}{2} \theta_2 \sin \frac{1}{2} \theta_3 \cos \frac{1}{2} \theta_1

\[ \theta_1 = \tan^{-1} \left( \frac{m_{21}}{m_{11}} \right) \]

\[ \theta_2 = \tan^{-1} \left( \frac{-m_{31}}{\sqrt{1-m_{31}^2}} \right) \]

\[ \theta_3 = \tan^{-1} \left( \frac{m_{32}}{m_{33}} \right) \]
(11) \( M = M(Z(\theta_1), X(\theta_2), Z(\theta_3)) = ZXZ \)

Axis Rotation Sequence: 3, 1, 3

\[
M = \begin{bmatrix}
-\sin\theta_1\cos\theta_2\sin\theta_3 & -\sin\theta_1\cos\theta_2\cos\theta_3 & \sin\theta_1\sin\theta_2 \\
\cos\theta_1\cos\theta_2\sin\theta_3 & -\cos\theta_1\cos\theta_2\cos\theta_3 & \cos\theta_1\sin\theta_2 \\
\sin\theta_2\sin\theta_3 & \sin\theta_2\cos\theta_3 & \cos\theta_2
\end{bmatrix}
\]

\[
q_1 = +\cos\theta_2\cos(\frac{1}{2}(\theta_1 + \theta_3))
\]
\[
q_2 = +\sin\theta_2\cos(\frac{1}{2}(\theta_1 - \theta_3))
\]
\[
q_3 = +\sin\theta_2\sin(\frac{1}{2}(\theta_1 - \theta_3))
\]
\[
q_4 = +\cos\theta_2\sin(\frac{1}{2}(\theta_1 + \theta_3))
\]

\[
\theta_1 = \tan^{-1}\left(\frac{m_{13}}{-m_{23}}\right)
\]

\[
\theta_2 = \tan^{-1}\left(\frac{\sqrt{1-m_{33}^2}}{m_{33}}\right)
\]

\[
\theta_3 = \tan^{-1}\left(\frac{m_{31}}{m_{32}}\right)
\]
12) \( M = M(Z(\theta_1), Y(\theta_2), Z(\theta_3)) = ZYZ \)

Axis Rotation Sequence: 3, 2, 3

\[
M = \begin{bmatrix}
\cos \theta_1 \cos \theta_2 \cos \theta_3 & -\cos \theta_1 \cos \theta_2 \sin \theta_3 & \cos \theta_1 \sin \theta_2 \\
-\sin \theta_1 \sin \theta_3 & -\sin \theta_1 \cos \theta_3 & \sin \theta_1 \sin \theta_2 \\
\sin \theta_1 \cos \theta_2 \cos \theta_3 & -\sin \theta_1 \cos \theta_2 \sin \theta_3 & \sin \theta_1 \sin \theta_2 \\
+\cos \theta_1 \sin \theta_3 & +\cos \theta_1 \cos \theta_3 & \cos \theta_2
\end{bmatrix}
\]

\[
q_1 = +\cos \frac{\theta_2}{2} \cos (\frac{\theta_1 + \theta_3}{2})
\]

\[
q_2 = -\sin \frac{\theta_2}{2} \sin (\frac{\theta_1 - \theta_3}{2})
\]

\[
q_3 = +\sin \frac{\theta_2}{2} \cos (\frac{\theta_1 - \theta_3}{2})
\]

\[
q_4 = +\cos \frac{\theta_2}{2} \sin (\frac{\theta_1 + \theta_3}{2})
\]

\[
\theta_1 = \tan^{-1} \left( \frac{m_{23}}{m_{13}} \right)
\]

\[
\theta_2 = \tan^{-1} \left( \frac{\sqrt{1-m_{33}^2}}{m_{33}} \right)
\]

\[
\theta_3 = \tan^{-1} \left( \frac{m_{32}}{-m_{31}} \right)
\]
APPENDIX B

COMPUTER SUBROUTINES FOR THE RELATIONSHIPS

The following subroutines with a brief description of their use are presented in this appendix.

(1) "EULMAT" - Generates the transformation matrix from a given set of Euler angles and an axis rotation sequence.

(2) "MATEUL" - Extracts the Euler angles from the given transformation matrix and an axis rotation sequence.

(3) "QMAT" - Generates the transformation matrix from a given quaternion.

(4) "MATQ" - Extracts the quaternion from a given transformation matrix.

