STIRLING ENGINE DESIGN MANUAL

William R. Martini
University of Washington

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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for
U.S. DEPARTMENT OF ENERGY
Office of Conservation and Solar Applications
Division of Transportation Energy Conservation
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1. SUMMARY

Interest in the application of Stirling engines to serve a variety of power producing needs has increased considerably over the past several years. This interest has been generated principally by the potentials for high efficiency and low emissions offered by the Stirling engine coupled with its inherent quietness and capability to operate with a variety of fuels or using a variety of heat sources.

The DOE Office of Conservation, Division of Transportation Energy Conservation, has established a number of broad programs aimed at reducing highway vehicle fuel consumption. The DOE Stirling Engine Highway Vehicle Systems Program is one such program. This program is directed at the development of the Stirling engine as a possible alternative to the spark-ignition engine.

Project Management responsibility for this project has been delegated by DOE to the NASA-Lewis Research Center. Support for the generation of this design manual was provided by a grant from the Lewis Research Center Stirling Engine Project Office.

For Stirling engines to enjoy widespread application and acceptance, not only must the fundamental operation of such engines be widely understood, but the requisite analytic tools for the simulation, design, evaluation and optimization of Stirling engine hardware must be readily available. At the present time, the most highly developed and verified analytic programs are proprietary to specific corporations.

The purpose of this design manual is to provide an introduction to Stirling cycle heat engines, to organize and identify the available Stirling engine literature, and to identify, organize, evaluate and, in so far as possible, compare non proprietary Stirling engine design methodologies. As such, the manual then represents a first step in the long process of making available comprehensive, well verified, economic-to-use, Stirling engine analytic programs.

The basic principles of heat engines are explained. A Stirling engine is defined as a heat engine that moves a body of gas around in such a way as to compress the gas principally in the cold part of the engine and expand it principally in the hot part of the engine. Heat is supplied and removed through the walls of the engine.

In introducing Stirling engines, the variety of Stirling engine types and their utility in comparison to other machines are discussed. Useful Stirling engines are or can be built from an output of a few watts to a megawatt. Power density is usually as high as a diesel engine and can approach a gasoline automobile engine. Efficiencies 30% higher than an automobile engine are projected. For complete engines designed to power vehicles experience shows that a current flame-heated Stirling engine powering all auxiliaries realized no more than 58% of the Carnot efficiency for the heat source and heat sink temperatures employed. Stirling engines can be designed to utilize almost any source of heat, are inherently quiet and can be made reversible.
The theory of Stirling engine is presented starting from simple cycle analysis. Important conclusions from cycle analysis are: 1) Compared to an engine with zero unswept gas volume (dead volume), the power available from an engine with dead volume is reduced proportional to the ratio of the dead volume to the maximum gas volume, and 2) At the usual dead volume ratios of greater than 50% used in Stirling engines the error in computing the work per cycle using one easy to compute isothermal spaces instead of the more realistic but more difficult to compute adiabatic spaces is 1 to 2%. 

Engine design methods are organized as first order, second order and third order with increased order number indicating increased complexity. 

First order design methods employ the classical Schmidt equation and are principally useful in preliminary systems studies to evaluate how well-optimized engines may perform in a given heat engine application. 

Second order design methods also utilize the Schmidt equation, but, in addition, incorporate engine loss relationships that apply generally for the full engine cycle. This method assumes that the different processes going on in the engine interact very little. The author's second order methods are given for several different types of Stirling engines. These methods are presented in detail by using work sheets that need only be filled out for the specific case. One sample problem is presented using these work sheets. 

The literature on third order methods is quite extensive. This method solves the equations expressing the conservation of energy, mass and momentum using numerical methods. The engine is divided into many nodes and short time steps are required for a stable solution. Some third order methods assume that at each instant in time the pressure is uniform. This assumption greatly reduces computation time. If pressure is not assumed uniform, then the time step can be no longer than the time it takes for sound to travel from one node to the next. Third order design methods compute the engine performance with much fewer assumptions but require thousands of times longer computation time. Both second and third order methods must be validated by agreement with measurement of the performance of an actual engine. 

The development and testing programs for engines greater than a few horsepower are summarized. Current engines by Philips, Ford and United Stirling are described. A 10 year old engine, the GPU-3, built by General Motors for the U.S. Army and now under test at NASA-Lewis, is described in enough detail so that predictions can be made about its indicated power output and efficiency. 

All the literature now available that compares engine measurements with calculated performance is presented. Qvale gives a partial description of a Stirling engine built by Allison and claims good agreement. Rios built and fully described a Stirling cooling machine and shows that his computation method agrees with his measurements. The performance of the GPU-3 engine presently under test at NASA-Lewis was to have been presented for certain agreed upon test points. Unfortunately, the data are as yet unavailable. The predictions of the indicated output power and efficiency have been made for these test points using both the second order analysis of Martini and the third order analysis of NASA-Lewis and are presented and compared.
The tasks undertaken in this grant proved larger than anticipated. The original objective of identifying and organizing all available information on Stirling engines has been met. Over 800 publicly available references on Stirling engines are given according to year of publication, personal author, corporate author and subject. However, a thorough evaluation of all available analysis methods could not be accomplished within the allotted resources. Nevertheless, it is felt that there is benefit to be gained by making available the progress to date. At this point, most of the design methods that are described in the literature in enough detail so that others may use them are given in this manual so that the reader can use them. All of the simple methods are given. For these methods to be of known utility, they must be compared with reliable engine test data over a range of operating conditions. This comparison has not been done because the data are not available at this time. Such data are being generated at at least two government laboratories. Future support, if forthcoming will enable incorporation of such data, completion of the design methods evaluation process and production of a more comprehensive design manual.
2. INTRODUCTION

2.1 Why Should Anybody Be Interested In Stirling Engines?

For many years during the last century, Stirling engines occupied a relatively unimportant role among the kinds of engines used during that period. They were generally called air engines and were characterized by high reliability and safety, but low specific power. They lost out in the dollars-per-horsepower race with other competing machines. In the 1930's some researchers employed by the Philips Company, in Holland, recognized some possibilities in this old engine, provided modern engineering techniques could be applied. Since then, this company has invested millions of dollars and has created a very commanding position in Stirling engine technology. Their developments have lead to smooth, and quiet-running demonstration engines which have very high efficiency and can use any source of heat. They may be used for vehicle propulsion to produce a zero or low level of pollution. A great variety of experimental Stirling engines have been built from the same general principles to directly pump blood, generate electricity, or directly generate hydraulic power. Many are used as heat pumps and some can be used as both heat pumps and heat engines depending upon the adjustment. With a few notable exceptions of independent individuals who have done very good work, most of the work on Stirling engines has been done by teams of engineers funded by the giant companies of the world. The vital details of this work are generally not available. The United States government is beginning to sponsor the development of an open technology on Stirling engines and is beginning to spend large sums of money in this area. DOE contracted with the Ford Motor Company to spend 160 million dollars over the next 8 years to bring about a commercial Stirling engine (77 ap)*. DOE will supply 110 million of this sum. About 4 million dollars will be spent in the first year to better assure both parties that 30% better than conventional engine gas mileage can be obtained with the Ford-Philips Stirling engine. Also DOE has announced that a second team composed of Mechanical Technology Incorporated, Latham, New York; United Stirling of Malmo, Sweden, and American Motors will be negotiated with to help establish the development base of component, subsystem and system designs, fabrication technology, test experience and assessment of cost and market ability necessary to support a decision by 1984 by the U.S. automobile industry to establish a production engineering program for the Stirling engine" (77 af). Since for many engineers interest must follow money, now for the first time a reason for beginning to become familiar with the interesting and varied properties of this class of thermal machines exists for a much larger group of engineers.

2.2 What Is A Stirling Engine?

Like any heat engine, the Stirling engine goes through the four basic processes of compression, heating, expansion, and cooling (See Figure 2-1). A couple of examples from everyday life may make this clearer. For instance, Figure 2-2 shows how an automobile internal combustion engine works. In this engine a gas-air mixture is compressed using work stored in the mechanical flywheel from a previous cycle. Then the gas mixture is heated by igniting it and allowing it to burn. The higher pressure gas mixture now is expanded.

* See references in Section 8
which does more work than was required for the compression and results in net work output. In this particular engine, the gas mixture is cooled very little. Nevertheless, the exhaust is discarded and a cool gas mixture is brought in through the carburetor.

Another example of the general process shown in Figure 2-1 is the closed cycle gas turbine engine (See Figure 2-3). The working gas is compressed, then it passes through a steady-flow regenerative heat exchanger to exchange heat with the hot expanded gases. More heat is added in the gas heater. The hot compressed gas is expanded which generates more energy than is required by the compressor and creates net work. To complete the cycle, the expanded gas is cooled first by the steady flow regenerative heat exchanger and then the additional cooling to the heat sink.

In the first example (Figure 2-2), the processes occur essentially in one place, one after the other in time. In the second example (Figure 2-3), these four processes all occur simultaneously in different parts of the machine. In the Stirling machine, the processes occur sequentially but partially overlapping in time. Also the processes occur in different parts of the machine but the boundaries are blurred. One of the problems which has delayed the realization of the potential of this kind of thermal machine is the difficulty in calculating with any real degree of confidence the complex processes which go on inside of a practical Stirling engine. The author has the assignment to present as much help on this subject as is presently freely available.
Figure 2-2. Example of Internal Combustion Engine.

Figure 2-3. Example of Closed Cycle Gas Turbine Engine.
A heat engine is a Stirling engine for the purpose of this book when:

1. The working fluid is contained in one body at nearly a common pressure at each instant during the cycle.

2. The working fluid is manipulated so that it is generally compressed in the colder portion of the engine and expanded generally in the hot portion of the engine.

3. Transfer of the compressed gas from the cold to the hot portion of the engine is done by manipulating the fluid boundaries without valves or real pumps. Transfer of the expanded hot gas back to the cold portion of the engine is done the same way.

4. A reversing flow regenerator (regenerative heat exchanger) may be used to increase efficiency.
The general process shown in Figure 2-1 converts heat into mechanical energy. The reverse of this process can take place in which mechanical energy is converted into heat pumping.

Figure 2-4 shows a generalized Stirling engine machine as described above. That is, a hot and a cold gas space is connected by a gas heater and cooler and regenerator. As the process proceeds to produce power, the working fluid is compressed in the cold space, transferred as a compressed fluid into the hot space where it is expanded again, and then transferred back again to the cold space. Net work is generated during each cycle equal to the area of the enclosed curve.

2.3 Major Types Of Stirling Engines

In this publication the author would like to consider the classification of Stirling engines from a more basic standpoint. Figure 2-5 shows the various design areas that must be addressed before a particular kind of Stirling engine emerges. First some type of external heat source must be determined. Heat must then be transferred through a solid into a working fluid. There must be a means of transporting this gas between the hot and cold portion of the engine and of compressing and expanding it. A regenerator is needed to improve efficiency. Power control is obviously needed as are seals to separate the working gas from the environment. Expansion and compression of the gas creates net indicated power which must be transformed by some type of linkage to create useful power. Also the waste heat from the engine must be rejected to a suitable sink.

![Figure 2-5. Stirling Engine Design Option Block Diagram.](image-url)
A wide variety of Stirling engines have been manufactured. These old engines are described very well by Finkelstein (59 c) and Walker (73 j). Usually these involve three basic types of Stirling engines. One, the alpha type, uses two pistons (See Figure 2-4 and 2-6). These pistons mutually compress the working gas in the cold space, move it to the hot space where it is expanded and then move it back. There is a regenerator and a heater and cooler in series with the hot and cold gas spaces. The other two arrangements use a piston and displacer. The piston does the compressing and expanding, and the displacer does the gas transfer from hot to cold space. The displacer arrangement with the displacer and the power piston in line is called the beta-arrangement, and the piston offset from the displacer, to allow a simpler mechanical arrangement, is called the gamma-arrangement. However, these arrangements concentrate on only one of the vital design choices which face a creative designer of Stirling engines. The other design choices under the categories shown in Figure 2-5 will now be enumerated to indicate the breadth of possible Stirling engines.

2.3.1 Heat Sources

Since heat is supplied to the outside of the Stirling engine, many kinds of heat sources can be incorporated. Almost all work so far has been toward the development of a liquid fuel burner which requires an air pre-heater to work effectively. Virtually any type of liquid fuel can be used. Some work is now being started using solid fuel burners which are mostly fluidized bed coal burning experiments in which a heat pipe transports the heat from the burning coal to the engine (76 f). For undersea application a system has been developed to react-lithium with SF$_6$ and transport heat to the engine with a heat pipe (75 f). Some researchers suggested that electrically heated thermal energy storage units coupled with Stirling engines might be better for electric vehicles than even the most advanced type of electrochemical batteries coupled with electric motors (76 c). Relatively small mirrors or lenses focusing on a small Stirling engine are being considered as potential power sources for the future (77 ac). The conclusion here is that there are many ways of using Stirling engines with less common heat sources which have definite possibilities for the future.

2.3.2 Solid-Gas Heat Transfer

The technology of how to add and remove heat from the working fluid in the Stirling engine is the most crucial of the entire Stirling engine design. There are two essential reasons for transferring heat into and out of the gas. The first is to supply the heat of expansion (heater) and remove the heat of compression (cooler). The second is to supply and remove the sensible heat as the working fluid oscillates between the hot and cold part of the engine (regenerator). It is quite clear from Figure 2-3 that the closed cycle gas turbine (Brayton Cycle) also shares these essential requirements. For both the Brayton Cycle as well as the Stirling Cycle engine frictional losses must be minimized. However, in the Brayton Cycle making the heat exchangers large to reduce friction has no effect on the capacity of the compressor-expander. However, the price that the Stirling engine pays for not having to have the cost and problems of compressors and expanders needed in the Brayton Cycle is the necessity of keeping the undisplaced or dead volume to a minimum for the heat exchangers and associated
ducts. Typically half of the engine volume is in the heat exchangers and ducts and this reduces the power output to about half of what it theoretically could be with no dead volume.

A number of early air engines (59 c) used a positive displacement compressor and expander. In these, dead volume was no problem but valving was. The valved hot gas engine at MIT (72 ar, 73 ay, 75 bf) is being used to investigate this alternative with modern engineering.

Typically in a Stirling engine the working fluid passes, as it goes from the cold space to the hot space, first, through a gas cooler made of many parallel small diameter tubes with the gas inside the tubes (See Figure 2-6), second, through a regenerator made of stacked fine mesh screens, and third, through a tubular gas heater like the cooler. Combining the heat exchangers with the variable volume spaces is theoretically a good way to greatly reduce dead volume. This is an old idea (1854 b) but has recently received renewed attention under the name thermalizers (73 p) or isothermalizers (77 h). One of the chief design challenges is to build efficient, low dead volume heat exchangers at a reasonable cost. Very high gas pressures are used because power density is proportional to average gas pressure. Losses increase only slowly with gas pressure. High quality ceramics are being considered because higher temperature and more complicated shapes might become economic compared to brazed super-alloy tubular heat exchangers. Higher temperature also greatly increases power density and adds to the efficiency.

2.3.3 Gas Transport and Power Take-Off (Seals)

Stirling engine gas transport is always positive displacement of some sort. Pistons are the usual means. The engine now receiving the most attention is a
combination of 4, 2 piston machines. However, since each piston is double acting, only four pistons are needed to implement the cycle. This is called the Rinia arrangement (See Figure 2-7) (46 d). Power take-off can be with a swashplate (See Figure 2-8) (69 f) or with a conventional crank and cross head (69 f) or with a hypocycloid crank (76 c). Piston seals cannot be oil lubricated since the oil will foul the heat exchangers quickly. Filled Teflon piston rings are usually used. The volume of gas inside the Stirling engine is very small so that any leakage greatly reduces power. Specially designed mechanical seals or
oil backed roll sock seals are used to almost eliminate leakage. Fabric reinforced roll sock seals are also a possibility (76 c), but have not received much testing. In some low power systems, leakage, sliding friction and mechanical wear have been eliminated by using diaphragms (75 l) or bellows (71 ao) instead of pistons. Finally, the free surface of a water column has been used as a power piston in some experimental water pumps (76 k) and heat pumps (75 g) to eliminate leakage and lower friction.

Rotary and shaft seals have been eliminated for a number of developments. Water pumping (71 g, 71 ap), oil pumping (77 x), and electric power output using a linear electric generator (75 n, 74 f), and gas pumping (76 al), and heat pumping (77 a, 77 bn, 73 b) are now being developed.

In the water, gas and heat pumping cases, pressure change may be due only to the transport of gas from the cold to the hot side of the displacer. If it pumps gas or liquid it is called a thermocompressor (69 x). This is equivalent to an internal combustion engine without compression before ignition of the gas mixture. Power is low but it is attractive because of its simplicity. If pressure surges are created by a single displacer operating between a hot and a warm zone, then a Vuilleumier cycle cooler is realized. This type of machine was patented by Rudolf Vuilleumier (18 a) and has recently received a lot of government development to produce reliable coolers for infrared sensors which must operate in the 20 K range.

These displacer-only type machines have been operated with a crank driven by an electric motor (76 i), and by a crank and flywheel driven by a displacer drive piston (74 n). A plug-in-orifice drive for the displacer has also been demonstrated (70 v). Latching electric solenoids can be made efficient and could be also used for the purpose of driving the displacer. Also, overcenter springs have been evaluated for this purpose.

All the displacer-only machines have their counterpart in a machine with a power piston. For instance, if the liquid or gas pump has some inertia added, then the mass-spring-damper system can be designed with a resonant frequency efficiently attainable by the Stirling engine displacer. A much larger displacement would then be possible for the same amount of gas processed by the displacer and regenerator. Therefore, more power would be generated without increasing losses. An oil pumper using a metallic inertia member and a bellows-sealed drive piston has been demonstrated to be self starting (77 x). Heat pumping using a fluid flywheel between two beta type Stirling engines is under development (77 h).

Finally, displacer-piston machines can act as thermally powered mechanical amplifiers. A small amount of power moving the displacer can control a large amount of power at the power piston. This could create a push or pull. In a multipiston engine a very even torque and a speed exactly matching the excitation frequency would be realized.

2.3.4. Power Control

The Stirling engines of the last century were controlled by heat input and by a hand operated vent valve (59 c). Heat was applied through burning wood
or coal, usually. When hot the engine was turned over once by hand and started. Water pumping was usually the task, so rapid control was not required. Modern engines are being used to generate electricity or operate vehicles. In both cases highly responsive speed control is needed. The standard method has been to rapidly add or remove working fluid (77 l). Other methods have been considered like varying the stroke of the power pistons (possible in the Philips swashplate engine (74 c)). Temporarily connecting the working gas space to the buffer space during a controlled part of each stroke also is shown to control power (73 v). Changing the phase angle between the piston and displacer during operation has been demonstrated (76 c). With this type of power control one can change during operation from positive torque to negative torque for regenerative braking and heat reclamation. For maximum efficiency at a particular speed and torque requirement one can arrange to change the stroke of the displacer at ± 90° phase angle (76 c). It is also theoretically possible to do the same thing with a Rinia swashplate machine by tilting the swashplate over center.

In a displacer-power piston machine (Beta or Gamma type) the displacer requires only a small fraction of the engine power output. Up until now we have talked about obtaining this power by mechanical linkage from the power piston through a crank mechanism which may be fixed or variable in phase angle and stroke. Engines have been built in which the displacer is driven by part of the pressure-volume energy generated by the engine but applied to the displacer drive piston instead of the power piston (72 j). Some of these engines are controllable by spoiling through a valve part of the energy applied to the displacer drive piston (77 x).

Displacers can be driven by electric, pneumatic or hydraulic means entirely independent of what happens at the power piston. A class of thermally powered actuators could be created. Also speed controlled engines analogous to a synchronous electric motor could be developed. That is, the engine would act as a heat engine or as a heat pump depending on whether the engine is driving the load or the load is driving the engine at the excitation frequency.

One can conclude that there are many useful ways of controlling Stirling engines some of which may be cheaper or more energy efficient than those now considered standard.

2.3.5 Heat Sinking

Compared to internal combustion engines, Stirling engines require a larger radiator. Compared to a Diesel engine, three times more heat must be dissipated through the radiator (75 w). A special thin corrugated radiator has been developed to allow a more powerful radiator to be installed in the same volume (74 c). The standard method is a pumped coolant loop and a heat exchanger to the air. Not much has been done with heat pipes, boiling condensing systems or fins. Possibly the cold space can be shaped with a large surface area so that direct cooling to the air through fins is a possibility.

In the heat sink area one difference between internal combustion machines and Stirling engines should not be overlooked. In an internal combustion engine the engine has to be kept warm to work well. In a Stirling engine the colder the heat sink the better the engine works. Another big advantage of a Stirling engine is that it can operate over the entire available temperature difference providing materials of construction can be found. Nobody has seriously looked at engines designed to operate between 2,000 C and ambient temperature.
2.3.6 Working Gas

Almost all the early Stirling engines used air at a minimum pressure of one atmosphere (59 c). As early as 1827 engines built by the Stirling brothers used pressurized air (59 c), but the idea did not catch on. The first engines built by Philips used pressurized air from a built-in air compressor (46 a). An analysis of all possible gases will show that hydrogen and helium are much better than any other gas. Hydrogen is best because it has the highest thermal conductivity, the lowest viscosity and a low heat capacity on a volume basis. Only a small amount of heat is needed to change its temperature. However, hydrogen permeates through metals and no container is completely impermeable. Hydrogen is also flammable, but the amount of gas employed is quite small. Also, some metals are embrittled by hydrogen. On the other hand, helium is inert and can be permanently contained in metal. It has an even lower volumetric heat capacity than hydrogen and almost as good a thermal conductivity, but the viscosity is twice that of hydrogen.Michels (76 e) showed that a Stirling engine can be designed to use either hydrogen, helium or nitrogen all with the same efficiency for the same temperature. However, the heater, cooler and regenerator of each engine would be designed quite differently. Helium and hydrogen can attain the same power density. However, an engine designed to run with hydrogen will run poorly with helium. Hydrogen has a broader range of high efficiency operation than does helium. A Stirling engine running on nitrogen or air appears to be limited to 20 to 25% of the power of a helium or hydrogen filled engine of the same displacement (76 e).

Dissociable gases like nitric oxide (67 h) have been proposed but there seems to have been no appreciation of the need for good heat transfer properties. Liquids like water have been used (31 a). Water-hydrogen has also been proposed for use in a Stirling engine (74 ao). The addition of water to the gas improves the power density. However, the water vaporizes at a high temperature and condenses at a low temperature. Little regeneration is possible for the water component of the working fluid. However, at moderate heat source temperatures, the simple Rankine cycle has a good efficiency compared to the maximum possible. Much more heat must be transferred throughout the same area and thickness of gas film. A better solution probably is to eliminate the permanent gas entirely to attain high rates of heat transfer. A successful displacer-power piston type-Stirling engine using gas-free water-steam was demonstrated in the artificial heart program (76 bc).

2.4 Present and Future Application Areas

At the present, the only Stirling engines that can be purchased on the open market demonstrate the principle but do not demonstrate the power density and efficiency possible with modern technology. However, the following are considered to be the future application areas.

2.4.1 Silent Electric Power

In the laboratory the application that has received the earliest attention by Philips (47 b) is the coupling of a Stirling engine to an electric generator to achieve a near silent electric power source. It appears that an engine made by FFV of Sweden will be marketed in 1979 in the United States just for that
purpose. Stirling Power Systems owned by Thetford Corp., Ann Arbor, Michigan, and FFW will be the marketing organization. Present small portable generators are unreliable and noisy. A premium priced Stirling engine machine may enjoy a good market among owners of yachts and large self-contained motor homes.

Relatively small solar heated electric generators are being studied with the idea of demonstrating their usefulness on a small scale (77 ac).

2.4.2 Reliable Electric Power

Super-reliable thermo-mechanical generators using a diaphragm Stirling engine and an oscillating electric alternator are beginning to supplant thermoelectric generators in remote power source applications (77 t, 75 z).

DOE is sponsoring two different developments for isotope powered electric power generation in remote locations. One uses the Philips Stirling engine (77 aj, 76 j). The other uses a free-piston engine and linear electric generator (76 az, 77 m).

It appears quite certain that super-reliable types of Stirling engines and electric generators will take the place of flame heated or radioisotope heated thermoelectric generators because they will be both cheaper to build and much more efficient and therefore cheaper to operate and have been demonstrated to be more durable with no degradation in efficiency that is always experienced with thermoelectric generators (77 t).

2.4.3 Motor Vehicle Power

Application of the Stirling engine to motor vehicles has to date received the most attention. A number of demonstration vehicles have been built and are in the process of being tested (77 am, 77 i, 77 aq). A Stirling engine is probably a better engine for automobiles and trucks as far as noise, performance and fuel economy is concerned. But, can the cost of the Stirling engine be reduced sufficiently to allow a saleable product to be offered at the auto showroom floor? As explained in Section 2.1, Ford Motor Co., Philips, MTI, United Stirling and American Motors are involved in development of the Stirling engine for motor vehicles.

A number of studies and proposals have shown that stored thermal energy coupled to a Stirling engine makes reasonable sense for vehicle propulsion (76 c, 74 c, 77 y). Using performance numbers from the 1975 and 1976 IECEC Records, the author performed a preliminary study of how a future propulsion system using thermal energy storage and a controlled Stirling engine would compare with future battery-electric motor propulsion systems. The table below shows the results.

<table>
<thead>
<tr>
<th>Propulsion System</th>
<th>Calculated Specific Energy</th>
<th>Calculated Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WHR(m)/Kg of System</td>
<td>WHR(m)delivered</td>
</tr>
<tr>
<td></td>
<td>at 20 W(m)/Kg System</td>
<td>WHR(e)supplied</td>
</tr>
<tr>
<td>Projected 800 C LiF Thermal Energy Storage + Controlled Stirling Engine</td>
<td>150</td>
<td>0.35 to 0.45</td>
</tr>
</tbody>
</table>
### Propulsion System Calculated Specific Energy Calculated Efficiency

<table>
<thead>
<tr>
<th>System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projected 500 C Lithium-Iron Sulfide Battery + Controls + Electric Motor</td>
</tr>
<tr>
<td>Projected Zinc-Nickel Oxide Battery + Controls + Electric Motor</td>
</tr>
<tr>
<td>Current Lead Acid Traction Battery + Controls + Electric Motor</td>
</tr>
</tbody>
</table>

The Stirling system is expected to be lighter and about as efficient as the advanced battery propulsion systems. In addition, if a practical way of rapidly transferring heat into the thermal storage can be developed then cheaper sources of heat from burning coal or wood or from concentrating solar energy could be employed.

A Stirling engine should be good for ship and boat propulsion. The low noise, reasonable power density and the higher performance due to the low heat sink temperature and especially its reliability should all be advantages to outweigh the probable higher cost. A good, reliable Stirling engine outboard should sell very well.

#### 2.4.4 Heat Pumping Power

Stirling engines in reverse, heat pumps, have enjoyed a good market in the cryogenic industry to produce liquified gases and to cool infrared sensors and the like (77 ax).

Stirling engines have also been tested to take the place of the electric motor in a common Rankine cycle heat pump for air conditioning (77 ad). One free-piston engine pump is being developed for this purpose (77 w). Engine driven heat pumps have the advantage of heating the building with both the waste heat from the engine and the product of the heat pump (77 j). Also being considered and undergoing preliminary testing are Stirling heat engine heat pumps. These could be two conventional Stirling engines connected together (73 x) or free-piston machines which eliminate much of the machinery and the seals (69 h). Using machines of this type it appears possible that the primary fuel needed to heat our buildings can be greatly reduced to less than 25% of that now being used (77 h). With this type of incentive Stirling engines for house heating and cooling may be very big in the future.
2.4.5 Biomedical Power

Miniature Stirling engines are now being developed to power an artificial heart (72, ak). Indeed this engine appears uniquely suited for this application since it is very reliable and can be made efficient in small sizes. One engine of this size has run continuously for 3.25 years and is still going (77 x). Once the blood pump compatibility with the body is improved to the order of years from the present six months then this application area will open up.

Between the tens to hundreds of horsepower required for automobiles and the few watts required for artificial hearts may be many other applications. For instance, powered wheel chairs now use a cumbersome lead-acid battery and control box between the wheels and an electric motor belt driving each large wheel. With a Stirling engine and thermal energy storage the same performance might be obtained, using a TES-Stirling engine, belt driving each wheel with the speed controlled electrically. The large battery box and controls could be dispensed with and the chair could become truly portable by being collapsible like an unpowered wheel chair. There may be many specialized applications like this.

2.4.6 Central Station Power

Many people have asked if Stirling engines are useful in the field of central station electric power. Very little has been published attempting to answer this question (68 k). R. J. Meijer (77 bc) calculates that Stirling engines can be made up to a capacity of 3,000 HP/cylinder and 500 HP/cylinder Stirling engines have been checked experimentally using part engine experiments (77 bc). Many simple but efficient machines could be used to convert heat to say hydraulic power. Then one large hydraulic motor and electric generator could produce the power. In the field of advanced electric power generation it should be emphasized that the Stirling engine can operate most efficiently over the entire temperature range available and could supplant many more complicated schemes for increasing the efficiency of electric power generation.

2.4.7 Power For Other Uses?

Who is to say whether the above list of uses is complete. As these machines come into use and many people become involved in perfecting them for their own purposes, many presently unforeseen uses may develop. A silent airplane engine may even be possible for small airplanes. The Stirling engine is still a heat engine and is limited to the Carnot efficiency as other heat engines are, but it appears to be able to approach it more closely than the others. Also the machine is inherently silent and uses fewer moving parts than most other engines. What more will inventive humans do with such a machine? Only the future can tell.
3. CURRENT LARGE ENGINES

The history of Stirling engines is fascinating. The reader is referred to Walker (73 j) or Finkelstein (59 c) for this type of information. In this section those current engines with power greater than 1 horsepower will be described. This selection leaves out model engines, and small free piston machines for pumping, refrigerant or blood or for producing electricity. The engines to be discussed are:

- N. V. Philips Co. (Netherlands)
  1-98
  4-215 (Ford testing & modifying)
- Ford Motor Company
  4-98 Double Acting Swashplate
- United Stirling of Sweden (USS)
  P-40
  P-75
  P-150
- General Motors (NASA-Lewis Testing)
  GPU-3
- FFV (A Swedish Government Owned Industrial Group)
  Auxiliary Power Unit Engine

3.1 Philips-Ford Programs

Current Philips engines use tubular gas heaters and gas coolers. The coolers are water cooled and the heaters may be heated by a flame or a heat pipe. Stacked screens with very fine wire mesh are used for the regenerator. Current types use rhombic drive with displacer and power piston (the 1-98 engine) and the 4 cylinder double acting Rinja configuration with a swashplate crank case (4-215 and the 4-98 engine). Power control is by adding and removing gas. The engines must be preheated and then cranked to start.

3.1.1 The 1-98 Engine (76 e)

About 30 of these engines were built. It has one cylinder and a piston swept volume of 98 cm³. Figure 3-1 shows one of those engines on test. Figure 3-2 shows a cross section of this type of Rhombic drive engine. The engine operates with a heater temperature from 250 C to 850 C and produces as much as 20 KW at reasonable efficiency. It is capable of delivering about 15 KW at 3,000 rpm and 220 atm gas pressure. With hydrogen working gas, with properly optimized heat exchangers and with a heater temperature of 850 C and a cooler temperature of 0 C, this engine will produce 10 KW at a shaft efficiency of about 43%. This does not include the heater efficiency (76 e).
Figure 3-1. Philips 1-98 Engine on a Test Bench.

Figure 3-2. Cross Section of a Rhombic Drive Engine.
3.1.2 The 4-215 Engine (77 k, 77 ag)

The development of the 4-215 engine started at Philips, Eindhoven, in mid 1972. Figure 3-3 shows a cross section and a picture of this engine. It uses the Rinia arrangement with 4 double-acting pistons with a swashplate drive. The major innovations incorporated into the 4-215 engine over previous swashplate engines are shown in Table 3-1. This engine was installed in a Ford Torino 4500 lb inertia weight vehicle. The best fuel economy in the vehicle was found to be 12.5 MPG on the Metro-Highway driving cycle (See Figure 3-4). This is significantly less than the 15.5 MPG that the conventional vehicle would get with emission controls. Ford has already made improvements in the engine and dynamometer tests have resulted in a simulated 14.4 MPG. Ford believes that with additional improvements now underway the objective of 15.7 MPG by October, 1978 will be reached. Ford has identified a series of modifications to the present engine which should realize a 30% improvement in gas mileage to 20.2 MPG by the end of the 160 million dollar program with DOE in 1985. Further out is the expectation that if ceramics like silicon carbide and silicon nitride can be substituted for the expensive nickel alloy hot ends, that the MPG would go to 23.2, the engine costs would drop and many more engines could be made with the available resources. Stephens, et al (77 at) calculate that 300,000 engines would consume as much cobalt as all the United States used in 1976. Scarce materials like cobalt and nickel and chromium must be used sparingly.

One of the big reasons DOE is interested in Stirling engines is that it can have high fuel economy and low pollution and low noise. Figure 3-5 shows that Ford has not been able to meet emissions standards with the vehicle engine. Previously conducted combuster rig tests at steady operating points showed very low pollution levels. Ford expects with further development to meet emission standards.

The results of other tests (Table 3-2) show that the engine is about 50 lbs over weight objective. The slower start-up and acceleration is attributed to a higher pressure drop through the combustion side of the engine than was anticipated.

Table 3-3 shows the present status and the objectives of the eight year Ford/NASA/DOE development program. Improvements in a number of categories are anticipated. However, for the program to be successful the big development has to be to lower cost. This aspect has been Ford's main concern from the beginning. Although nothing has been published on this most vital aspect, Ford must see a way to make Stirling engines economically attractive or they would not continue the program. Like any other development program, Ford has encountered quite a number of problems. Table 3-4 shows the major technical problems they have encountered and resolved. Table 3-5 shows the major technical problems but not yet resolved. These certainly are major problems but there is nothing of a basic nature. During this development program, Ford intends to utilize the same basic engine concept but with improvements in components, auxiliaries, and drive to achieve the stated objectives.

3.1.3 The 4-98 Engine (77 k)

The 4-98 Stirling engine is so far a design study of a down-sized version
4-215 170 H.P. ENGINE

- LENGTH — 36.5 INCHES
- WIDTH (MAX) — 26.0 INCHES
- HEIGHT (MAX) — 27.3 INCHES
- WEIGHT (LBS) — 713
- DISPLACEMENT (PER CYLINDER) — 215 CC
- DESIGN SPEED — 4500 RPM (MAX)

a. Cross Section

b. Assembled

Figure 3-3. Philips-Ford 2-215 Engine
Table 3-1
Major Innovations in the 4-215 Engine
(Reference 77 aq)

- 200 atmospheres working gas pressure vs. 150 atmospheres for previous engines.
- First engine with rotary ceramic preheater system.
- New air/fuel control system to satisfy dynamic requirements.
- New power control system for automobile demands.
- Three times larger than previous swash plate engines.
- Half the specific weight of previous Stirling engines.
- Packageable within existing engine compartments.
- 4000 rpm capability vs. 2000-3000 rpm of rhombic drives.
- First engine with exhaust gas recirculation.
- Unique coolant flow through cooling units.
- New lubrication system.
- First engine designed to drive full range of automotive type accessories.

Figure 3-4. Stirling Engine Fuel Economy.
Figure 3-5. Stirling Engine Emissions.

Table 3-2.
Other Ford Vehicle Test Results
(Reference 77 aq)

Acceleration (0-60 MPH Time)
Dynamometer Test = 16.0 Sec.
Objectives = 12.7 Sec.

Vehicle Weight (Curb)
Measured = 106 lbs. over baseline
Objectives = 50 lbs. over baseline

Start Up Time (Key-on To Driveaway)
Measured = 24 Sec.
Objectives = 15 Sec.
### Table 3-3

Ford Phase I Functional Status Vs. Objectives  
(Reference 77 aq)

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<td>0.04</td>
<td>0.20</td>
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</tr>
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<td>CO (Max GM/Mile)</td>
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<td>2.90</td>
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<td>NOX (Max GM/Mile)</td>
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<td>0.35</td>
<td>0.20</td>
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<td>Fuel Economy (MPG-M/H)</td>
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<td>14.4</td>
<td>15.4</td>
<td>20.6</td>
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<td>Performance (0-60 mph Time, sec.)</td>
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<td>16.0</td>
<td>14.0</td>
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<td>70</td>
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<td>5</td>
<td>Warm-up Time (Key on to Driveaway sec.)</td>
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<tr>
<td>6</td>
<td>Driveability (Jury Rating)</td>
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<td></td>
<td>6.0</td>
<td>6.0</td>
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<tr>
<td>7</td>
<td>Curb Weight (Max lbs. over Baseline)</td>
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<td>106</td>
<td>8</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Hydrogen Leakage Rate (% Loss Per Yr.)</td>
<td>15.0</td>
<td></td>
<td></td>
<td>15.0</td>
<td></td>
</tr>
</tbody>
</table>
Table 3-4
Major Technical Problems Encountered and Resolved on Philips-Ford Program (Reference 77 aq)

- Swashplate surface galling.
- Drive system noise due to non-concentric crossheads.
- Regenerator end-plate bending.
- Crankcase failure.
- Engine out of balance.
- Piston attachment failure.
- Insufficient exhaust gas recirculation.
- Unstable air/fuel control system.
- Power control contamination.

Table 3-5
Problems Encountered Yet To Be Resolved on Philips-Ford Program (Reference 77 aq)

- Roll sock seal system failure.
- Preheater leakage.
- Preheater binding.
- Fuel burning on preheater core.
- Heaterhead temperature distribution.
- Excessive warm-up time.
- Insufficient burner air supply.
- Power control instability.
- Heater head cracking.
of the 4-215 engine to be used for smaller cars which will be more common in the future. Figure 3-6 shows a section of this engine and gives some pertinent parameters. Other dimensions like the number and size of the heater head tubes, the number and size of the cooler tubes, piston diameter, regenerator wire diameters, etc., were also chosen. Many cases were evaluated to select an optimum design. The calculated performance of this optimum design is shown in Figure 3-7. Note that although the design point is at 5,400 rpm the best torque and efficiency is at 2,000 rpm. Note that the torque is exactly proportional to mean pressure. This engine performance map is used along with the characteristics of the vehicle and the requirements of the particular driving cycle to compute the vehicle performance characteristics like fuel economy, rate of acceleration, etc.

A new engine must usually fit into the engine compartment of the existing vehicle. Figure 3-8 shows the result of a design study by Ford on how the 4-98 engine would fit in a 1976 Pinto. This engine in this arrangement required that the car be lengthened 3.2 inches to accommodate it. Therefore, a front wheel drive arrangement was studied. Figure 3-9 shows that this way the engine fits well under the hood and permits a shorter front end overhang than the conventionally powered 1976 Pinto. The accessory drive arrangement is somewhat more complex, however, due to the fact that neither the fan nor the preheater drive shaft is parallel to the engine crank axis.

3.2 United Stirling Engines (77 i, 77 j, 77 a1, 77 am)

KB United Stirling (Sweden) AB & Co., was organized in 1968 as a research and development company jointly owned by Kockums and FFV. Kockums is a publicly owned company having its main business in shipbuilding and lumber industry. FFV is a government-owned industrial group. United Stirling is a licensee of NV Philips Company. United Stirling started by building rhombic drive machines. They then started on Rinia arrangement machines but have not used the swashplate of Philips but have used more nearly conventional crank drives. They also do not now use the roll-sock seal that Philips employs but have developed their own mechanical seal.

3.2.1 Application Plan

United Stirling is planning a product line of 3 engines (See Table 3-6). All three engines are intended to be available as direct flame heated versions as well as heat pipe heated versions.

United Stirling has evaluated the market for Stirling engines. They feel that the market will be penetrated starting from the upper left hand corner of Figure 3-10 and proceeding through the applications in "waves of attack". Note that in this plan Taxis and Cars would be the last application. However, if a large amount of assistance is forthcoming from DOE based upon application of the United Stirling engine to cars, this segment of the market will obviously receive early attention.

United Stirling publications indicate that field testing of preproduction prototypes will start in 1979 with mine vehicles in a Swedish iron ore mine. Figure 3-11 shows how their P-150 engine will occupy essentially the same space as the Diesel engine it replaces. United Stirling plans to give high priority to total energy systems which use 40-140 KW Stirling engines to power
4-98 84 H.P. ENGINE

- LENGTH — 25.5 INCHES
- WIDTH (MAX) — 20.6 INCHES
- HEIGHT (MAX) — 23.0 INCHES
- WEIGHT (LBS) — 374
- DISPLACEMENT (PER CYLINDER) — 98 CC
- DESIGN SPEED — 5400 RPM (MAX)

MAJOR CYCLE & COMPONENT DESIGN PARAMETERS FOR THE 4-98 STIRLING ENGINE

CYCLE PARAMETERS

- MAX MEAN PRESSURE — 200 ATM
- WORKING GAS — HYDROGEN
- HEATER INSIDE WALL TEMP. — 1023 K (1382 F)
- COOLER INSIDE WALL TEMP. — 353 K (176 F)
- MAX. ENGINE SPEED — 5400 RPM

COMPONENT PARAMETERS

- NUMBER OF CYLINDERS — 4
- SWASHPLATE ANGLE — 18
- SWEPT VOLUME/CYLINDER — 98 CM³
- VOLUMETRIC RATIO OF EXPANSION — COMPRESSION — 1.10
- REGENERATOR FILLING FACTOR — 38%

Figure 3.6. The 4-98 Engine Partial Description
Figure 3-7.
4-98 STIRLING ENGINE
NET TORQUE VS.
ENGINE SPEED

Figure 3-8. Packaging of the Ford 4-98 engine in a 1976 Pinto, Rear Wheel Drive.
Figure 3-9. Packaging of the Ford 4-98 Engine in a 1976 Pinto, Front Wheel Drive.

Table 3-6. Performance Targets for United Stirling Product Line

<table>
<thead>
<tr>
<th>Engine Number</th>
<th>P40</th>
<th>P75</th>
<th>P150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>40 KW</td>
<td>75 KW</td>
<td>150 KW</td>
</tr>
<tr>
<td>RPM</td>
<td>4000</td>
<td>2400</td>
<td>2400</td>
</tr>
<tr>
<td>Number of cyl.</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Max efficiency (installed in vehicles)</td>
<td>35%</td>
<td>37%</td>
<td>37%</td>
</tr>
<tr>
<td>Weight (with auxiliaries)</td>
<td>180 kg</td>
<td>350 kg</td>
<td>650 kg</td>
</tr>
</tbody>
</table>
electric generators and heat pumps. These would be used in shopping centers or apartment houses. United Stirling expects to realize a 45% fuel saving compared to a conventional furnace.

After this, United Stirling sees the next important application will be the city bus. Traffic authorities in many metropolitan areas are reportedly interested in obtaining engines for testing as soon as possible. United Stirling's plans would allow this to happen in 1979. At present, United Stirling has four P75 engines of the V type on test in their laboratory. They have their V4X35 experimental engine installed in a Ford Taunus station wagon for the purpose of testing engine control.

3.2.2 Engine Design

United Stirling's production engines are now expected to look like Figure 3-12. The P-40 and P-75 will have two cranks on each of two crankshafts geared to a common drive shaft. The P-150 will be two P-75's combined into one block. The four connecting rods drive the four double-acting pistons through cross heads to take the side thrust. An oil pump pressure lubricates these working parts. The different components of the engine design will now be discussed starting with the seal which is the principle design problem.
COMBUSTOR BLOWER AND AIR FILTER

EXHAUST PIPE

RADIATOR

P-150

ORIGINAL DIESEL ENGINE

FAN

Figure 3-11. United Stirling P-150 Engine in a Truck.
Figure 3-12. Concept for United Stirling Production Engines.
3.2.2.1 Seals

The seal for this engine is presented in Figure 3-13. United Stirling reports that this seal has been tested for several thousand hours in component and in engine test rigs and has gradually been developed to its present satisfactory state. The three elements of the seal are:

1. The top seal element. This seal is dry. The shaft passing through the seal has not been wet with oil. The seal seals between a varying engine pressure and the minimum engine pressure below the seal. This minimum cycle pressure is maintained by a check valve that also brings leaking working gas back into the cycle.

2. The scraper ring. This element removes the excess lubricating oil from the piston rod. Oil scraped away drains to an oil-gas separator and thence back to the crankcase.

3. The Hydrogen seal. This element maintains the pressure difference between minimum cycle pressure of the working gas and atmospheric pressure air and lubricating oil in the crank case.
Nothing is said about the piston seals but they are probably filled Teflon piston rings since these have worked well for others particularly when the piston liner is cast iron.

3.2.2.2 Gas Cooler

The gas cooler consists of many small tubes the same as the Philips engines. The regenerator is a porous mass probably also the same as in the Philips engine. These parts have to transfer the heat with little flow friction and little dead volume. An optimum design is very important to engine performance as well as cost.

3.2.2.3 Gas Heater

The gas heater to be used on United Stirling production engines is of the involute type (See Figure 3-12 and 3-14). Each of the V tubes in the ring is the same. Its inner leg is bare and straight up. Its outer leg is finned, slanted and curved so that the tubes maintain an even spacing. This gas heater was considered the best by USS from the standpoint of performance and production cost.
3.2.2.4 **Burner and Air Preheater**

The burner and air preheater is different than the Philips engine previously described. This air preheater is the counterflow type (See Figure 3-15). This substitution eliminates the machinery needed to rotate the reversing flow matrix and seal the matrix as it rotates as done in the Ford-Philips design. On the other hand, it is difficult to pack in as much surface area for heat transfer in the limited space available using the counter flow heat exchanger. No real information is yet available on the United Stirling air preheater.

The burner burns atomized fuel with swirling preheated air. Possibly secondary air is added. Figure 3-15 shows that the hot gases from the burner enter the gas heater at 2,000 C and leave at 750 C. This exhaust leaves the air preheater at 190 C. This drop from 750 C to 190 C raises incoming combustion air from 50 C to 680 C.

This burner system can get the engine started rapidly from a cold start. Figure 3-16 shows how this is done. "The heater tubes in a Stirling engine must be heated up before the starter motor is engaged. The burner blower, normally driven by the engine, is driven by a separate electric motor during the heating-up period. When the heater tube temperature has reached 400-
300 C, the starter is engaged. According to Figure 3-16, after 12 seconds of heating-up, the starter is engaged for about 2 seconds. The engine now runs at idle speed and a driver would have been able to drive away. After and additional 30 seconds the heater temperature has reached its normal level and the engine can deliver full power. This start represents 20 C ambient temperature. Tests at -32 C have been made with a slight increase in cranking time due to higher hydrodynamic losses in the drive mechanism." (77 i).

The burner system must supply heat rapidly as is seen, but it also must not overheat the working gas heater. A control system to do this is shown in Figure 3-17. An explanation from reference 77 i is given below.

With varying demand for heat in the working cycle of the engine, the air/fuel flow is controlled in such a way that heater temperature is kept constant. Thus the air/fuel control is indirectly governed by the power control. In addition the air/fuel ratio is controlled with regard to emissions. Figure 3-17 shows a system, where a Bosch K-Jetronic unit is used.

The temperature of the heater tube is measured by a thermocouple 1. The signal of the thermocouple is amplified and converted in the electronic control unit 2 to a signal controlling the position of the air throttle 3. Thus the right amount of air is delivered to the combustor via the burner air blower 4. In a slightly modified Bosch K-Jetronic unit a sensor plate 6 is installed inside a conical air

Figure 3-16. Cold Starting Sequence for the United Stirling V4X35 Engine.
passage provides a position indication of air flow rate.

The fuel from the tank 5 passes an electric pump 7 and a filter 8. The fuel pressure is held constant by a relief valve 9. The position of the sensor plate controls via a plunger 10 the amount by which a fuel metering port is opened.

The differential pressure across the metering port is maintained at a constant value by a valve 11 so that the fuel flow to the atomizer depends upon the amount the port is opened only.

The air/fuel ratio depends upon the hydraulic counter pressure controlled by a pressure regulating valve 12. Adjustment of the ratio over the load range can be achieved by a modification of the shape of the conical air passage.

3.2.2.5 Power Control

Power control of the engine is now done by changing the average gas pressure in the engine. This is the same way Philips does it although United Stirling had used dead volume control on their engine they put into a Pinto for Ford. United Stirling uses one hydrogen gas compressor operating as an auxiliary, and Philips uses two pistons on each of the four power pistons as part of an internal gas compressor. Otherwise the process is very similar. Quoting again from reference 77 i:
A simplified diagram of the power control system is shown in Figure 3-18. Main parts of the system are hydrogen storage vessel, hydrogen compressor, control valve block and a servo-system (not shown) which controls the position of the control valve.

To increase power, the control valve slide in Figure 3-18 is moved to the right. Thereby hydrogen flows from the high pressure storage vessel via the control valve to a timed supply system built into the engine. This timed supply system mainly supplies hydrogen into the cylinders when the cycle pressure is near its maximum value. A gas flow into the cylinders without the timed supply system results in an undesirable torque drop during increase of pressure.

To decrease power the slide is moved to the left. During the first part of the movement dumping of hydrogen from the engine via the compressor to the storage vessel lowers the power output. At the second part short circuiting of hydrogen between the cylinders is added, thus giving a quick decrease of power...

The link between accelerator pedal and control valve is a servosystem which for different accelerator levels moves the control valve slide in such a way that an engine pressure corresponding to desired power output will be reached and maintained.

Low idling speed is maintained by control of appropriate working pressure, using a speed sensor and the short circuiting valve.

The hydrogen compressor is an oil-free, single
stage, double-acting compressor with piston rings acting as suction valves. The displacement is 10 cubic cm and the pressure ratio is 1:10.

To unload the compressor during increase of power and steady state conditions, the suction and pressure sides of the compressor are connected to each other by a compressor short circuiting valve.

To illustrate how rapidly this power control system operated, the V4X35 engine, equipped with all auxiliaries, was disengaged from the dynamometer and speed increase and pressure response were measured. Figure 3-19 shows speed and pressure increase versus time for the free running V4X35 engine when the accelerator is suddenly depressed.

(Note: A slight short circuiting effect is maintained at low idling. Depressing the accelerator will close the short circuiting valve and start engine acceleration before pressure increases, see Figure 3-19.)

The tests show that the power control system acts fast and accurately. No torque drop could be measured and the lag time between accelerator depression and valve response was short." (from 77 1)
3.2.3 Engine Performance

Finally, some performance measurements have been published for the P150V4 engine which is also sometimes called the P75 engine since it is half the P150V8 engine. Figure 3-20 shows a cross section of this engine and Figure 3-21 shows this engine with its auxiliaries attached. Figure 3-22 shows the measured and calculated power output and efficiency for two different mean pressure levels. Note that the calculated and measured values agree closely. The maximum efficiency is 32% at 70°C cooling water. If the ambient temperature is 30°C then a 0.8 m² area radiator would be needed (77 ft²). If it is used as a marine engine, efficiency increases to 36% at 20°C cooling water.

3.3 General Motors Engine

3.3.1 History

General Motors started a cooperative effort with N.V. Philips in 1958. In 1965, GM was able to state that "The Allison Division, the Electro-Motive Division and the Research Laboratories have operated full-size, modern, practical Stirling engines for a total of 6500 hours," (65 t). Eventually, about 31,000 hours of operating time were accumulated (74 bc). The last paper from GM (69 f) talked about a 4 cylinder inline Rinea type engine and reported on the initial swashplate drive tests. About this time the GM program was cancelled never to be revived after an expenditure, reportedly, of $13 million from GM. From about 1960 to about 1966 GM Research conducted a program for the U.S. Army to produce a silent electric power source. This Ground Power Unit (GPU) development went through three different models.
Figure 3-21. P150 V4 Engine Module with Auxiliaries.

Figure 3-22. Performance Results for P150 V4 Module of P150 Engine Showing Present Development Status (United Stirling).
3.3.2 NASA-Lewis Testing

Two of the last model GPU-3 were preserved and are now being used by NASA-Lewis to obtain reliable measurements of a more or less modern type of Stirling engine. The first report on this effort (77 av) indicates that the machine is almost ready for detailed testing. Figure 3-23 shows the GPU-2. Tests have shown that the brake specific fuel consumption is about the same as that obtained by the Army in their acceptance testing (See Figure 3-24). However, the engine output falls short of that originally obtained by the Army (See Figure 3-25). The difference is suspected to be due to excessive leakage of gas past the power piston. This leak is being fixed and with minor repairs and additional instrumentation it will soon be ready for detailed testing (as of December 1977).

Figure 3-23. The General Motors GPU-3-2 Stirling Electric Ground Power Unit for Near Silent Operation (ref. 68 p). Picture Courtesy General Motors Research.
Figure 3-24. Measured Specific Fuel Consumption for the GPU-3 Engine.

Figure 3-25. Measured Engine Horsepower for the GPU-3.
3.3.3 Engine Measurements (77 bd)

Presently available physical characteristics of the GPU are given in Table 3-7. Volume displacement tests at LeRC indicate that max. gas volume exceeds calculated volume by about 20 percent. Table 3-8 gives the test points that NASA-Lewis intends to use to compare with various analytical models. The heater temperature is measured by therocouples in the working gas stream half way through the gas heater. It is considered to be a gas temperature rather than a metal temperature. Note that there is no good way of measuring the cooler metal temperature and the inlet water temperature is as close as they can get. The experimental data, when available, will be provided in an addendum to and/or a second edition of this manual along with updated engine dimensions.

3.4 FFV Engine

FFV is a Swedish government owned industrial group which is 50% owner of United Stirling. They have produced an engine to power an auxiliary electric power unit for commercial and military applications. A number of these engines are being demonstrated. The engine will be marketed in the United States by Stirling Power Systems of Ann Arbor, Michigan. This company is owned by FFV (80.5%) and Thetford Co. (19.5%), a recreational vehicle equipment supply firm of Ann Arbor, Michigan. The first technical paper on this machine will be in late 1978, and the engine generator is planned to be for sale to the general public in 1979.

![Diagram](image_url)

Figure 3-30. Rhombic Drive Schematic.
Table 3-7 GPU 3-2 Engine Dimensions and Parameters
(See Figure 3-26 for Overall Engine Schematic)

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<tr>
<td>Working Fluid</td>
<td>H₂</td>
</tr>
<tr>
<td>Design Speed</td>
<td>3000 RPM</td>
</tr>
<tr>
<td>Design Pressure</td>
<td>1000 PSIA</td>
</tr>
<tr>
<td>Design Output</td>
<td>8.0 Net Brake HP</td>
</tr>
<tr>
<td>Design Efficiency</td>
<td>26.5%</td>
</tr>
<tr>
<td>Bore</td>
<td>2.75 in.</td>
</tr>
<tr>
<td>Stroke</td>
<td>1.208 in.</td>
</tr>
<tr>
<td>Displacement</td>
<td>7.175 in.³</td>
</tr>
</tbody>
</table>

Cooler (See Figure 3-29)

<table>
<thead>
<tr>
<th></th>
<th>OLD</th>
<th>NEW COOLERS*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube Length</td>
<td>1.76 in</td>
<td>1.813 in</td>
</tr>
<tr>
<td>Heat Transfer Length</td>
<td>1.37 in</td>
<td>1.408 in</td>
</tr>
<tr>
<td>Tube I.D.</td>
<td>0.040 in</td>
<td>0.0425 in</td>
</tr>
<tr>
<td>Tube O.D.</td>
<td>0.060 in</td>
<td>0.0625 in</td>
</tr>
<tr>
<td>No of Tubes/Cylinder</td>
<td>312 (or 39 tubes/regenerator)</td>
<td>5 tubes/regenerator</td>
</tr>
<tr>
<td>Design Water Flow</td>
<td>10 GPM/Cylinder</td>
<td>100°F</td>
</tr>
<tr>
<td>Design Water Inlet Temp.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Heater

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube Length</td>
<td>9.539 in.</td>
</tr>
<tr>
<td>Heat Transfer Length</td>
<td>6.12 in.</td>
</tr>
<tr>
<td>Tube I.D.</td>
<td>.119 in.</td>
</tr>
<tr>
<td>Tube O.D.</td>
<td>.19 in.</td>
</tr>
<tr>
<td>No of tubes/cylinder</td>
<td>40 (or 5 tubes/regenerator)</td>
</tr>
<tr>
<td>Design inside wall temp.</td>
<td>1400°F</td>
</tr>
</tbody>
</table>

* New coolers will be used in the baseline tests provided they are completed in time.

** See heater tubes in Figure 3-27 for definition of locations 1 → 4.
Cold End Connecting Ducts (See Figure 3-27)

<table>
<thead>
<tr>
<th>Description</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>0.625 in. plus 0.0170 in.³</td>
</tr>
<tr>
<td>Duct I.D.</td>
<td>0.235 in.</td>
</tr>
<tr>
<td>No. of ducts/cylinder</td>
<td>8</td>
</tr>
</tbody>
</table>

Regenerators (See Figure 3-28)

<table>
<thead>
<tr>
<th>Description</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (inside)</td>
<td>0.89 in.</td>
</tr>
<tr>
<td>Diameter (inside)</td>
<td>0.89 in.</td>
</tr>
<tr>
<td>No. per cylinder</td>
<td>8</td>
</tr>
<tr>
<td>Material</td>
<td>304 Stainless Steel Wire Cloth</td>
</tr>
<tr>
<td></td>
<td>(308 Layers of 213 x 213 wires/inch X. 0.0016 in. diameter mesh)</td>
</tr>
<tr>
<td>Filler Factor</td>
<td>28.6</td>
</tr>
</tbody>
</table>

Drive (See Figure 3-30)

<table>
<thead>
<tr>
<th>Description</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connecting Rod Length</td>
<td>1.812 in.</td>
</tr>
<tr>
<td>Crank - Radius</td>
<td>0.55 in.</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.813 in.</td>
</tr>
</tbody>
</table>

Miscellaneous

<table>
<thead>
<tr>
<th>Description</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacer rod diameter</td>
<td>0.375 in.</td>
</tr>
<tr>
<td>Piston rod diameter</td>
<td>0.875 in.</td>
</tr>
<tr>
<td>Piston diameter</td>
<td>2.740 in. buffer side, 2.748 in. comp. side</td>
</tr>
<tr>
<td>Displacer diameter</td>
<td>2.740 in.</td>
</tr>
<tr>
<td>Expansion Space Clearance Volume</td>
<td>0.252 in.³</td>
</tr>
<tr>
<td>Compression Space Clearance Volume</td>
<td>0.353 in.³</td>
</tr>
<tr>
<td>Buffer Space Minimum Volume</td>
<td>17.0 in.³</td>
</tr>
<tr>
<td>Transfer Ring Dead Volume</td>
<td>0.405 in.³</td>
</tr>
<tr>
<td>Dead Volume Due to Drill Holes at top of Engines Cylinder</td>
<td>0.099 in.³</td>
</tr>
</tbody>
</table>

* This is total volume of cold end connecting ducts. No clearance volume is assumed between the displacer and the power piston.
Table 3-8
Test Points for GPU-3 for
Comparison to Various Analytical Models

Working Fluid: H₂

<table>
<thead>
<tr>
<th>Test Point</th>
<th>Engine Speed (rpm)</th>
<th>Mean Pressure (psi)</th>
<th>Heater Temp.** (F)</th>
<th>Cooler Temp.* (F)</th>
<th>Cooler Water Flow (GPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1500</td>
<td>300</td>
<td>1300</td>
<td>70</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>300</td>
<td>1300</td>
<td>70</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>2500</td>
<td>300</td>
<td>1300</td>
<td>70</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>3000</td>
<td>300</td>
<td>1300</td>
<td>70</td>
<td>6</td>
</tr>
</tbody>
</table>

Working Fluid: He

<table>
<thead>
<tr>
<th>Test Point</th>
<th>Engine Speed (rpm)</th>
<th>Mean Pressure (psi)</th>
<th>Heater Temp.** (F)</th>
<th>Cooler Temp.* (F)</th>
<th>Cooler Water Flow (GPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3000</td>
<td>600</td>
<td>1300</td>
<td>70</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>1500</td>
<td>600</td>
<td>1300</td>
<td>70</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>3000</td>
<td>400</td>
<td>1300</td>
<td>70</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>1500</td>
<td>400</td>
<td>1300</td>
<td>70</td>
<td>6</td>
</tr>
</tbody>
</table>

* Cooling Water Inlet Temp. - Is Not Controlled - Will Be At
  Ambient Temp.

** Temperature of the working gas stream at Point 5 in Figure 3-27.
Figure 3-26. Schematic of Single Cylinder Stirling Engine with Rhombic Drive.
Figure 3-27. Schematic of Working Space. (Indicates Different Regions of Heater Tubes, Compression Space Clearance Volume and Location of Heater Temp. Measurement for Baseline Tests.)
Figure 3-28. Schematic Showing Dimensions Needed for Calculating Heat Conduction. (Black Dots on Regenerator and Cylinder Indicate Where Temperatures will be Measured.)
Figure 3-29. Schematic Showing Arrangement of Regenerator-Cooler Unit Around Cylinder. (Also Indicates Path of Cooling Water Flow and Number of Heater and Cooler Tubes Per Regenerator.)
4. REVIEW OF ENGINE DESIGN METHODS

The main purpose of this publication is to teach an understanding of Stirling engines and compare and present in understandable form the available ways for designing Stirling engines. First, in cycle analysis the basic thermodynamics of the Stirling engine will be explained and the effect of important design parameters will be discussed using a theoretical stepwise engine model. Next, the methods of calculating loss-free engine output power when the engine is crank operated will be presented and compared. These methods are usually called the Schmidt analysis. This analysis when combined with experience factors is defined herein as first order analysis.

Next, second order engine analyses from a number of different sources will be presented and compared. Second order analysis starts with the Schmidt analysis or something similar. Various power losses are calculated and deducted from the Schmidt power. Various heat losses are calculated and added to the Schmidt heat. All these engine processes are assumed to proceed in parallel and independent of each other.

Finally, third order analyses will be presented. These analyses divide the engine into a number of nodes and solve the basic differential equations that govern this engine by numerical methods. Third order methods are much more laborious, but since fewer assumptions are made, prediction of engine performance is expected to be more accurate.

The task undertaken in this grant proved larger than anticipated. A thorough evaluation of all available analysis methods could not be accomplished within the scheduled resources and time allotments. However there is benefit to be gained by making available the progress to date, incomplete as it is. It is expected that future support will enable all available analytical methods to be described. It is also expected that performance measurements of a number of fully described engines will also be available. With all the analytical methods available and with reliable engine measurements to compare them against also available, the designer will finally be able to choose with confidence the design method that meets his requirements.

4.1 Stirling Engine Cycle Analysis

In this section on cycle analysis the basic thermodynamics of a Stirling engine will be explained and the effect of some necessary complications will be assessed. The thermodynamic definition of a Stirling cycle is isothermal compression and expansion and constant volume heating and cooling, 1, 2, 3, 4, 1 in Figure 4-1.

The thermodynamic definition of an Ericsson cycle is isothermal compression and expansion and constant pressure heating and cooling, 1, 2', 3, 4', 1 in Figure 4-1. This Ericsson cycle encompasses more area than the Stirling cycle and therefore, produces more work. However, the volumetric displacement is larger, therefore, the engine is larger. There is a modern pumping engine concept
which approximates this cycle (73 p). The early machines built by John Ericsson used valving to attain constant pressure heating and cooling (59 c), thus the cycle name.

The thermodynamic definition of the Otto cycle is adiabatic compression and expansion and constant volume heating and cooling, 1, 2", 3, 4", 1 in Figure 4-1. The reason this cycle is mentioned is that the variable volume spaces in a Stirling engine are usually of such size that their compression and expansion is essentially adiabatic since little heat can be transferred to the walls during the process of compression or expansion. An internal combustion engine approximates the Otto cycle. In real Stirling machines, a large portion of the gas is in the dead volume which is compressed nearly isothermally so the loss of work per cycle is not as great as shown.

![Figure 4-1. Theoretical Stirling, Ericsson and Otto Cycles.](image)

In Section 4-1, discrete processes of compression, heating, expansion and cooling will always be employed. Numerical examples will be used to make the processes clearer. The section starts with the simplest case and proceeds through some of the more complicated cases. The conclusions from this section carry over into the next section where the volume of the variable volume spaces change sinusoidally.
4.1.1 Stirling Cycle, Zero Dead Volume, Perfect Regeneration

The Stirling cycle is defined as a heat power cycle using isothermal compression and expansion and constant volume heating and cooling. Figure 4-2 shows such a process. Specific numbers are being used to make the explanations easier to follow and allow the reader to check to see if he is really getting the idea. Let us take 100 cm$^3$ of hydrogen at 10 MPa (~100 atm) and compress it isothermally to 50 cm$^3$. The path taken by the compression is easily plotted because PV is a constant. Thus, at 50 cm$^3$ the pressure is 20 MPa (~200 atm). The area under this curve is the work required to compress the gas and it is also the heat output from the gas for the cycle. If the pressure is expressed in Pascals (Newton/sq. meter) (1 atm = $10^5$N/m$^2$) and if the volume is expressed in m$^3$, then the units of work are (N/m$^2$)(m$^3$) = N·m = Joules = watt seconds. For convenience, megapascals (MPa) and cm$^3$ will be used to avoid very large and very small numbers.

The equation of the line is

$$PV = 100 \times 10^5 \text{Pa} \left(100 \times 10^{-6} \text{m}^3\right) = 1000 \text{ Joules}$$

$$= 10 \text{ MPa} \left(100 \text{ cm}^3\right) = 1000 \text{ Joules}$$

The work increment is

$$dw = PdV = \frac{1000}{V} dV$$

Integrating

$$w = 1000 \int_{V_1}^{V_2} \frac{dV}{V} = 1000 \left[ \ln V \right]_{V_1}^{V_2}$$

$$= 1000 \ln \left(\frac{V_2}{V_1}\right)$$

Thus

$$w = 1000 \ln \left(\frac{50}{100}\right) = -693.14 \text{ Joules}$$

The answer is negative because work is being supplied. Also by the perfect gas law,

$$PV = nRT_c$$

where $P$ = gas pressure, N/m$^2$ or MPa

$V$ = gas volume, m$^3$ or cm$^3$

$n$ = number of moles of hydrogen, g mol
Figure 4-2. Theoretical Cycles.
R = universal gas constant
    = 8.134 Joule/K (g mol)

T_c = cold side temperature, K

Thus

\[(10 \text{ MPa})(100 \text{ cm}^3) = n (8.314)(300)\]
\[n = 0.4009 \text{ g mol}\]

Therefore, the formula for work normally given in text books is

\[w = nRT_c \ln \left(\frac{V_2}{V_1}\right) = -693.14 \text{ Joules}\]  \hspace{1cm} (4-3)

This quantity is also the negative of heat of the compression of the gas or the heat removed from the cycle.

Next from state 2 to 3 the gas is heated at constant volume from 300 to, say, 900 K. Assume for the moment that the regenerator that supplies this heat has no dead volume and is 100\% effective. The heat that must be supplied to the gas by the regenerator matrix is:

\[q(r) = nC_v (T_H - T_c)\]  \hspace{1cm} (4-4)

where

\[C_v = \text{Heat capacity of hydrogen at constant volume,}\]
\[= 21.030 \text{ J/K (g mol) at 600 K average temperature}\]

Therefore \(q(r) = 0.4009 (21.030)(900-300)\)
\[= 5059 \text{ Joules}\]

Note that the heat transfer required in the regenerator is 7.3 times more than the heat rejected as the gas is compressed.

The pressure at state 3 after all gas has attained 900 K is:

\[P = nRT_H/V_2\]
\[= 0.4009 (8.314)(900)/50\]
\[= 60 \text{ MPa}\]

Isothermal expansion of the gas from state 3 to state 4 (Figure 4-1) is governed by the same laws as the compression.
\[ w(\text{out}) = nRT_H \ln \frac{V_1}{V_2} \]
\[ = 0.4009 \times (8.314) \times (900) \ln \frac{100}{50} = 2079.4 \text{ Joules} \]

This quantity is also the heat input to the engine. The expansion line is easily plotted when it is noted that \( PV = (60 \text{ MPa})(50 \text{ cm}^3) \)
\[ = 3000.0 \text{ Joules} \]

Finally the return of the expanded gas from state 4 to state 1 back through the regenerator finishes the cycle. The same formula applies as for heating.

\[ q(r) = nC_v(T_H - T_c) \]
\[ = 0.4009 \times (21.030)(900-300) \text{ Joules} \]
\[ = 5059 \text{ Joules} \]

The net work generated per cycle is

\[ w(\text{net}) = w(\text{in}) + w(\text{out}) = -693.14 + 2079.4 = 1386.3 \text{ Joules} \]

The efficiency of the cycle therefore is

\[ \eta = \frac{w(\text{net})}{q(\text{in})} = \frac{1386.3}{2079.4} = 0.6667 \]

In general the efficiency is

\[ \eta = \frac{w(\text{in}) + w(\text{out})}{q(\text{in})} = \frac{nRT_c \ln \frac{V_2}{V_1} + nRT_H \ln \frac{V}{V_2}}{nRT_H \ln \frac{V_1}{V_2}} \]  
(4-5)

\[ \eta = \frac{T_H - T_c}{T_H} = \frac{900 - 300}{900} = 0.6667 \]  
(4-6)

This efficiency formula is recognized as the Carnot efficiency formula. Therefore, the limiting efficiency of the Stirling cycle is as high as is possible.

### 4.1.2 Stirling Cycle, Zero Dead Volume, Imperfect Regenerator

One of the chief engine inefficiencies is the regenerator. Consider an annular gap around the displacer which acts as gas heater, regenerator and cooler. (See Figure 4-3) Assume that this engine operates in a stepwise manner and that this annular gap has negligible dead volume. Let \( E \) be the regenerator effectiveness during the transfer. For the transfer from cold space to hot space:
Figure 4-3. Simple Stirling Engine with Annular Gap Regenerator.

\[ E = \frac{T_R - T_C}{T_H - T_C} \]  

(4-7)

Now during transfer the heat from the regenerator is:

\[ q(r) = nC_v (T_R - T_C) \]  

(4-8)

and the heat from the gas heater is:

\[ q(b) = nC_v (T_H - T_R) \]  

(4-9)

Therefore, the efficiency becomes:

\[ n = \frac{nRT_H \ln \frac{V_1}{V_2} - nRT_C \ln \frac{V_1}{V_2}}{nRT_H \ln \frac{V_1}{V_2} + nC_v (T_H - T_R)} \]  

(4-10)

Which reduces to:

\[ n = \frac{T_H - T_C}{T_H + \frac{C_v}{V_1} (T_H - T_C)(1 - E)} \left( \ln \frac{V_1}{V_2} \right) \]  

(4-11)
For the numerical example being used here:

\[
\eta = \frac{900 - 300}{900 + \frac{21.03 \cdot (900 - 300)}{8.314 \ln \frac{100}{50}} (1 - E)} = \frac{600}{900 + 2189.5 (1 - E)}
\]

Figure 4-4 shows how the engine efficiency is affected by regenerator effectiveness for this numerical example. Some of the early Stirling engines worked with the regenerator removed. Figure 4-4 shows that at low regenerator effectiveness the efficiency is still reasonable. How close it pays to approach 100% effectiveness depends on a trade-off which will be discussed under Section 4.3.

Rallis (77 ay) has worked out a generalized cycle analysis in which the compression and expansion is isothermal but the heating and cooling can be at constant volume or at constant pressure or a combination. The heating process does not need to be the same as the cooling process. He assumes no dead volume, but allows for imperfect regeneration. For a Stirling cycle he derives the formula:
where
\[ \eta = \frac{(\gamma - 1)(\tau - 1) \ln v}{(1 - \varepsilon)(\tau - 1) + \tau(\gamma - 1) \ln v} \] (4-12)

\[ \eta = \text{cycle efficiency} \]
\[ \gamma = \frac{C_p}{C_v} \]
\[ \tau = \frac{T_H}{T_C} \]
\[ v = V_1/V_2 \]
\[ \varepsilon = E \]

Equations 4-12 and 4-11 are the same, just different nomenclature. Note that for \( \varepsilon = E = 1 \) both Equation 4-11 and 4-12 reduce to the Carnot equation, Equation 4-6.

Rallis (77 ay) also derived a formula for the Ericsson cycle efficiency:
\[ \eta = \frac{(\gamma - 1)(\tau - 1) \ln v}{\gamma(1 - \varepsilon)(\tau - 1) + \tau(\gamma - 1) \ln v} \] (4-13)

Equation 4-13 also reduces to Equation 4-6 when \( \varepsilon = 1 \), that is for perfect regeneration. To attain Carnot efficiency, the compression and expansion ratio must be the same. Rallis shows this using cycles which will not be treated here.

Rallis also gives a useful formula for the net work per cycle for the Stirling cycle:
\[ \frac{W}{\Delta V(P_1)} = \frac{v(\tau - 1) \ln v}{(v - 1)} \] (4-14)

For instance, for the numerical example being used here:
\[ W = (50 \text{ cc})(10 \text{ MPa}) 2 (3-1) \ln 2/(2-1) \]
\[ = 1386.3 \text{ Joules} \]

which is the same as obtained previously.

4.1.3 Otto Cycle, Zero Dead Volume, Perfect or Imperfect Regeneration

The variable volume spaces in Stirling engines are usually shaped so that there is little heat transfer possible between the gas and the walls during the time the gas is expanded or compressed. Analyses have been made by Rallis (77 az) and also by Martini (69 a) which assume adiabatic compression and expansion with the starting points being the same as for the Stirling cycle. For instance for the numerical example in Figure 4-2, compression goes from 1 to 2" instead of from 1 to 2. Expansion goes from 3 to 4" instead
of from 3 to 4. It appears that considerable area and therefore work per cycle is lost.

However, this process is not correct because the pressure at point 3 is not the same as for the isothermal case. For the numerical example after compression to point 2" the pressure of the gas is 26.39 MPa and the gas temperature is 396 K. As this gas moves into the hot space through a cooler, regenerator and heater all of negligible dead volume, it is cooled to 300 K in the cooler, heated to 900 K in the heater. As the gas is transferred at zero total volume change from the cold space to the hot space the pressure rises. This pressure rise results in a temperature increase in the gas due to adiabatic compression. Therefore, at the end of the transfer process the mixed mean gas temperature in the hot space will be higher than 900 K. Point 3 is calculated for all the gas to be exactly 900 K. Adiabatic expansion then takes place. Then by the same process as just described, the transfer of the expanded gas back into the cold space results in a lower gas temperature than 300 K at the end of this stroke. The computational process must be carried through for a few cycles until this process repeats accurately enough. One way of computing this process will be described in Section 4.1.5 when the effect of dead volume will also be considered.

4.1.4 Stirling Cycle, Variable Dead Volume, Perfect or Imperfect Regeneration

An inefficient regenerator backed up by an adequate gas heater and gas cooler will not change the work realized per cycle but will increase the heat required per cycle. It will now be shown that addition of dead volume which must be present in any real engine decreases the work available per cycle.

Assume that the annulus between displacer and cylinder wall (see Figure 4-3) has a dead volume of 50 cm³, that the temperature gradient from one end of the displacer to the other is uniform and that the pressure is essentially constant. The gas contained in this annulus is:

\[
n = \frac{P}{R} \int_{X=0}^{X=X} \frac{dv}{v} \tag{4-15}
\]

where

- \( v \) = total volume of annulus
- \( dv = A dx \) = differential volume of the annulus
- \( A \) = flow area of annulus
- \( x \) = distance along annulus
- \( X \) = total length of annulus
\[ T_x = T_H - \frac{X}{X} (T_H - T_C) \]  

By substituting and integrating one obtains:

\[ n = \frac{PV \ln \left( \frac{T_H}{T_C} \right)}{R (T_H - T_C)} \]  

(4-17)

Thus the effective gas temperature of the regenerator dead volume is

\[ T_R = \frac{(T_H - T_C)}{\ln \left( \frac{T_H}{T_C} \right)} \]  

which is the log mean temperature. Thus for the numerical example:

\[ T_R = \frac{900 - 300}{\ln \left( \frac{900}{300} \right)} = 546.1 \, K \]

Quite often it is assumed that \( T_R = \frac{T_H + T_C}{2} = \frac{900 + 300}{2} = 600 \, K \).

For the large dead volumes which will almost always result, it is important to have the right gas temperatures for the regenerator and heat exchangers.

Assume for the moment that the hot and cold gas spaces can be maintained at 900 K and 300 K and that the pressure at the end of the expansion stroke, 30 MPa (300 atm) is maintained. It is sometimes concluded that one should compare engine cycles that have the same peak pressure because this pressure is used to size the engine wall thickness. However, the wall thickness should be sized on the basis of creep. The time averaged pressure would be more appropriate. Thus, the above assumption. The gas inventory now is:

\[ n = \left( \frac{V_H}{T_H} + \frac{V_R}{T_R} \right) \]  

(4-19)

\[ n = \frac{30}{8.314} \left[ \frac{100}{900} + \frac{50}{546.1} \right] \]

\[ = 0.7313 \, g \, mol. \]

The equation for the gas expansion is:

\[ P = \frac{n \frac{R}{V_H} + \frac{R}{V_R}}{\frac{V_H}{T_H} + \frac{V_R}{T_R}} = \frac{(0.7313)(8.314)}{5472 + \frac{50}{546.1}} \]

(4-20)

\[ P = \frac{A}{V_H + B} \]  

where \( A = 5472 \).  \( B = 82.4 \)
The work output by expanding from $V_{H1} = 50 \text{ cm}^3$ to $V_{H2} = 100 \text{ cm}^3$ is:

\[
w_{(out)} = \int_{V_{H1}}^{V_{H2}} P \, dV_H = \int_{V_{H1}}^{V_{H2}} \frac{A \, dV_H}{V_H + B} = A \ln \left( \frac{V_{H2} + B}{V_{H1} + B} \right)
\]

\[
= 5472 \ln \left( \frac{100 + 82.4}{50 + 82.4} \right)
\]

\[
= 1753 \text{ Joules}
\]

The equation for gas compression is

\[
P = \frac{n R}{\frac{V_C}{T_C} + \frac{V_R}{T_R}} = \frac{(0.7313)(8.314)}{300 + 545.1}
\]

\[
= \frac{C}{V_C + D} \quad \text{where } C = 1824.02, D = 27.4
\]

Analogously the work of compression is

\[
w_{(in)} = C \ln \left( \frac{V_{C2} + D}{V_{C1} + D} \right)
\]

\[
= 1824.02 \ln \left( \frac{50 + 27.4}{100 + 27.4} \right)
\]

\[
= -908.37 \text{ Joules}
\]

Therefore the net work is

\[
w_{(net)} = w_{(out)} + w_{(in)} = 1753.08 - 908.37 = 844.71 \text{ Joules}
\]
Figure 4.5 shows how dead volume as % of maximum total gas volume affects the work per cycle. For more generality the work per cycle is expressed as a % of the work per cycle at zero dead volume. Note that the relationship is almost linear. This curve differs from that published by Martini (77 h) in that in Figure 4-5 the pressure at the end of the expansion stroke was made the same (average pressure). In the previous Figure 2 of reference 77 h, the minimum pressure was made the same. This caused the average pressure to decrease more rapidly as dead volume increased. Figure 4-5 is more truly representative of the effect of dead volume on work per cycle.