(5) "YPRQ" - Generates the quaternion directly from the yaw-pitch-roll Euler angles.

(6) "POSNOR" - Computes the positive-normalized quaternion from the given quaternion.
NAME: EULMAT

PURPOSE: Generates a 3 x 3 transformation matrix from a given sequence and Euler angle set.

INPUT: ISEQ - Rotation Sequence (Integer Array (3); i.e., 1, 2, 3)
        EUL - Euler Angles in radians, in "ISEQ"
        Order; ARRAY...(3)

OUTPUT: A - The 3 x 3 transformation matrix

ALGORITHM reference: Appendix A; Euler Sequences (1) thru (12).
EULER ANGLES TO THE TRANSFORMATION MATRIX

FOR IS = 1,15, FULMAT = EULMAT
FOR SSD3 - 02/19/77 - 06:24:23 L.01.

SUBROUTINE EULMAT
ENTRY POINT DCL237

STORAGE USF: CODE(1) DCL250; DATA(G) DCL104; BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)

<table>
<thead>
<tr>
<th>Block</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>103</td>
<td>SIN</td>
</tr>
<tr>
<td>104</td>
<td>COS</td>
</tr>
<tr>
<td>105</td>
<td>TAN</td>
</tr>
</tbody>
</table>

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

<table>
<thead>
<tr>
<th>Block</th>
<th>Type</th>
<th>Relative Location</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>R</td>
<td>1</td>
<td>SINA</td>
</tr>
<tr>
<td>102</td>
<td>R</td>
<td>2</td>
<td>COSA</td>
</tr>
<tr>
<td>103</td>
<td>R</td>
<td>3</td>
<td>TANA</td>
</tr>
</tbody>
</table>

SUBROUTINE EULMAT(SSL,CLU4)
DIMENSION ISED(31,UL1,1,A3,3)
DIMENSION ND(6,3,3,3),PL(5,3)
DO 10 I=1,5
DO 10 J=1,5
DO 10 K=1,7
IF(15.ge.J) CONTINUE
10 CONTINUE
IF(15.ge.K) CONTINUE
IF(15.ge.J) CONTINUE
SINA = SINA + FUL(M)
COSA = COSA + EUL(M)
IF(15.ge.K) CONTINUE
IF(15.ge.J) CONTINUE
X14 = X14 + SINA
X15 = X15 + COSA
X16 = X16 - SINA
X17 = X17 - COSA
60 TO 10
20 X11 = X11 + SINA
X12 = X12 + COSA
X13 = X13 + SINA
X14 = X14 + COSA
X15 = X15 - SINA
X16 = X16 - COSA
X17 = X17 + SINA
X18 = X18 - COSA
X19 = X19 + SINA
X20 = X20 + COSA
X21 = X21 - SINA
X22 = X22 - COSA
X23 = X23 + SINA
X24 = X24 + COSA
X25 = X25 - SINA
X26 = X26 - COSA
X27 = X27 + SINA
X28 = X28 + COSA
X29 = X29 - SINA
X30 = X30 - COSA
X31 = X31 + SINA
X32 = X32 + COSA
X33 = X33 - SINA
X34 = X34 - COSA
X35 = X35 + SINA
X36 = X36 + COSA
X37 = X37 - SINA
X38 = X38 - COSA
X39 = X39 + SINA
X40 = X40 + COSA
X41 = X41 - SINA
X42 = X42 - COSA
X43 = X43 + SINA
X44 = X44 + COSA
X45 = X45 - SINA
X46 = X46 - COSA
X47 = X47 + SINA
X48 = X48 + COSA
X49 = X49 - SINA
X50 = X50 - COSA
X51 = X51 + SINA
X52 = X52 + COSA
X53 = X53 - SINA
X54 = X54 - COSA
X55 = X55 + SINA
X56 = X56 + COSA
X57 = X57 - SINA
X58 = X58 - COSA
X59 = X59 + SINA
X60 = X60 + COSA
X61 = X61 - SINA
X62 = X62 - COSA
X63 = X63 + SINA
X64 = X64 + COSA
X65 = X65 - SINA
X66 = X66 - COSA
X67 = X67 + SINA
X68 = X68 + COSA
X69 = X69 - SINA
X70 = X70 - COSA
X71 = X71 + SINA
X72 = X72 + COSA
X73 = X73 - SINA
X74 = X74 - COSA
X75 = X75 + SINA
X76 = X76 + COSA
X77 = X77 - SINA
X78 = X78 - COSA
X79 = X79 + SINA
X80 = X80 + COSA
X81 = X81 - SINA
X82 = X82 - COSA
X83 = X83 + SINA
X84 = X84 + COSA
X85 = X85 - SINA
X86 = X86 - COSA
X87 = X87 + SINA
X88 = X88 + COSA
X89 = X89 - SINA
X90 = X90 - COSA
X91 = X91 + SINA
X92 = X92 + COSA
X93 = X93 - SINA
X94 = X94 - COSA
X95 = X95 + SINA
X96 = X96 + COSA
X97 = X97 - SINA
X98 = X98 - COSA
X99 = X99 + SINA
X100 = X100 + COSA
X101 = X101 - SINA
X102 = X102 - COSA
X103 = X103 + SINA
X104 = X104 + COSA
X105 = X105 - SINA
X106 = X106 - COSA
X107 = X107 + SINA
X108 = X108 + COSA
X109 = X109 - SINA
X110 = X110 - COSA
X111 = X111 + SINA
X112 = X112 + COSA
X113 = X113 - SINA
X114 = X114 - COSA
X115 = X115 + SINA
X116 = X116 + COSA
X117 = X117 - SINA
X118 = X118 - COSA
X119 = X119 + SINA
X120 = X120 + COSA
X121 = X121 - SINA
X122 = X122 - COSA
X123 = X123 + SINA
X124 = X124 + COSA
X125 = X125 - SINA
X126 = X126 - COSA
X127 = X127 + SINA
X128 = X128 + COSA
X129 = X129 - SINA
X130 = X130 - COSA
X131 = X131 + SINA
X132 = X132 + COSA
X133 = X133 - SINA
X134 = X134 - COSA
X135 = X135 + SINA
X136 = X136 + COSA
X137 = X137 - SINA
X138 = X138 - COSA
X139 = X139 + SINA
X140 = X140 + COSA
X141 = X141 - SINA
X142 = X142 - COSA
X143 = X143 + SINA
X144 = X144 + COSA
X145 = X145 - SINA
X146 = X146 - COSA
X147 = X147 + SINA
X148 = X148 + COSA
X149 = X149 - SINA
X150 = X150 - COSA
X151 = X151 + SINA
X152 = X152 + COSA