![Graph](image)

**Figure 4-5.** Effect of Dead Volume on Work Per Cycle for Isothermal Spaces and Constant Average Pressure.
4.1.5 Combined Stirling and Otto Cycle Variable Dead Volume, Perfect or Imperfect Regeneration

If the hot and cold spaces of the Stirling engine are free of heat exchange surface as they usually are, then compression and expansion in these spaces takes place essentially adiabatically. Assume that the heat exchangers and regenerator are placed as shown in Figure 4-3 so that gas entering the hot space is at hot space temperature. Assume further that: 1) the gas in the heater is at heat source temperature, 2) the gas in the cooler is at heat sink temperature and the gas in the regenerator is at the mean temperature between these two. Assume that the dead volume is distributed as follows:

<table>
<thead>
<tr>
<th>Space</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heater</td>
<td>10 cm$^3$</td>
</tr>
<tr>
<td>Regenerator</td>
<td>30 cm$^3$</td>
</tr>
<tr>
<td>Cooler</td>
<td>10 cm$^3$</td>
</tr>
<tr>
<td>Total</td>
<td>50 cm$^3$</td>
</tr>
</tbody>
</table>

Assume as before that the cold space is compressed from 100 cm$^3$ to 50 cm$^3$ while the hot space is zero. Transfer takes place to the hot space with the total volume held at 100 cm$^3$. Expansion takes place in the hot space from 50 to 100 cm$^3$. Finally, transfer occurs back to the cold space with the total volume held at 150 cm$^3$. We will now follow through this cycle and keep track of pressures. Using a gas inventory of 0.7313 g mol$^{-1}$, as before, the initial common pressure is:

$$P_1 = \frac{V_{HS}R}{V_{HS} + V_H + V_R + V_C + V_{CS}}$$

Assume:

$$P_1 = \frac{0.7313(8.314)}{900 + 10 + 30 + 10 + 100}$$

Therefore:

$$P_1 = 14.051 \text{ MPa}$$

Now let $V_{CS}$ go from 100 to 50 cc, $V_T$ from 150 to 100 cc.

During the compression stroke the gas in the cold space is compressed adiabatically and the gas in the heater, regenerator and cooler is compressed isothermally. Thus:

$$P_2 = \frac{0.7313(8.314)}{900 + 900 + 546.1 + 300 + T_{CS}}$$

Also the adiabatic compression law applies to the cold space.
\[
\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}}, \quad T_{CS} = \left( \frac{P_2}{C_{P}/C_v} \right)^{0.286}
\]

(4-24)

where \( k = 1.40 = C_\rho/C_v \) for hydrogen. These two equations in two unknowns are solved. Equation 4-24 is solved for \( T_{CS} \) and substituted in Equation 4-23. Then \( P_2 \) is determined by a secant method of approximation using a programmable calculator. Thus:

\[
T_{CS} = 354.92 \text{ K}, \quad P_2 = 25.31 \text{ MPa}
\]

From state 2 to 3 (see Figure 4-2) gas is transferred at no change in overall volume. The gas from the cold space is cooled down to 300 K as it enters the heat exchanger. It heats to 900 K in the regenerator and gas heater and enters the hot space at 900 K. As elements of gas enter the hot space, pressure increases. This pressure increase causes the first elements to attain a temperature higher than the heat source temperature. Assume that the gas in the hot space is thoroughly mixed at each stage. Assume that 5 steps are accurate enough to define the process. This will be checked later. By the gas law:

\[
P_{2.2} = \frac{0.7313 (8.314)}{\frac{10}{T_{HS2.2}} + \frac{10}{900} + \frac{30}{546.1} + \frac{10}{300} + \frac{40}{T_{CS2.2}}}
\]

\[
= \frac{6.08003}{\frac{10}{T_{HS2.2}} + 0.09938 + \frac{40}{T_{CS2.2}}}
\]

also by the adiabatic compression law

\[
\frac{T_{CS2.2}}{T_{CS2.0}} = \left( \frac{P_{2.2}}{P_{2.0}} \right)^{0.286}, \quad T_{CS2.2} = 354.92 \left( \frac{P_{2.2}}{25.31} \right)^{0.286}
\]

(4-26)

also

\[
T_{HS2.2} = 900 \left( \frac{P_{2.2}}{25.31} \right)^{0.286}
\]

combining

\[
P_{2.2} = \frac{6.08003}{\frac{10}{0.286} + 0.09938 + \frac{40}{0.286}}
\]

\[
900 \left( \frac{P_{2.2}}{25.31} \right)^{0.286} + 354.92 \left( \frac{P_{2.2}}{25.31} \right)^{0.286}
\]
Solving as before:

\[ P_{2.2} = 27.62 \text{ MPa} \quad T_{HS2.2} = 922.73 \text{ K} \quad T_{CS2.2} = 363.88 \text{ K} \]

Then during the next increment the next 10 cc starts into the hot space at 900 K and the first 10 cc continues on from 922.73 K. Thus:

\[
P_{2.4} = \frac{6.08003}{10} + \frac{6.08003}{922.73 \left( \frac{p_{2.4}}{27.62} \right)} + \frac{6.08003}{900 \left( \frac{p_{2.4}}{27.62} \right)} + \frac{6.08003}{30} + \frac{6.08003}{363.88 \left( \frac{p_{2.4}}{27.62} \right)}
\]

\[ P_{2.4} = 30.23 \quad T_{HS2.4} = 947.06 \text{ K} \quad T_{CS2.4} = 923.73 \text{ K} \]

The two parts of gas in the hot space have different temperatures. A mean temperature is found by adding the masses and finding the effective temperature. Thus:

\[
\frac{v_1}{T_1} + \frac{v_2}{T_2} = \frac{v_1 + v_2}{T_3}
\]

\[ \frac{10}{947.06} + \frac{10}{923.73} = \frac{20}{T_3} \]

\[ T_3 = T_{HS2.4} = 935.25 \text{ K} \]
A computer program was written to do the above calculation nearly automatically. The results are given in Table 4-1. The work diagram for this specific example is plotted in Figure 4-6. The work diagram is compared with the same engine cycle and gas inventory only assuming the hot and cold gas spaces are isothermal at the heat source and heat sink temperature. For 33.3% dead volume assumed here the effect of having adiabatic gas spaces instead of isothermal creates 10% more pressure swing but only 2.7% more work per cycle. Current practice in Stirling engine design is to have about 58% dead volume (see Section 7). Therefore, the error in assuming isothermal variable volume spaces instead of the more realistic adiabatic variable volume spaces would be less than 2.7% when the work per cycle is computed. It is not expected that this conclusion will change when crank operated engines are considered. Therefore, it is concluded that figuring the variable volume gas spaces as isothermal at the heat source and heat sink temperature will give the right work per cycle within one or two percent for practical engines. The isothermal assumption greatly reduces the labor of computation.

Other formulations are given in the literature for solving the above problem. Hoffman (77 be) and Rios (69 o, 69 ar) present equations which probably get to the same end point a different way. The author has not been able to fully understand and compare their methods with that given above. However, it is known that the Rios method uses a computer program working on 720 time increments per revolution. However this is not mandatory but the effect of reducing the number is not known. Rios did not actually use such fine divisions.

4.1.6 Conclusions from Cycle Analysis

1. Stirling and Ericsson cycles have the same limiting efficiency as the well known Carnot cycle.

2. A good regenerator is needed to attain high efficiency but the cycle has some efficiency without one.

3. An inefficient regenerator backed up by an adequate gas heater and gas cooler will not affect the work realized per cycle but will add to the heat required per cycle.

4. Dead volume has an almost linear effect on work available per cycle. That is, if half the maximum gas volume in the engine is dead volume, about half the work per cycle would be realized compared to the same displacements and average pressure with no dead volume. Some dead volume is inescapable.

5. The most correct effective temperature for the regenerator is the log mean temperature.

6. At the usual dead volume ratios used in Stirling engines the error in computing the work per cycle using isothermal spaces instead of the more realistic adiabatic spaces is 1 or 2%.
Table 4-1
Effect of Adiabatic Spaces on a Discrete Stirling Cycle with Dead Volume

VHD = 10 cm³, VRD = 30 cm³, VCD = 10 cm³
TH = 900 K, TR = 546.1 K, TC = 300 K

<table>
<thead>
<tr>
<th>VHL cm³</th>
<th>VCL cm³</th>
<th>P MPa</th>
<th>THS K</th>
<th>TCS K</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>14.051</td>
<td>900</td>
<td>300</td>
<td>Initial Conditions</td>
</tr>
<tr>
<td>0</td>
<td>50</td>
<td>25.31</td>
<td>1064</td>
<td>355</td>
<td>First Compression</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>27.61</td>
<td>923</td>
<td>364</td>
<td>Gas Transfer to</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
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<td>936</td>
<td>374</td>
<td>Hot Space at</td>
</tr>
<tr>
<td>30</td>
<td>20</td>
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<td>949</td>
<td>384</td>
<td>ΔV = 10 cm³</td>
</tr>
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<td>40</td>
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</tr>
<tr>
<td>50</td>
<td></td>
<td>40.38</td>
<td>975</td>
<td>406</td>
<td>P by using ΔV =</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>26.41</td>
<td>911</td>
<td>359</td>
<td>10 cm³ which is</td>
</tr>
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<tr>
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</tr>
<tr>
<td>25</td>
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<td>935</td>
<td>379</td>
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</tr>
<tr>
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<td>942</td>
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</tr>
<tr>
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<td>948</td>
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</tr>
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</tr>
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<td>766</td>
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</table>
Table 4-1, Page 2

<table>
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<th>VCL cm$^3$</th>
<th>P MPa</th>
<th>THS K</th>
<th>TCS K</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>729</td>
<td>276</td>
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</tr>
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<td>75</td>
<td>14.95</td>
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<td>Space $\Delta V = 5 \text{ cm}^3$.</td>
</tr>
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<td>705</td>
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<td>716</td>
<td>279</td>
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<tr>
<td>0</td>
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<td>Compression in</td>
</tr>
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<td>970</td>
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</tr>
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<td>50</td>
<td>0</td>
<td>37.39</td>
<td>929</td>
<td>375</td>
<td>of $P$ at this point.</td>
</tr>
<tr>
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<td>0</td>
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<td>922</td>
<td>359</td>
<td></td>
</tr>
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<td>352</td>
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</tr>
<tr>
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<td>345</td>
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<tr>
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<td>0</td>
<td>28.37</td>
<td>870</td>
<td>339</td>
<td>$0.3%$ error in</td>
</tr>
<tr>
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<td>0</td>
<td></td>
<td></td>
<td></td>
<td>closure of $P$ at</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>this point.</td>
</tr>
</tbody>
</table>

Satisfactory.
Figure 4-6. Comparison of Adiabatic and Isothermal Hot and Cold Gas Spaces for 33% Dead Volume.
4.2 First Order Design Methods

In the preceding section the thermodynamic principles were explained and the effect of dead volume, regenerator efficiency and adiabatic versus isothermal variable volume spaces were discussed using a Stirling engine model in which the 4 processes of compression, heating, expansion and cooling are entirely separated.

In almost all Stirling engines the displacer and the power piston or the two power pistons are moved with a crank. Therefore, the four processes overlap. The heart of the first order design method is the computation of the output power when the parts move sinusoidally.

There are basically two ways to attack this problem, numerically and analytically. In the numerical method the hot and cold volumes of the engine under consideration are computed for a number of times during the cycle - say every 30° of crank angle. The dead volume is also computed. The effective temperatures of the hot, cold and dead volume spaces are specified. Also the gas inventory is specified. It is assumed that at each crank angle the pressure throughout the engine is the same. Since the temperature and volume of each gas space is specified, the common pressure at each crank angle is calculated using the perfect gas law. The gas pressure is then plotted against the total gas volume and the area of the closed curve is measured to give work output per cycle. The maximum and minimum pressures are also noted.

In the analytical method the movement of the machine parts are specified sinusoidal with a specified phase angle difference between them. In the same way as in the numerical method, gas temperatures in the different parts of the engine are specified and are assumed to be constant. Then using the methods of calculus the pressure-volume diagram for the engine is integrated for the general case. Gustaf Schmidt (1871 a) was the first to do this and publish his results. Since then, a number of authors have presented formulas based upon the Schmidt analysis.

In this section the analysis will be divided into piston-displacer engines and dual piston engines since some formulas work for one type and some for the other. Within each subdivision the numerical method will be explained and a sample problem will be worked out showing the work diagram and an approximation of the integral. Next the analytical equations will be presented and the same engines will be calculated using these equations. If the equation is valid the same numerical result should be obtained. Finally, a survey will be presented of published comments which relate the Carnot efficiency to the actual efficiency of real engines and which relate the indicated power output to the power calculated by the Schmidt analysis.

4.2.1 Piston - Displacer Engines

4.2.1.1 Engine Definition

The nomenclature for engine internal volumes and motions is described in Figure 4-7 and 4-8. The following equations describe the volumes and pressures:
AHD = area of hot face of displacer, cm².
VHD = hot dead volume, cm³.
STD = stroke of displacer, cm.
VRD = regenerator dead volume, cm³.
VCD = cold dead volume, cm³.
ACD = area of cold face of displacer, cm².
VPL = power piston live volume, cm³.
TH = effective hot gas temperature, K
TR = effective regenerator gas temperature, K
TC = effective cold gas temperature, K
M = engine gas inventory, g mol
R = universal gas constant, 8.314 J/g mol·K
P = common gas pressure, MPa.
PHI = crank angle, degrees.
ALPH = phase angle, degrees.

Figure 4-7. Piston Displacer Engine Nomenclature.

\[
V_{HL} = AHD(\text{STD}) \tag{4-28}
\]
\[
V_{CL} = ACD(\text{STD}) \tag{4-29}
\]

Hot volume,
\[
V_H = \frac{V_{HL}}{2} \left[ 1 - \cos(\text{PHI}) \right] + V_{HD} \tag{4-30}
\]

Cold volume,
\[
V_C = \frac{V_{CL}}{2} \left[ 1 + \cos(\text{PHI}) \right] + V_{CD} + \frac{V_{PL}}{2} \left[ 1 - \cos(\text{PHI} - \text{ALPH}) \right] \tag{4-31}
\]
Figure 4-8. Phasing of Displacer and Power Piston.

Total volume

\[ VT = VH + VC + VRD \]  

(4-32)

Engine pressure

\[ P = \frac{M(R)}{\frac{VH}{TH} + \frac{VC}{TC} + \frac{VRD}{TR}} \]  

(4-33)
VCD includes the dead volume in the cooler as well as the dead volume between the strokes of the displacer and the power piston. According to the classification of engines given in Figure 2-6, the gamma type machine must have some volume between the strokes to allow for clearance and the flow passages between. In the beta type engine the strokes of the displacer and the power piston overlap so that they almost touch at one point in the cycle. This overlap volume is subtracted from the dead volume in the cold heat exchanger. For a beta type engine with this type of stroke overlap and ALPH = 90° and VCL = VPL then VCD = VCDHX - \[ \frac{VPL}{2} (2 - \sqrt{2}) = VCDHX - VPL \left(1 - \frac{\sqrt{2}}{2}\right) \text{ where } VCDHX = \text{cold dead volume in heat exchanger.}

4.2.1.2 Sample Engine Specifications

In order to check equations which look quite different, it was decided to specify a particular engine and then determine if the work integral checks. The specification decided upon was:

\[
\begin{align*}
M(R) &= 10.518 \text{ J/K} \\
TH &= 600 \text{ K} \\
TC &= 300 \text{ K} \\
VHL &= VCL = VPL = VRD = 40 \text{ cm}^3 \\
VHD &= VCD = 0 \\
ALPH &= 90°
\end{align*}
\]

TR is defined a number of ways, depending how it is defined in the analytical equation that is being checked. It may be:

1. Arithmetic mean (Walker)
   \[ TR = \frac{(TH + TC)}{2} = 450 \text{ K} \]
2. Log mean, most realistic
   \[ TR = \frac{(TH - TC)}{\ln(TH/TC)} = 432.8 \text{ K} \]
3. Half volume hot, half volume cold (Mayer)
   \[
   \frac{1}{TR} = \frac{1}{2(TH)} + \frac{1}{2(TC)} \Rightarrow TR = 400 \text{ K}
   \]

The above sample engine specification is for a gamma engine. For a beta engine assume in addition that VCDHX = 0 then:

\[
VCD = 0 - 40 \left(1 - \frac{\sqrt{2}}{2}\right) = -11.715 \text{ cm}^3
\]

4.2.1.3 Numerical Analysis

Using the numbers given in Section 4.2.1.2, Equations 4-28 to 4-33 can be evaluated for PHD = 0, 30, 60, . . . , 360, P can be plotted against VT and the result closed curve can be integrated graphically and the maximum and min-
immun gas pressure can be noted. The author's experience with a number of different examples gives a result which is 4.5% low when compared with valid analytical equations and with numerical calculations with very small crank angle increments. If the investigator has access to a programmable calculator or a computer then the computation can be made with any degree of precision desired. Figure 4-9 shows the flow diagram which was used for programming. The author has used both an HP-65 and an HP-67 for this purpose. He has also used this method as part of a larger second order calculation written in FORTRAN.

Using the 400 K effective regenerator temperature the following results were obtained for the numerical example.

<table>
<thead>
<tr>
<th>DELPHI</th>
<th>$\int p,dv$</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>314.36 Joules</td>
<td>4.5</td>
</tr>
<tr>
<td>20°</td>
<td>322.56</td>
<td>2.0</td>
</tr>
<tr>
<td>10°</td>
<td>327.53</td>
<td>0.50</td>
</tr>
<tr>
<td>5°</td>
<td>328.78</td>
<td>0.13</td>
</tr>
<tr>
<td>0.25°</td>
<td>329.1994570</td>
<td>0.0003</td>
</tr>
<tr>
<td>Mayer equation</td>
<td>329.2005026</td>
<td>0</td>
</tr>
</tbody>
</table>

The Mayer equation will be given in Section 4.2.1.4 and discussed more fully there. It uses the same assumptions as were employed in the numerical analysis. One can see from the above table that the result by numerical analysis approaches the Mayer equation result as DELPHI approaches zero. The two check.

If the arithmetic average is used TR = 450 K then:

DELPHI $\int p\,dv$ $p_{\text{max}}$ PHI for $p_{\text{max}}$

1° 360.45 58.10 MPa 117°

If the log mean average is used TR = 432.8 K then:

DELPHI $\int p\,dv$ $p_{\text{max}}$ PHI for $p_{\text{max}}$

1° 350.04 56.99 MPa 117°

For the case of the Beta engine with essentially touching displacer and power piston at one point in the cycle VCD = -11.715 cm³. For the arithmetic average Third volume temperature TR = 450 K. Then:

DELPHI $\int p\,dv$ $p_{\text{max}}$ PHI for $p_{\text{max}}$

1° 516.32 74.0862 117°

Precision in calculating this work integral is mainly of academic interest because the result will be multiplied in first order analysis by an experience factor like 0.5 or 0.6 (one figure precision). Even in second or third order analysis no more than two figure accuracy in the final power output and
Figure 4-9. Flow Diagram for Work Integral Analysis.
efficiency should ever be expected. Thus errors less than 1% should be considered insignificant. Therefore, DELPHI = 15° would be adequate for all practical purposes.

4.2.1.4 Schmidt Equations

At McDonnell Douglas, Mort Mayer reduced the Schmidt Equation to the following relatively simple form (68 c).

\[
W = \frac{M(R)(TC)(2\pi B(VP))}{B^2 + C^2} \left[ \frac{A}{(A^2 - B^2 - C^2)^{3/2}} - 1 \right] 
\]

(4-34)

where:

\[
W = \text{work per cycle, J} \\
M = \text{gas inventory, g mol} \\
R = \text{gas constant} = 8.314 \text{ J/g mol} \cdot \text{K} \\
TC = \text{effective cold gas temperature, K} \\
TH = \text{effective hot gas temperature, K} \\
A = \frac{VCO + \frac{TC}{TH} (VHO)}{VPL} \\
VCO = \frac{VPL}{2} - VDC + \frac{VCL}{2} + \frac{VRD}{2} \\
VHO = \frac{VHD + \frac{VHL}{2} + \frac{VRD}{2}}{2} \\
VI = \frac{VPL}{2} \\
B = \frac{VHL}{2} (1 - \frac{TC}{TH}) \sin(ALPH) \\
C = \frac{[VPL - VHL(1 - \frac{TC}{TH}) \cos(ALPH)]}{2} \\
ALPH = \text{phase angle between displacer and power piston, normally 90°}
\]

From the sample engine specifications:

\[
VCO = \frac{40}{2} + 0 + \frac{40}{2} + \frac{40}{2} = 60 \text{ cm}^3 = 60 \times 10^{-6} \text{ m}^3 \\
VHO = 0 + \frac{40}{2} + \frac{40}{2} = 40 \text{ cm}^3 = 40 \times 10^{-6} \text{ m} \\
A = 60 \times 10^{-6} \times \frac{300}{600} (40 \times 10^{-6}) = 8 \times 10^{-5} \text{ m}^3 \\
B = \frac{40 \times 10^{-6}}{2} (1 - \frac{300}{600}) = 1 \times 10^{-5} \text{ m}^3 \\
C = \frac{40 \times 10^{-6}}{2} = 2 \times 10^{-5} \text{ m}^3
\]

Using these inputs the Mayer equation gives:

\[W = 329.2005026 \text{ Joules}\]
The Mayer equation evaluates the integral exactly given the assumptions that were used in its derivation, like sinusoidal motion and half the dead space at hot temperature and half at cold temperature. The numerical method (Section 4.3.1.3) approaches this same value as the angle increment approaches zero. The Mayer equation must have VHL = VCL. That is, it cannot consider the effect of the displacer drive rod.

J. R. Senft (76 n) presents a Schmidt equation for finding the energy generated per cycle. His assumptions are the same as have been made so far with the temperature of the dead space gas having the arithmetic mean between the hot and cold gas spaces. This equation is for a beta type engine with the displacer and power piston essentially touching at one point during the cycle. His equation is:

\[
W = \frac{\pi(1 - \tau) P_{\text{max}} V_d k \sin \alpha}{Y + (Y^2 - X^2)^{1/4}} \left[ \frac{Y - X}{Y + X} \right]^{1/2}
\]

(4-35)

where:

\[
X = \left[ (\tau - 1)^2 + 2(\tau - 1) k \cos \alpha + k^2 \right]^{1/2}
\]

\[
Y = \tau + 4\tau \tau/(1 + \tau) + D
\]

\[
D = (1 + k^2 - 2k \cos \alpha)^{1/2}
\]

In order to illustrate and check this equation it is evaluated for a specific case previously computed by numerical methods. (See Section 4.2.1.3 for TR = 450 K and VCD = -11.715 cm³.)

\[
\tau = \frac{TC}{TH} = 0.5
\]

\[
V_d = \text{volume swept by displacer} = \text{VHL} = \text{VCL} = 40 \text{ cm}^3
\]

\[
V_p = \text{volume swept by piston} = \text{VPL} = 40 \text{ cm}^3
\]

\[
V_D = \text{volume of all dead space} = \text{VRD} + \text{VHD} + \text{VCD} = 40 \text{ cm}^3
\]

\[
x = \frac{V_d}{V} = 40/40 = 1
\]

\[
k = \frac{V_p}{V} = 40/40 = 1
\]

\[
\alpha = \text{phase angle} = \text{ALPH} = 90^0
\]

\[
P_{\text{max}} = \text{maximum pressure attained during each cycle} = 74.0862 \text{ MPa}
\]

\[
D = (1 + 1 - 2(1)\cos 90^0)^{1/2} = \sqrt{2}
\]

\[
Y = 0.5 + \frac{4(1)(0.5)}{(1.5)} + \sqrt{2} = 3.247547
\]

\[
X = \left[ (0.5 - 1)^2 + 2(0.5 - 1)(1)(\cos 90^0) + 1 \right]^{1/2} = 1.118034
\]
\[
\left[\frac{Y - X}{Y + X}\right]^2 = 0.698424
\]

\[
Y + (Y^2 - X^2)^{\frac{1}{2}} = 6.296573
\]

\[
W = \pi \left(1 - 0.5 \right) (74.0862) (40) (1) \sin 90^\circ (0.698424)
\]

\[
= 516.33 \text{ Joules}
\]

This answer agrees very well with results obtained by numerical methods of 516.32 joules. Senft (77 ak) also has adapted his equation for a gamma type engine (without stroke overlap). In this case the equations for \(W\) and \(X\) are the same and the equation for \(Y\) is:

\[
Y = \frac{4X}{(1 + \tau)} + 1 + \tau + k
\]

(4-36)

Therefore:

\[
Y = \frac{4(1)(0.5)}{1.5} + 1 + 0.5 + 1 = 3.833333
\]

\[
\left(\frac{Y - X}{Y + X}\right)^2 = 0.740513
\]

\[
Y + (Y^2 - X^2)^{\frac{1}{2}} = 7.50000
\]

To agree with the numerical analysis of Section 4.2.1.3 for \(TR = 450 K\), \(P_{\text{max}} = 58.10 \text{ MPa}\).

Thus:

\[
W = \frac{\pi (1 - 0.5)(58.10)(40) \sin 90^\circ (0.740513)}{7.50000}
\]

\[W = 360.45 \text{ Joules}\]

This result agrees exactly with the numerical analysis for \(\text{DELPHI} = 1^\circ\), \(TD = 450 K\) and \(P_{\text{max}} = 58.10 \text{ MPa}\). (See Section 4.2.1.3.)

This new Senft equation is also correct.

Cooke-Yarborough (74 i) has published a simplified expression for power output which makes the approximation that not only the volume changes but also the pressure changes are sinusoidal. The regenerator is treated as being half at the hot volume temperature and half at the cold volume temperature. His equation is:

\[
\text{Output Power} = \frac{P_a}{B} \frac{V_E}{V_M} \frac{V_E V_0}{V_M} - \Delta T \sin \frac{T_C + V_C}{V_M} \Delta T
\]

(4-37)
where:

\[ \bar{p} = \text{mean pressure of working gas, or pressure with both displacer and power piston at mid-stroke. (With the approximations used, these two pressures can be regarded as identical.) If the mean pressure is known, it can be used directly in Equation (4-37). Otherwise the mid-stroke pressure can be calculated as follows:} \]

\[
\bar{p} = \frac{MR}{2(TH)} + \frac{VPL}{2 TC} + \frac{VLC}{2 TC} + \frac{VRD}{TR} + \frac{VHL}{600} + \frac{VRD}{432.8} + \frac{VPL}{300} + \frac{VLC}{300} \\
= \frac{10.518}{20} + \frac{40}{432.8} + \frac{20}{300} + \frac{20}{300} \\
= 40.59 \text{ MPA} \\
= \text{operating frequency, radians/sec} \\
= 2\pi \text{ so that output power in watts is numerically equal to power per cycle, Joules} \\
\]

\[ V_E = VHL = 40 \text{ cm}^3 \]
\[ V_0 = VPL = 40 \text{ cm}^3 \]
\[ V_M = \text{total gas volume of system when output piston is at midstroke} \\
= VHL + VRD + \frac{VPL}{2} \\
= 40 + 40 + 20 = 100 \text{ cm}^3 \]
\[ \Delta T = T_E - T_C = 600 - 300 = 300 \text{ K} \]
\[ \theta = \text{phase angle} = \alpha = 90^\circ \]
\[ V_C = \text{cold gas volume with both piston and displacer at midstroke and regenerator volume split between hot and cold volumes} \\
= VRD + VCL + VPL \\
= \frac{VRD}{2} + \frac{VCL}{2} + \frac{VPL}{2} \\
= \frac{40}{2} + \frac{40}{2} + \frac{40}{2} = 60 \text{ cm}^3 \]

output power = \frac{40.59(\frac{\pi}{8}) 40(40) 40(40) (300)1}{100 300 + 60 300 (300)} \\
= 318.79 \text{ Joules/cycle} \]
Because of how $V_o$ is determined this result should be compared to the Mayer equation, that is, to 329.20 joules. Therefore, the Cooke-Yarborough equation appears to be a reasonably good approximation (3.2% error). The accuracy improves as the dead volume is increased, because the pressure waveform is then more nearly sinusoidal.

4.2.2 Dual Piston Engines

4.2.2.1 Engine Definition and Sample Engine Specifications

The nomenclature for engine internal volumes and motions are described in Figure 4-10. Also given in Figure 4-10 are the assumed values for the sample case. The following equations describe the volumes and pressures.

Hot Volume
\[ VH = \frac{VHL}{2} \left[ 1 - \sin(PHI) \right] + VHD \]  \hspace{1cm} (4-38)

Cold Volume
\[ VC = \frac{VCL}{2} \left[ 1 - \sin(PHI - \alpha PH) \right] + VCD \]  \hspace{1cm} (4-39)

Total Volume
\[ VT = VH + VC + VRD \]  \hspace{1cm} (4-40)

Engine Pressure
\[ P = \frac{M(R)}{\frac{VH}{TH} + \frac{VC}{TC} + \frac{VRD}{TR}} \]  \hspace{1cm} (4-41)

4.2.2.2 Numerical Analysis

Using the assumed values given in Figure 4-10, Equations 4-38 to 4-41 were evaluated for PHI = 0, 30, 60, ... 360. The results were:
Figure 4-10. Dual Piston Engine Nomenclature and Assumptions for Sample Case.
This data is graphed in Figure 4-11 and graphically integrated. A value of 695.3 J was obtained. As before, a numerical integration was carried along as the points were calculated. This was 668.8 joules, a 3.8% error which indicates the accuracy of the graphical integration procedure. To approach the answer that should be obtained by valid Schmidt equations, DELPHI should be reduced toward zero. The results obtained were:

<table>
<thead>
<tr>
<th>PHI Degrees</th>
<th>VT cm³</th>
<th>P MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100.0</td>
<td>-41.2</td>
</tr>
<tr>
<td>30</td>
<td>87.3</td>
<td>45.7</td>
</tr>
<tr>
<td>60</td>
<td>72.7</td>
<td>54.4</td>
</tr>
<tr>
<td>90</td>
<td>60.0</td>
<td>67.6</td>
</tr>
<tr>
<td>120</td>
<td>52.7</td>
<td>83.0</td>
</tr>
<tr>
<td>150</td>
<td>52.7</td>
<td>91.9</td>
</tr>
<tr>
<td>180</td>
<td>60.0</td>
<td>86.1</td>
</tr>
<tr>
<td>210</td>
<td>72.7</td>
<td>71.2</td>
</tr>
<tr>
<td>240</td>
<td>87.3</td>
<td>57.0</td>
</tr>
<tr>
<td>270</td>
<td>100.0</td>
<td>47.3</td>
</tr>
<tr>
<td>300</td>
<td>107.3</td>
<td>41.9</td>
</tr>
<tr>
<td>330</td>
<td>107.3</td>
<td>39.9</td>
</tr>
<tr>
<td>360</td>
<td>100.0</td>
<td>41.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DELPHI</th>
<th>$\phi_{VdP}$ Joules</th>
<th>$P_{\text{max}}$</th>
<th>TR K</th>
<th>% Error Due to DELPHI</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>668.8</td>
<td>91.87</td>
<td>450</td>
<td>4.5</td>
</tr>
<tr>
<td>10</td>
<td>696.8</td>
<td>91.98</td>
<td>450</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>700.324</td>
<td>91.98</td>
<td>450</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>641.284</td>
<td>89.121</td>
<td>432.8</td>
<td>4.5</td>
</tr>
<tr>
<td>1</td>
<td>671.517</td>
<td>89.220</td>
<td>432.8</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>587.9</td>
<td>83.831</td>
<td>400</td>
<td>4.5</td>
</tr>
<tr>
<td>1</td>
<td>615.619</td>
<td>83.831</td>
<td>400</td>
<td>0</td>
</tr>
</tbody>
</table>
Note the difference in the result depending on what is used for the effective temperature of the gas in the regenerator. If the regenerator has a uniform temperature gradient from hot to cold, which it usually does, then the log mean temperature \( TR = 432.8 \text{ K} \) is correct. The arithmetic mean \( TR = 450 \text{ K} \) gives a result for this numerical example 4.3% high. The assumption that the regenerator is half hot and half cold \( TR = 400 \text{ K} \) gives a result 9.1% low.

![Figure 4-11. Work Diagram for Dual Piston Sample Case (DELPHI = 30°).](image)

4.2.2.3 Schmidt Equations

Walker (73 j) gives a Schmidt equation most adaptable to the two piston engine. Using his nomenclature it is:

---
\[ P = (P_{\text{max}} - V_T) \frac{\delta \sin \theta}{1 + (1 - \delta^2)^{\frac{1}{2}}} \]

where

\[ P = \text{work per cycle, Joules} \]
\[ P_{\text{max}} = \text{maximum pressure during cycle, MPa} \]
\[ V_T = V_E + V_C = (1 + \kappa)V_E \]
\[ V_E = \text{swept volume in expansion space} = VHL \]
\[ V_C = \text{swept volume in compression space} = VCL \]
\[ \kappa = \frac{V_C}{V_E}, \text{swept volume ratio} \]
\[ \tau = \frac{T_C}{T_E} \]
\[ T_C = \text{compression space gas temperature} = T_C \]
\[ T_E = \text{expansion space gas temperature} = T_H \]
\[ T_D = \text{dead space gas temperature} = T_R \]
\[ \delta = \frac{(\tau^2 + 2\tau \kappa \cos \alpha + \kappa^2)^{\frac{1}{2}}}{(\tau + \kappa + 2S)} \]
\[ \alpha = \text{angle by which volume variations in expansion space lead those in compression space, degrees} \]
\[ S = \frac{2X}{(\tau + 1)} \] (This is where the arithmetic average temperature for the regenerator enters.)
\[ X = \frac{V_D}{V_E}, \text{dead volume ratio} \]
\[ V_D = \text{total dead volume, cm}^3 = VHD + VRD + VCD \]
\[ \theta = \tan^{-1} (\kappa \sin \alpha / (\tau + \kappa \cos \alpha)) \] (Note that \( \theta \) defined incorrectly in Walker's table of nomenclature and on page 36, but is right on page 28 of reference 73j)

Now in order to check this equation against numerical analysis it should give a work per cycle of slightly greater than 700.324 Joules when 91.98 MPa is used as the maximum pressure. TD = 450 K is the same assumption for both. (See Section 4.2.2.2.)
Therefore:

\[ V_T = 40 + 40 = 80 \text{ cm}^3 \]
\[ \kappa = \frac{V_C}{V_E} = \frac{40}{40} = 1 \]
\[ P_{\text{max}} = 91.98 \text{ MPa} \]
\[ \tau = \frac{T_C}{T_E} = \frac{300}{600} = 0.5 \]
\[ X = \frac{V_D}{V_E} = \frac{40}{40} = 1 \]
\[ S = 2(1)(0.5)/(0.5 + 1) = 2/3 \]
\[ \delta = (0.5^2 + 12)^{1/2}/(0.5 + 1 + 2(2/3)) = 0.39460 \]
\[ \theta = \tan^{-1}(1/0.5) = 63.43^\circ \]
\[ P = -700.37 \text{ joules} \]

Thus the formula checks to 4 figure accuracy except for the si.

Walker obtained the above equation along with most of the nomenclature from the published Philips literature. Meijer's thesis (60 c) contains the same formula on page 12 of reference 60 c, except he uses \((1 - \tau)\) instead of \((\tau - 1)\) and a positive result would therefore be obtained.

In Meijer's thesis (60 c), the quantity \(S\) is defined so that dead spaces in heaters, regenerator and coolers and clearance spaces in the compression and expansion spaces, all of which have different temperatures associated with them, can be accommodated.

Thus:

\[ S = \sum_{s=1}^{s=n} \frac{V_s}{V_E} \frac{T_C}{T_s} \]  (4-43)

where \(V_s\) and \(T_s\) are the volumes and absolute temperatures of the dead spaces. Using this formula it would be possible to use the more correct log mean temperature for the regenerator. Thus:

\[ S = \frac{40(300)}{40(432.8)} = 0.693 \]

The above equation then evaluates to:

\[ P = 671.537 \text{ Joules} \]

This is within 0.003\% of the value of 671.517 computed numerically for 1 degree increments (see Section 4.2.2.2).

Finkelstein (60 v) independently of Meijer derived the following formula for the work per cycle:

\[ P = \frac{2\pi n(1 - \tau)\sin\alpha}{(\tau + \eta + \phi)^2} \text{ WRT } (f_w(\rho)) \]  (4-44)
where

$$f_w(\rho) = 1/(\sqrt{1 - \rho^2}(1 + \sqrt{1 - \rho^2}))$$

From his derivation it is apparent that his nomenclature runs parallel to that used by Walker. Thus:

$$n = k, \tau = \tau, \alpha = \alpha, V = X, \nu = 2S, \rho = \frac{\delta}{T} = T_c$$

and $$V = V_E$$ also Finkelstein's WR is equivalent to $$M(R)$$ used in the numerical analysis. When these transformations are made:

$$p = \frac{2\pi\kappa(1 - \tau)(\sin\alpha)M(R)T_c}{(\tau + \kappa + 2S)^2\sqrt{1 - \delta^2}(1 + \sqrt{1 - \delta^2})}$$

(4-45)

Using the last numerical example:

$$S = \frac{40(300)}{40(32.8)} = 0.693$$

$$\tau = 0.5$$

$$\kappa = 1$$

$$\alpha = 90^\circ$$

$$M(R)T_c = 10.518(300) = 3155.4$$

$$\delta = \sqrt{1.25/\left(1.5 + 2S\right)} = 0.38735$$

$$P = 671.55$$

This compares to 671.537 by the Meijer formula and to 671.517 by numerical analysis with 1 degree increments. Therefore, the above formula also is useful in computing the work output per cycle. Note that this formula employs the gas inventory instead of the maximum pressure.

Zarinchang (75 d) presents a formula for the work output per cycle of a dual piston engine. It bears a superficial resemblance to the Walker equation but is not identical or easily converted to it. It will now be evaluated numerically to determine if it gives the right answer.

$$W = \frac{\pi k(1 - t) P_{\text{max}} V_{1,\text{max}} \sqrt{(1 - F)} \sin\alpha}{(t + k + S) \sqrt{(1 + F)} (1 + \sqrt{1 - F^2})}$$

(4-46)

where

- $$W =$$ work output per cycle, joules
- $$k =$$ ratio of swept volume of compression space to expansion space, $$= 1$$ for the sample problem being used.
- $$t =$$ temperature ratio $$300/600 = 0.5$$
- $$P_{\text{max}} =$$ maximum pressure $$= 91.98 \text{ MPa}$$
- $$V_{1,\text{max}} =$$ assumed to be swept volume of expansion space, not defined in reference 75 d $$= 40 \text{ cm}^3$$
I: \[ F = \frac{t^2 + k^2 + 2tk \cos \alpha}{(t + k + S)^3} \]

\( \alpha \) = phase lag of the crank for the compression space piston

\( S \) = proportional dead space referring to swept volume of the expansion and reduction to lower temperature

\( W \) = 669.408 Joules - 4.4% low compared to the Walker formula and to numerical analysis

However, if \( F \) is defined the same as \( R \) of the Walker formula, that is, put a square root sign over the entire numerator, then \( F = 0.39460 \) and \( W = 700.37 \), exactly the same magnitude as obtained with the Walker formula. Thus it is concluded that the Zarinchang formula is also valid after it is corrected as shown above.

4.2.3 Experience Factors

Once the Schmidt and the Carnot equations have been evaluated for an engine, a rough estimate of its power output and efficiency can be made if it is designed similar to an engine for which test data exists.

Figure 4-12 shows how experience factors can be applied after relatively simple calculations to obtain ball park estimates of the size, weight, and efficiency for a particular output power level desired.

To illustrate, assume one has an engine for which you know the dimension, the speed of operation, average operating pressure, type of engine (\( a, k \), or \( y \)) and input and output metal temperature. From this information one can calculate using the appropriate Schmidt equation from the preceding sections the calculated power output, \( W_c \). If the engine were perfectly designed with no losses then the Carnot equation would apply:

\[ n_c = 1 - \frac{W}{Q_c} \]

where \( Q_c \) = basic heat required before losses are added.

By measuring instantaneous values of engine pressure and power piston displacement, one can plot a work diagram. The area of this diagram times engine speed is the measured indicated power, \( W_i \). The heat input to the engine, \( Q \), can be most conveniently measured by adding the heat absorbed by the cooling water to the power output. \( Q \) is considerably greater than \( Q_c \). The indicated
efficiency, \( \eta \), is \( \frac{W_i}{Q} \). Some of the measured indicated power is absorbed by friction in the seals and cranks of the engine. Brake power, \( W_B \), is the power that can be measured on the output shaft by measuring the braking torque and the speed. The mechanical efficiency is \( \frac{W_m}{W_i} \). The brake efficiency is \( \eta_B = \frac{W_B}{Q} \). To operate independently most engines require auxillaries. For a typical Stirling engine a fuel pump, a combustion air blower, a water pump, a cooling air fan and a working fluid pump and a generator would all draw power from the engine. The net brake power, \( W_{NB} \), is what is left. Net brake efficiency is \( \eta_{NB} = \frac{W_{NB}}{Q} \).

The efficiency experience factor is of three kinds:
1. indicated, \( \eta/\eta_C \)
2. brake, \( \eta_B/\eta_C \)
3. net brake, \( \eta_{NB}/\eta_C \)

These efficiency factors are expressed in the following tables as a percent of Carnot efficiency.

Power efficiency factors relate the power realized to that calculated by a Schmidt equation or equivalent.

All available information that would allow this efficiency and power experience factors to be calculated will now be given.

### 4.2.3.1 Efficiency Experience Factors

The most extensive information is available from publications authorized by N.V. Philips Co., and their licensees which generally give efficiency as a function of output power for a given displacement engine. For each point on the curves given the size of the gas heater, regenerator, gas cooler, the gas pressure and engine speed are chosen to give the best efficiency for the desired power. No dimensions except for the displacement are usually given but this information indicates what a well designed engine can do. Table 4-2 shows indicated efficiencies calculated by Michels for a 1-98 rhombic drive Stirling engine. The 1-98 has a 98 cc displacement displacer and power piston.

Michels shows that for optimized engines the indicated efficiency depends upon heater temperature and cooler temperature and not upon the working gas used. Figure 4-13 shows Michels' curve correctly labeled.
Figure 4-12. Engine Experience Factor Nomenclature.

Figure 4-13. Indicated Efficiencies vs. Heater Temperature $T_h$ at Two Different Cooler Temperatures $T_c$. Engine Displacement $98\, \text{cm}^3$. 
Table 4-2
Indicated Efficiencies of a
1-98 Rhombic Drive Philips Engine
(Reference 76 e)

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<td>100</td>
<td>0.35</td>
<td>18</td>
<td>63</td>
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<tr>
<td>He</td>
<td>850</td>
<td>100</td>
<td>6</td>
<td>50</td>
<td>75</td>
</tr>
<tr>
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<td>400</td>
<td>100</td>
<td>1</td>
<td>30</td>
<td>67</td>
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<td>100</td>
<td>0.18</td>
<td>17</td>
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Another way of correlating the calculations is to relate the indicated efficiency to the Carnot efficiency for the particular heater and cooler temperature employed. Figure 4-14 shows this indicated efficiency factor, \( \eta / \eta_c \). Note that it goes from 65 ± 6% at 250°C heat temperature to 75 ± 2% at 800°C. Based upon much experimental data, Michels calculates the brake efficiency factor, \( \eta_b / \eta_c \), for an engine without powering necessary auxiliaries. Table 4-3 shows the maximum.
Figure 4-14. Indicated and Brake Efficiency Factors for Optimized Philips 1-98 Engines (76 e).
Table 4-3
Computed Brake (Shaft) Efficiencies for
a 1-98 Rhombic Drive Philips
Engine Optimized for Each Operating Point
(Reference 76 e)

<table>
<thead>
<tr>
<th>Working Fluid</th>
<th>Heater Temp. C</th>
<th>Cooler Temp. C</th>
<th>Shaft Power at Max. Eff. K. watt</th>
<th>Brake Eff. ( \frac{W_B}{Q} )</th>
<th>% of Carnot Eff. ( \eta_B / \eta_C )</th>
<th>Mechanical Efficiency ( \frac{W_B}{W_I} )</th>
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<td>0.67</td>
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<tr>
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<td>4</td>
<td>40</td>
<td>60</td>
<td>0.80</td>
</tr>
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<td>100</td>
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<td>100</td>
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<td>-------------</td>
<td>-----------------------------------</td>
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</tr>
<tr>
<td></td>
<td>Manufacturer</td>
<td>(MPa)</td>
<td>(°C)</td>
<td>(°C)</td>
<td>(KW) BHP RPM Brake Eff. % of Carnot</td>
<td>wt, kg</td>
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<td>Prototype H₂</td>
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<td>691</td>
<td>71</td>
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<td>71</td>
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<td>°C (°F)</td>
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<td>Dimension</td>
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<tr>
<td></td>
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<td>Brake % of Carnot</td>
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<td>816 1500</td>
<td>10 50</td>
<td>78 105     1500 44 60</td>
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<td></td>
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<td>816 1500</td>
<td>10 50</td>
<td>75 100     1800 44 60</td>
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<td>816 1500</td>
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<td>816 1500</td>
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<td>30 40      2000 40 54</td>
<td>94 x 50 x 84</td>
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<td>24 75</td>
<td>-- 1800    26.3 28</td>
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<td>593 1100</td>
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*Bare engine with preheater.  ** Without flywheel.
Table 4-6

Maximum Net Brake Efficiencies for Various Stirling Engines

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<th>Manufacturer</th>
<th>Working Fluid</th>
<th>Engine Designation</th>
<th>Mean Pressure</th>
<th>Heater Temp</th>
<th>Cooler Temp</th>
<th>Maximum Efficiency</th>
<th>Operating Point</th>
<th>Dimension cm</th>
<th>Engine Type</th>
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<td>Philips</td>
<td>He</td>
<td>4-215 (Ref. 75 t)</td>
<td>19.6</td>
<td>71</td>
<td>75</td>
<td>56</td>
<td>32</td>
<td>50</td>
<td>340</td>
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<tr>
<td>Anal. Opt. Des.</td>
<td>H2</td>
<td>Philips (Ref. 75 T)</td>
<td>22.1</td>
<td>~760</td>
<td>71</td>
<td>75</td>
<td>600</td>
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<tr>
<td>General Motors</td>
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<td>~7</td>
<td>1900</td>
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<td>960</td>
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<td>40</td>
<td>0.0375 cycles</td>
<td>16.9</td>
<td>26.5</td>
<td>---</td>
</tr>
</tbody>
</table>
brake efficiency points for these calculations. Figure 4-14 shows how the brake efficiency factor depends on heater temperature. The mechanical efficiency for this machine is generally about 80% (see Table 4-3).

The size, weight, power and efficiency for a number of other engines mentioned in the literature are presented in Tables 4-4 and 4-5. It should be emphasized that the powers given are for the maximum efficiency operating point not the maximum power point. Note that the brake efficiencies range from 46 to 69% of Carnot.

Finegold and Vanderbrug (77 ae) using data from the Philips-Ford 4-215 engine conclude that the maximum brake efficiency is 52% of the Carnot efficiency. This factor is based upon 1975 data. Improvements have been made since then.

Net Brake Efficiency - The information presented in Tables 4-3 to 4-5 is for engines without auxiliaries. In Table 4-6 performance and efficiencies are given for the engine powering all auxiliaries needed to have the engine stand alone. This includes the cooling fan, the blower and atomizer and fuel pump for the burner, and the water pump for the radiator. Table 4-6 shows that the maximum net brake efficiency is from 38 to 65% of ideal.

Carlqvist, et. al (77 al) give the following formula for well optimized engines operating on hydrogen at their maximum efficiency points.

\[
\eta_{\text{eff}} = \frac{P_{\text{net}}}{E_F} = (1 - \frac{T_C}{T_H}) \cdot C \cdot \eta_H \cdot \eta_M \cdot f_A
\]

(4-47)

where

- \(\eta_{\text{eff}}\) = overall thermal or effective efficiency
- \(P_{\text{net}}\) = net shaft power with all auxiliaries driven
- \(E_F\) = fuel energy flow
- \(T_C, T_H\) = compression-expansion gas temperature, K
- \(C\) = Carnot efficiency ratio of indicated efficiency to Carnot efficiency, normally from 0.65 to 0.75. Under special conditions 0.80 can be reached.
- \(\eta_H\) = heater efficiency, ratio between the energy flow to the heater and the fuel energy flow. Normally between 0.85 and 0.90.
- \(\eta_M\) = mechanical efficiency, ratio of indicated to brake power. Now about 0.85 should go to 0.90.
- \(f_A\) = auxiliary ratio. At maximum efficiency point \(f_A = 0.95\).

Thus the most optimistic figures:

\[
\eta_{\text{eff}} = (1 - \frac{T_C}{T_H}) \cdot 0.75(0.90)(0.90)(0.95) = (1 - \frac{T_C}{T_H})(0.58)
\]
4.2.3.2 Power Experience Factors

Experience factors based upon the Schmidt Equation - Only a very few references give numbers relating the output brake power to that predicted by application of the Schmidt equation. The available data are:

<table>
<thead>
<tr>
<th>Author</th>
<th>Reference</th>
<th>( \frac{W_B}{W_C}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urieii</td>
<td>1977 af</td>
<td>0.3 to 0.4</td>
</tr>
<tr>
<td>Zarinchang</td>
<td>1975 d</td>
<td>0.3 to 0.4</td>
</tr>
<tr>
<td>Finegold &amp; Vanderbrug</td>
<td>1977 ae</td>
<td>0.32</td>
</tr>
<tr>
<td>Martini (calc)</td>
<td>Table 5.5</td>
<td>0.6</td>
</tr>
</tbody>
</table>

\*See Figure 4-12 for nomenclature, at maximum efficiency point.

This power experience factor is the product of two factors. One factor of about 0.8 is the mechanical efficiency for the machine. The other factor expresses the fraction of the basic power remaining after internal flow friction is deducted. This second factor is larger at the maximum efficiency point than it is at the maximum power point. Until more is known about this factor, it should be used as a very rough guide.

4.2.4 First Order Design Procedure

In a first order design procedure the following steps are used:

1. Establish \( T_H \) and \( T_C \).
2. Calculate \( n_c \).
3. Determine, indicated, brake and net brake efficiencies using efficiency experience factors.
4. Establish type of engine, piston displacement and fraction dead volume and speed. (See Section 3.3.3 for description of a typical engine.)
5. Compute \( W_C \) with appropriate Schmidt equation.
6. Predict brake power using the power experience factor.

As a sample of this procedure the following problem and solution is offered.

Problem: Your lab reports a new material has been discovered which will allow an engine like the GPU-3 (see Section 3.3) to operate at a 1500 C. Estimate power output and efficiency based upon first order methods.

Solution:

1) \( T_H = 1500 + 273 = 1773 \) K
   \( T_C = 20 + 273 = 293 \) K

2) \( n_c = 1 - \frac{T_C}{T_H} = 1 - \frac{293}{1773} = 0.835 \)
3) From extrapolation of Figure 4-14 the brake efficiency factor is 0.65. Therefore \( \eta_B = 0.65(0.835) = 0.54 \).

4) Use the Cooke-Yarborough Equation (4-37) and further establish that:

\[
\begin{align*}
\rho &= 4.137 \text{ MPa (600 psia)} \\
\omega &= 100\pi \text{ (50 Hz)} \\
V_E &= (7.01)^2(\pi/4)(3.068) = 118.4 \text{ cm}^3 \\
V_O &= 118.4 \text{ cm}^3 \\
\frac{V_M}{V} &= \text{VHD + VRD + VCD + 0.854V_E + 0.354V_O} \\
\text{V_M} &= \text{81.8 + 51.8 + 17.2 + 101.1 + 41.9 = 293.8 cm}^3 \\
\Delta T &= 1773 - 293 = 1480 \text{ K} \\
\theta &= 90^\circ \\
V_C &= \frac{\text{VRD}}{2} + \text{VCD} + (V_O + V_E)(0.354) \\
&= \frac{51.8}{2} + 17.2 + (118.4 + 118.4)(0.354) = 126.9 \text{ cm}^3 \\
\end{align*}
\]

5) Calculated output power, \( W_C = \frac{\rho \omega V_E V_O}{8} \frac{\Delta T \sin \theta}{V_M} + \frac{V_C}{V_M} \Delta T \)

\[
\begin{align*}
W_C &= \frac{4.135(100\pi)(118.4)(118.4)}{8} \frac{(1480)(\sin 90^\circ)}{293.8 + 126.9} (1480) \\
&= 12.3 \text{ KW} \\
\end{align*}
\]

6) Choose a power experience factor of \( W_B/W_C = 0.6 \) so brake power, \( W_B = 0.6(12.3) = 7.38 \text{ KW} \), heat input, \( \eta_B = 7.38/0.54 = 13.7 \text{ KW} \)

4.2.5 Conclusions on First Order Design Methods

1. First order design methods are good for preliminary system analysis.
2. Current flame-heated Stirling engines powering all auxiliaries can realize no more than 58% of the Carnot efficiency figured between the heater metal temperature and the cooling water temperature.
3. Published Schmidt equations to compute basic output power (Figure 4-12) from engine dimensions are generally correct and can be checked easily by numerical methods.
4. Reliable experience factors relating the basic power to the brake power for a well-designed Stirling engine do not exist at this time.

4.3 Second Order Design Methods

Second order design methods are relatively simple computational procedures that are particularly useful for optimizing the design of a Stirling engine from scratch. An equation or a brief computational procedure is used to determine the basic power output and heat input. These then are modified by the various identifiable energy loss terms. For instance, the net output power is the
The net heat input is the basic heat input plus reheat loss, shuttle conduction, gas and solid conduction, pumping loss, temperature swing loss, internal temperature swing loss, heat exchanger loss, and minus the frictional heating in the hot end. Then the net efficiency is the net output power divided by the net heat input.

The methods of computing these basic heat inputs and power outputs and the losses are presented by drawing on the available literature. As in Sections 4.2.1.4 and 4.2.1.3, more than one equation or method may be available. For lack of time some of the more complicated methods will only be referred to in this edition. Eventually a trade-off needs to be made between cost of computation and accuracy. However, many more computations need to be done and much more reliable measurements need to be made on wellknown engines before this kind of judgement can be made.

This section reviews the literature. Section 7.1 presents an engine design form using some of the simpler equations. Section 7.2 presents a worked out example for the GPU-3 engine. This section will use an all-capital-letter nomenclature so that there will not be a change in nomenclature when the equations are coded in FORTRAN or another machine language.

4.3.1 Capital Letter Nomenclature for Section 4.3

\[ A = \sqrt{(LCR)^2 - (ECC - RC)^2} \]  
(See Figure 4-15.)

\[ AC = \text{Effective free flow area for matrix, cm}^2 \]

\[ AF = \text{Average free flow area through regenerator, cm}^2 \]

\[ AHT = \text{Heat transfer area for matrix or cylinder wall, cm}^2 \]

\[ AHTA = \text{Conduction heat transfer area of one regenerator cylinder at level A, cm}^2 \]

\[ AHTB = \text{Conduction heat transfer area of one regenerator cylinder at level B, cm}^2 \]

\[ AHTC = \text{Conduction heat transfer area of one regenerator cylinder at cold end, cm}^2 \]

\[ AHTH = \text{Conduction heat transfer area of one regenerator cylinder at hot end, cm}^2 \]

\[ ALPH = \text{Phase angle, degrees} \]

\[ B = \sqrt{(LCR)^2 - (ECC + RC)^2} \]  
(See Figure 4-15.)

\[ BET = \text{Half angle of interleaving cones, degrees} \]

BP = Basic output power from Stirling engine before losses are deducted, watts

\[ B(M'',N'') = \frac{1}{2} \sin 2M'' - 1 \cos 2N'' - 1 \] 

\[ M'' = \frac{3 - NX}{2} \]

\[ N'' = \frac{v}{v} \]

(for wires and screens, \( NX = 0.59 \) and \( B(M'',N'') \) is 1.79)
\begin{align*}
C &= \sqrt{(LR - RC)^2 - (ECC)^2} \\
CP &= \text{Heat capacity of gas at constant pressure, J/g K} \\
CPH &= \text{Heat capacity of gas at constant pressure when flowing from hot end to cold end of regenerator, J/g K} \\
CPM &= \text{Heat capacity of regenerator matrix, J/g K} \\
CPO &= \text{Heat capacity of gas at constant pressure when flowing from cold end to hot end of regenerator, J/g K} \\
CPD &= \text{Heat capacity of piston or displacer, J/g K} \\
CPW &= \text{Heat capacity of cylinder wall, J/g K} \\
CV &= \text{Heat capacity of gas at constant volume, J/g K} \\
C1 &= \text{Al(GV)/(CP)} \\
C2L &= (Xk)(AHK)/(CP)(MDA)^{1 - N(AF)} \text{ with } Xk, MDA, \text{ and } N \text{ evaluated at outlet, cold, conditions} \\
C2O &= (Xk)(AHK)/(CP)(MDA)^{1 - N(AF)} \text{ with } Xk, MDA, \text{ and } N \text{ evaluated at inlet, hot conditions} \\
C3 &= \text{Geometry constant (see Equation 4-130)} \\
D &= \sqrt{1 + K^2 - 2K \cos(\alpha)} \\
DC &= \text{Diameter of compression, cold piston, cm} \\
DCY &= \text{Diameter of engine cylinder, cm} \\
DCB &= \text{Diameter of cone at base, cm} \\
DD &= \text{Diameter of displacer cylinder, cm} \\
DDR &= \text{Diameter of displacer drive rod, cm} \\
DE &= \text{Diameter of expansion, hot space, cm} \\
DEL &= \sqrt{(TAU)^2 + 2(TAU)(KAP)\cos(ALPH) + (KAP)^2} \\
DEL1 &= \text{Pressure drop, MPa} \\
DELTM &= \text{Temperature swing of regenerotor matrix material, K (see Equation 4-127)} \\
DELW &= \text{Work for one increment} \\
DIC &= \text{Inside diameter of cooler tubes, cm} \\
DID &= \text{Internal diameter of displacer, cm} \\
DIH &= \text{Inside diameter of heater tubes, cm} \\
DOC &= \text{Outside diameter of cooler tubes cm} \\
DOH &= \text{Outside diameter of heater tubes, cm} \\
DP &= \text{Diameter of power piston cylinder, cm} \\
DR &= \text{Diameter of each regenerator, cm} \\
DM &= \sqrt{(LCR + RC)^2 - (ECC)^2} \\
E &= \text{RC sin(\beta)} \\
ECC &= \text{Crank eccentricity, cm (see Figure 4-15)} \\
LT &= \text{Regenerator efficiency} \\
F &= \text{Friction factor} \\
FA &= \text{Area factor for radiant heat transfer, Equation 4-122 and 4-123} \\
FC &= \text{Fraction of gas inventory in cold space} \\
FCMAX &= \text{Maximum in FC during cycle} \\
FCMIN &= \text{Minimum in FC during cycle} \\
FC = \text{Fraction of cycle time for gas flow into hot space} \\
FC = \text{Effective fraction of the total cycle time steady flow passes in one direction through the cooler (heater).} \\
\end{align*}
Effective fraction of cycle time steady mass flow moves out of (into)
the hot space

Effective fraction of cycle time steady mass flow moves out of (into)
the cold space

Cooling water flow, w sec

Emmissivity factor for radiant heat transfer

Filler factor, fraction of regenerator volume filled with solid

Fraction of gas inventory in hot space

Maximum in H during cycle

Minimum in H during cycle

Radiation shield factor

Fraction of gas inventory in regenerator

\[ \frac{V_{H}}{V_{LR}} = \frac{1}{V_{LR} - V_{R}(\cos \theta)} \]

Mass velocity, based upon flow area, g/cm²/sec

Single annulus cooler gap thickness, cm

Mass velocity when gas flows from hot end of regenerator to cold end

Mass velocity when gas flows from cold end of regenerator to hot end

Conversion constant

\( 10^{-5} (\text{MPa} \cdot \text{sec}^{-2}) \cdot \text{cm} \)

Single annulus heater gap thickness, cm

Displacer gap thickness, cm

Time averaged gap thickness, cm

Heat transfer coefficient, w cm² K⁻¹

Heat transfer coefficient when gas flows from hot end to cold end of
regenerator, w cm² K⁻¹

Heat transfer coefficient when gas flows from cold end to hot end of
regenerator, w cm² K⁻¹

K

KAP

KAP VIII

KAP VIII

Gas thermal conductivity, w cm⁻¹ K⁻¹

Thermal conductivity of metal, w cm K⁻¹

Metal thermal conductivity at level A, w cm K⁻¹

Metal thermal conductivity at level B, w cm K⁻¹

Metal thermal conductivity at cold metal temperature, w cm K⁻¹

Metal thermal conductivity at hot metal temperature, w cm K⁻¹

Thermal conductivity of regenerator matrix, w cm K⁻¹

Thermal conductivity of piston or displacer, w cm K⁻¹

Thermal conductivity of cylinder wall, w cm K⁻¹

Length of matrix, cm

Length wall from level A to cold end (See Figure 4.23), cm

\[ L_B = \frac{1}{K_B} \left( \frac{\text{K}_A}{K_T} - K_T \right) \]

Length wall from level B to A (See Figure 4.23), cm

Length of each cooler tube, cm

Cylindrical length of each cooler tube, cm

Connecting rod length for Rhombic drive, cm (See Figure 4.25)

Length of displacer connecting rod, cm
LCRP = Length of power piston connecting rod, cm
LCL = Length of cone, cm
LP = Length of displacer, cm
LH = Total length of each heater tube, cm
LRB = Length of cylinder or regenerator wall from hot end to level B (See Figure 4-3A), cm
LHHL = Heated length of each heater tube, cm
LNX = Half thickness for sheets and radius for wires of regenerator material, cm
LR = Length of regenerator, cm
LT1 = Temperature wave length in displacer wall, cm
\[ \frac{\lambda_1}{D1} \]
LT2 = Temperature wave length in cylinder wall, cm
\[ \frac{\lambda_2}{D2} \]
L1 = \( KL \sqrt{\frac{OMG}{GR}} \)
M = Working gas inventory, g mol
MN = Amplitude of sinusoidal mass velocity at inlet, hot (\( x=0 \)) of regenerator, g/sec cm
ML = Mechanical friction loss, watts
MMX = Mass of all matrix material, g
MSH = Mesh size, wires/cm
MG = Gas viscosity, g cm sec
MW = Molecular weight of working gas, g/mol
N = Number of power units per engine
NC = Number of cones on piston or displacer
NP = Net power, watts
NR = Number of regenerators per power unit
NS = Number of screen layers per regenerator
NTC = Number of cooler tubes per power unit
NTII = Number of heater tubes per power unit
NTUC = Number of transfer units for gas cooler
\( H(AHT) \)
\( \frac{2F1(HCS)}{(CV)} \)
NTUH = Number of transfer units for gas heater
\( H(AHT) \)
\( \frac{2F1(TWH)}{(CV)} \)
NTUP = Number of transfer units for regenerator using constant pressure heat capacity
\( H(AHT)/\left(WHS(CP)\right) \)
NTUV = Number of transfer units for regenerator using constant volume heat capacity
\( H(AHT)/\left(WHS(CV)\right) \)
NU = Engine frequency, Hz, cycles/sec
NX = Exponent in correlation for heat transfer coefficient
\( H \cdot X_0^{K(N)} \)
P = Pressure at angle PHI for MARG = 1, MPa, see Equation 4-68
OMG = Frequency of operation, radians/sec
\( \omega \) (rad/sec)
PFA = Amplitude of sinusoidal pressure swing, MPa
PAVG = Time averaged mean pressure, MPa
PC  = Calculated pressure for given PAVG at given angle PHI, MPa
PHI  = Crank angle, degrees
PM  = Mean pressure for cycle (see Equation 4-69), MPa
PMAx  = Maximum engine pressure, MPa
PMIN  = Minimum cycle pressure, MPa
PR  = Prandtl number

Pavg = \frac{\text{CP} \cdot \text{MU}}{\text{KG}}

QC  = Conduction heat transfer, watts
QGC  = Heat from gas cooler, watts
QGH  = Heat to gas heater, watts
QITS  = Internal temperature swing loss, watts
QN  = Net heat input, watts
QR  = Heat transport by radiation, watts
QRH  = Reheat loss, watts
QS  = Static heat conduction loss, watts
QT  = Temperature swing loss, watts
R  = Universal gas constant
\approx 8.314 \text{ J/g mol K}
RC  = Crank radius, cm (see Figure 4-15.)
RCD  = Crank radius for displacer, cm
RCP  = Crank radius for power piston, cm
RE  = Reynolds number
\approx 4 \text{ (RH)G/MU}
RH  = Hydraulic radius, cm
\approx AC(L)/(AHT)
RHOM  = Mean gas density, g/cm³
RH01  = Gas density at entrance, g/cm³
RH02  = Gas density at exit, g/cm³
RM  = Gas constant in mass units, J/g K
ROM  = Density of regenerator matrix material, g/cm³
ROI  = Density of piston or displacer wall, g/cm³
RO2  = Density of cylinder wall, g/cm³
R1 = Thermal resistance of hot third of regenerator cylinder, K/watt
\text{(see Equation 4-114)}
R2 = Thermal resistance of middle third of regenerator cylinder, K/watt
\text{(see Equation 4-115)}
R3 = Thermal resistance of cold third of regenerator cylinder, K/watt
\text{(see Equation 4-116)}
S = \frac{\text{TC}}{\text{VHD}} \left( \frac{\text{VHD} + \text{VRD} + \text{VCD}}{\text{VHL}} \right)
\text{VHL} \left( \frac{1}{\text{TH} + \text{TR} + \text{TC}} \right)
\text{VHL} \left( \frac{1}{\text{TH} + \text{TR} + \text{TC}} \right)
\text{Reduced dead volume}
SC  = Stroke of compression, cold piston, cm
SCL  = Stroke clearance, cm
SD  = Stroke of displacer, cm
SE  = Stroke of expansion, hot piston, cm
SGM  = \frac{(KU)(SD)}{(GK)(OMG)} \left( (\text{RH}01)(\text{CP}1)(\text{WTT}) + (\text{RH}02)(\text{CP}2)(\text{WTT}) \right)
SIG  = Stefan-Boltzman constant
\approx 5.67 \times 10^{-8} \text{ W/(cm²K⁴)}
SP = Stroke of power piston, cm
TA = Temperature at level A, K
TB = Temperature at level B, K
TAU = TC/TH
TC = Effective temperature in cold, compression space, K
TCM = Heat sink metal temperature, K
TCS = Temperature of cold space, K
TCW1 = Temperature of cooling water into engine, K
TD1 = Thermal diffusivity of piston or displacer wall, cm²/sec
   = K1/((RH01)(CPI))
TD2 = Thermal diffusivity of cylinder wall, cm²/sec
   = K2/((RH02)(CD2))
TH = Effective temperature in hot, expansion, space, K
TRET = tan⁻¹[(KAPsin(ALPH))/(TAU + KAPcos(ALPH))], degrees
THF = Thickness of foil separating gaps in slot regenerator, cm
THM = Heat source metal temperature, K
THS = Temperature of wire in screens of regenerator, cm
THD = Effective entering gas temperature to gas heater, K
TR = Effective regenerator temperature, K
   = (THM - TCM)/(THM/TCM)
T1 = Gas temperature at inlet, hot end, of regenerator, K
T2 = Gas temperature at outlet, cold end of regenerator, K
VC = VCLX + VCD
VCD = Cold dead volume, cm³
VCDX = Extra cold dead volume besides that in gas cooler, cm³
VCL = Maximum cold live volume, cm³
VCLX = Cold live volume at a particular angle PHI, cm³
VCL1 = Cold live volume at beginning of increment, cm³
VCL2 = Cold live volume at end of increment, cm³
VD = Total dead volume, cm³
   = VHD + VRD + VCD
VH = VHLX + VHD
VHD = Hot dead volume, cm³
VHDX = Extra hot dead volume besides that in gas heater, cm³
VHL = Maximum hot live volume, cm³
VHLX = Hot live volume at a particular angle PHI, cm³
VHL1 = Hot live volume at beginning of increment, cm³
VHL2 = Hot live volume at end of increment, cm³
VPL = Power piston live volume, cm³
VRD = Regenerator dead volume, cm³
VT = Total gas volume at angle PHI
   = VH + VC + VRD
VTL = VH + VCL
W = Mass rate of flow, g/sec
WCS = Effective flow rate of gas into cold space, g/sec
WHS = Effective flow rate of gas into hot space, g/sec
WP = Total windage power, watts
WPC = Gas cooler windage power, watts
WPH = Gas heater windage power, watts
WPR = Regenerator windage power, watts
WRS = Effective flow rate of gas through regenerator, g/sec
WL = Wall thickness of displacer or hot cap wall, cm
Wc = Wall thickness of cylinder wall, cm

\[ X = \frac{(TAU - 1)^2}{4(TAU - 1)(K)cos(ALPH) + K^2} \]

\[ XK = \text{Coefficient in heat transfer correlation, } H = XK(W)NY \]

\[ Y = TAU + 4\left(\frac{VI}{VHL}\right)\frac{1}{TAU + K} \]

\[ YK = \text{Factor in Equation 4-106 defined by Equations 4-107, 4-108, or 4-109} \]

\[ Z = 4\left(\frac{VI}{VHL}\right)(TAU + K) + \frac{1}{TAU + K} \]

\[ ZK = \text{Factor in Equation 4-106 defined in Table 4-10} \]

\[ ZI = \text{Compressibility factor} \]

= 1 except for temperatures less than 70 K

4.3.2 Basic Power Output

Basic power output is the Schmidt equation or a numerical equivalent of the Schmidt equation which is discussed in Section 4.3. The basic assumptions for the Schmidt equation are as follows:

1. Temperature in each gas space is known and stays constant.
2. Variable volumes change sinusoidally with a fixed phase angle between the hot and cold variable volumes.
3. There is no pressure difference between the gas spaces.
4. Ideal gas law applies.
5. There is no leakage into or out of the working gas space.

Assumption 5 is not trivial because a small leak can make a big difference in Stirling engine performance. Assumption 4 is good for heat engines. Below 70 K a compressibility factor needs to be used. Assumption 3 is not serious for engine designs that would normally be used. Assumption 2 is exactly true for some engines like the Renia type swashplate but it is quite far off for the Rhombic drive machine with its short connecting rods. It is important to know the phase angle between the live cold volume and the live hot volume.

Assumption 1 is not true. The gas temperatures in the hot and cold variable volume spaces in the usual type of Stirling engine vary over a wide range during each cycle (see Section 4.1.6). However, for the dead volume fractions normally encountered in Stirling engines, calculations based on assumption 1 give the right power output (see Figure 4-6). This basic power is determined by the formulas given below for the different types of Stirling engines.

Basic power formulas are now given in the uniform capital letter nomenclature for the following different cases:

1. Schmidt equation - sinusoidal, isothermal
   1.1 alpha engine form
   1.2 beta engine form
   1.3 gamma engine form
2. Sinusoidal, non-isothermal
3. Non-Sinusoidal, isothermal
   3.1 rhombic drive
   3.2 double crank
4. Non-sinusoidal, adiabatic variable volume spaces
   4.1 rhombic drive

4.3.2.1 Schmidt Equations-Sinusoidal-Isothermal

4.3.2.1.1 Alpha. Dual Piston Form of Schmidt Equation

Equation 4-49 below is an adaption of Equation 4-45 to give basic power instead of work per cycle. It was selected because the average pressure normally specified can be used to compute the gas inventory by assuming that the displacer and the power piston are both at the mid-point of their stroke. That is:

\[
M = \frac{PAVG}{K} \left( \frac{VHL \times TH + VRD \times TR + VCD \times TC}{2TC} \right)
\]  

Thus the basic power is:

\[
BP = \frac{\nu(2\pi)(KAP)(1 - \tau\alpha)(\sin(\alpha)) M(R) TC}{(\tau\alpha + KAP + 25)(1 - (\Delta e)^2)(1 + \sqrt{1 - (\Delta e)^2})}
\]  

Alternately the basic power can be determined by first noting that the maximum pressure is related to the mean pressure by the formula (73 j):

\[
PMAX = \frac{PAVG}{\sqrt{1 - (\Delta e)}/(1 + \Delta e)}
\]  

Also it is instructive to know that (73 j):

\[
PMAX = 1 + \Delta e
\]

Thus Equation 4-42 is transformed to:

\[
BP = \frac{\nu(PAVG)(\nu)e(1 - \tau\alpha)(\Delta e)\sin(\theta\epsilon)}{(KAP + 1)(1 + \sqrt{1 - (\Delta e)^2})}
\]

4.3.2.1.2 Beta. Engine Form, Schmidt Equation

For engines in which the displacer and the power piston are in the same cylinder and have the same diameter the stroke of the displacer and the power piston overlap and the displacer and power piston come very close at one point in the cycle. The basic power for this type of machine is given by the following equation:

\[
BP = \frac{\nu(\pi)(1 - \tau\alpha)(PMAX)(VHL)\sin(\alpha)(\sin(\pi))}{\sqrt{Y + (Y' - X')^2}} \left[ \frac{Y - X}{Y + X} \right]^2
\]
There is a problem here in that Equation 4-53 calls for PMAX but PAVG is usually specified. Use Equation 4-50 to obtain PMAX from PAVG. Also, in Equation 4-53, the dead volume is taken to have a temperature that is the arithmetic mean of the hot and cold volume temperatures instead of the more correct log mean temperature.

4.3.2.1.3 Gamma Engine Form - Schmidt Equation

Note that Equation 4-54 has the same qualifications as Equation 4-53. Thus:

$$BP = \frac{(NU)(\pi)(1 - \tau \lambda)(PMAX)(VHL)(K)(sin(ALPH))}{Z + (Z^2 - X^2)^{\frac{1}{2}}} \left[ \frac{Z - X}{Z + X} \right]^{\frac{1}{2}}$$

(4-54)

4.3.2.2 Basic Power Assuming Sinusoidal, Non-Isothermal Processes

Most Stirling engines have open hot and cold variable volume spaces. These spaces have so much volume for the surrounding wall area that they act like adiabatic spaces. E. B. Qvale (68 m, 67 n) takes pressure, temperature and mass as independent variables. First the basic performance is calculated; second, the required displacements are found; and finally, the basic performance is corrected for frictional flow losses and finite heat transfer rates. His basic performance is based on adiabatic variable volume spaces, no frictional flow losses and no temperature difference for heat transfer in the heat exchangers and in the regenerator, but with dead volume within the heat transfer components. Because of the way Qvale has chosen the independent variables, the analysis is directly applicable to engine synthesis rather than the prediction of the performance of a given engine.

The author has diligently studied the above references but he has not been able to follow it through well enough to use these references or explain them to others. Qvale claims close agreement with experimental measurements on the Allison PD-67A Stirling engine (see Section 4.1). Qvale assumes sinusoidal variations in mass.

4.3.2.3 Non-Sinusoidal, Isothermal

Practical Stirling engines quite often have short cranks that lead to piston motions quite far from sinusoidal. Also, the rhombic drive used in many Stirling engines is even more complicated because the cranks are eccentric and the dead center is not 180° from bottom dead center. Means for calculating the basic power for two important types of Stirling engines will be given in this section:

1. Rhombic Drive - Beta Engine (Philips)
2. Crank Drive - Alpha Engine (United Stirling)
4.3.2.3.1 Rhombic-Rota (Philips Engine)

The rhombic drive is commonly used on Philips Stirling engines. Any other scheme that uses short connecting rods would also deviate significantly from simple harmonic motion. Figure 4-15 shows the position of the rhombus when the cold live volume is at zero and at a maximum. Zero crank angle PHI is when the cranks are inward. The rhombus is fully extended vertically and the power piston and displacer are closest together. From Figure 4-15:

$$A = (LCR)^2 - (ECC - RC)^2$$  \hspace{1cm} (4-55)
$$B = (LCR)^2 - (ECC + RC)^2$$  \hspace{1cm} (4-56)

Therefore:

$$VCL = 2(A - B) \frac{\pi}{4} ((DCY)^2 - (DDR)^2)$$  \hspace{1cm} (4-57)

In general for any angle PHI:

$$VCLX = \frac{\pi}{2} ((DCY)^2 - (DDR)^2) \sqrt{[(LCR)^2 - (ECC - RC)^2] - [(LCR)^2 - (ECC + RC \cos (PHI))^2]}$$  \hspace{1cm} (4-58)

Figure 4-16 shows the position of the displacer and cylinder wall and cranks at zero and at maximum hot volume. From Figure 4-16:

$$D1 = (LCR + RC)^2 - (ECC)^2$$  \hspace{1cm} (4-59)
$$C = (LCR - RC)^2 - (ECC)^2$$  \hspace{1cm} (4-60)

Therefore:

$$VHL = (D1 - C) \frac{\pi}{4} (DCY)^2$$  \hspace{1cm} (4-61)

In general:

$$E = RC \sin (PHI)$$  \hspace{1cm} (4-62)
$$F1 = (LCR)^2 - (ECC - RC \cos (PHI))^2$$  \hspace{1cm} (4-63)
$$VHLX = \frac{\pi}{4} (DCY)^2 (F1 + E - C)$$  \hspace{1cm} (4-64)

These more complicated relationships for the hot and cold volumes make an analytic solution impossible. Fortunately, a numerical solution is not very difficult. From the desired operating conditions for the engines set TH, TC and TR which is usually the log mean of TH and TC. Also set PAVG as the time average pressure for the cycle. From the dimensions of the engine evaluate and simplify the above equations for VHLX and VCLX. If this calculation is to be performed by hand, evaluate VHLX and VCLX for PHI = 0, 30, 60, ..., 330, 360°. Then compute VT and P by the following equations:

$$VH = VHL + VHD$$  \hspace{1cm} (4-65)
$$VC = VCLX + VCD$$  \hspace{1cm} (4-66)
\[ V_1 = V_H + V_C + V_{RBD} \]  
\[ p = \frac{1}{V_H + V_{RBD} + V_C} M(R) \]  
(4-62)

Calculate the mean pressure by:

\[ P_M = \left( \sum_{P_{HI:300}} P \right) \cdot \frac{1}{12} \]  
(4-69)

The desired \( M(R) \) to meet the specified \( P_{AVG} \) is given by:

\[ M(R) = \frac{P_{AVG}}{P_M} \]  
(4-70)

The pressures in the engine, \( P_C \), at each \( P_{HI} \) to cause the specified \( P_{AVG} \) to be correct are determined by:

\[ P_C = \frac{P(AVG)}{P_M} \]  
(4-71)

\( V_I \) is plotted against \( P_C \). This work curve is integrated numerically to obtain the work output. 4.5% should be added to the calculated volume to make up for the error involved in using only 12 points for integration (see Section 4.2).