B-3
### Euler Angles to the Transformation Matrix

(Continued)

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>00153</td>
<td>00 460 L = 1, 2</td>
</tr>
<tr>
<td>00155</td>
<td>00 460 M = 3-L</td>
</tr>
<tr>
<td>00161</td>
<td>00 350 I = 1, 3</td>
</tr>
<tr>
<td>00164</td>
<td>00 350 J = 1, 3</td>
</tr>
<tr>
<td>00167</td>
<td>TEMPO.</td>
</tr>
<tr>
<td>00172</td>
<td>DO C0. K = 1, 3</td>
</tr>
<tr>
<td>00173</td>
<td>IF (L.EQ.1) HOLD = X(K, J, 3)</td>
</tr>
<tr>
<td>00175</td>
<td>IF (L.LT.2) HOLD = U(K, J1)</td>
</tr>
<tr>
<td>00177</td>
<td>IF (ABS(HOLD) - (1, 1, 1, 1) .GE. 1) GO TO 250</td>
</tr>
<tr>
<td>00201</td>
<td>IF (CASES(X(K, M - 1), L) + 1) .GE. 12) GO TO 250</td>
</tr>
<tr>
<td>00223</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>00224</td>
<td>END</td>
</tr>
</tbody>
</table>

End of Compilation: No Diagnostics.

---

Original page is of poor quality.
NAME: MATEUL

PURPOSE: Extracts the Euler angles from the given transformation matrix and the required Euler rotational sequence.

INPUT: ISEQ - Rotation sequence, (Integer Array (3), i.e., 1,2,3.)

A - The 3 x 3 transformation

OUTPUT: EUL - The Euler angles, in "ISEQ" order, ARRAY(3).