During the calculation of the basic power, it is also important to calculate the mass distribution of gas between the hot spaces, regenerator and cold spaces assuming a constant pressure during each instant of the cycle. From the perfect gas equation:

\[ 1 = \frac{P(V_H)}{M(R) V_H} + \frac{P(V_{RBD})}{M(R) V_{RBD}} + \frac{P(V_C)}{M(R) V_C} \]  
(4-71.1)

in in in

hot regenerator cold
space space space

\[ 1 = H + R + R \]  
(4-71.2)

The first and third of these fractions are evaluated and are used to compute the mass flow through the regenerator, heater and cooler.

4.3.2.3.2 Crank Drive-Alpha Engine

The current United Stirling engine (see Section 3.2) uses 4 double-acting pistons in a Rinta arrangement with cranks using short connecting rods operating each piston. The analysis below will be for one quarter of the total engine. Figure 4-17 shows what is being analyzed. For any angle \( P_{HI} \) the height of the piston above bottom dead center is:

\[ h = (L_C R)^2 - ((R_C) \sin (P_{HI})^2) - (R_C)^2 \cos (P_{HI}) \]  
(4-72)

\[ - L_C R + R_C \]
Therefore the hot live volume is:

\[ VHLX = \frac{n}{4} (DCY)^2 (2(RC) - X1) \]

and the cold live volume is:

\[ VCLX = \frac{n}{4} ((DCY)^2 - (DDR)^2)(X2) \]

where:

\[ X2 = (LCR)^2 - ((RC) \sin (PHI + ALPH))^2 - RC \cos (PHI + ALPH) - LCR + RC \]
Next, Equations 4-65 to 4-71 are employed to calculate the work diagram in the same way as was done for the rhombic drive. A numerical integration of the work diagram gives the work per cycle. A 12 point per cycle integration is 4.5% low. To get the power for one power unit of the four in the engine, one must multiply this integral by the engine frequency in cycles per second to obtain the basic power in watts.

Here also the mass fractions in the hot, regenerator and cold spaces are computed by Equation 4-71.1. Note that these fractions depend upon the geometry and the temperatures but do not change with pressure.

4.3.2.4 Non-Sinusoidal, Non-Isothermal

After Qvale's work for Professor J. L. Smith at M.I.T. mentioned in Section 4.3.2.2, Rios (69 am, 69 o) expanded the work using the same general assumptions. However, he is able to start with a specific engine with a crank-connecting rod drive mechanism. He was able to solve the differential equations and integrate numerically to an overall steady state. He built his own cooling engine and was able to closely predict its performance. The author has studied this work at some length and he has a listing of the associated computer program but has not yet put it into operation.

An extension of the computational method described in Section 4.1.5 will now be presented as it applies to a rhombic drive beta type engine. This calculation procedure is not practical to compute by hand, but it can be done in a few minutes with programmable hand-held calculators.

The computational procedures given in Section 4.3.2.3.1 are followed up to Equation 4-64. Then the pressure for $M(R) = 1$ at any angle PHI during the cycle is given by:

$$P = \frac{1}{\frac{VHLX}{THS} + \frac{VHD}{TH} + \frac{VRD}{TR} + \frac{VCD}{TC} + \frac{VCLX}{TCS}}$$

(4-76)

where THS and TCS are the hot space and cold space temperatures. Start at PHI = 0 with THS1 = TH and TCS1 = TC. Compute P by Equation 4-76 and set P1. Let VHLX, and VCLX at PHI = 0 by VHL1 and VCL1. At, say, PHI = 30° compute VHL2 and VCL2 using Equations 4-64 and 4-58, respectively. In other words, for the first calculation:

$$P1 = \frac{1}{\frac{VHL1}{TH} + \frac{VHD}{TH} + \frac{VRD}{TR} + \frac{VCD}{TC} + \frac{VCL1}{TC}}$$

(4-77)

For the second calculation:

$$P2 = \frac{1}{\frac{VHL2}{THS2} + K1 + \frac{VCL2}{TCS2}}$$

(4-78)

Where:

$$K1 = \frac{VHD}{TH} + \frac{VRD}{TR} + \frac{VCD}{TC}$$

(4-79)

which is always constant. Also for the adiabatic spaces:
\[
\frac{\text{THS}_2}{\text{TH}} = \left(\frac{P_2}{P_1}\right)^E \quad \text{also} \quad \frac{\text{TCS}_2}{\text{TC}} = \left(\frac{P_2}{P_1}\right)^E
\]

Where \( E = k - \frac{1}{2} \) and \( k = c_p/c_v = 1.40 \) for hydrogen so \( E = 0.296 \).

Substituting Equations 4-80 into 4-78:

\[
P_2 = \frac{1 - \frac{\text{VHL}_2}{\text{TH}}\left(\frac{P_2}{P_1}\right)^E + \frac{\text{VCL}_2}{\text{TC}}\left(\frac{P_2}{P_1}\right)^E}{\text{TH} \left(\frac{P_2}{P_1}\right)^E}
\]

(4-81)

The only unknown in Equation 4-81 is \( P_2 \). A solution is made by the secant method of approximation or some other successive approximation method like Newton's method. \( P_2 \), \( \text{THS}_2 \) and \( \text{TCS}_2 \) are calculated. The third pressure in the series is calculated by:

\[
P_3 = \frac{1}{XH + K1 + XC}
\]

where if \( \text{VHL}_3 > \text{VHL}_2 \):

\[XH = \frac{\text{VHL}_2}{\text{THS}_2\left(\frac{P_2}{P_1}\right)^E} + \frac{\text{VHL}_3 - \text{VHL}_2}{\text{TCS}_2\left(\frac{P_3}{P_2}\right)^E} \text{TH}\left(\frac{P_3}{P_2}\right)^E\]

or if \( \text{VHL}_3 < \text{VHL}_2 \):

\[XH = \frac{\text{VHL}_3}{\text{THS}_2\left(\frac{P_2}{P_1}\right)^E}\]

In the same way, if \( \text{VCL}_3 > \text{VCL}_2 \):

\[XC = \frac{\text{VCL}_2}{\text{TCS}_2\left(\frac{P_3}{P_2}\right)^E} + \frac{\text{VCL}_3 - \text{VCL}_2}{\text{TC}\left(\frac{P_3}{P_2}\right)^E} \text{TC}\left(\frac{P_3}{P_2}\right)^E\]

or if \( \text{VCL}_3 < \text{VCL}_2 \):

\[XC = \frac{\text{VCL}_3}{\text{TCS}_2\left(\frac{P_3}{P_2}\right)^E}\]

Now in Equation 4-82, \( P_3 \) is the only unknown and it is solved as before. Then the mixed mean gas temperatures are found by:

\[\text{THS}_3 = \frac{\text{VHL}_3}{XH} \quad \text{and} \quad \text{TCS}_3 = \frac{\text{VCL}_3}{XC}\]

(4-83)

where \( XH \) and \( XC \) are calculated by whichever equation above applies.

From increment to increment in PHI as the calculation progresses this method takes into account that when gas is leaving a variable volume space the temperature in that space simply obeys the adiabatic compression and expansion law. On the other hand if gas is entering the variable volume space the old gas
temperature changes according to the adiabatic law from the previous gas temperature. The new gas temperature changes according to the same law but from the entering gas temperature. Then there is a mixing at the end of each increment to get the mixed mean temperature for the next increment.

This computational process is continued for two or more cycles until the pressures and temperatures start to repeat with adequate accuracy. This computational procedure is stable at any angle increment. Smaller angle increments give more accurate results. For a particular case the effect of smaller angle increments should be evaluated to determine at what point adequate accuracy is obtained.

4.3.3 Fluid Friction Loss

The basic power is computed as if there is no fluid friction. Energy loss due to fluid friction is deducted from the basic power as a small perturbation on the main engine process. If fluid friction consumes a large fraction of the basic power the following methods will not be accurate but then one would not choose a design to be built unless the fluid friction were less than 10% of the basic power.

Fluid friction inside the engine can be computed by published correlations for fluid flow through porous media and in tubes. These flow friction correlations are applicable for steady, fully-developed flow. If the fraction of the gas inventory found in the hot spaces and in the cold spaces is plotted against crank angle, it is apparent that to a good approximation this periodic flow can be approximated by (1) steady flow, in one direction, (2) no flow for a period of time (3) then steady flow back in the other direction, and (4) then no flow to complete the cycle. (See Figure 7-1.) The mass flow into and out of the regenerator is not quite in phase due to accumulation and depletion of mass in the regenerator. Note that the mass flow at the cold end is much more than the mass flow at the hot end mostly due to gas density change. The average mass flow rate and the average fraction of the total cycle time that gas is flowing in one direction at the hot end of the regenerator is used for the heater flow friction and heat transfer calculations. The average mass flow rate and the average fraction of the total cycle time flowing in one direction at the cold end of the regenerator is used for the cooler flow friction and heat transfer calculations. For the regenerator the mean of the above two flows and of the above two fractions will be used.

The above decisions are approximations. In the future the author hopes to determine how good these approximations are by comparing them with more laborious but more exact calculations.

4.3.3.1 Regenerator Pressure Drop

4.3.3.1.1 Screens

Kays and London (vol 1, p. 33) give the formula for pressure drop through a matrix as would be used for a regenerator:
The flow acceleration term can be ignored in computing windage loss for the full cycle because the flow acceleration for flow into the hot space very nearly cancels the flow acceleration for flow out of the hot space. With this simplifying assumption, the pressure drop due to regenerator friction is:

\[
\text{DELP} = \frac{(G)^2}{2(GC)(\text{RHO1})} \left[ \frac{1}{(AF)^3} \left( \frac{\text{RHO1}}{\text{RHO2}} - 1 \right) + \frac{F(L)\text{RHO1}}{(\text{RH})\text{RHOM}} \right] \quad (4-84)
\]

In the above equation the friction factor \( F \) is a function of the Reynolds number, \( \text{RE} = 4(\text{RH})G/\text{MU} \). Figure 4-18 shows the correlation for stacked screens usually used in Stirling engines. Note that the relationship is dependent somewhat on the porosity. Since this calculation is already an approximation it is recommended that a simpler relationship be used more adapted to use in simple computer programs. For \( \text{RE} < 60 \) let:

\[
\log F = 1.73 - 0.93 \log(\text{RE}) \quad (4-86)
\]

For \( 60 < \text{RE} < 1000 \):

\[
\log F = 0.714 - 0.365 \log(\text{RE}) \quad (4-87)
\]

For \( \text{RE} > 1000 \):

\[
\log F = 0.015 - 0.125 \log(\text{RE}) \quad (4-88)
\]

This relationship is shown by a dashed line in Figure 4-18.

Figure 4-18. Flow Through an Infinite Randomly Stacked Woven-Screen Matrix, Flow Friction Characteristics; a Correlation of Experimental Data from Wire Screens and Crossed Rods Simulating Wire Screens. Perfect Stacking, i.e., Screens Touching, is Assumed. (64 l, p. 130)
Finally, the viscosity of the gas must be evaluated. For hydrogen:

$$\mu = 88.13 \times 10^{-6} + 0.200 \times 10^{-6}(T - 293) + 0.118 \times 10^{-6}(PAVG)$$  (4.89)

For helium:

$$\mu = 196.14 \times 10^{-6} + 0.464 \times 10^{-6}(T - 293) - 0.093 \times 10^{-6}(PAVG)$$  (4.90)

For air:

$$\mu = 181.94 \times 10^{-6} + 0.536 \times 10^{-6}(T - 293) + 1.72 \times 10^{-6}(PAVG)$$  (4.91)

This data is from American Institute of Physics Handbook, 2nd Edition, pp. 2-227. Table 4-7 tabulates the data also.

### Table 4-7

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<th>TR K</th>
<th>Hydrogen $\mu$</th>
<th>Helium $\mu$</th>
<th>Air $\mu$</th>
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</tr>
</tbody>
</table>
4.3.3.1.2 Slots

Besides screens, the other type of surface sometimes used is the annular gap or the slot. This type of surface is practical at flow velocities always in the laminar region, \( RE < 2000 \). For this case (641, p. 103):

\[
F = 24/RE \quad (4-92)
\]

4.3.3.2 Heater and Cooler Pressure Drop - Tubular

Heater and cooler pressure drops are usually small in comparison with the regenerator. Heaters and coolers are usually small diameter round tubes although an annular gap is practical for small engines (see Section 4.3.3.1.2). Pressure drop through these heaters and coolers are determined by Equation 4-85 with \( F \) determined from the fanning friction factor plot (see Figure 4-19) and \( \text{Rhom} \) being evaluated at heat source or heat sink temperature and at \( \text{Pavg} \). The length-to-diameter ratio is usually very large so for simple programs let:

- For \( RE \leq 2000 \):
  \[
  F = 16/RE \quad (4-93)
  \]
- For \( RE > 2000 \):
  \[
  \log F = -1.34 - 0.20 \log(RE) \quad (4-94)
  \]

Figure 4-19. Gas Flow Inside Circular Tubes with Abrupt Contraction Entrances; a Summary of Experimental and Analytical Data. (641, p. 123)
4.3.3.3 Heater and Cooler Pressure Drop - Interleaving Fins (see Ref. 77 h)

One of the advantages of this type of heat exchanger is that the gas flows into it rather than through it. Also, it is rather complicated because the flow passage area changes with the stroke. Experimental data are needed. But before these can be obtained, an approximate theory is presented in the interim. One of the best types of interleaving fins is the nesting cone because the cone like the tube can have a thin wall and heat can be added and removed directly from the outside of the cone. Assume that the effective average flow rate takes place in the gap between the cone and its mate at mid-stroke at the point where the volume of the gap beyond this point is equal to the volume of the gap up to this point. Also assume that the friction factor is the same as for slots.

4.3.3.4 Heater, Cooler and Regenerator Windage Loss

Since the gas flows through these parts twice per cycle, the windage loss in the heater, regenerator or cooler is determined, approximately from the computed pressure drop by the approximate formula:

\[ WP = (DELP)(VHL)^2(NU) \text{ watts} \]  (4-95)

4.3.4 Mechanical Friction Loss

Mechanical friction due to the seals and the bearings is hard to compute reliably. It essentially must be measured. However, if the engine itself were used, the losses due to mechanical friction would be combined with power required or delivered by the engine. If indicated and brake power are determined then mechanical friction loss is the difference, that is, \( W_B - W_I \) using the nomenclature from Figure 4-12. The friction loss should be measured directly by having the engine operate at the design average pressure with a very large dead volume so that very little engine action is possible. The engine need not be heated but the seals and bearing need to be at design temperature.

4.3.5 Basic Heat Input

The basic heat input of an engine using the second order approximations is the basic power output divided by the Carnot efficiency for the assumed gas temperatures for the heater and cooler spaces. Therefore:

\[ BHI = BP/(1 - TC/TH) \]  (4-96)

This basic heat input must be transferred through the gas heater of the engine. Also the heat needed to supply the reheat loss must also be transmitted through this heater. Therefore, after the computational process is gone through once there must be an adjustment downward in the effective hot gas temperature and upward in the effective cold gas temperature to allow for some effective temperature drop in the gas heater and gas cooler. These new temperatures for the hot space and cold space would change many of the calculations performed the first time around. Particularly it would change the basic heat input because of their direct effect on the Carnot efficiency. It has been found that this procedure is rapidly convergent.
4.3.6 Reheat Loss

One way that extra heat is required at the heat source is due to the inefficiency of the regenerator. The regenerator reheats the gas as it returns to the hot space. The reheat not supplied by the regenerator must be supplied by the heater as extra heat input. Figure 4-20 shows how the gas temperatures vary in the heater, regenerator, and cooler during flow out of the hot space as well as flow into it. Note that at inflow, the gas attains cooler temperature, then is heated up in the regenerator part way. The temperature difference, A, is then multiplied by the heat capacity, the effective flow rate, and the fraction of time that this gas is flowing, to obtain the reheat loss. The methods derived from the literature and from the author's own practice are given below. The formula for reheat used by the author is:

\[
QRH = \frac{\text{Effective Regenerator Flow Rate}}{\text{Regenerator Ineffectiveness}} = \frac{FCT \times (WRS) \times (CV) \times (THM - TCM)}{\frac{2}{NTUV + 2}} \times \frac{\text{Fraction Heat Temp}}{\text{Time Flowing}} \times \frac{\text{Capacity Into Hot Space}}{A}
\]

(4-97)

Figure 4-20. Reheat Loss.
Each element in Equation 4-97 is a type of an approximation. The fraction of time flowing into the hot space is estimated by extrapolating the maximum flow into the hot space to the total flow to find the fraction of the total cycle time that this process would occupy, if the flow rate were always at its maximum value. This fraction, ICT, turns out to be about one-third. ICT will be taken as 1/3 if an analytical Schmidt equation is used. If a numerical procedure is used, ICT is computed by Equation 4-107.5. The effective flow rate then is determined by the flow through the regenerator, WRS (see Equation 4-101 or 4-102.6). Neither heat capacity CV or CP is strictly correct. More complicated analyses can take into account more rigorously the effect of pressure change during gas flow through the regenerator (75 ac, 77 bl). The rationale for using CV in Equation 4-97 is that the transfer of gas takes place when the total volume is relatively constant. However only a small amount of the total volume is in the regenerator at any one time. A better equation suggested by LeRC during review is probably:

\[ Q_{RH} = FCT \left( \frac{(WRS)(CP)(THM - TCM)}{N - TCI} \right) \left( \frac{V_{RD}(CV)(P_{MAX} - P_{MIN})}{N - TCI} \right) \left( \frac{N_{TON} + 2}{N - TCI} \right) \]  

(4-97.1)

This point deserves further study because QRH is quite often the chief loss term.

The temperature difference \( A \) in Figure 4-20 is represented by the total temperature difference between the hot metal and the cold metal times the regenerator ineffectiveness. This ineffectiveness is one minus the effectiveness of the regenerator material (see Equation 4-7). This formula for ineffectiveness agrees with the simple equations in earlier standard references on regenerators such as Saunders and Smoleniec (51 q).

The idea of separating power output and the heat losses into a number of superimposed processes has been used by a number of investigators of the Vuilleumier cycle. The details of this analysis have been given in a number of government reports. The Vuilleumier cycle is a heat operated refrigeration machine which uses helium gas and regenerators very similar to the way the Stirling engine is constructed. This super position analysis has worked well in VM cycle machines. In an RCA report (69 ac, pp. 3-37) the measured cooling power using this method of analysis was found to be within 8.9% of that calculated. Crouthamel and Shelpuk (75 ac) give the following formula for the reheat loss after it is translated into the nomenclature used in this section:

\[ Q_{RH} = \left( \frac{1}{4} \right) (WRS)(CP)(THM - TCM) \left( \frac{N_{TON} + 2}{N - TCI} \right) \]  

(4-98)

Equation 4-98 is written in the same order as Equation 4-97 and therefore can be directly compared. The first term, one quarter, is specific for their particular machine and therefore needs to be evaluated for another type of machine. The flow rate is evaluated in the same way, but the heat capacity is different. Probably this can be justified to be CP instead of CV because the VM cycle machine undergoes a relatively small change in pressure during its cycle. Also, the distinction between metal temperatures is also relatively small at this stage of analysis.
Ronald White (76-1) gives an equation for reheat loss which he obtains from B. Lee (71-5). It is generally of the same form as the above except that the regenerator efficiency is determined by using the heat transfer coefficient for the flow out as well as for the flow in. The Lee equation is:

\[ Q_{RH} = (1 - LE)CP(THM - 1CM) \left( \frac{PMAX - PMIN}{2R(TCM)^{1/2}} \right) \]  

Where:

\[ LE = \frac{(H1)(H2)(LR)}{\left( H1 \right)^2} + H2(LR)(CP1)(GI)(RH) + H1(LR)(CP0)(80)R1 \]  

Lee states in recent correspondence that these equations are used for small crank angle increments and then summed. They cannot be used as an overall equation for one full cycle.

In Equations 4-97 and 4-98 the number of transfer units in the regenerator must be evaluated. Now:

\[ NTUV = \frac{H(AH1)}{(WRS)(CV)} \] and \[ NTUP = \frac{H(AHT)}{(WRS)(CP)} \]  

This useful dimensionless group is made from a heat transfer coefficient, H, a heat transfer area, AHT, a mass flow rate, WRS, and a heat capacity, CV or CP. Means for evaluating these components will now be given.

Heat capacity is a function of temperature. In the units being used in this manual, heat capacities are given in Table 4-8.

Heat transfer area for screens is determined by the formula:

\[ AHT = \frac{H(MSH)(THM)(DR)}{N(THM)(DR)} \text{ cm}^2 \]  

The heat transfer area for a single or multiple annulus or another type of regenerator can be calculated similarly.

In reality the mass flow rates through the regenerator are continuously varying and are quite different at one end of the regenerator than at the other. As explained in Section 4,3,3 a good approximation of the flow rate through the regenerator is two periods of steady flow between two periods of no flow. For design procedures that use a Schmidt equation, it is assumed that the flow time in one direction is one third of the cycle time. The following formula is used to calculate WRS in previous equations.

\[ WRS = \frac{VHL(3 NH)RROM}{2g/sec} \]  

where the mean gas density is evaluated by the formula:

\[ RROM = 0.042(PAVG)/(TR) \]  

For computations using a numerical integration, it is possible to compute the mass distribution at each point in the cycle. Then graphically or numerically the steady mass flows and times can be determined for both ends of the regenerator.

The effective fraction of the total cycle time steady flow passes in one direction through the heater is:
Table 4-8

Heat Capacities for Working Gases, J/g K

<table>
<thead>
<tr>
<th>Temperature K</th>
<th>Hydrogen</th>
<th>Hehium</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CP</td>
<td>CV</td>
<td>CP</td>
</tr>
<tr>
<td>298.15</td>
<td>14.31</td>
<td>10.18</td>
<td>5.20</td>
</tr>
<tr>
<td>400</td>
<td>14.50</td>
<td>10.37</td>
<td>5.20</td>
</tr>
<tr>
<td>500</td>
<td>14.52</td>
<td>10.39</td>
<td>5.20</td>
</tr>
<tr>
<td>600</td>
<td>14.56</td>
<td>10.43</td>
<td>5.20</td>
</tr>
<tr>
<td>700</td>
<td>14.62</td>
<td>10.49</td>
<td>5.20</td>
</tr>
<tr>
<td>800</td>
<td>14.70</td>
<td>10.57</td>
<td>5.20</td>
</tr>
<tr>
<td>1000</td>
<td>14.99</td>
<td>10.86</td>
<td>5.20</td>
</tr>
<tr>
<td>1200</td>
<td>15.43</td>
<td>11.30</td>
<td>5.20</td>
</tr>
<tr>
<td>1500</td>
<td>16.03</td>
<td>11.90</td>
<td>5.20</td>
</tr>
<tr>
<td>2000</td>
<td>17.03</td>
<td>12.90</td>
<td>5.20</td>
</tr>
<tr>
<td>2500</td>
<td>17.86</td>
<td>13.73</td>
<td>5.20</td>
</tr>
<tr>
<td>3000</td>
<td>18.40</td>
<td>14.27</td>
<td>5.20</td>
</tr>
</tbody>
</table>


\[
FCTH = (FCT1 + FCT2)/2 \quad (4-102.1)
\]

The effective steady mass flow rate for the heater is:

\[
WHS = \frac{(FHMAX - FHMIN)M(MW)}{FCTH/NU} \quad (4-102.2)
\]

The effective fraction of the total cycle time steady flow passes in one direction through the cooler is:

\[
FCTC = (FCT3 + FCT4)/2 \quad (4-102.3)
\]

The effective steady mass flow rate for the cooler is:

\[
WCS = \frac{(FCMAX - FCMIN)M(MW)}{FCTC/NU} \quad (4-102.4)
\]

Thus for the regenerator:

\[
FCT = (FCTH + FCTC)/2 \quad (4-102.5)
\]

and

\[
WRS = (WHS + WCS)/2 \quad (4-102.6)
\]

The heat transfer coefficient is derived from Figure 4-21. For instance, for a porosity of \((1 - 0.286) = 0.717\) the equation is:
$\log\left(\frac{H}{G'(CP)} \cdot (PR)^3\right) = -0.13 - 0.412 \log(RE)$

Figure 4-21.—Gas Flow Through an Infinite Randomly Stacked Woven-Screen Matrix, Heat Transfer Characteristics; a Correlation of Experimental Data from Wire Screens and Crossed Rods Simulating Wire Screens. Perfect Stacking, i.e., Screens Touching, is Assumed (64 1, p. 129).

where the Reynolds number, $RE$, is the same as that used in Section 4.3.3.1.1 and the Prandtl number $PR = CP(MU)/KG$. $CP$ and $MU$ have been given previously. KG and PR are given in Table 4-9.

In a Stirling engine the regenerator is subjected to an important gas pressure oscillation out of phase with the gas flow oscillation that has been evaluated in the above equations. Bjorn Qvale (69 n) has developed an equation that takes this pressure wave into account. He assumes that pressure and mass flow variations are sinusoidal. He assumes that the matrix temperature is expressible by a second order polynomial in $X$, the distance along the regenerator. He also concludes that, 1) fluid friction has negligible effect on the state of the gas in the regenerator, 2) the gas and matrix temperatures at one location are practically constant with time, 3) the difference between the gas- and matrix temperature is small compared to the longitudinal temperature change and, 4) the effect of longitudinal conduction is negligible on the heat transfer process. He starts out with equations representing conservation of momentum, mass, and energy and the equation of state. He concludes that:
Table 4-9
Thermal Conductivities and Prandtl Numbers
for Working Gases
Thermal Conductivity, KG, W/cm K
Prandtl Number, PR, dimensionless
101 atm pressure

<table>
<thead>
<tr>
<th>Temperature K</th>
<th>Kg(^1) Hydrogen KG(^1)</th>
<th>PR</th>
<th>Kg(^1) Helium KG(^1)</th>
<th>PR</th>
<th>Kg(^1) Air KG(^1)</th>
<th>PR</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>18.15 x 10(^{-4})</td>
<td>0.720</td>
<td>14.99 x 10(^{-4})</td>
<td>0.688</td>
<td>2.614 x 10(^{-4})</td>
<td>0.761</td>
</tr>
<tr>
<td>400</td>
<td>22.12 x 10(^{-4})</td>
<td>0.730</td>
<td>17.95 x 10(^{-4})</td>
<td>0.709</td>
<td>3.345 x 10(^{-4})</td>
<td>0.772</td>
</tr>
<tr>
<td>500</td>
<td>25.64 x 10(^{-4})</td>
<td>0.744</td>
<td>21.14 x 10(^{-4})</td>
<td>0.717</td>
<td>3.95 x 10(^{-4})</td>
<td>0.796</td>
</tr>
<tr>
<td>600</td>
<td>29.1 x 10(^{-4})</td>
<td>0.757</td>
<td>24.7 x 10(^{-4})</td>
<td>0.711</td>
<td>4.56 x 10(^{-4})</td>
<td>0.830</td>
</tr>
<tr>
<td>700</td>
<td>32.5 x 10(^{-4})</td>
<td>0.771</td>
<td>27.8 x 10(^{-4})</td>
<td>0.718</td>
<td>5.13 x 10(^{-4})</td>
<td>0.864</td>
</tr>
<tr>
<td>800</td>
<td>36.0 x 10(^{-4})</td>
<td>0.781</td>
<td>30.7 x 10(^{-4})</td>
<td>0.729</td>
<td>5.69 x 10(^{-4})</td>
<td>0.899</td>
</tr>
<tr>
<td>1000</td>
<td>42.8 x 10(^{-4})</td>
<td>0.810</td>
<td>36.3 x 10(^{-4})</td>
<td>0.749</td>
<td>6.72 x 10(^{-4})</td>
<td>0.974</td>
</tr>
<tr>
<td>1200</td>
<td>49.5 x 10(^{-4})</td>
<td>0.846</td>
<td>41.6 x 10(^{-4})</td>
<td>0.770</td>
<td>7.59 x 10(^{-4})</td>
<td>1.057</td>
</tr>
<tr>
<td>1500</td>
<td>59.7 x 10(^{-4})</td>
<td>0.890</td>
<td>49.4 x 10(^{-4})</td>
<td>0.795</td>
<td>8.70 x 10(^{-4})</td>
<td>1.189</td>
</tr>
<tr>
<td>2000</td>
<td>79.6 x 10(^{-4})</td>
<td>0.923</td>
<td>62.0 x 10(^{-4})</td>
<td>0.828</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td></td>
<td></td>
<td>73.9 x 10(^{-4})</td>
<td>0.858</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td></td>
<td></td>
<td>85.1 x 10(^{-4})</td>
<td>0.887</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^1\) Touloukian, Y.S., et. al., Thermophysical Properties of Matter, Volume 3.
\[ QRH = \frac{c_p(MDA)(T_1)(AF)}{(OMG)^2} \times \frac{2B(M'', N'')}{(C20)(YL)^2 + (C2L)(YL)^{NX}} \]

\[ = \frac{2(YL)(T_2^2/T_1 - 1) + (C1 - A1)((YL)^2 \cos (THT) + \cos (THT) - (A1)(TAUL))}{(YLI)^2} \]

where:

- \( MDA = \) amplitude of the sinusoidal mass velocity at inlet, \((X = 0)\) of regenerator, \(g/sec \ cm^2\)
- \( T1 = \) gas temperature at inlet, hot, end, \(K\)
- \( T2 = \) gas temperature at outlet, cold, end, \(K\)
- \( OMG = \) frequency of operation, radians/sec
- \( \pi/2 \sin 2M'' - 1 \theta'' \cos 2N'' - 1 \theta'' d\theta'' \)
- \( B(M'', N'') = \)
- \( M'' = (3 - NX)/2 \)
- \( N'' = \)
- \( NX = \) exponent in correlation for the heat transfer coefficient
- \( W = \) mass flow rate, \(g/sec\)
- \( C2 = \frac{(XX)(AHT)}{(CP)(MDA)^{1-N}(AF)} \)
- \( C20 = C2 \) with \( XX, MDA \) and \( N \) evaluated at inlet, hot, conditions
- \( C2L = C2 \) with \( XX, MDA \) and \( N \) evaluated at outlet, cold, conditions
- \( YL = \) ratio between the maximum mass flow rate at the end of the regenerator to that at the beginning of the regenerator
- \( QVALE\) compares his theory with the experimental results on a cooling engine.

There is some doubt that Equation 4-105 is interpreted correctly. At a number of places quantities were undefined and guesses had to be made. Also Qvale's thesis (67 n) gives almost the same formula but would predict \( QRH \) one half that in Equation 4-105. A letter from Qvale says Equation 4-105 is correct.

If the power output for a particular Stirling engine were evaluated by a numerical method which also gives the pressure, and the mass flow at the hot and cold ends of the regenerator, then one would have the information necessary to substitute into Equation 4-105 and obtain an answer.

Qvale compared his theory with the experimental results on a cooling engine.
done by Rea (66 h) and predicts the ineffectiveness and therefore the QRH within ± 20%.

Rios (69 ar), as was mentioned previously, calculates a work diagram assuming adiabatic hot and cold spaces and any form of volume change with time that can be specified. His reheat loss uses 4 quantities that are calculated when the work diagram is calculated by the computer program.

4.3.7 Shuttle Conduction

Figure 4-22 shows how shuttle conduction works. Shuttle conduction happens anytime a displacer or a hot cap oscillates across a temperature gradient. It is usually not frequency dependent for the speeds and materials used in Stirling engines. The displacer absorbs heat during the hot end of its stroke and gives off heat during the cold end of its stroke. Usually neither the displacer nor the cylinder wall change temperatures appreciably during the process. Shuttle conduction depends upon the area involved, the thickness of the gas filled gap, GR, the temperature gradient \((TH-TC)/L\), the gas thermal conductivity, KG, and the displacer stroke, SD. It is also dependent on the wave form of the motion and in some cases, upon the thermal properties of the displacer and of the cylinder wall. All formulas in the literature are of the form:

\[
Q_{SH} = \frac{(Y_K)(Z_K)(SD)^2(K_G)(THM-TCM)(DCY)}{(GR)(LD)} \tag{4-106}
\]

The quantity \(Z_K\) depends upon the type of displacer or hot cap motion, and \(Y_K\) depends upon the thermal properties of the walls and the frequency of operation. Table 4-10 shows the results of a literature survey for \(Z_K\). Note that there is a substantial disagreement about what \(Z_K\) should be for the sinusoidal case. The author has derived the lower value and he would recommend it. This value, \(\pi/8\), agrees with Rios but does not agree with Zimmerman. However, there are no data that would lay the matter to rest.

Rios has published values for \(Y_K\) to take into account the effect of frequency or wall thermal properties which are sometimes important.

In Rios' Ph.D. thesis (69 ar) he gives:

\[
Y_K = \frac{2(L_1)^2 - L_1}{2(L_1)^2 - 1} \tag{4-107}
\]

where

\[
L_1 = \frac{K_1}{K_G} \left(\frac{OMG}{GR}\right)^2 \sqrt{\frac{2(TD_1)}{K_1}}
\]

\(K_1\) = thermal conductivity of piston or displacer, \(\text{W/cm·K}\)

\(TD_1\) = thermal diffusivity of piston or displacer, \(\text{cm}^2/\text{sec}\)

\[
R_01 = \frac{(K_1)}{(R_01)(C_P1)}
\]

\(R_01\) = density of piston or displacer, \(\text{g/cm}^3\)

\(C_P1\) = heat capacity of piston or displacer, \(\text{J/g·K}\).
Later Rios modified his theory to take into account the thermal properties of the cylinder wall as well (71 an). The new theory gives:

\[ Y_K = \frac{1}{1 + (LB)^2} \]

(4-108)

Figure 4-22. Shuttle Conduction.
Table 4-10
Coefficient for Shuttle
Heat Conduction Equation
(Ignoring Effect of Walls)

<table>
<thead>
<tr>
<th>Motion</th>
<th>Investigator</th>
<th>Ref.</th>
<th>ZK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square wave (t_1)</td>
<td>Zimmerman</td>
<td>71 be</td>
<td>(\frac{\pi}{4} = 0.785)</td>
</tr>
<tr>
<td>time at one end, (t_2) time at other</td>
<td>Crouthame &amp;</td>
<td>75 ac</td>
<td>(\frac{\pi}{4} = 0.785)</td>
</tr>
<tr>
<td></td>
<td>Shelpuk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sinusoidal</td>
<td>Martini</td>
<td>(1)</td>
<td>(\frac{\pi}{8} = 0.393)</td>
</tr>
<tr>
<td>(effect of walls ignored)</td>
<td>Zimmerman</td>
<td>71 be</td>
<td>(\frac{\pi}{8} = 0.582)</td>
</tr>
<tr>
<td></td>
<td>Rios</td>
<td>71 an</td>
<td>(\frac{\pi}{8} = 0.393)</td>
</tr>
<tr>
<td></td>
<td>White</td>
<td>71.1</td>
<td>(0.186\pi = 0.584)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>69 aa</td>
<td>(0.186\pi = 0.584)</td>
</tr>
</tbody>
</table>

(1) McDonnell Douglas Reports, never published
where:

\[ LB = 1 + \frac{1}{2\pi} \frac{KG}{GR} \left( \frac{LT1}{K1} + \frac{LT2}{K2} \right) \]

LT1 = Temperature wave length in displacer

\[ = 2\pi \sqrt{\frac{Z(TD1)}{OMG}} \]

LT2 = Temperature wave length in cylinder wall

\[ = 2\pi \sqrt{\frac{Z(TD2)}{OMG}} \]

TD2 = thermal diffusivity of the cylinder wall, cm²/sec (defined same as TD1)

The above factor applies for simple harmonic motion and for engines in which LT1 is smaller than the thickness of the displacer wall and LT2 is smaller than the thickness of the cylinder wall. Rios gives equations for solving the problem for any periodic motion by using of Fourier series expansion. To help determine whether the above factor applies, Rios gives some typical values of LT at room temperature (see Table 4-11).

Table 4-11
Typical Temperature Wave Lengths, LT, at Room Temperature Conditions

Reference: Rios, 71 an

<table>
<thead>
<tr>
<th>Centimeters</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild Steel</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.21</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>0.74</td>
</tr>
<tr>
<td>Phenolic</td>
<td>0.85</td>
</tr>
<tr>
<td>Pyrex Glass</td>
<td>0.26</td>
</tr>
</tbody>
</table>

If the wall thickness is considerably smaller than the temperature wave length, then it may be assumed that radial temperature distribution in the walls is uniform. Rios (71 an) proposes the following definition of YK for this case:

\[ YK = \frac{1}{1 + (SGM)^2} \]  \hspace{1cm} (4-109)
where:

$$SGM = \frac{(KG)(SD)}{(GR)(OMG)} \left( \frac{1}{(RO1)(CP1)(WT1)} + \frac{1}{(RO2)(CP2)(WT2)} \right)$$

and:

$$WT1 = \text{wall thickness of displacer, cm}$$
$$WT2 = \text{wall thickness of cylinder wall, cm}$$

Note that when the thermal properties of the wall do not matter, YK whether evaluated by Equation 4-107, 4-108 or 4-109 would all evaluate to nearly 1. There is not any published formula that treats the case of cylinder and displacer wall thickness of the order of the temperature wave length. There are also no published formulas for the case of a thick cylinder wall and a thin displacer or visa-versa. For horsepower size engines Equation 4-108 will apply. For model engines or artificial heart engines Equation 4-109 will apply. Therefore, for horsepower size, high pressure engines the recommended equation for shuttle heat conduction is:

$$QSH = \frac{(1 + (LB)^2/8)}{(1 + (LB)^2/8)} \pi \frac{(SD)^2 (KG)(THM - TCM)(DCY)}{(GR)(LD)} \quad (4-110)$$

For model size engines using low gas pressure and very thin walls:

$$QSH = \left( \frac{1}{1 + (SGM)^2} \right)^{1/2} \pi \frac{(SD)^2 (KG)(THM - TCM)(DCY)}{(GR)(LD)} \quad (4-111)$$

It also should be emphasized that Equation 4-110 and 4-111 are for nearly sinusoidal motion of the displacer or hot cap. Square wave motion would double this result. Ramp motion should reduce this result some.

**4.3.8 Gas and Solid Conduction**

This heat loss continues while the engine is hot, independent of engine speed. It is simply the heat transferred through the different gas and solid members between the hot portion and the cold portion of the engine. Heat can be transferred by conduction or radiation. In the regenerator the gas moves, but under this heading the heat loss is computed as if the gas were stagnant. In Section 4.3.6, the reheat loss is computed assuming there is no longitudinal conduction.

The uncertainty about what thermal conductivities and what emissivities to use to evaluate this loss makes its measurement with the engine desirable. In some engines the hot and cold spaces are heated and cooled directly. In this case measuring the heat absorbed by the cooling water with the engine heated to temperature but stopped will give this heat loss directly. However, all the horsepower-size engines described in Section 3 have indirectly heated and cooled hot and cold gas spaces. For this case the sum of the gas and solid conduction and the shuttle conduction can be determined by measuring the heat absorbed by the cooling water for a number of slow engine speeds with the engine heater at temperature and then extrapolating to zero engine speed.
4. Radiation along a cylinder with radiation shields.

Solutions to each one of these problems will now be given.

4.3.8.1 Constant Area Conduction

Heat loss by conduction of this type is computed by the formula:

$$QC = \frac{KG(AHT)(THM - TCM)}{LD}$$  \hspace{1cm} (4-112)

where the thermal conductivities areas and lengths are germain to Path.3 and 4a above, KG is evaluated at mid-point temperature. (See Table 4-9.)

4.3.8.2 Variable Area, Variable Thermal Conductivity

For one dimensional heat conduction where the heat transfer area varies continually and the thermal conductivity changes importantly, the heat conduction path is divided into a number of zones. The average heat conduction area for each zone is calculated. The temperature in each zone is estimated and from this estimate a thermal conductivity is assigned. Figure 4-23 gives the thermal conductivities for some probable construction materials in the units used in this manual. It should be noted that there is quite a variability in some common materials like low carbon steel. Measured thermal conductivity differs by a factor of 3 is shown. Differences are due to heat treatment and the exact composition. With commercial materials having considerable variability, it is strongly recommended that the static heat loss be checked by extrapolating the heat requirement for the engine to zero speed. This number would then need to be analyzed to determine how much shuttle heat loss is also being measured and how much is static heat loss.

For purposes of illustration, assume 3 zones are chosen along a tapered cylinder wall. (See Figure 4-24.) Temperatures $TA$ and $TB$ must be estimated between $TH$ and $TC$ to start. The heat transfer areas $AHTH$, $AHTB$, $AHTA$ and $AHTC$ are computed based upon engine dimensions. The heat through each segment is the same. Thus:

$$QC = \frac{(KMH + KMB)(AHTH + AHTB)}{2} \left(\frac{THM - TB}{LHB}\right)$$

$$= \frac{(KMB + KMA)(AHTB + AHTA)}{2} \left(\frac{TB - TA}{LBA}\right)$$

$$= \frac{(KMA + KMC)(AHTA + AHTC)}{2} \left(\frac{TA - TCM}{LAC}\right)$$

Let:

$$R1 = \frac{LHB}{(KMH + KMB)(AHTH + AHTB)}$$ \hspace{1cm} (4-114)

$$R2 = \frac{LBA}{(KMB + KMA)(AHTB + AHTA)}$$ \hspace{1cm} (4-115)

$$R3 = \frac{LAC}{(KMA + KMC)(AHTA + AHTC)}$$ \hspace{1cm} (4-116)
Usually the following conduction paths are identified and should be evaluated for each engine:

<table>
<thead>
<tr>
<th>Path No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Engine cylinder wall.</td>
</tr>
<tr>
<td>2.</td>
<td>Displacer or hot cap wall.</td>
</tr>
<tr>
<td>3.</td>
<td>Gas annulus between cylinder and hot cap.</td>
</tr>
<tr>
<td>4.</td>
<td>Gas space inside displacer or hot cap.</td>
</tr>
<tr>
<td></td>
<td>a. gas conduction</td>
</tr>
<tr>
<td></td>
<td>b. radiation</td>
</tr>
<tr>
<td>5.</td>
<td>Regenerator cylinders.</td>
</tr>
<tr>
<td>6.</td>
<td>Regenerator packing.</td>
</tr>
</tbody>
</table>

The engine cylinder, the displacer and regenerator cylinders must be designed strong enough to withstand the gas pressure for the life of the engine without changing dimension appreciably. However, extra wall thickness contributes unnecessarily to the heat loss. For this reason the cylinder walls of most high powered engines are much thinner at the cold end where the creep strength is high than they are at the hot end. This, of course, complicates evaluation of this type of heat loss.

The following types of heat transfer problems need to be solved to evaluate these heat losses:

1. Steady, one dimensional conduction, constant area, variable thermal conduction.
2. Steady, one dimensional conduction, variable area, variable thermal conductivity.
3. Steady, one dimensional conduction through a composite material (wire screens).
Figure 4-23. Thermal Conductivities of Probable Construction Materials for Stirling Engines.
Then:

\[ QC = \frac{THM - TCM}{RI + R2 + R3} \]  \hspace{1cm} (4-117)

Once QC is computed, then:

\[ TB = THM - T1(QC) \]  \hspace{1cm} (4-118)

\[ TA = TB - R2(QC) \]  \hspace{1cm} (4-119)

TB and TA are compared with the original guesses. If they are appreciably different so that the thermal conductivities would be different, then new thermal conductivities based upon these computed values of TB and TA would be determined and the process repeated. Once more is usually sufficient.

This same procedure is used for the engine cylinder and the displacer if the walls are tapered.

Figure 4-24. Computation of Tapered Cylinder Wall Conduction.
4.3.8.3 Conduction Through Regenerator Matrices

Usually the regenerator of a Stirling engine is made from many layers of fine screen that is lightly sintered together. The degree of sintering would have a big bearing on the thermal conductivity of the screen stack since the controlling resistance is the contact between adjacent wires. Some cryogenic regenerators use a bed of lead spheres.

In the absence of data, Goring (61 n) gives the following formula for conduction through a square array of uniformly sized cylinders.

\[
K_{MX} = KG \left( \frac{1 + (KM/KG)}{1 - (KM/KG)} - FF \right)
\]

(4-120)

The thermal conductivity of the gas KG and the metal KM are evaluated at TR. The heat loss through the screens is then determined using an equation like 4-112.

Sometimes the regenerator is made from slots in which metal foils run continuously from hot to cold ends. The conductivity of the matrix in this case is:

\[
K_{MX} = \frac{KG(GR) + KM(THF)}{GR + THF}
\]

(4-120a)

Then the heat loss through the matrix is then determined using an equation like 4-112.

4.3.8.4 Radiation Along a Cylinder with Radiation Shields

The engine displacers or the hot cap for a dual piston machine is usually hollow. Heat transport across this gas space is by gas conduction and by radiation. Radiation heat transport follows the standard formula:

\[
QR = (FA)(FE)(FN)(/4)(DID)^2(SIG)(TTHM) - (TCM)^4
\]

(4-121)

The area factor, FA, is usually determined by a graph computed by Hottel (McAdams, Heat Transmission, 3rd Ed., p. 69). For the case of two discs separated by non-conducting but reradiating walls his curve is correlated by the simple formula:

\[
FA = 0.50 + 0.20 \ln \left( \frac{DID}{LD} \right)
\]

(4-122)

Equation 4-122 is good for values of DID/LD from 0.2 to 7. For (DID/LD) < 0.2 use:

\[
FA = \frac{DID}{LD}
\]

(4-123)

Emissivity factor, FE, is the product of the emissivity at the hot end and at the cold end. Thus:

\[
FE = (EH)(EC)
\]

(4-124)

The hot and cold emissivities can be obtained from any standard text on heat
transfer. This emissivity depends upon the surface finish, the temperature and the material. It is not easy to know what the emissivity in a particular case is.

If the emissivity of the radiation shields is intermediate between the emissivity of the hot and cold surfaces, then from the number of radiation shields, NRS, the radiation shield factor, FN, is calculated approximately.

\[ FN \approx \frac{1}{1 + NRS} \]  

(4-125)

4.3.9 Pumping Loss

A displacer or a hot cap has a radial gap between the ID of the engine cylinder and the OD of the displacer. This gap is sealed at the cold end. As the engine is pressurized and depressurized, gas flows into and out of this gap. Since the closed end of the gap is cold, extra heat must be added to the gas as it comes back from this gap. Leo (70 ac) gives the formula:

\[ QPU = \frac{2(\Delta CY)^{0.6}(LD)(PMAX - PMIN)^{1.6}(NU)^{1.6}(CP)^{1.6}(THM - TCM)(GR)^{2.6}}{1.5(71)(RM)^{1.6}(KG)^{0.6}((THM + TCM)/2)^{1.6}} \]  

(4-126)

4.3.10 Temperature Swing Loss

In computing the reheat loss (see Section 4.3.6) it was assumed that the regenerator matrix temperature oscillates during the cycle a negligible amount. In some cases the temperature oscillation of the matrix will not be negligible. The temperature swing loss is this additional heat that must be added by the gas heater due to the finite heat capacity of the regenerator. The temperature drop in the regenerator matrix temperature all along the line due to a single flow of gas into the hot space is:

\[ \text{DELMX} = \frac{\text{WHS}(CV)FCT(THM - TCM)}{\text{NU}(MAX)(CPM)} \]  

(4-127)

Half of this, (DELMX)/2, is equivalent to A in Equation 4-97 and Figure 4-20 since DELMX starts at zero at the start of the flow and grows to DELMX. Thus the temperature swing loss is:

\[ QTS = (FCT)(WHS)(CV)(DELMX)/2 \]  

(4-128)

Creuthamel and Shelpek (75 ac) point out this loss but their equation is:

\[ QTS = (FCT)(WHS)(CP)(DELMX) \]  

(4-128)

Their equation substitutes CP for CV as was done also in Section 4.3.6. The reason for division by 2 seems to be recognized in their text but is not reflected in their formula. Equation 4-128 was used in Section 7. Based upon the discussion in Section 4.3.6, it is now recommended that an effective gas heat capacity based upon Equation 4-97.1 be used in Equations 4-127 and 4-128.
4.3.11 Internal Temperature Swing Loss

Some types of regenerator matrices could have such low thermal conductivity (for example, glass rods) that all the mass of the matrix would not undergo the same temperature swing. The interior would undergo less swing and the outside would undergo more swing than would be calculated by Equation 4-127. This additional swing would result in an additional heat loss. Crouthamel and Shelpuk (75 ac) give this loss as:

\[
Q_{ITS} = QTS \left[ C3 \left( \frac{\text{ROM}}{\text{CPM}} \times \frac{\text{LMX}}{\text{KM}} \text{NU} \right) \right] \cdot (4-130)
\]

The geometry constant \( C3 \) is given as 0.32 by Crouthamel and Shelpuk (75 ac) who refer to page 112 of Carslaw and Jaeger (59 o). This constant is for a slab. The constant for a cylinder or a wire is 0.25 (59 o, p. 203).

4.3.12 First Round Engine Performance Summary

At this point it is necessary to take stock of the first estimate of the net power out and the total heat in based upon the first estimate of the effective hot and cold gas temperature. The total heat requirement will be used along with the characteristics of the heat exchangers to compute the effective hot and cold gas temperatures. These new computed temperatures will be used to determine a better estimate of the basic output power and basic heat input. Heat losses and power losses will remain the same. The net power output is:

\[
NP = BP - WP - MFL \quad (4-131)
\]

The net heat input is:

\[
QN = BHI + QRH + QSH + QS + QPU + QTS + QITS - WPH - \frac{WPR}{2} \quad (4-132)
\]

4.3.13 Heat Exchanger Evaluation

Once the first estimate of the net heat input, \( QN \), is computed, the duty of the gas heater and gas cooler are determined:

\[
Q_{GH} = QN \quad (4-133)
\]

\[
Q_{GC} = QN - NP \quad (4-134)
\]

Next, the heat transfer coefficient for the gas heater and gas cooler is computed. The most common type is the tubular heat exchanger. Small machines can use an annular gap heat exchanger. Isothermalizer heat exchangers are possible. How to compute the heat transfer coefficient for each one of these types of heat exchangers will be presented. Then the way of estimating the temperature offset will be shown.

4.3.13.1 Tubular Heat Exchangers

The Reynolds number for the gas heater is:

\[
RE = \frac{\text{WHS}}{\frac{\pi}{4}(\text{DIH})(\text{NTH})(\text{MU})} \quad (4-135)
\]
A similar equation would be used for the cooler. The heat transfer coefficient is derived from the correlation on Figure 4-13. Use the solid lines because the surface temperature is controlled. Note that there is an important effect of length to diameter, LU/NUH, for the heater. The ordinate of interest in Figure 4-13 is \(H/(C_P(G))(PR)^{1/4}\). The last factor shown on Figure 4-13 is ignored because the heat exchanger will operate with small temperature differences. In evaluating the above ordinate for \(H\), the heat transfer coefficient, let \(G = WHS/(\frac{1}{4}(DIH)(NTU))\) for the heater and a similar relationship for the cooler.

4.3.13.2 Annular Gap Heat Exchangers

Small Stirling engines can use annular gap heat exchangers effectively. The heat is applied from one side and the surface temperature is assumed constant. From reference 64, page 103:

\[
\frac{H(4)(GH)}{KG} = 4.86
\]  
(4-136)

4.3.13.3 Isothermalizer Heat Exchangers

The gas in the isothermalizer is chiefly heated or cooled by compression or expansion. Under these conditions the gas in the hot space is uniformly cooled by expansion and at the same time heated from both surfaces of the gas layer. In the cold space an analogous process goes on. Under these conditions the gas layers are effectively in the form of a slab. Appendix A shows that:

\[
TH = THM - \frac{QGH(GTA)}{6(KG)(AHT)}
\]  
(4-137)

and:

\[
TC = TCM + \frac{QGC(GTA)}{6(KG)(AHT)}
\]  
(4-138)

4.3.14 Iteration To Find Effective Gas Temperature

4.3.14.1 Flow Heat Exchangers

In tubular or annular gap heat exchangers most of the heat is transferred during times of gas flow. Two thirds of the time, \(2(FCT)\), gas can be assumed to flow one way or the other through the heat exchangers. Thus:

\[
QGH = 2(FCT)(WHS)(CV)(TH - THO)
\]  
(4-138.1)

\[
= H(AHT) - \frac{(TH - THO)}{\ln(\frac{THM - THO}{TH - TH})}
\]  
(4-138.2)

or:

\[
TH = THM - \frac{QGH}{2(FCT)(WHS)(CV)(\exp(NTUH) - 1)}
\]  
(4-139)

where:

\[
NTUH = \frac{H(AHT)}{2(FCT)(WHS)(CV)}
\]
similarly:

\[ TC = TCM + \frac{QGC}{2(FCT)(WCS)(CV)(\exp(NTUC) - 1)} \]  

(4-140)

Crouthamel and Shelpuk (75 ac) present a similar calculation but reason that most of the heat transfer in the heater happens during outflow from the hot space and that most of the heat transfer in the gas cooler happens during outflow from the cold space. If this reasoning proves true the 2 in Equation 4-139 and 4-140 and in the definition of NTUH and NTUC would be changed to 1. More exact third order calculations might be used to show what factor should be used in Equations 4-139 and 4-140. Possibly 1.5 would be a good choice. Also, the question of whether to use CP or CV in the above two equations can be settled by more exact third order calculations. Equations 4-139 and 4-140 were used in Section 7. As discussed in Section 4.3.6 it is now recommended that CP be used instead of CV in the above equation because the pressure change term in Equation 4-97.1 would cancel out when one full cycle is considered.

Equation 4-139 and 4-140 give a much better estimate of TH and TC than was used the first time the basic power, BP, and the basic heat input, BHI, were calculated. If there is a significant change in TH and TC the calculation for BP and BHI are repeated. All other heat loss equations do not use TH and TC and therefore would not change. A new QGH and QGC would be computed. Again by Equations 4-139 and 4-140 a new estimate of TH and TC would be calculated. These temperatures will probably be essentially the same as the previous estimate, and the computation will be finished.

In the case of the GPU-3 engine the cooler is water cooled. Since the heat transfer coefficient for the water is much greater than for the gas, the cold metal temperature is very close to the cooling water temperature and it can be assumed to be the same. On the other hand, the thermocouple measuring the heater temperature is in the gas stream half way through the gas heater. During the time gas is flowing into the hot space the heater must heat the gas to make up for regenerator reheat loss. In addition, during the first half of the inflow, the heater must cool the gas due to some compression heating. During the last half of the inflow the heater must heat the gas due to some expansion cooling. When the inflow is complete the gas in the hot space is cooled by expansion. The main heat load for the heater comes during outflow from the hot space when the expanded gas is heated. During the first half of the outflow some additional expansion cooling occurs as the gas traverses the heater. During the second half of the outflow some compression heating occurs as the gas traverses the heater. If the thermocouple is small enough to follow the change in gas temperature, it should change considerably each cycle. Since the change reportedly does not occur, the thermocouple is large and registers the average gas temperature at the mid-point of the heat exchanger. Assume, therefore, for the case of the GPU-3 heater that the effective hot space temperature, TH, is that measured by the thermocouple and that for the purpose of computing shuttle heat conduction, static conduction and reheat loss, THM = TH. Nevertheless, the temperature of the flame heated metal tubes will be hotter than the measured temperature. Equation 4-139 can be used to estimate this temperature.
4.3.14.2 Isothermalizer Heat Exchangers

The same kind of iterative process is used as above except Equations 4-137 and 4-138 are used instead of 4-139 and 4-140.

4.3.15 Conclusions on Second Order Design Methods

1. Second order is good for practical engine design and engine optimization.
2. Second order design methods identify and quantify the losses. This makes it easier to determine what must be done to minimize the sum of all the losses.
3. Much basic work needs to be done to extend the theory and experimentally validate the equations used for second order analysis.
4. The required degree of complication in the analysis of a Stirling engine design to adequately predict performance has not been determined at this stage of public knowledge.

4.4 Third Order Design Methods

Third order design methods start with the premise that the many different processes assumed to be going on simultaneously and independently in the second order design method (see Section 4.3) do in reality importantly interact. Whether this premise is true or not is not known and no papers have been published in the open literature which will definitively answer the question. Qvale (68 m, 69 n) and Rios (70 z) have both published papers claiming good agreement between their advanced second order design procedures and experimental measurements (see Sections 5.1 and 5.2). Third order design methods are an attempt to compute the complex process going on in a Stirling engine all of a piece. Finkelstein pioneered this development (62 a, 64 b, 67 d, 75 a1) and in the last year or so a number of other people have taken up the work. If the third order method is experimentally validated then much can be learned about the workings of the machine from the computation that can not be measured reliably.

Third order design methods start by writing down the differential equations which express the ideas of conservation of energy, mass and momentum. These equations are too complex for a general analytical solution so they are solved numerically. The differential equations are reduced to their one dimensional form. Then depending on just what author's formulation is being used, additional simplifications are employed.

In this design manual the non-proprietary third order design methods will be discussed. It will not be possible to describe these methods in detail. However, the basic assumptions that go into each calculation procedure will be given.
4.4.1 Basic Design Method

In broad outline the basic design method is as follows (see Figure 4-25):
1. Specify dimensions and operating conditions, i.e., temperatures, charge
   pressure, motion of parts, etc. Divide engine into control volumes.
2. Convert the differential equations expressing the conservation of mass,
   momentum, and energy into difference equations. Include the kinetic
   energy of the gas. Include empirical formulas for the friction factor
   and the heat transfer coefficient.
3. Find a mathematically stable method of solution of the engine parameters
   after one time step given the conditions at the beginning of that time
   step.
4. Start at an arbitrary initial condition and proceed through several engine
   cycles until steady state is reached by noting that the work output per
   cycle does not change.
5. Calculate heat input.

4.4.2 Fundamental Differential Equations

Following the explanation of Urieli (77 d), there are 4 equations that must
be satisfied for each element. They are:
1. Continuity
2. Momentum
3. Energy
4. Equation of state

These relationships will be given in words and then in the symbols used by
Urieli.

4.4.2.1 Continuity Equation

The continuity equation merely expresses the fact that matter can neither be
created nor destroyed. Thus:

\[
\left. \frac{\partial m}{\partial t} + \nabla \cdot \mathbf{v} \rho \mathbf{g} \right|_{\text{control volume}} = \left. \text{net mass flux convected} \right|_{\text{outwards through surface}} \right|_{\text{of control volume}} \quad (4-141)
\]

Urieli (77 d) expresses this relationship as:

\[
\frac{\partial m}{\partial t} + \mathbf{v} \cdot \nabla \rho \mathbf{g} = 0 \quad (4-142)
\]
Figure 4-25. Sample Division of Engine Working Gas Space into Control Volumes for a Third Order Design Method.
where:

\[ m = \frac{m}{M} \]
\[ m = \text{mass of gas in control volume, Kg} \]
\[ M = \text{mass of gas in engine, Kg} \]
\[ t = \text{time, seconds} \]
\[ V = \frac{V}{Vs} \]
\[ V = \text{volume of control volume, m}^3 \]
\[ Vs = \text{total power stroke volume of machine, m}^3 \]
\[ g = \frac{g}{(M(RT_k)/Vs)} \]
\[ g = \text{mass flux density, kg/m\textsuperscript{sec}} \]
\[ R = \text{gas constant for working gas, J/Kg\cdot K} \]
\[ T_k = \text{cold sink absolute temperature, K} \]
\[ x = \frac{x}{(Vs)^{1/3}} \]
\[ x = \text{distance, meters} \]

4.4.2.2 **Momentum Equation**

\[
\begin{align*}
\frac{\partial}{\partial t} & \left( \text{rate of changes of} \right) \\
& \left( \text{net momentum flux convected} \right) \\
& \left( \text{outwards through control surface A} \right) \\
= & \left( \text{net surface force acting} \right) \\
& \left( \text{on the fluid in the control volume} \right)
\end{align*}
\]

Urieli (77 d) expresses this relationship as:

\[
\frac{3}{3t} (gV) + V \frac{3}{3x} (g^2v) + V \frac{3p}{3x} + F = 0
\]

(4-144)

where in addition:

\[ v = \frac{v}{(Vs/M)} \]
\[ V = \text{specific volume, m}^3/\text{Kg} \]
\[ P = \frac{P}{(M(RT_k)/Vs)} \]
\[ P = \text{pressure, N/m}^2 \]
\[ F = \frac{F}{M(RT_k)/(Vs)^{1/3}} \]
\[ F = \text{frictional drag force, N} \]

4.4.2.3 **Energy Equation**

\[
\begin{align*}
\text{Rate of heat transfer} & \quad = \quad \text{Rate of energy} \\
\text{to the working gas} & \quad \text{accumulation} \\
\text{from the environment} & \quad \text{within the control} \\
\text{through control surface A} & \quad \text{volume V} \\
+ & \quad \text{Net energy flux convected} \\
\text{outwards by the working} & \quad \text{Net rate of flow work} \\
\text{gas crossing the control} & \quad \text{in pushing the mass of} \\
\text{surface A} & \quad \text{working gas through} \\
\end{align*}
\]

(4-145)
Net rate of mechanical work done 
by the working gas on the environ-
ment by virtue of the rate of change 
of the magnitude of the control volume \( V \).

Urieli (77 d) expresses this relationship finally as:

\[
\frac{dQ}{dt} = \rho \frac{gT}{\gamma - 1} + \rho \frac{\partial g}{\partial \gamma} \frac{\partial g}{\partial T} - g(v) \left( V \frac{\partial p}{\partial x} + \frac{dW}{dt} \right)
\]  
(4-146)

where in addition:

\[
\begin{align*}
Q &= \frac{\dot{Q}}{MR(T_k)} \\
\dot{Q} &= \text{heat transferred, } J \\
\gamma &= \text{ratio of specific heat capacity of working gas } = \text{CP/CV} \\
T &= \frac{T}{Tk} \\
\uparrow &= \text{working gas temperature in control volume, } K \\
W &= \frac{\ddot{W}}{MR(T_k)} \text{ mechanical work done, } J
\end{align*}
\]

4.4.2.4 Equation of State

Due to the normalizing parameters Urieli uses the equation of state merely is:

\[
p(v) = m(T)
\]  
(4-147)

4.4.3 Comparison of Third Order Design Methods

It is generally beyond the scope of this first edition of the design manual to give the reader a complete explanation of how to compute by third order methods. A number of them are or soon will be in the literature. These methods will now be described briefly.

4.4.3.1 Urieli

This design method is described fully in Israel Urieli's thesis (77 af). A good short explanation is given in this IEECEC paper (77 d). He applies his method to an experimental Stirling engine of the two piston type. The hot cylinder is connected to the cold cylinder by a number of tubes in parallel. Sections of each one of these tubes are heated, cooled or allowed to seek their own temperature level in the regenerator part (see Figure 4-26). This type of engine was chosen because of ease in programming, and because heat transfer and fluid flow correlations for tubes are well known. Also an engine like this is built and is operating at the University of Witwatersrand in Johannesburg, South Africa. The intention is to obtain experimental confirmation of this design method. Urieli converts the above partial differential equations to a system of ordinary differential equations by converting all differentials to difference quotients except for the time variable. Then he solves these ordinary differential equations using the 4th order Runge-Kutta method starting from a stationary initial condition. The thesis contains the FORTRAN program. The first copies of this thesis have three errors in the main program. Later copies of the thesis will probably have these corrections added. Corrections should be obtained from I. Urieli, Ormat Turbines, E. O., Box 68, Yavne, Israel.
Urieli is now attempting to calculate the data points for the GPU-3 engine by appropriately modifying his program.

![Diagram of engine model]

Figure 4-26. Urieli Engine Model.

4.4.3.2 Schock

Al Schock, Fairchild Industries, Germantown, Maryland, presented some results of calculations using his 3rd order design procedure at the Stirling Engine Seminar at the Joint Center for Graduate Study in Richland, Washington, August 1977. His calculation started with the same differential equations as Urieli but his method of computer modeling was different but undefined. He confirmed what Urieli had said at the same meeting that the time step must be smaller than the time it takes for sound to travel from one node to the next through the gas. Al Schock's assignment was to develop an improved computer program for the free displacer, free piston Stirling engine built by Sunpower for DOE. The engine had a very porous regenerator. Although the pressures in the expansion and compression space of the engine were different, they were not visibly different when the gas pressure vs. time was plotted.

This program is as yet not publicly documented. Schock is awaiting good experimental data with which to correlate the model.

4.4.3.3 Vanderbrug

In reference 77 ae, Finegold and Vanderbrug present a general purpose Stirling engine systems and analysis program. The program is explained and listed in a 42 page appendix. Quoting from 77 ae:

The technical approach used in the Stirling Cycle Analysis Model (SCAM) is based on obtaining system transient response by lumped parameter, or nodal, numerical integration. The integration technique assumes that the thermodynamic processes are quasi-static during a small time interval for each system node or control volume. The Stirling analysis programs developed at LeRC (77 bl), the recent Finkelstein models (67 d), and most thermal analyzer programs employ this method to avoid direct integration of many non-linear uncoupled differential equations.
The lumped parameter method is desirable because either empirical or theoretical definition of component performance characteristics is easily achieved without use of transform functions. Also, discontinuities such as those produced by flowing from laminar to turbulent, or from subsonic to choked, are readily simulated.

One disadvantage of this approach is the extremely small computing time interval which is required to satisfy the quasi-static assumption for pneumatic systems. The small computing interval unavoidably translates into a high computer dollar charge.

The SCAM program was designed to provide extreme flexibility due to the modular structure and variety of user input options. The SCAM consists of a compilation, or library, of individual modeling routines. Each routine contains the logic and generalized equations, such as chamber, volume, duct, temperature controller, heat exchanger, etc., required to simulate a pneumatic system component. This allows the user to select components from the library and assemble them into fluid flow and logic control functional segments ("legs" or "loops") representative of the physical system being modeled. The user specifies initial conditions, boundary conditions, and other key parameters describing the performance characteristics of each component. All input parameters can be either constants, curve data, or forced to vary as a function of time, temperature, or some other control-value, such as crank angle.

At this point, there is nothing available to show how well this computer model works.

4.4.3.4 Finkelstein

Ted Finkelstein has made his computer analysis program (75 al) available through Cybernet at about $25 per case. Instructions and directions for use are obtainable from TCA, P. O. Box 643, Beverly Hills, California 90213. One must become skilled in the use of this program since as the engine is optimized it is important to adjust the temperature of some of the metal parts so that the temperature at the end of the cycle is nearly the same as at the beginning.

Urieli and Finkelstein use the same method in handling the regenerator nodes in that the flow conductance from one node to the next depends upon the direction of flow. Finkelstein solves the same equations as Urieli presents but he neglects the kinetic energy of the flowing gas. By so doing, he is able to increase his time step substantially. Neglecting kinetic energy will cause errors in predicting pressures during the cycle. However, it is not clear what effect this simplifying assumption has upon power output and efficiency calculations. To make a comparison one would have to use the same correlations for friction factor and heat transfer coefficient and be certain that the geometries are identical. The author has directions for using this program. Possibly a comparison of this program with the GPU-3 measurements
and other calculation procedures can be made in the future.

Finkelstein claims that his program has been validated experimentally but that the results are proprietary.

4.4.3.5 Lewis Research Center (LeRC)

The author has attempted to formulate a design procedure based upon some computation concepts originally used by M. Mayer at McDonnell Douglas. A simplified version was presented (75 ag). However, an attempt failed to extend the method to include a real regenerator with dead volume and heat transfer as a function of fluid flow. The procedure was computationally stable and approached a limiting value as the time step decreased. But when the heat transfer coefficients were set very high, there should have been no heat loss through the regenerator, but the computation procedure did not allow this to happen because gas was always entering the hot space at the temperature of the hottest regenerator element. There was also the problem of finding the proper metal temperature for the regenerator elements.

Parallel and independently of the author, Roy Tew, Kent Jefferies and Dave Miao at LeRC have developed a computer program which is very similar to the author's (77 bl). In addition, they have come up with a way of handling the regenerator which gets around the problem the author found.

The LeRC method assumes that the momentum equation need not be considered along with the equations for continuity, energy and equation of state. They assume that the pressure is uniform throughout the engine and varies with time during the engine cycle. LeRC combines the continuity, energy equation and equation of state into one equation.

\[
\frac{dT}{dt} = \frac{hA}{mcp} (T_w - T) + \frac{W_i}{m} (T_i - T) + \frac{W_o}{m} (T_o - T) + \frac{V}{mcp} \frac{dp}{dt} \tag{4-148}
\]

This equation indicates that the temperature change in a control volume depends upon heat transfer, flow in and out and pressure change. Equation 4-148 could be solved by first order numerical integration or by higher order techniques such as 4th order Runge-Kutta. LeRC did not use this approach.

LeRC used an approach of separating the three effects and considering them successively instead of simultaneously. From a previous time step they have the masses, temperature and volumes for all 13 gas nodes used. From this they calculate a new common pressure. Using this new pressure and the old pressure and assuming no heat transfer during this stage, they calculate a new temperature for each gas node using the familiar adiabatic compression formula. Next, the volumes of nodes 1 and 13, the expansion and compression space, are changed to the new value based upon the rhombic drive. New masses are calculated for each
control volume. Once the new mass distribution is known, the new flow rates between nodes are calculated from the old and new mass distributions. The new gas temperature is now modified to take into account the gas flow into and out of the control volumes during the time step. During this calculation it is assumed that each regenerator control volume has a temperature gradient across it equal to the parallel metal temperature gradient and that the temperature of the fluid that flows across the boundary is equal to the average temperature of the fluid before it crossed the boundary; heater and cooler control volumes are at the bulk or average temperature throughout. Next, local heat transfer coefficients are calculated based upon the flows. Temperature equilibration with the metal walls and matrix is now calculated for the time of one time step and at constant pressure. An exponential equation is used so that no matter how large the heat transfer coefficient, the gas temperature cannot change more than the \( \Delta T \) between the wall and the gas. Heat transfer during this equilibration is calculated. In the regenerator nodes heat transfer is used to change the temperature of the metal according to its heat capacity. In the other nodes where the temperature is controlled, the heat transfers are summed to give the basic heat input and heat output. This final temperature set after temperature equilibration along with the new masses and volumes calculated during this time step are now set to be the old ones to start the process for the next time step.

The model is set up to take into account leakage between the buffer space and the working gas volume. LeRE has developed an elaborate method of accelerating convergence of the metal nodes in the regenerator to the steady state temperature. On the final cycle LeRE considers the effect of flow friction to make the pressure in the compression and expansion space different from each other in a way to reduce indicated work per cycle.

To quote Tew (77 bl):

Typically it takes about 10 cycles with regenerator temperature correction before the regenerator metal temperatures steady out. Due to the leakage between the working and buffer spaces, a number of cycles are required for the mass distribution between working and buffer space to settle out. The smaller the leakage rate, the longer the time required for the mass distribution to reach steady-state. For the range of leakage rates considered thus far it takes longer for the mass distribution to steady-out than for the regenerator metal temperatures to settle out. Current procedure is to turn the metal temperature convergence scheme on at the 5th cycle and off at the 15th cycle. The model is then allowed to run for 15 to 25 more cycles to allow the mass distribution to settle out. When a sufficient number of cycles have been completed for steady operation to be achieved, the run is terminated.

Current computing time is about 5 minutes for 50 cycles on a UNIVAC 1100 or 0.1 minute per cycle. This is based on 1000 iterations per cycle or a time increment of \( 2 \times 10^{-7} \) seconds when the engine frequency is 50 Hz. The number of iterations per cycle (and therefore computing time) can be reduced by at
least a factor of 5 at the expense of accuracy of solution; on the order of 10% increase in power and efficiency results when iterations per cycle are reduced to 200.

4.4.4 Conclusions on Third Order Design Methods

1. A number of well constructed third order design methods are or will soon be available.
2. A choice is available between rigorous 3rd order (Urieli, Schock, Vanderbrug), 3rd order ignoring fluid inertia (Finkelstein) and 3rd order assuming a common pressure (LeRC).
3. There is a spectrum of design methods reaching from the simplest first order through simple and complex second order culminating in rigorous 3rd order analysis. However, all these methods depend upon heat transfer and fluid flow correlations based upon steady flow instead of periodic flow, because correlations of periodic flow heat transfer and flow friction which should be used have not been generated.
4. Third order analysis can be used to compute flows and temperatures inside the engine which cannot be measured in practice.
5. Third order analysis can be used to develop simple equations to be used in second order analysis.
6. Eventually when all calculation procedures are perfected to agree as well as possible with valid tests of Stirling engines, third order design methods will be the most accurate and also the longest. The most rigorous formulations of third order will be much longer and more accurate than the least rigorous formulations.
5. COMPARISON OF THEORY WITH EXPERIMENT

Among conventional engines with tubular heaters and coolers and porous regenerators, there are only three engines that are well enough known and accurately measured to be considered:

1. The Allison Stirling engine
2. The MIT cooling engine
3. The GPU-3 engine

Each one of these will now be described and the comparisons given in the literature will be made.

5.1 Allison Engine

Qvale (67 n) gives the most complete specification of the Allison Model PD-6ZA engine (62 n). These are:

- Phase angle between two volumes $\phi = -118^0$
- Engine speed 3000 rpm (50 Hz)
- $V_{AH} =$ amplitude of sinusoidal volume variation in hot space
  $= 2.475 \text{ in}^3 = 40.56 \text{ cm}^3$
- $V_{AC} =$ amplitude of sinusoidal volume variation in cold space
  $= 2.33 \text{ in}^3 = 38.18 \text{ cm}^3$
- $V_C =$ dead volume of cold heat exchanger and ducting and clearance
  $= 1.715 \text{ in}^3 = 28.10 \text{ cm}^3$ (ducting and clearance accounts for
  $1.215 \text{ in}^3 = 19.91 \text{ cm}^3$)
- $V_R =$ regenerator dead volume
  $= 4.388 \text{ in}^3 = 71.91 \text{ cm}^3$ (this
  includes 0.488 in$^3$ on one side and 0.55 in$^3$ on the other side
  of the regenerator)
- $V_H =$ dead volume of hot heat exchanger and ducting and clearance
  $= 2.59 \text{ in}^3 = 42.44 \text{ cm}^3$ (includes 1.29 in$^3 = 21.14 \text{ cm}^3$ from
  the hot volume)
- Surface Temperature - Hot = 1680 R = 933.3 K
  Cold = 628 R = 348.9 K
- Working gas - Helium
- Mean pressure = 1544 psia = 10.64 MPa
- Cold heat exchangers
  number of tubes = 152
  ID of tubes = 0.040 in = 0.102 cm
  length of tubes = 2.6 in = 6.6 cm
- Regenerators
  matrix: screen stack
  wire diameter = 0.0016 in = 0.0041 cm
  mesh = 250/in$^2 = 98.4/cm$
  filler factor = 0.31
  length = 0.8 in = 2.0 cm
  cross sectional area of all regenerators = 6.24 in$^2 = 40.26 \text{ cm}^2$

number of regenerators not known
Hot heat exchanger
number of tubes = 76
inner tube diameter = 0.060 in = 0.152 cm
length = 6.0 in = 15.2 cm

Allison built two engines which differed only in the phase angle between the two volumes. One was 118°, and one was 112°. Qvale analyzed the engine performance according to his methods (67 n, 68 m) and showed the agreement given in Figure 5-1 and 5-2. Qvale (67 n) also gives a breakdown of the predicted losses and the experimental uncertainties. These are given in Table 5-1.

5.2 MIT Cooling Engine

Rios (69 ar, 69 o) built a cooling engine and made performance measurements on it. Since it used tubular heat exchangers and porous solid regenerators, it is of interest to large Stirling engine designers.