ALGORITHM REFERENCE: Appendix A; Euler angles as a function of the matrix elements, sequences (1) thru (12).
TRANSFORMATION MATRIX TO THE EULER ANGLES

FOR IS15 MATEUL MATEUL
FOR IS13-02/19/77-06/7/729 (b:)

SUBROUTINE MATEUL ENTRY POINT U2335

STORAGE USED: CODE(1) 20353; DATA(1) 20352; BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)
103 SORT
105 USB
1054 AIANC
1065 MRPR3

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

1131 X00354 ESL
1141 X00354 ESL
1131 X00354 ESL
1141 X00354 ESL
1131 X00354 ESL
1141 X00354 ESL
1131 X00354 ESL
1141 X00354 ESL

;

SUBROUTINE MATEUL (ISEL, IP, HFUL)
DIMENSION A(4,21,7), IS(13)
IS=043
IF(EQ.0.11) IF(IN=4.95
IF(IN=0.5) IF(IN=0.5
IF(IN=1.5) IF(IN=1.5
IF(IN=2.5) IF(IN=2.5
IF(IN=3.5) IF(IN=3.5
IF(IN=4.5) IF(IN=4.5
GO TO 5
55
GO TO 3
35
GO TO 15
GO TO 3
GO TO 3
GO TO 3
GO TO 3
GO TO 3

B-6
TRANSFORMATION MATRIX TO THE EULER ANGLES

(Continued)

| 30152 | 29* | IF (iEQk.NE.0) L=1  |
| 30153 | 30* | GO TO 30          |
| 30154 | 31* | 25. C$IGN=-1.C   |
| 30155 | 32* | IF (iEQk.NE.0) L=3 |
| 30156 | 33* | GO J=2 N=13       |
| 30157 | 34* | FNSGN=1+3        |
| 30158 | 35* | FDSGN=11*2       |
| 30159 | 36* | IF (N.EQ.2) GO TO 70 |
| 30160 | 37* | IF (N.EQ.1) GO TO 50 |
| 30161 | 38* | IF (I EQK.NE.0) GO TO 40 |
| 30162 | 39* | FNSGN=BSIGN       |
| 30163 | 40* | JJ=1             |
| 30164 | 41* | GO TO 45         |
| 30165 | 42* | JJ=7             |
| 30166 | 43* | IF (FNSGN.GT.0) FDSGN=1.0 |
| 30167 | 44* | FNUM=FNSGN*A(I,J) |
| 30168 | 45* | FDCN=FDSGN*A(I,J) |
| 30169 | 46* | GO TO 90         |
| 30170 | 47* | IF (IEQK.NE.0) GO TO 55 |
| 30171 | 48* | FNSGN=BSIGN      |
| 30172 | 49* | JJ=K             |
| 30173 | 50* | GO TO 60         |
| 30174 | 51* | 55 FDSGN=BSIGN  |
| 30175 | 52* | JJ=1             |
| 30176 | 53* | GO TO 45         |
| 30177 | 54* | IF (FNSGN.GT.0) FDSGN=1.0 |
| 30178 | 55* | FNUM=FNSGN*A(J,K) |
| 30179 | 56* | FDCN=FDSGN*A(I,J) |
| 30180 | 57* | GO TO 90         |
| 30181 | 58* | 7. IF (I EQK.NE.0) GO TO 80 |
| 30182 | 59* | FNUM=CSIGN*A(I,K) |
| 30183 | 60* | FDCN=SQRT(I=0-AT(A1,(1*K)) ) |
| 30184 | 61* | GO TO 90         |
| 30185 | 62* | 8. FNUM=SQRT(I=0-AT(A1,(1*K)) ) |
| 30186 | 63* | FDCN=AT(I) |
| 30187 | 64* | 9. COL1=ATAN2(FNUM,FDCN) |
| 30188 | 65* | CONTINUE         |
| 30189 | 66* | RETURN           |
| 30190 | 67* | END OF COMPIILATION, NO DIAGNOSTICS. |

END OF COMPIILATION, NO DIAGNOSTICS.
NAME: QMAT

PURPOSE: Generates the transformation matrix from the given quaternion.

INPUT: Q - The quaternion; ARRAY(4).

OUTPUT: A - The 3 x 3 transformation matrix

ALGORITHM REFERENCE: Equation (15) from Section 2.2.
QUATERNION TO THE TRANSFORMATION MATRIX

FOR I=1S...0MAT, QMAT
FOR S-23-02-19/77-08-024=19 (5, J)

SUBROUTINE QMAT ENTRY POINT G00077

STORAGE USED: CODE(21), CODE(33), DATA(I) BLOCK BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)
...3 NERRIS

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)
...QCO R QCO77 INJQ1 UQ R SAP009 P2 QCO0 P COQ071
...QCO R QCO77 INJQ1 UQ R QCO009 TEMP

END OF COMPILED... NO DIAGNOSTICS.
NAME: MATQ

PURPOSE: Extracts the positive quaternion from the given transformation matrix.