The dimensions of this engine are as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>Warm cylinder</th>
<th>Warm exchanger</th>
<th>Regenerator</th>
<th>Cold exchanger</th>
<th>Cold cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>2.0 in</td>
<td>0.047 in</td>
<td>2.065 in</td>
<td>0.047 in</td>
<td>1.625 in</td>
</tr>
<tr>
<td>Stroke</td>
<td>3.0 in</td>
<td>0.008 in</td>
<td>0.030 in</td>
<td>0.008 in</td>
<td>2.5 in</td>
</tr>
<tr>
<td>Con. rod length</td>
<td>14.4 in</td>
<td>0.008 in</td>
<td>3.4 in</td>
<td>0.008 in</td>
<td>12.0 in</td>
</tr>
<tr>
<td>End clearance</td>
<td>0.010 in</td>
<td>0.020 in</td>
<td>0.030 in</td>
<td>0.010 in</td>
<td>0.004 in</td>
</tr>
<tr>
<td>Number of tubes</td>
<td>210</td>
<td>231</td>
<td>304 stainless steel</td>
<td>231</td>
<td></td>
</tr>
<tr>
<td>Outside diameter</td>
<td>0.047 in</td>
<td>0.047 in</td>
<td>0.047 in</td>
<td>0.047 in</td>
<td>1.625 in</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.008 in</td>
<td>0.008 in</td>
<td>0.030 in</td>
<td>0.008 in</td>
<td>2.5 in</td>
</tr>
<tr>
<td>Tube material</td>
<td>304 stainless steel</td>
<td>copper-nickel/spheroidal powder</td>
<td>304 stainless steel</td>
<td>1.625 in</td>
<td></td>
</tr>
<tr>
<td>Average length</td>
<td>21.5 in</td>
<td>21.5 in</td>
<td>3.4 in</td>
<td>21.5 in</td>
<td>9.4 in</td>
</tr>
<tr>
<td>(bent in quarter circle)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shell inside diam</td>
<td>2.065 in</td>
<td></td>
<td>2.065 in</td>
<td></td>
<td>1.625 in</td>
</tr>
<tr>
<td>Shell wall thickness</td>
<td>0.030 in</td>
<td></td>
<td>0.030 in</td>
<td></td>
<td>2.5 in</td>
</tr>
<tr>
<td>Shell length</td>
<td>3.4 in</td>
<td></td>
<td>3.4 in</td>
<td></td>
<td>12.0 in</td>
</tr>
<tr>
<td>Shell material</td>
<td>stainless steel</td>
<td></td>
<td>stainless steel</td>
<td></td>
<td>7.19 in</td>
</tr>
<tr>
<td>Matrix</td>
<td></td>
<td></td>
<td>copper-nickel/spheroidal powder</td>
<td></td>
<td>304 stainless steel</td>
</tr>
<tr>
<td>Ave. diam. of powder</td>
<td>0.010 in</td>
<td></td>
<td>0.010 in</td>
<td></td>
<td>0.004 in</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.39</td>
<td></td>
<td>0.39</td>
<td></td>
<td>0.010 in</td>
</tr>
<tr>
<td>Matrix retainers</td>
<td>100 mesh screens at each end</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of tubes</td>
<td>231</td>
<td></td>
<td>231</td>
<td></td>
<td>231</td>
</tr>
<tr>
<td>Outside diameter</td>
<td>0.047 in</td>
<td></td>
<td>0.047 in</td>
<td></td>
<td>1.625 in</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.008 in</td>
<td></td>
<td>0.008 in</td>
<td></td>
<td>2.5 in</td>
</tr>
<tr>
<td>Tube material</td>
<td>304 stainless steel</td>
<td></td>
<td>304 stainless steel</td>
<td></td>
<td>12.0 in</td>
</tr>
<tr>
<td>Length</td>
<td>9.4 in</td>
<td></td>
<td>9.4 in</td>
<td></td>
<td>7.19 in</td>
</tr>
<tr>
<td>Cold cap length</td>
<td>7.19 in</td>
<td></td>
<td>7.19 in</td>
<td></td>
<td>7.19 in</td>
</tr>
<tr>
<td>Cold cap clearance</td>
<td>0.004 in</td>
<td></td>
<td>0.004 in</td>
<td></td>
<td>0.004 in</td>
</tr>
<tr>
<td>Cold cap material</td>
<td>linen-filled micarta</td>
<td></td>
<td>linen-filled micarta</td>
<td></td>
<td>0.010 in</td>
</tr>
<tr>
<td>End clearance</td>
<td>0.010 in</td>
<td></td>
<td>0.010 in</td>
<td></td>
<td>0.010 in</td>
</tr>
</tbody>
</table>
Figure 5-1. Heat Input and Work Output for Allison PD-67A Stirling Engine.

Figure 5-2. Internal Efficiency Data for Allison PD-67A Stirling Engine.
Table 5.1
Breakdown of Losses and Powers for the Allison Model PD-67A Engine with 118° Phase Angle

<table>
<thead>
<tr>
<th>Operating Conditions</th>
<th>Speed</th>
<th>3000 rpm</th>
<th>50 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heat source temperature</td>
<td>1680 R</td>
<td>933.3 K</td>
</tr>
<tr>
<td></td>
<td>Heat sink temperature</td>
<td>628 R</td>
<td>348.9 K</td>
</tr>
<tr>
<td></td>
<td>Mean gas pressure</td>
<td>1544 psia</td>
<td>10.64 MPa</td>
</tr>
<tr>
<td>Work Output</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic work, hot cylinder</td>
<td>285 ft lbf/(\pi)</td>
<td>386.5 J/(\pi)</td>
<td></td>
</tr>
<tr>
<td>corrected for heat exchanger performance</td>
<td>283.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic work, cold cylinder</td>
<td>122.5</td>
<td>166.1</td>
<td></td>
</tr>
<tr>
<td>corrected for heat exchanger performance</td>
<td>125.5</td>
<td>170.2</td>
<td></td>
</tr>
<tr>
<td>Net basic work</td>
<td>157.5</td>
<td>213.6</td>
<td></td>
</tr>
<tr>
<td>Flow losses - cooler</td>
<td>8.8</td>
<td>11.9</td>
<td></td>
</tr>
<tr>
<td>- regenerator</td>
<td>4.1</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>- heater</td>
<td>22.6</td>
<td>30.6</td>
<td></td>
</tr>
<tr>
<td>Predicted net work (neglecting motoring and leakage losses)</td>
<td>122.0</td>
<td>165.4</td>
<td></td>
</tr>
<tr>
<td>Experimental net work</td>
<td>108.5</td>
<td>147.1</td>
<td></td>
</tr>
<tr>
<td>Heat Input</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic work at hot cylinder</td>
<td>283.0</td>
<td>383.7</td>
<td></td>
</tr>
<tr>
<td>corrected for heat exchanger</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat conduction of metal parts</td>
<td>12.6 ft lbf/(\pi)</td>
<td>17.1 J/(\pi)</td>
<td></td>
</tr>
<tr>
<td>Regenerator loss</td>
<td>25.6</td>
<td>34.7</td>
<td></td>
</tr>
<tr>
<td>Flow loss in hot control volume</td>
<td>-24.7</td>
<td>-33.5</td>
<td></td>
</tr>
<tr>
<td>Predicted heat input</td>
<td>296.5</td>
<td>402.1</td>
<td></td>
</tr>
<tr>
<td>Experimental heat input</td>
<td>287.0</td>
<td>389.2</td>
<td></td>
</tr>
<tr>
<td>Predicted Efficiency</td>
<td>= 122/296.5 = 0.411</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental Efficiency</td>
<td>= 108.5/287.0 = 0.378</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Predicted Pressure Ratio</td>
<td>= 1.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental Pressure Ratio</td>
<td>= 1.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental Accuracies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net work</td>
<td>± 4%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat Input</td>
<td>± 4%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency</td>
<td>± 6%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The warm end crank was driven by an electric motor and it in turn drove the cold end crank by a timing belt. The warm heat exchanger was water cooled. The cold heat exchanger was covered by a brass shell. An electric heater was soldered to the cold exchanger shell to adjust the load to the refrigerator. Refrigerant boiling out of this space was measured with a rotameter.

Helium was used as a working gas. The filling of the shell of the cold heat exchanger was either nitrogen, Freon 12 or Freon 13. In steady operation the power to the cold-exchanger-shell heater was adjusted to boil the liquid at the same rate that the vapor was condensed on the exchanger tubes. A steady pressure in the cold exchanger shell indicated a steady operating temperature.

The basic data taken by Rios were as follows:
1. Indicator diagram for the cold and warm end cylinders and indicator diagrams for the pressure drop. These diagrams provided values for comparison against the model with perfect components and pressure drop losses.
2. Cold exchanger and warm exchanger temperatures. The warm-end temperature was the cooling water outlet, while the cold end temperature was that of the condensed fluid. Difference between inlet and outlet cooling water was negligible.
3. Cold-exchanger-heater power. Provides a direct measurement of the net refrigeration.
4. Refrigerator RPM.
5. Volume variation phase angle.
6. Rate at which gas is vented from cold-exchanger shell.

Rios published the results of 20 data points. These are copied from (69 ar) for the convenience of the reader (see Table 5.2). The following definitions are used in Table 5.2.

Windage Power = \( \frac{\omega}{2\pi} \int \delta p dV_c \)

\( \omega \) = angular velocity, radians/sec

\( \delta p \) = pressure drop through heater regenerator and cooler

\( dV_c \) = differential change in cold volumes

\( W_c = \frac{\int \delta p dV_c}{(P_w)_{max}}V_{AC} \)

\( P_w \) = instantaneous pressure measured in the warm space

\( V_c \) = instantaneous volume of cold space

\( V_{AC} = \frac{(V_{max} - V_{min})}{2} \)

\( W_w = \frac{P_w dV_w}{(P_w)_{max}}V_{AW} \)

where in addition:

\( V_w \) = instantaneous volume of warm space, cm\(^3\)

\( V_{AW} \) = warm cylinder volume amplitude, cm\(^3\)
Table 5.2
Summary of Rios' Data (69 ar)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>T&lt;sub&gt;C&lt;/sub&gt; F</th>
<th>T&lt;sub&gt;W&lt;/sub&gt; F</th>
<th>Phase Angle deg</th>
<th>Vapor around cold HX</th>
<th>Speed RPM</th>
<th>Refrig. Load Watts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-14.5</td>
<td>42.0</td>
<td>62.0</td>
<td>Freon 12</td>
<td>326</td>
<td>91.0</td>
</tr>
<tr>
<td>2</td>
<td>-6.0</td>
<td>41.5</td>
<td>89.5</td>
<td>Freon 12</td>
<td>326</td>
<td>111.6</td>
</tr>
<tr>
<td>3</td>
<td>-11.0</td>
<td>42.0</td>
<td>102.0</td>
<td>Freon 12</td>
<td>326</td>
<td>98.0</td>
</tr>
<tr>
<td>4</td>
<td>-12.5</td>
<td>41.0</td>
<td>76.0</td>
<td>Freon 12</td>
<td>326</td>
<td>86.0</td>
</tr>
<tr>
<td>5</td>
<td>-163.3</td>
<td>38.5</td>
<td>75.0</td>
<td>Freon 13</td>
<td>325</td>
<td>86.6</td>
</tr>
<tr>
<td>6</td>
<td>-163.3</td>
<td>37.5</td>
<td>88.5</td>
<td>Freon 13</td>
<td>325</td>
<td>93.0</td>
</tr>
<tr>
<td>7</td>
<td>-167.3</td>
<td>38.5</td>
<td>101.5</td>
<td>Freon 13</td>
<td>325</td>
<td>101.5</td>
</tr>
<tr>
<td>8</td>
<td>-162.0</td>
<td>38.5</td>
<td>62.0</td>
<td>Freon 13</td>
<td>325</td>
<td>76.0</td>
</tr>
<tr>
<td>9</td>
<td>-162.0</td>
<td>38.5</td>
<td>61.0</td>
<td>Freon 13</td>
<td>483</td>
<td>93.6</td>
</tr>
<tr>
<td>10</td>
<td>-164.7</td>
<td>37.5</td>
<td>74.0</td>
<td>Freon 13</td>
<td>483</td>
<td>120.0</td>
</tr>
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<td>-164.0</td>
<td>37.5</td>
<td>87.5</td>
<td>Freon 13</td>
<td>483</td>
<td>125.0</td>
</tr>
<tr>
<td>12</td>
<td>-164.0</td>
<td>37.5</td>
<td>101.0</td>
<td>Freon 13</td>
<td>483</td>
<td>108.0</td>
</tr>
<tr>
<td>13</td>
<td>-317.2</td>
<td>38.3</td>
<td>87.0</td>
<td>Nitrogen</td>
<td>480</td>
<td>48.5</td>
</tr>
<tr>
<td>14</td>
<td>-315.4</td>
<td>38.3</td>
<td>73.5</td>
<td>Nitrogen</td>
<td>480</td>
<td>44.0</td>
</tr>
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<td>15</td>
<td>-314.2</td>
<td>38.3</td>
<td>101.0</td>
<td>Nitrogen</td>
<td>481</td>
<td>0.0</td>
</tr>
<tr>
<td>16</td>
<td>-311.6</td>
<td>36.3</td>
<td>61.5</td>
<td>Nitrogen</td>
<td>485</td>
<td>*</td>
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<td>17</td>
<td>-313.0</td>
<td>36.0</td>
<td>87.5</td>
<td>Nitrogen</td>
<td>325</td>
<td>29.5</td>
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<td>18</td>
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<td>101.0</td>
<td>Nitrogen</td>
<td>325</td>
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<td>75.0</td>
<td>Nitrogen</td>
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<td>0.0</td>
</tr>
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<td>20</td>
<td>-311.2</td>
<td>38.3</td>
<td>62.0</td>
<td>Nitrogen</td>
<td>325</td>
<td>0.0</td>
</tr>
</tbody>
</table>
### Table 5.2 Continued

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$\frac{\pi}{2} \int_{P_{in}}^{P_{out}} F \cdot PdV_c \left(\text{watts}\right)$</th>
<th>$W_c$</th>
<th>$W_W$</th>
<th>$p_{\text{max}}$</th>
<th>$p_{\text{min}}$</th>
<th>$r_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.1</td>
<td>0.521</td>
<td>0.432</td>
<td>176.4</td>
<td>73.6</td>
<td>2.41</td>
</tr>
<tr>
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<td>20.0</td>
<td>0.601</td>
<td>0.496</td>
<td>177.1</td>
<td>91.8</td>
<td>1.93</td>
</tr>
<tr>
<td>3</td>
<td>24.4</td>
<td>0.637</td>
<td>0.525</td>
<td>160.6</td>
<td>83.7</td>
<td>-1.92</td>
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<td>4</td>
<td>15.7</td>
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<td>0.486</td>
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<td>75.8</td>
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<td>0.441</td>
<td>0.625</td>
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<td>124.1</td>
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<td>21.8</td>
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<td>0.667</td>
<td>262.1</td>
<td>123.8</td>
<td>2.12</td>
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<td>28.6</td>
<td>0.496</td>
<td>0.641</td>
<td>257.6</td>
<td>131.5</td>
<td>-1.95</td>
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<tr>
<td>8</td>
<td>22.9</td>
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<td>0.580</td>
<td>279.9</td>
<td>117.2</td>
<td>2.38</td>
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<tr>
<td>9</td>
<td>31.4</td>
<td>0.377</td>
<td>0.589</td>
<td>228.0</td>
<td>87.5</td>
<td>2.61</td>
</tr>
<tr>
<td>10</td>
<td>42.0</td>
<td>0.439</td>
<td>0.638</td>
<td>242.7</td>
<td>100.0</td>
<td>2.43</td>
</tr>
<tr>
<td>11</td>
<td>57.6</td>
<td>0.499</td>
<td>0.681</td>
<td>226.4</td>
<td>103.5</td>
<td>2.18</td>
</tr>
<tr>
<td>12</td>
<td>72.2</td>
<td>0.519</td>
<td>0.619</td>
<td>220.9</td>
<td>108.7</td>
<td>2.04</td>
</tr>
<tr>
<td>13</td>
<td>74.6</td>
<td>0.325</td>
<td>0.806</td>
<td>313.0</td>
<td>131.7</td>
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<td>14</td>
<td>56.4</td>
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<td>0.750</td>
<td>321.7</td>
<td>125.3</td>
<td>2.56</td>
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<td>15</td>
<td>76.9</td>
<td>0.366</td>
<td>0.827</td>
<td>224.2</td>
<td>96.5</td>
<td>2.33</td>
</tr>
<tr>
<td>16</td>
<td>26.5</td>
<td>0.279</td>
<td>0.710</td>
<td>169.8</td>
<td>62.0</td>
<td>2.74</td>
</tr>
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<td>17</td>
<td>23.4</td>
<td>0.320</td>
<td>0.722</td>
<td>301.0</td>
<td>133.3</td>
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<td>18</td>
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<td>0.799</td>
<td>269.9</td>
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<td>0.286</td>
<td>0.760</td>
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<td>20</td>
<td>38.6</td>
<td>0.280</td>
<td>0.672</td>
<td>308.7</td>
<td>120.5</td>
<td>2.56</td>
</tr>
</tbody>
</table>

Static heat leak at 1 atm = 39 watts
Static heat leak at 2.0 torr = 75 watts
* Venting nitrogen vapor from cold exchanger shell with no load
and finally:

\[
\frac{(p_w)}{\text{max}} \geq \frac{(p_w)}{\text{min}}
\]

Explanations of the Rios theory were not given in Section 4. During the current effort the author was not able to completely review and become familiar with it. However, he now has the computer program and one might be able to use it without really understanding it. Rios claims very close agreement between his theory and experiment. In Figures 5-3, 5-4 and 5-5 the points are the experimental points from Table 5-2. The lines are calculated by his computer program. These results show very good agreement at almost every point. The parameters used in the captions in Figures 5-3 to 5-5 are defined as follows:

- \( r_{VT} \) displaced-mass ratio = \( V_{AC}/V_{AW} \)
- \( T \) heat exchange temperature, absolute degrees
- \( V_D \) reduced dead volume
- \( m_D \) mass of gas in dead space, grams
- \( R \) gas constant, J/g K
- \( p \) instantaneous pressure, MPa
- \( r_{cs} \) ratio of connecting rod length to one-half the stroke

The quantity \( V_D \) is given as a constant although both \( m_D \) and \( p \) vary during the cycle. If the dead volume gas temperatures do not change, \( m_D \) is proportional to \( p \) and \( V_D \) would be a constant. This needs to be discussed. The quantity \( r_{cs} \) is given as 4.8. But in appendix G of the Rios thesis (69 ar) the ratio for the connecting rod length to the full stroke is 4.8. Rios writes that the latter is correct.

Measured and calculated pressure drops are given for Rios' 20 data points (see Table 5.3 and Figure 5.6). Good agreement is shown although there is increased scatter at the lower values.

5.3 The GPU-3 Engine

This engine is described in detail in Section 3.3. Figures 3-24 and 3-25 show that the efficiency and engine power now at NASA-Lewis are about as good as they were when the engine was first tested at Ft. Belvoir by the U.S. Army. Eight test points are given in Table 3-8. It is planned that the various ways of computing Stirling engine performance will be compared with experimental measurements now being made with the engine.

Table 5-4 shows the computed results using the NASA-Lewis 3rd order calculation procedure explained in Section 4.4.

Table 5-5 shows the computed results using the 2nd order calculation procedure given in Section 7. Both Table 5-4 and Table 5-5 give a detailed breakdown of the basic powers and heat inputs along with an itemized list of the losses in approximately the same format. Table 5-6 compares the indicated power output and the indicated efficiency obtained from the two methods of computation.
Fig. 5-3. Dimensionless cold work $W_C$, warm work $W_W$, and pressure ratio $r_p$, for $r_{VT} = 0.62$, $U_D = 2.15$, $r_{CS} = 4.8$ at 325 rpm using helium.

Fig. 5-4. Dimensionless cold work $W_C$, warm work $W_W$, and pressure ratio $r_p$, for $r_{VT} = 0.92$, $U_D = 2.54$, $r_{CS} = 4.8$ using helium.

Fig. 5-5. Dimensionless cold work $W_C$, warm work $W_W$, and pressure ratio $r_p$, for $r_{VT} = 1.87$, $U_D = 3.48$, $r_{CS} = 4.8$ using helium.
### Table 5-3. Pressure Drop Loss $\frac{w}{2T} \int \delta p dy$

<table>
<thead>
<tr>
<th>No.</th>
<th>$\theta$ (deg F)</th>
<th>$\phi$ (rpm)</th>
<th>Speed (ft/min)</th>
<th>Calculated Loss (Watts)</th>
<th>Measured Loss (Watts)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>-17.5</td>
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<td>11.1</td>
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<td>320</td>
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<td>20.6</td>
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<td>3</td>
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<td>122.0</td>
<td>320</td>
<td>25.5</td>
<td>24.4</td>
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<tr>
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<td>325</td>
<td>27.5</td>
<td>28.6</td>
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<tr>
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<td>102.0</td>
<td>325</td>
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<td>9</td>
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<td>61.0</td>
<td>400</td>
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<td>21.4</td>
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<td>37.6</td>
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<td>400</td>
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<td>450</td>
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<td>67.5</td>
<td>450</td>
<td>21.9</td>
<td>23.4</td>
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<tr>
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<td>101.0</td>
<td>450</td>
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<td>320</td>
<td>11.7</td>
<td>12.6</td>
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</tbody>
</table>

Fig. 5-6. Pressure-drop loss $\frac{w}{2T} \int \delta p dy$
<table>
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<tr>
<th>Test Point</th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>2000</td>
<td>2500</td>
<td>3000</td>
<td>3000</td>
<td>1500</td>
<td>3000</td>
<td>1500</td>
</tr>
<tr>
<td>Mean Pressure, MPa psia</td>
<td>2.068</td>
<td>2.068</td>
<td>2.068</td>
<td>2.068</td>
<td>4.136</td>
<td>4.136</td>
<td>2.758</td>
<td>2.758</td>
</tr>
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<td>Power, watts (HP)</td>
<td>Basic (Indicated + windage)</td>
<td>2163</td>
<td>2968</td>
<td>3758</td>
<td>4541</td>
<td>8650</td>
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<td>5824</td>
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<td>Total Flow Friction</td>
<td>44.7</td>
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<td>179</td>
<td>298</td>
<td>1044</td>
<td>157</td>
<td>783</td>
</tr>
<tr>
<td></td>
<td>(heater + regenerator + cooler + end effects)</td>
<td>(.06)</td>
<td>(.13)</td>
<td>(.24)</td>
<td>(.40)</td>
<td>(1.4)</td>
<td>(.21)</td>
<td>(.06)</td>
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<td>2871</td>
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<td>4253</td>
<td>7606</td>
<td>4191</td>
<td>5041</td>
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<td>Mech. Friction (2 × Basic)</td>
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<td>753</td>
<td>910</td>
<td>1730</td>
<td>872</td>
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<td>2826</td>
<td>3333</td>
<td>5876</td>
<td>3318</td>
<td>3878</td>
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<td>Heat Input, watts (HP)</td>
<td>Basic (includes everything except shuttle loss, static conduction and windage credit)</td>
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<td>5391</td>
<td>6786</td>
<td>8158</td>
<td>16229</td>
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<td>410</td>
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<td>8151</td>
<td>9463</td>
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<td>33.6</td>
<td>34.5</td>
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<td>Indicated Efficiency (%)</td>
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<td>Time averaged gas temperature in heater (Point 5, Fig. 3-27)</td>
<td>979</td>
<td>979</td>
<td>979</td>
<td>979</td>
<td>978</td>
<td>975</td>
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<td>Cold Metal</td>
<td>294</td>
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</table>

Table 5-4
LeHC Computed Performance for GPU-3 Test Points
Heater Average Gas Temperature 978 K (1760 R)
Cold Metal Temperature 295 K (530 R)
## Table 5-5

Computed Performance for GPU-3 Test Points  
**Hot Gas Temp.** 978 K (1300 F)  
**Cooler Temperature** 295 K (70 F)  
**Cooling Water Flow, 6 GPM**

<table>
<thead>
<tr>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
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<tbody>
<tr>
<td>Working Fluid</td>
<td>H₂</td>
<td>H₂</td>
<td>H₂</td>
<td>H₂</td>
<td>He</td>
<td>He</td>
<td>He</td>
<td>He</td>
</tr>
<tr>
<td>Engine Speed, RPM</td>
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<td>2000</td>
<td>2500</td>
<td>3000</td>
<td>3000</td>
<td>1500</td>
<td>3000</td>
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<tr>
<td>Mean Pressure, MPa</td>
<td>2.068</td>
<td>2.068</td>
<td>2.068</td>
<td>2.068</td>
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<td>14.36</td>
<td>2.758</td>
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<td>82</td>
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<td>209</td>
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<td>389</td>
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<td>788</td>
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<td>1802</td>
<td>947</td>
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<td>Static Conduction, QS</td>
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<td>104</td>
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Notes: Calculations not calculated.
Table 5-6
Comparisons of Indicated Power Output and Indicated Efficiency for GPU-3 Test Points

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<tr>
<th>Test Point</th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>Working Fluid</td>
<td>H₂</td>
<td>H₂</td>
<td>H₂</td>
<td>H₂</td>
<td>He</td>
<td>He</td>
<td>He</td>
<td>He</td>
</tr>
<tr>
<td>Engine Speed, RPM</td>
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<tr>
<td>Hz</td>
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<td>2.068</td>
<td>2.068</td>
<td>2.068</td>
<td>4.136</td>
<td>4.136</td>
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<tr>
<td>LeRC</td>
<td>2118</td>
<td>2871</td>
<td>3579</td>
<td>4253</td>
<td>7606</td>
<td>4191</td>
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<td>LeRC</td>
<td>39.0</td>
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<td>Sec. 7</td>
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<td>46.2</td>
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<td>53.8</td>
<td>52.5</td>
<td>49.8</td>
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that agreement is usually good between the two calculation methods for low engine speed (25 Hz). As speed increases the second order design method described in Section 7 predicts a higher rate of increase in power than the third order calculation procedure formulated by LeRC and described in Section 4-4.

At this point, no experimental measurements have been received. It will be very interesting to see how they compare with these predictions.
6. AUXILIARY STIRLING ENGINE DESIGN PROBLEMS

More than is usually the case for engine design, the design of a proper Stirling engine requires the close and interactive cooperation among 1) the engine thermodynamicists who figure the power output and heat input for a particular engine configuration, 2) the mechanical designer who works out the seals and the mechanisms that operate the pistons and the displacer, 3) the designers who add required components, such as the burner and air preheater, starting motor, power control system, transmission system, etc., and 4) the manufacturing engineer who determines whether the machine as designed can be built so as to be sold for an attractive price in the marketplace.

Discussion of the various Stirling engine component design problems falls outside the scope of the present effort. Some of these problems were discussed briefly in Section 2 and Section 3. The subject index to the references (Table 8-5) can give the reader access to pertinent literature.
7. **SAMPLE DESIGN PROCEDURE**

As was discussed in Section 4, first order design procedures are useful to give a preliminary indication of how a Stirling engine designed by experts can be applied in a new situation.

A good second order design procedure can be used to design a Stirling engine from scratch. The simplifying assumptions that are made have been found by comparison with some small engine tests to lead to reasonable accurate predictions over a usable range of engine operating conditions. A second order design procedure is simple enough to be done a few times by hand. The procedure can be incorporated in a computerized search routine to search out the best design out of many thousands of possible designs.

Third order design procedures are much more realistic but also much more labor-intensive. Even the simplest procedure requires the use of a large computer. Third order procedures will be useful for studying in detail the performance of a given engine. If proved valid by agreement with experimental measurements it can provide valuable insight to how the engine functions by providing much information that cannot be measured reliably. Specifically, third order procedures have the following uses:

1. Third order model predictions of how internal engine variable change over the cycle could help improve simple equations used to calculate losses and powers in second order procedures.

2. Third order calculations are needed to assess the limits of the observation that for normal engines, the assumption of isothermal hot and cold spaces gives the same basic power as the assumption of adiabatic hot and cold spaces.

3. A third order kind of mathematical model would be needed to calculate the effect of engine imperfections such as non-uniform temperature distributions in heater head and flow maldistributions.

4. The most rigorous third order model, produced by Urieli (77 af), by Schock, (no reference), and by Vanderbrug (77 ae) can be used to show how significant the effects of fluid inertia and pressure wave dynamics are in a particular case. Puzzling performance might be explained only when the most rigorous third order model is employed. Comparison of these rigorous models with simpler third-order models like the LeRC model (77 bl) and to second order models could lead to rules to determine the limits for each method.

The procedure outlined in this section is second order. A less complete version of this procedure has been used by the author to calculate artificial heart engines. Equations have been added to quantify loss mechanisms found important in cooling engines. The procedure has been broadened to apply to crank operated machines and machines using many different types of heat exchangers and regenerators. This procedure is not published elsewhere. At this point it is not known whether this procedure is a "good" procedure because it has not as yet been checked out by comparison with experimental measurements of large engines.
The procedures outlined in this section will start with engine dimensions and operating conditions and proceed to calculate the input heat requirement and the output power. The procedure uses the equations described in Sections 4.2 and 4.3. Section 7.1 gives the blank design forms adapted to a number of different kind of engine designs. It is also adapted to the use of helium, hydrogen or air as the working fluid. Air has not been used recently in large power engines, however, some designers may want to consider it for simple engines. Section 7.2 uses the design form from Section 7.1 to calculate the performance of operating point 1 on Table 3-8.

7.1 Stirling Engine Design Form -- W. R. Martini, October, 1977

1. Type of Piston Arrangement

1.1 Alpha: Dual Piston

DE = diameter of expansion, hot, piston ________ cm.
SE = stroke of expansion piston ________ cm.
DC = diameter of compression, cold, piston ________ cm.
SC = stroke of compression piston ________ cm.

1.2 Beta: Displacer-Piston Overlapping Strokes

DCY = diameter of engine cylinder ________ cm.
SD = stroke of displacer ________ cm.
SP = stroke of power piston ________ cm.
DDR = diameter of displacer drive rod ________ cm.

1.3 Gamma: Displacer and Piston in Separate Cylinders

DD = diameter of displacer cylinder ________ cm.
SD = stroke of displacer ________ cm.
DP = diameter of power piston cylinder ________ cm.
SP = stroke of power piston ________ cm.
DDR = diameter of displacer drive rod ________ cm.

2. Dead Volumes and Heat Exchangers and Wall Thicknesses

2.1 Volumes and Heat Exchangers at Heat Source Temperature
VHDX = extra hot dead volume besides that in the gas heater cm$^3$.

2.1.1 Tubular Gas Heater

NTH = number of heater tubes per power unit.

LH = total length of each heater tube cm.

LHHT = heated length of each heater tube cm.

DIH = inside diameter of heater tubes cm.

DOH = outside diameter of heater tubes cm.

2.1.2 Annular Gap Gas Heater

LHHT = heated length of heater surface cm.

GH = single annulus heater gap thickness cm.

2.1.3 Nesting Cone Isothermalizer

NC = number of cones on piston or displacer.

DC1 = diameter of cone at base cm.

LC1 = length of cone cm.

SCL = stroke clearance cm.

2.2 Volumes at Regenerator Temperature

2.2.1 Screen Regenerators

LR = length of regenerator cm.

NR = number of regenerators/unit.

DR = diameter of each regenerator cm.

NS = number of screen layers.

MSH = mesh size wires/cm.

THW = thickness of wire in screens cm.

FF = filler factor, fraction of regenerator volume filled with wires.
2.2.2 Slot Regenerators

LR = length of regenerator \( \text{cm} \).

GR = regenerator gap thickness \( \text{cm} \).

AF = free flow area through regenerator \( \text{cm}^2 \).

THF = thickness of foil separating gaps \( \text{cm} \).

2.2.3 Displacer or Hot Cap Gap Volume

LD = length of displacer \( \text{cm} \).

DCY = diameter of cylinder around displacer \( \text{cm} \).

GR = gap between displacer and cylinder wall \( \text{cm} \).

Displacer wall material

Cylinder wall material

WT1 = wall thickness of displacer, \( \text{cm} \).

WT2 = wall thickness of cylinder wall \( \text{cm} \).

EH = emissivity of hot surface inside of displacer or hot cap

EC = emissivity of cold surface inside of displacer or hot cap

NRS = number of radiation shields inside the displacer or hot cap

2.3 Volumes and Heat Exchangers at Heat Sink Temperature

VCDX = extra cold dead volume besides that in the gas cooler \( \text{cm}^3 \).

2.3.1 Tubular Gas Cooler

NTC = number of cooler tubes per power unit

LC = total length of each cooler tube \( \text{cm} \).
LCHT = cooled length of each cooler tube cm.
DIC = inside diameter of cooler tubes cm.
DOC = outside diameter of cooler tubes cm.

2.3.2 Annular Gap Gas Cooler
LCHT = cooled length of cooler surface cm.
GC = single annulus cooler gap thickness cm.

2.3.3 Nesting Cone Isothermalizer
NC = number of cones on piston or displacer.
DCI = diameter of cone at base cm.
LCI = length of cone cm.
SCL = stroke clearance cm.

2.4 Regenerator Wall Dimensions (see Figure 4-24)
AHTH = heat conduction area at hot end cm².
AHTB = heat conduction area at level B cm².
AHTA = heat conduction area at level A cm².
AHTC = heat conduction area at cold end cm².
LHB = hot length regenerator wall cm.
LBA = middle length regenerator wall cm.
LAC = cold length regenerator wall cm.

Regenerator wall material

<table>
<thead>
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<th>Temp K</th>
<th>Thermal Conductivity</th>
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<tbody>
<tr>
<td>THM</td>
<td>KMH = w/cm·K</td>
</tr>
<tr>
<td>TB</td>
<td>(est.) KMB =</td>
</tr>
<tr>
<td>TA</td>
<td>(est.) KMA =</td>
</tr>
<tr>
<td>YCM</td>
<td>KMC =</td>
</tr>
</tbody>
</table>
2.5 Cylinder Wall Dimensions (see Figure 4-23)

AHTH = heat conduction area at hot end cm².
AHTB = heat conduction area at level B cm².
AHTA = heat conduction area at level A cm².
AHTC = heat conduction area at cold end cm².
LHB = hot length regenerator wall cm.
LBA = middle length regenerator wall cm.
LAC = cold length regenerator wall cm.

Cylinder wall material

<table>
<thead>
<tr>
<th>Level Temp. K</th>
<th>Thermal Conductivity</th>
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</thead>
<tbody>
<tr>
<td>THM</td>
<td>KMH = KMH w/cm·K</td>
</tr>
<tr>
<td>TB</td>
<td>(est) KMB</td>
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<tr>
<td>TA</td>
<td>(est) KMA</td>
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<tr>
<td>TCM</td>
<td>KMC</td>
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</tbody>
</table>

3. Drives

N = Number of power units/engine

3.1 Alpha - Swashplate (Ford-Philips)

ALPH = phase angle (usually 90°) °.

3.2 Alpha - Crank (United Stirling)

LCR = connecting rod length cm
RC = crank radius cm.

ALPH = phase angle (usually 90°) °.

3.3 Beta-Rhombic Drive

LCR = connecting rod length cm.
RC = crank radius cm.
ECC = crank eccentricity (see Figure 4-15) ______ cm.

3.4 Beta or Gamma - Crank

LCRP = length of power piston connecting rod ______ cm.

RCP = crank radius for power piston ______ cm.

LCRD = length of displacer connecting rod ______ cm.

RCD = crank radius for displacer ______ cm.

ALPH = crank angle ______°, usually about 90°.

3.5 Beta or Gamma - Scotch Yoke or other-SHM Linkage

ALPH = crank angle ______°.

4. Given Operating Conditions

TC = effective temperature in cold, compression space ______ K.

\[ K = C + 273.1 \text{ Make initial estimate} \]

\[ K = \frac{F + 460}{1.8} \text{ initial estimate or measurement} \]

TH = effective temperature in hot, expansion space, ______ K.

FCW = cooling water flow ______ g/sec.

TCWI = Temperature of cooling water into engine ______ K.

NU = engine frequency ______ HZ.

(HZ = RPM/60)

PMAX = maximum engine pressure ______ MPa.

MPa = 0.006894 (psia)

= 0.1013 (atm)

PAVG = time averaged mean pressure ______ MPa.

THM = heat source metal temperature ______ K.

TCM = heat sink metal temperature ______ K.

Working Gas ______________.
5. Computation of Engine Volumes

5.1 Live Volumes

VHL = hot live volume, cm\(^3\) for alpha dual piston, SHM,
(simple harmonic motion)

\[
VHL = \frac{\pi}{4} (DE)^2 (SE) = \frac{\pi}{4} (\text{...})^2 (\text{...}) = \text{_____ cm}^3.
\]

for beta, SHM

\[
VHL = \frac{\pi}{4} (DCY)^2 (SD) = \frac{\pi}{4} (\text{...})^2 (\text{...}) = \text{_____ cm}^3.
\]

for gamma, SHM

\[
VHL = \frac{\pi}{4} (DD)^2 (SD) = \frac{\pi}{4} (\text{...})^2 (\text{...}) = \text{_____ cm}^3.
\]

Note: Live volumes for crank and rhombic drives will need not be calculated.