INPUT: A - The 3 x 3 transformation matrix

OUTPUT: Q - The positive quaternion; ARRAY(4).

ALGORITHM REFERENCE: See Reference 2.
TRANSFORMATION MATRIX TO THE QUATERNION

FOR SI

MATH-MAHO
FOR SNC3-02/19/77-06:24:21 (C)

SUBROUTINE MATHO ENTRY POINT 024269

STORAGE USED: CODE(1-5000); DATADO(5000); BLANK COMMON(T)

EXTERNAL REFERENCES (BLOCK, NAME)

SUBROUTINE MATH(A,0)
DIMENSION A(3,3),C(1,1,1,1)

GO TO 00
IF(J.EQ.0) GO TO 50
IF(J.EQ.3) GO TO 50
IF(J.EQ.4) GO TO 50

TEMP=C(1,1)+A(1,2)+A(1,3)+1
T(J)=V
GO TO 50

1: TEMPC(1,1)-A(2,2)-A(3,3)+1
T(J)=A(2,3)-A(3,3)
GO TO 50

2: TEMPC(1,1)+A(2,2)-A(3,3)+1
T(J)=A(2,3)-A(3,3)
GO TO 50

3: TEMPC(1,1)-A(2,2)+A(3,3)+1
T(J)=A(2,3)+A(3,3)
GO TO 50

4: CONTINUE
IF(I.EQ.0) GO TO 50
A(I,0)=A(I,0)
IF(I.EQ.1) A(I,1)=A(I,1)+T(11)*T(11)
TEMP=0.5*T(11)
J0 = J.5 J-214
O(I,J)=TEMP*T(J)
CONTINUE

5 J RETURN

END
NAME: YPRQ

PURPOSE: Generates the quaternion directly from the yaw-pitch-roll Euler angles, i.e., a 3, 2, 1 Euler sequence.

INPUT: YPR - The yaw-pitch-roll Euler angles; ARRAY (3).

OUTPUT: QO - The positive quaternion, ARRAY (4).

ALGORITHM REFERENCE: Appendix A, the quaternion equations for Euler sequence (10), a 3, 2, 1 sequence.

NOTE: This subroutine calls "POSNOR" to take the normal of the positive quaternion.
SUBROUTINE YPRO
ENTRY POINT YPRO

STORAGE USED: CODE(1) MOD121: DATA(D) MOD229: BLANK COMMON(2) CU

EXTERNAL REFERENCES, (BLOCK, NAME)

JO03 POSNOR
JO04 COS
JO05 SIN
JO06 NEPR31

STORAGE ASSIGNMENT, (BLOCK, TYPE, RELATIVE LOCATION, NAME)

JO00 R D111: CP
JO00 R D111: HY
JO00 R D111: SY

END OF COMPILATION: NO DIAGNOSTICS.
NAME: POSNOR

PURPOSE: To output the positive and normalized quaternion from the given quaternion.

INPUT: Q - The quaternion; ARRAY (4).

OUTPUT: QO - The positive-normalized quaternion; ARRAY (4).

ALGORITHM REFERENCE:

1. If the sign of Q(1) is negative:
   Set QO(I) = -Q(I) for I = 1, 2, 3, 4.

2. Set QO(I) = QO(I)/TEMP
   where TEMP = \sqrt{QQ_1^2 + QQ_2^2 + QQ_3^2 + QQ_4^2}
SELECTIONS THE POSITIVE QUATERNION AND NORMALIZES

FOR IS = 0 TO 18
FOR J = 0 TO 18

SUBROUTINE POSNOR ENTRY POINT 000005

STORAGE USED: CODE(11), DATA(2), BLANK COMMON(2, ) C

EXTERNAL REFERENCES (BLOCK, NAME)

2000 SUBRI
2004 VLOGRS

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

3001 SUBRIV 111 6361 . JD036 1216 1900 I. DORV
3002 END

SUBROUTINE POSNOR(I, J)

DIMENSION Q14(0), G0(0)

T = C1(I) + C(I,J) TEMP = 1.0

SUM = 50 SUM = SUM + G0(I)*G0(J)

DO 1=1, 4

CONTINUE

TEMP = 0.0/ SQRT(SUM)

END

END OF COMPILED: NO DIAGNOSTICS.

ORIGINAL PAGE IS OF POOR QUALITY