VCL = cold live volume, cm\(^3\)

for alpha, SHM

\[
VCL = \frac{\pi}{4} (DC)^2 (SC) = \frac{\pi}{4} (\text{...})^2 (\text{...}) = \text{_____ cm}^3.
\]

for beta or gamma, SHM

\[
VCL = \frac{\pi}{4} (DD^2 - DDR^2)SD = \frac{\pi}{4} (\text{...}^2 - \text{...}^2) x (\text{...}) = \text{_____ cm}^3.
\]

VPL = power piston live volume, cm\(^3\)

for gamma, SHM

\[
VPL = \frac{\pi}{4} (DP)^2 (SP) = \frac{\pi}{4} (\text{...})^2 (\text{...}) = \text{_____ cm}^3.
\]

5.2 Dead Volumes

VHD = hot dead volume, cm\(^3\)

for tubular gas heater

\[
VHD = VHDX + \frac{\pi}{4} (DIH)^2 (LH) (NTH)
\]
\[
\begin{align*}
\text{for annular gap gas heater} & \quad \frac{\pi}{4} \left( \phi \right)^2 \left( \frac{d}{2} \right) = \text{cm}^3 \\
& \quad \text{VHD} + \pi \left( \text{DCY or DD} \right) \left( \text{LHHT} \right) \left( \text{GH} \right) = \text{cm}^3.
\end{align*}
\]

\[
\begin{align*}
\text{for nesting cone isothermalizer} & \quad \frac{\pi}{4} \left( \text{DE or DCY or DD} \right)^2 \left( \text{SCL} \right) = \text{cm}^3 \\
& \quad \text{VHD} + \pi \left( \text{DE or DCY or DD} \right) \left( \text{LHC} \right) \left( \text{GCH} \right) = \text{cm}^3.
\end{align*}
\]

\[
\begin{align*}
\text{VRD} & = \text{regenerator dead volume, cm}^3 \\
& \text{for screen regenerators, short calculation} \\
& \quad \frac{\pi}{4} \left( \text{DR} \right)^2 \left( \text{LR} \right) \left( 1 - FF \right) = \text{cm}^3
\end{align*}
\]

\[
\begin{align*}
\text{VRD} & = \text{regenerator dead volume, cm}^3 \\
& \text{for screen regenerators, long calculation} \\
& \quad \frac{\pi}{4} \left( \text{DR} \right)^2 \left[ \left( \text{LR} \right) - \frac{\pi}{2} \left( \text{NS} \right) \left( \text{MSH} \right) \left( \text{THW} \right)^2 \right] = \text{cm}^3
\end{align*}
\]

\[
\begin{align*}
\text{VRD} & = \text{regenerator dead volume, cm}^3 \\
& \text{for slot regenerators} \\
& \quad \frac{\pi}{4} \left( \text{LR} \right) = \text{cm}^3
\end{align*}
\]

\[
\begin{align*}
\text{VRD} & = \text{regenerator dead volume, cm}^3 \\
& \text{for alpha or gamma type} \\
& \text{VCD} = \text{cold dead volume} \\
& \text{for tubular gas coolers} \\
& \quad \frac{\pi}{4} \left( \text{DCY or DD} \right)^2 \left( \text{LCHT} \right) \left( \text{GC} \right) = \text{cm}^3
\end{align*}
\]
\[ \text{VCDX} + \frac{n}{4}(\text{DE or DCY or DD})^2(\text{SCL}) \]
\[ = \frac{\pi}{4} \left( \frac{VCD}{D} \right)^2(\text{SCL}) = \text{cm}^3. \]

For beta type, \( \text{ALPH} = 90^\circ \)
\( \text{VCDHX} = \text{VCD}, \) calculated above
\( \text{VCD} = \text{VCDHX} - \text{VCL} \left(1 - \frac{\sqrt{2}}{2}\right) \)

6. General Intermediate Parameters

\( \text{TAU} = \frac{\text{TC}}{\text{TH}} = \left(\frac{\text{VCD}}{\text{TH}}\right) = \frac{\text{VCD}}{\text{TH}} \)

\( \text{TR} = \left(\frac{\text{THM} - \text{TCM}}{\ln(\text{THM}/\text{TCM})}\right) = \left(\frac{\text{THM} - \text{TCM}}{\ln(\text{THM}/\text{TCM})}\right) = \text{K} \)

\( \text{KAP} = \frac{\text{VCL}}{\text{VHL}} = \left(\frac{\text{VCL}}{\text{VHL}}\right) \)

\( \text{S} = \frac{\text{TC}}{\text{VHL}} \left(\frac{\text{VHD}}{\text{TH}} + \frac{\text{VRD}}{\text{TR}} + \frac{\text{VCD}}{\text{TC}}\right) \)

\[ = \text{S} \left(\frac{\text{VHD}}{\text{TH}} + \frac{\text{VRD}}{\text{TR}} + \frac{\text{VCD}}{\text{TC}}\right) \]

\( \text{DEL} = \frac{\sqrt{\left(\frac{1}{\text{TAU}^2} + 2(\text{TAU})(\text{KAP})\cos(\text{ALPH}) + (\text{KAP})^2\right)}}{\left(\text{TAU} + \text{KAP} + 2\text{S}\right)} \)

\[ = \frac{\sqrt{\left(\text{TAU}^2 + 2(\text{TAU})(\text{KAP})\cos(\text{ALPH}) + (\text{KAP})^2\right)}}{\left(\text{TAU} + \text{KAP} + 2\text{S}\right)} \]

\( \text{PMAX} = \frac{\text{PAVG}}{\sqrt{(1 - \text{DEL})/(1 + \text{DEL})}} = \frac{\text{PAVG}}{\sqrt{(1 - \text{DEL})/(1 + \text{DEL})}} \)

\[ = \text{MPa} \]

\( \text{PMIN} = \frac{\text{PMAX}(1 - \text{DEL})}{(1 + \text{DEL})} = \frac{\text{PMAX}(1 - \text{DEL})}{(1 + \text{DEL})} \)

\[ = \text{MPa} \]

7. Basic Power Outputs

7.1 Alpha Engine - Schmidt Equation

...
\[ \theta = \tan^{-1}\left( \frac{K \sin(\alpha) / (\tau + K \cos(\alpha))}{\tan^{-1}\left( \frac{\sin(\alpha) / (1 + \cos(\alpha))}{\cos(\alpha)} \right)} \right) \]

= \tan^{-1}\left( \frac{\sin(\theta)/1}{\cos(\theta)} \right) \text{ degrees} \]

The basic power, BP, is

\[ BP = \frac{N(1 - \tau)(\delta \sin(\theta))}{(K + 1)(1 + \sqrt{1 - (\delta)^2})} \]

= \left( \frac{\ldots}{\ldots} \right)^{\pi(1 - \ldots)} \left( \frac{\ldots}{\ldots} \right) \text{ watts.} \]

7.2 Beta Engine, Schmidt Equation

\[ K = \frac{V_{PL}}{V_{HL}} \]

= \frac{\ldots}{\ldots} = \ldots \]

\[ D = \sqrt{1 + K^2 - 2K \cos(\alpha)} \]

= \sqrt{1 + (\ldots)^2 - 2(\ldots) \cos(\ldots)} = \ldots \]

\[ V_D = V_{HD} + V_{RD} + V_{CD} \]

= \ldots + \ldots + \ldots = \ldots \text{ cm}^3. \]

\[ Y = \tau + 4\left( \frac{V_D}{V_{HL}} \right)\left( \frac{\tau}{1 + \tau} \right) + D \]

= \ldots + 4\left( \frac{\ldots}{\ldots} \right) \left( \frac{\ldots}{\ldots} \right) = \ldots \]

\[ X = \sqrt{(\tau - 1)^2 + 2(\tau - 1)K \cos(\alpha) + K^2} \]

= \sqrt{(\ldots - 1)^2 + 2(\ldots - 1)(\ldots) \cos(\ldots) + (\ldots)^2} = \ldots \]
BP = \(\frac{\pi(1 - TAU)(PMAX)(VHL)(K)\sin(ALPH)}{Y + \sqrt{Y^2 - X^2}}\) \(\frac{Y - X}{Y + X}\)

= \(\frac{\pi(1 - \frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})\sin(\frac{1}{2})}{\sqrt{\frac{1}{4}} - \left(\frac{1}{4}\right)^2}\) \(\sqrt{\frac{1}{4}} - \left(\frac{1}{4}\right)^2\)

= _____ watts.

7.3 Gamma Engine - Schmidt Equation
Evaluate K, D, VD and X in 7.2

\[ Z = 4 \left(\frac{VD}{VHL(1 + TAU)}\right) + 1 + TAU + K \]

= 4(1 + \frac{1}{2}) + 1 + _____ + _____

Z = _____.

Then:

BP = \(\frac{(NU)\pi(1 - TAU)(PMAX)(VHL)(K)\sin(ALPH)}{Z + \sqrt{Z^2 - X^2}}\) \(\frac{Z - X}{Z + X}\)

= \(\frac{\pi(1 - \frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})\sin(\frac{1}{2})}{\sqrt{\frac{1}{4}} - \left(\frac{1}{4}\right)^2}\) \(\sqrt{\frac{1}{4}} - \left(\frac{1}{4}\right)^2\)

= _____ watts.

7.4 Rhombic Drive Philips Engine (See Figure 4-15 and 4-16)

\[ A = \sqrt{(LCR)^2 - (ECC - RC)^2} \]

= \(\sqrt{(\text{cm})^2 - (\text{cm})^2}\) = _____ cm

\[ B = \sqrt{(LCR)^2 - (ECC + RC)^2} \]
\[
= \sqrt{\left( \frac{a}{b} \right)^2 - \left( \frac{c}{d} \right)^2} = \text{_______ cm}
\]

For a Check:
\[
VCL = (A - B) \frac{\pi}{2} \left( \frac{(DCY)^2 - (DDR)^2}{2} \right)
= \left( \frac{a}{b} \right) \frac{\pi}{2} \left( \frac{c}{d} \right)^2 - \left( \frac{e}{f} \right)^2 = \text{_______ cm}^3.
\]
\[
VCLX = \frac{\pi}{2} \left( \frac{(DCY)^2 - (DDR)^2}{2} \right) \left\{ A - \sqrt{(LCR)^2 - \left( \frac{ECC - RC \cos (PHI)}{2} \right)^2} \right\}
= - \left( \frac{a}{b} - \sqrt{\frac{c}{d} - \left( \frac{e}{f} \cos (PHI) \right)^2} \right)
\]
\[
VC = VCD + VCLX = \text{_______ + } \text{_______}
\]
\[
VC = \text{_______ + } \sqrt{\text{_______} - \left( \text{_______ - } \text{_______} \cos (PHI) \right)^2}
\]

VC is calculated for PHI = 0, 30, 60, . . . . . . . 360° and entered on the next page.

\[
D1 = \sqrt{(LCR + RC)^2 - (ECC)^2}
= \sqrt{\left( \frac{a}{b} + \frac{c}{d} \right)^2 - \left( \frac{e}{f} \right)^2} = \text{_______ cm}.
\]
\[
C = \sqrt{(LCR - RC)^2 - (ECC)^2}
= \sqrt{\left( \frac{a}{b} - \frac{c}{d} \right)^2 - \left( \frac{e}{f} \right)^2} = \text{_______ cm}
\]
\[
VHL = \left( D1 - C \right) \frac{n}{4} (DCY)^2
= \left( \frac{a}{b} - \frac{c}{d} \right) \frac{n}{4} \left( \frac{e}{f} \right)^2 = \text{_______ cm}^3.
\]
\[
VH = VHLX + VHD = \frac{n}{4} (DCY)^2 \left\{ \sqrt{(LCR)^2 - \left( \frac{ECC - (RC) \cos (PHI)}{2} \right)^2} \right.
+ \left. RC \sin (PHI) - C \right\} + VHD
\]
\[
VH = \frac{n}{4} \left( \frac{a}{b} \right)^2 \left\{ \sqrt{\left( \frac{c}{d} \right)^2 - \left( \frac{e}{f} \cos (PHI) \right)^2} \right.
+ \left. \sin (PHI) - \text{_______} \right\} + \text{_______}
\]
### COMPUTATION OF BETA PHILIPS BASIC POWER

<table>
<thead>
<tr>
<th>PHI</th>
<th>VH</th>
<th>VC</th>
<th>VT</th>
<th>P</th>
<th>PC</th>
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12

\[
P_M = P_{AVG} = \frac{PM}{12} = J/K
\]
\[ \text{VH} = \sqrt{\left(\frac{1}{\text{TH}} + \frac{1}{\text{TR}} + \frac{1}{\text{TC}}\right) - \left(\cos(\text{PHI})\right)^2 + \sin(\text{PHI})} \]

VH is calculated for PHI = 0 \rightarrow 360 and entered on previous page.

\[ \text{VT} = \text{VH} + \text{VC} + \text{VRD} = \text{VH} + \text{VC} + \]

VT is calculated for PHI = 0 \rightarrow 360 and entered on previous page.

\[ \frac{1}{\text{VT}} \]

P is calculated for PHI = 0 \rightarrow 360 and entered on previous page.

\[ \text{PM} = \sum_{\text{PHI} = 30}^{360} \frac{1}{12} \text{P} \]

(see previous page)

Note: Do Not Add in P at PHI = 0.

\[ \text{M(R)} = \frac{\text{PAVG}}{\text{PM}} = \text{J/K} \]

\[ \text{PC} = \text{P} \left(\frac{\text{PAVG}}{\text{PM}}\right) \]

PC is calculated for PHI = 0, 30, ..., 360 and entered on previous pg.

PC is integrated verses VT using the trapezoidal rule, i.e.

\[ \text{DELW} = \frac{\text{PC}_1 + \text{PC}_2}{2} (\text{VT}_2 - \text{VT}_1) \]

For the first increment the subscript 1 stands for PHI = 0° and the subscript 2 stands for PHI = 30°. For the second increment the subscript 1 stands for PHI = 30° and the subscript 2 stands for PHI = 60°. DELW is summed for all 12 increments. (See previous page).
BP = NU(1.045) \sum_{1}^{12} DELW

= (1.045) \frac{\text{DElw}}{1.045} = \text{watts.}

The mass flow rates are now computed

FH = \frac{P(VH)}{M(R)(TH)} = \frac{P(VH)}{M(R)} (\frac{1}{\text{K}})

M(R) = \text{J/K} \quad \frac{M(R)}{8.314} = \frac{\text{mol}}{8.314}

FH = \frac{M(R)}{8.314} g \text{ mol}

FH is calculated for PHI = 30 to 360 and entered 2 pages back.

FHMAX = \text{FHMIN = } \text{g mol}

FH is graphed and effective times for steady flow in and out are determined

FCT1 = \text{FCT2 = } \text{sec}

FCTH = \frac{FCT1 + FCT2}{2} = \text{sec}

WHS = \frac{(FHMAX - FHMIN)M(MW)}{FCTH/NU}

= \left( \frac{P(VH)}{M(R)(TH)} \right) (\frac{1}{\text{K}}) = \text{g/sec}

FC = \frac{P(VC)}{M(R)(TC)} = \frac{P(VC)}{M(R)} (\frac{1}{\text{K}})

FC is calculated for PHI = 30 to 360° and entered 2 pages back.

FCMAX = \text{FCMIN = } \text{g/sec}

FC is graphed and effective times for steady flow in and out are determined

FCT3 = \text{FCT4 = } \text{sec}

FCTC = \frac{FCT3 + FCT4}{2} = \text{sec}

WCS = \frac{(FCMAX - FCMIN)M(MW)}{FCTC/NU}

= \left( \frac{P(VC)}{M(R)(TC)} \right) (\frac{1}{\text{K}}) = \text{g/sec}
For the regenerator:

\[ \text{FCT} = \frac{(\text{FCTH} + \text{FCTC})}{2} = \left( \frac{\text{FCTH} + \text{FCTC}}{2} \right) \]

\[ \text{WRS} = \frac{(\text{WHS} + \text{WCS})}{2} = \left( \frac{\text{WHS} + \text{WCS}}{2} \right) \]

7.5 Crank Drive - Alpha Engine

\[ \text{VH} = \frac{\pi}{4} \left( \text{DCY}^2 - \text{DDR}^2 \right) \left\{ \sqrt{\left( \text{LCR} \right)^2 - \left( \text{RC sin (PHI)} \right)^2} \right. \]
\[ + \text{RC cos (PHI) + LCR} \left\} + \text{VHD} \]

\[ = \frac{\pi}{4} \left( \text{DCY}^2 - \text{DDR}^2 \right) \left\{ \sqrt{\left( \text{LCR} \right)^2 - \left( \text{RC sin (PHI)} \right)^2} \right. \]
\[ + \text{RC cos (PHI) + LCR} \left\} + \text{VHD} \]

\[ \text{VC} = \frac{\pi}{4} \left( \text{DCY}^2 - \text{DDR}^2 \right) \left\{ \sqrt{\left( \text{LCR} \right)^2 - \left( \text{RC sin (PHI + ALPH)} \right)^2} \right. \]
\[ - \text{RC cos (PHI + ALPH) - LCR + RC} \left\} + \text{VCD} \]

\[ = \frac{\pi}{4} \left( \text{DCY}^2 - \text{DDR}^2 \right) \left\{ \sqrt{\left( \text{LCR} \right)^2 - \left( \text{RC sin (PHI + ALPH)} \right)^2} \right. \]
\[ - \text{RC cos (PHI + ALPH) - LCR + RC} \left\} + \text{VCD} \]

\[ \text{VT} = \text{VH} + \text{VC} + \text{VRD} = \text{VH} + \text{VC} + \text{VRD} \]

\[ P = \frac{1}{\text{TH} + \frac{\text{VRD}}{\text{TR}} + \frac{\text{VC}}{\text{TC}}} = \frac{1}{\text{VH} + \text{VC}} \]
VH, VC, VT and P are calculated for PHI = 0, 30, 60, ... 360°
and entered on the next page.

\[
PM = \left( \sum_{PHI = 30}^{PHI = 360} P \right) / 12
\]

PM = 

Note: Do Not Add in P at PHI = 0

\[
M(R) = \frac{PAVG}{PM} = \left( \frac{P_{AVG}}{PM} \right) = \text{___________} \text{J/K}
\]

PC = \[ P \left( \frac{PAVG}{PM} \right) = P\left( \right) \]

Calculate PC and enter on next page.
PC is integrated vs VT using the trapezoidal rule as explained
in previous section.

\[
BP = NU (1.045) \sum_{1}^{12} \text{DELW}
\]

= \[ (1.045) \text{_______} = \text{_______} \text{watts} \]

Effective flow rates and effective fractions of the cycle times these flows
are assumed to occur in are computed using the method given in 7.4.

7.6 Rhombic Drive Beta with adiabatic hot and cold spaces.

Calculate VCLX and VHLX as in 7.4.

\[
VT = VCLX + VCD + VRD + VHD + VHLX
\]

= VCLX + _______ + _______ + _______ + VHLX

= VCLX + _______ + VHLX
<table>
<thead>
<tr>
<th>PHI</th>
<th>VC $\text{cm}^3$</th>
<th>VH $\text{cm}^3$</th>
<th>VT $\text{cm}^3$</th>
<th>P MPa</th>
<th>PC MPa</th>
<th>DELW</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>
Enter VCLX, VHLX and VT for PHI = 0, 30, . . . . . . 360° on
next page.

\[
K_1 = \frac{VHD}{TH} + \frac{VRD}{TR} + \frac{VCD}{TC} = \frac{\ldots}{\ldots} + \ldots + \ldots
\]

\[
= \ldots \text{cm}^3/\text{K} \quad \text{gas} \quad \text{hydrogen} \quad \text{helium} \quad \text{air}
\]

\[
E \quad 0.286 \quad 0.400 \quad 0.286
\]

For X = 1 first calculate \(1H\) (check 1)

if \(VHL_1 > VHL_{12}\) \(\square\)

\[
1H = \frac{VHL_{12}}{TH} \left(\frac{P_1}{P_{12}}\right)_E + \frac{VHL_1 - VHL_{12}}{TH} \left(\frac{P_1}{P_{12}}\right)_E
\]

if \(VHL_1 < VHL_{12}\) \(\square\)

\[
1H = \frac{VHL_1}{TH} \left(\frac{P_1}{P_{12}}\right)_E
\]

For the first time around, THS12 and PI2 will not be known. Assume
THS12 = TH and PI2 = P1, so no matter which inequality is true:

\[
1H = \frac{VHL_1}{TH} = \ldots = \ldots \text{cm}^3/\text{K}
\]

Enter at top of XH column on next page.

Next, calculate \(1C\) (check 1)

if \(VCL_1 > VCL_{12}\) \(\square\)

\[
1C = \frac{VCL_{12}}{TCS_{12}} \left(\frac{P_1}{P_{12}}\right)_E + \frac{VCL_1 - VCL_{12}}{TC} \left(\frac{P_1}{P_{12}}\right)_E
\]

if \(VCL_1 < VCL_{12}\) \(\square\)

\[
1C = \frac{VCL_1}{TCS_{12}} \left(\frac{P_1}{P_{12}}\right)_E
\]
### Computation of Work Diagram - Rhombic Beta - Adiabatic Hot & Cold Spaces

<table>
<thead>
<tr>
<th>PHI</th>
<th>VCLX cm³</th>
<th>VHLX cm³</th>
<th>VT</th>
<th>X</th>
<th>XH</th>
<th>XC</th>
<th>PX</th>
<th>THSX</th>
<th>TCSX</th>
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</table>
For the first time around, TCS12 and P12 will not be known. Assume
TCS12 = TC and P12 = P1. So no matter which inequality is true:

$$1C = \frac{VCL1}{TC} = \frac{1}{TC} \text{ cm}^3/K$$

Enter at top of XC column on previous page.

Now P1 is determined by:

$$P1 = \frac{1}{TH + K1 + TC}$$

In general, solve for P1 by a successive approximation method.

Enter P1 on previous page at top of column PX. Then knowing P1, calculate 1C and 1H and enter top of columns XC and XH on previous page. For the first time around P1 can be solved directly.

$$P1 = \frac{1}{TH+K1+TC} = \text{MPa}$$

Then:

$$\text{THS1} = \frac{VHL1}{TH} = \frac{1}{TH} \text{ K}$$
$$\text{TCS1} = \frac{VCL1}{TC} = \frac{1}{TC} \text{ K}$$

Enter these at the top of their columns on previous page.

For $X = -2$

$\text{VHL2} > \text{VHL1}$ □ OR $\text{VHL2} < \text{VHL1}$ □

$$2H = \frac{\text{VHL1}}{\text{THS1} \left( \frac{P2}{PT} \right)} + \frac{\text{VHL2} - \text{VHL1}}{\text{TH} \left( \frac{P2}{PT} \right)}$$

$$2H = \frac{\text{VHL2}}{\text{THS1} \left( \frac{P2}{PT} \right)}$$

$$2H = \frac{\left( \frac{P2}{PT} \right)}{\left( \frac{P2}{PT} \right)} + \frac{\left( \frac{P2}{PT} \right)}{\left( \frac{P2}{PT} \right)}$$

$$2H = \frac{\left( \frac{P2}{PT} \right)}{\left( \frac{P2}{PT} \right)}$$
\[
VCL_2 > VCL_1 \quad \text{or} \quad VCL_2 < VCL_1
\]

\[
2C = \frac{VCL_1}{\text{TCS}_1 \left(\frac{P_2}{P_1}\right)^E} + \frac{VCL_2 - VCL_1}{\text{TC} \left(\frac{P_2}{P_1}\right)^E}
\]

\[
2C = \frac{\left(\frac{P_2}{P_1}\right)}{2H + K_1 + 2C}
\]

\[
P_2 = \frac{1}{2H + K_1 + 2C}
\]

\[
2H = \frac{\text{VHL}_2}{\text{THS}_2} = \frac{\text{TCS}_2}{2C}
\]

P2, 2H, 2C, THS2 and TCS2 are now entered on the second line of the table 2 page back. The process continues as is outlined for the rest of the cycle back to PHI = 360°, X = 1. The pressure at 360°, P1, will be different than the pressure at 0°, P1. The calculation procedure must continue until the hot space and cold space temperature and the cycle pressure repeat with adequate accuracy. This procedure must be programmed on a hand-held computer, at least, to be practical.

8. Fluid Friction loss

8.1 Regenerator Windage

8.1.1 Screens -- \[
\text{VHL} = \frac{\text{cm}^3}{\text{tr}} \quad \text{from 5.1 or 7.4}
\]

\[
\text{TR} = \frac{\text{OK}}{\text{from 6}}
\]

\[
\text{PAVG} = \frac{\text{MPa}}{\text{from 4}}
\]
for hydrogen:
\[ \mu = 88.73 \times 10^{-6} + 0.2 \times 10^{-6} (T_R - 293) + 0.118 \times 10^{-6} (P_{AVG}) \]
\[ = \ ]
\[ = \text{______ g/cm sec} \]

for helium:
\[ \mu = 196.14 \times 10^{-6} + 0.464 \times 10^{-6} (T_R - 293) - 0.093 \times 10^{-6} (P_{AVG}) \]
\[ = \ ]
\[ = \text{______ g/cm sec} \]

for air:
\[ \mu = 181.94 \times 10^{-6} + 0.536 \times 10^{-6} (T_R - 293) + 1.22 \times 10^{-6} (P_{AVG}) \]
\[ = \ ]
\[ = \text{______ g/cm sec} \]

\[ \rho_m = \text{mean gas density} \]
\[ \mu = 2.02 \quad 4.00 \quad 29 \]
\[ = \frac{\mu}{22.4 \times 0.1013 \times 273} = \frac{\mu(P_{AVG})}{T_R} \]
\[ = 0.1202 \left( \frac{\mu}{\mu(P_{AVG})} \right) = \text{______ g/cm}^3 \]

For analytical Schmidt Analysis
\[ WRS = (VHL)(\mu)(3)(\rho_m) = 3(\mu)(\rho_m) \]
\[ = \text{______ g/sec} \]

For numerical Schmidt analysis (from 7.4)
\[ WRS = \text{______ g/sec} \]
\[ AC = \frac{VRD}{LR} = \text{______ cm}^2 \]
\[ G = \frac{WRS}{AC} = \text{______ g/sec cm}^2 \]
AHT = \( \frac{2}{(\pi)^2} (MSH)(THW)(DR)^2(NR)(NS) \)

\[
= \frac{2}{(\pi)^2}( ) ( ) ( )^2 ( ) ( )
\]

\[
= \text{cm}^2
\]

RH = \( \frac{AC(LR)}{AHT} = \frac{()}{()} = \text{cm} \)

RE = \( \frac{4(RH)G}{MU} = 4\frac{()}{()} = \text{__________} \)

If RE < 60:

\( \log F = 1.73 - 0.93 \log (RE) \)

If 60 < RE < 1000:

\( \log F = 0.714 - 0.365 \log (RE) \)

If RE > 1000:

\( \log F = 0.015 - 0.125 \log (RE) \)

DELP = \( \frac{F(G)^2L}{2(10^7)(RH)(RHom)} \)

\[
= \frac{()}{(2 \times 10^7)( ) ( )} = \text{MPa}
\]

For analytical Schmidt Analysis

WPR = \( (DELP)(VHL)(2)(NU) = 2( ) ( ) ( ) \)

\[
= \text{__________} \text{watts}
\]
For numerical Schmidt analysis

\[
WPR = \frac{(DELP) \cdot (WRS) \cdot (2) \cdot (FCT)}{(RHOM)}
\]

\[
= \left( \frac{\text{---}}{\text{---}} \right) \cdot \left( \frac{\text{---}}{\text{---}} \right) \cdot 2 \cdot \left( \frac{\text{---}}{\text{---}} \right) = \text{---} \text{ watts}
\]

8.1.2 Slot or Annulii Regenerator

\[
RH = \frac{GR}{2} = \frac{\text{---}}{\text{2}} = \text{---} \text{ cm}
\]

\[RH = \text{GR/2} = \frac{\text{---}}{\text{2}} = \text{---} \text{ cm}\]

\[MU, \text{ RHOM, and WRS calculate in 8.1.1.}\]

\[
G = \frac{\text{WRS}}{AP} = \text{---} = \text{---} \text{ g/sec cm}^2
\]

\[
RE = \frac{4(RH)G}{\text{MU}} = \frac{4}{\left( \frac{\text{---}}{\text{---}} \right)} \left( \frac{\text{---}}{\text{---}} \right) = \text{---}
\]

\[RE = \frac{4(RH)G}{\text{MU}} = \frac{4}{\left( \frac{\text{---}}{\text{---}} \right)} \left( \frac{\text{---}}{\text{---}} \right) = \text{---}\]

\[
\text{If } RE \leq 2000
\]

\[
\log F = -1.34 - 0.20 \log(RE)
\]

\[
\text{If } RE > 2000
\]

\[
\log F = -1.34 - 0.20 \log(RE)
\]

\[
WPR = \frac{F(G)^2 \cdot L(VHL) \cdot (NU)}{10^7 (RH) \cdot (RHOM)} = \text{---}
\]

\[
WPR = \frac{F(G)^2 \cdot L(VHL) \cdot (NU)}{10^7 (RH) \cdot (RHOM)} = \text{---}
\]

\[
WPR = \text{---} \text{ watts}
\]

8.2 Gas Heater -- Windage

8.2.1 Tubular Heater

for Hydrogen:

\[
MU = 10^{-6} \left\{ 88.73 + 0.2(\text{THM} - 293) + 0.118 (PAVG) \right\}
\]

\[
= \text{---}
\]

\[
= \text{---} \text{ g/cm*sec}
\]
for Helium:

\[ \mu_L = 10^{-6} \left\{ 196.14 + 0.464(THM - 293) - 0.093(\text{PAVG}) \right\} \]

\[ = \quad \text{g/cm•sec} \]

for Air:

\[ \mu_A = 10^{-6} \left\{ 181.94 + 0.536(THM - 293) + 1.22(\text{PAVG}) \right\} \]

\[ = \quad \text{g/cm•sec} \]

\[ \mu = \quad \text{g/cm•sec} \]

\[ \frac{H_2}{He} \quad \text{air} \]

\[ \text{MW} = 2.02 \quad 4.00 \quad 29 \]

\[ \text{RHOM} = 0.1202 \cdot \text{MW(\text{PAVG})/(THM)} \]

\[ = 0.1202 \frac{\text{g}}{\text{cm}^3} \]

For analytical Schmidt analysis

\[ \text{WHS} = (\text{VHL})(\mu)(3)(\text{RHOM}) = 3( ) ( ) ( ) \]

\[ = \quad \text{g/sec} \]

For numerical Schmidt analysis (from 7.4)

\[ \text{WHS} = \quad \text{g/sec} \]

\[ \text{AC} = (\text{NTH})(\text{DIH})^2 \quad \text{g/sec} = ( ) ( )^2 \quad \text{g/sec} = \quad \text{cm}^2 \]

\[ G = \text{WHS}/\text{AC} = \left( \frac{\text{WHS}}{\text{AC}} \right) = \quad \text{g/cm}^2 \quad \text{sec} \]

\[ \text{RE} = \frac{(\text{DIH})G}{\mu} = \left( \frac{\text{DIH}G}{\mu} \right) = \quad \text{cm}^2 \quad \text{sec} \]

If \text{RE} < 2000, \text{F} = \frac{16}{\text{RE}} = \frac{16}{( )} \]

If \text{RE} > 2000, \log \text{F} = -1.34 = -0.20 \log (\text{RE})
For analytical Schmidt analysis

\[ \text{WPH} = \frac{4F(G)^2(LH)(VHL)(\text{NU})}{10^7(DIH)(\text{RHM})} = \frac{4(\quad )^2(\quad )(\quad )(\quad )}{10^7(\quad )(\quad )} \]

\[ \text{WPH} = \text{watts} \]

For numerical Schmidt analysis

\[ \text{DELP} = \frac{4'(F)(G)^2(LH)}{10^7(DIH)(\text{RHM})} = \frac{4'(\quad )^2(\quad )}{10^7(\quad )(\quad )} = \text{MPa} \]

\[ \text{WPH} = \frac{\text{DELP}(\text{WHS})2(FCHI)}{\text{RHM}} \]

\[ (\quad ) \{ \quad \} \{ \quad \} = \text{watts} \]

8.2.2 Annular Gap Heater

Find \( \text{MU}, \text{RHOM} \) and \( \text{WHS} \) in 8.2.1

\[ \text{AC} = \pi(DCY)(GH) = \pi(\quad )(\quad ) = \text{cm}^2 \]

\[ \text{G} = \frac{\text{WHS}}{\text{AC}} = \quad = \text{g/sec cm}^2 \]

\[ \text{RE} = \frac{2(GH)G}{\text{MU}} = \frac{2(\quad )(\quad )}{\quad } = \]

If \( \text{RE} < 2000 \), \( F = 24/\text{RE} = 24/(\quad ) = \)

If \( \text{RE} > 2000 \), \( \log F = -1.34 - 0.20 \log (\text{RE}) \)

\[ \text{WPH} = \frac{2F(G)^2(LHHT)(VHL)(\text{NU})}{10^7(GH)(\text{RHM})} \]

\[ \text{WPH} = \frac{2(\quad )^2(\quad )(\quad )(\quad )}{10^7(\quad )(\quad )} = \text{watts} \]
8.2.3 Nested Cone Isothermalizer

Find MU, RHOM and WHS in 8.2.1

\[ \text{BET} = \tan^{-1} \left( \frac{\text{DC1}}{2 \text{(LC1)}} \right) = \tan^{-1} \left( \frac{2}{2(\text{ )})} \right) \]
= \text{degrees}

\[ \frac{\text{GTA} = \left( \text{SCL} + \frac{\text{SE or SD}}{2} \right) \sin \left( \text{BET} \right) = \left( \frac{\text{ ( )}}{2} \right) \sin ( \text{ )} )}{\text{cm}} \]

\[ \frac{\text{AC} = \frac{\text{DC1}}{\sqrt{2}} \frac{\pi \text{(GTA)(NC)}}{\sqrt{2}} = \frac{\pi}{\sqrt{2}} (\text{ )} (\text{ )} (\text{ )}) = \text{cm}^2}{\text{ }} \]

\[ \frac{\text{G} = \frac{\text{WHS}}{	ext{AC}} = \text{g/sec cm}^2}{\text{ }} \]

\[ \frac{\text{RE} = \frac{2 \text{(GTA)} \text{G}}{\text{MU}} = \frac{2(\text{ )} (\text{ )})}{\text{ }} = \text{ }} \]

If RE < 2000, F = 24/RE = 24/(\text{ }) = \text{ }

If RE > 2000, \log F = -1.34 - 0.20 \log (\text{RE}), \text{F} = \text{ }

\[ \frac{\text{L} = \sqrt{(\text{LC1})^2 + \left( \frac{\text{DC1}}{\sqrt{2}} \right)^2 (1 - \frac{1}{\sqrt{2}})} = \text{ cm}}{\text{ }} \]

\[ = \sqrt{(\text{ )}^2 + \left( \frac{\text{ )}}{2} \right)^2 (0.2929 \text{ )}} = \text{ cm} \]

\[ \frac{\text{WPH} = \frac{2 \text{(G)} \text{2(L)(VHL)(NU)}}{10^4 \text{(GH)(RHOM)}} = \frac{2(\text{ )} (\text{ )} (\text{ )} (\text{ )})}{10^4(\text{ )}) (\text{ }) \text{ watts}}}{\text{ }} \]
8.3 Gas Cooler Windage

8.3.1 Tubular Cooler

for Hydrogen:

$$\mu = 10^{-6} \left\{ 88.73 + 0.2(TCM - 293) + 0.113(PAVG) \right\}$$

$$= \mu_\text{g/cm sec}$$

for Helium:

$$\mu = 10^{-6} \left\{ 196.14 + 0.464(TCM - 293) - 0.093(PAVG) \right\}$$

$$\mu = \mu_\text{g/cm sec}$$

for Air:

$$\mu = 10^{-6} \left\{ 181.94 + 0.536(TCM - 293) + 1.22(PAVG) \right\}$$

$$\mu = \mu_\text{g/cm sec}$$

$$\frac{H_2}{He} \quad \frac{H_2}{air}$$

$$MW = 2.02, 4.00, 29$$

$$\rho_\text{H2} = 0.1202(MW)(PAVG)/TCM$$

$$= 0.1202( )/( )/( ) = \rho_\text{g/cm}^3$$

For analytical Schmidt analysis

$$WCS = VCL(\mu)(3)(\rho_\text{H2}) = 3( )/( )/( )$$

$$= \rho_\text{g/sec}$$

For numerical Schmidt analysis (from 7.4)

$$WCS = \rho_\text{g/sec}$$
\[ AC = (NTC)(DIC)^2 \]
\[ 4 = ( ) ( )^2 \]
\[ 4 = \text{cm}^2 \]

\[ G = \frac{WCS}{AC} = \text{g/sec cm}^2 \]

\[ RE = \frac{(DIC)G}{MU} = ( ) ( ) \]

If \( RE < 2000 \), \( F = 16/RE = 16/( ) = \)

If \( RE > 2000 \), \( \log F = -1.34 - 0.20 \log(\text{RE}) \)

For analytical Schmidt analysis

\[ WPC = \frac{4F(G)^2(LC)(VCL)(MU)}{10^7(DIC)(RHOM)} \]

\[ WPC = \frac{4( )^2( ) ( ) ( )}{10^7( ) ( )} \]

\[ WPC = \text{watts} \]

For numerical Schmidt analysis

\[ Delp = \frac{2(F)(G)^2(LC)}{10^7(DIC)(RHOM)} - \frac{2( ) ( )^2 ( )}{10^7( ) ( )} \]

\[ = \text{MPa} \]

\[ WPC = \frac{Delp(WCS)^2(FCTC)}{RHOM} \]

\[ \{ ( ) \} \{ ( ) \}^2 ( ) = \text{watts} \]

8.3.2 Annular Gap Cooler

Find \( MU \), \( RHOM \), and \( WCS \) in 8.3.1

\[ AC = n(\text{DCY})(\text{GC}) = n( ) = \text{cm}^2 \]

\[ G = \frac{WCS}{AC} = \text{g/cm}^2 \text{ sec} \]
If $\text{RE} < 2000$, $F = 24/\text{RE} = 24/($

If $\text{RE} > 2000$, $\log F = -1.34 - 0.20 \log(\text{RE})$

$$WPC = \frac{2F(G)^2(LCHT)(VCL)(NU)}{10^7(GC)(RHOM)}$$

$$= \frac{2(\quad)(\quad)^2(\quad)(\quad)}{10^7(\quad)(\quad)}$$

$WPC =$ ___________ watts

8.3.3 Nested Cone Isothermalizer

Find $\mu$, $\text{RHOM}$, and $WCS$ in 8.3.1

$\text{BET} = \tan^{-1}\left(\frac{\text{DCI}}{2(\text{LCI})}\right) = \tan^{-1}\left(\frac{\quad}{2(\quad)}\right) =$ ___________ degrees

$\text{GTA} =$

$$= \left(\frac{\text{SCL} + \frac{\text{SC} \text{ ur SLD}}{2}}{\quad + \quad} \right) \sin(\text{BET})$$

$\text{AC} =$

$$= \frac{\text{DCI}}{\sqrt{2}} \left(\frac{\pi}{\sqrt{2}}\right)(\text{GTA})(\text{NC}) = \frac{\pi}{\sqrt{2}} (\quad)(\quad)(\quad) = \quad \text{cm}^2$$

$G =$

$$= \frac{WCS}{\text{AC}} = \quad = \quad \text{g/sec cm}^2$$

$\text{RE} =$

$$= \frac{2(\text{GTA})G}{\mu} = 2(\quad)(\quad)(\quad)$$

If $\text{RE} < 2000$, $F = 24/\text{RE} = 24/$

If $\text{RE} > 2000$, $\log F = -1.34 - 0.20 \log(\text{RE})$
\[ L = \sqrt{(LCL)^2 + \left( \frac{\text{DC} L}{2} \right)^2 \left( 1 - \sqrt{\gamma} \right)} \]

\[ = \sqrt{(\ldots)^2 + \left( -\frac{\text{DC} L}{2} \right)^2 \left( 0.2929 \right)} \quad \text{cm.} \]

\[ \text{WPC} = \frac{2F(G)^2(L)(VCL)(\text{NU})}{10^7(GC)(\text{RHOM})} \]

\[ \times \frac{2\left( \ldots \right)^2 \left( \ldots \right)^2 \left( \ldots \right)^2 \left( \ldots \right)}{10^7 \left( \ldots \right) \left( \ldots \right)} \]

\[ \text{WPC} = \ldots \text{watts} \]

8.4 Fluid Friction Loss Summary

Gas Heater, WPH = \ldots \text{watts}.

Regenerator, WPR = \ldots \text{watts}.

Gas Cooler, WPC = \ldots \text{watts}.

Total, WP = \ldots \text{watts}.

9. Mechanical Friction Loss

MFL = \ldots \text{based upon experimental measurement or}

\[ \text{MFL} = 0.2 \text{ BP.} = 0.2 \left( \ldots \right) = \ldots \text{watts} \]

10. Basic Heat Input

\[ \text{BHI} = \frac{\text{BP.}}{\left( 1 - \frac{\gamma}{\text{TH}} \right)} = \frac{\ldots}{\left( 1 - \ldots \right)} \quad \text{watts} \]

11. Reheat Loss:

11.1 Constant Volume Assumption (Equation 4-97):

\[ \text{FCT} = \ldots \text{ (usually 1/3) (from 7.4)} \]

\[ \text{WRS} = \ldots \text{g/sec (from 8.1)} \]

12.
CV = ____________ for TR = ____________ K (see Table 4-8)
CP = ____________ for TR = ____________ K (see Table 4-8)
AHT = ____________ cm² (from 8.1)
G = ____________ g/cm² sec (from 8.1)
PR = ____________ for TR = ____________ °K (see Table 4-9)
RE = ____________ (from 8.1)

\[
\log \left( \frac{H}{G(CP)} \right) = -0.13 - 0.412 \log (RE)
\]

\[
\log \left( \left( \frac{H}{G(CP)} \right)^{2/3} \right) = -0.13 - 0.412 \log (RE)
\]

\[
H = \frac{\text{___________}}{\text{w/cm}^2 \text{°K}}
\]

\[
\frac{H(AHT)}{(WRS)(CV)} = \left( \frac{\text{___________}}{\text{___________}} \right)
\]

\[
\text{QRH} = \frac{(FCT)(WRS)(CV)(THM - TCM)}{\left( \frac{2}{\text{NTUV} + 2} \right)}
\]

\[
= \left( \frac{\text{___________}}{\text{___________}} \right) \left( \frac{2}{\text{NTUV} + 2} \right)
\]

\[
= \text{___________ watts}
\]

11.2 Constant Pressure Assumption (Equation 4-98)

Find WRS, CP, AHT, G, PR, RE and H in 11.1

\[
\text{NTUP} = \frac{H(AHT)}{(WRS)(CP)} = \left( \frac{\text{___________}}{\text{___________}} \right)
\]

\[
\text{QRH} = \frac{1}{4} \left( \frac{(WRS)(CP)(THM - TCM)}{\left( \frac{2}{\text{NTUP} + 2} \right)} \right)
\]

\[
= \frac{1}{4} \left( \frac{\text{___________}}{\text{___________}} \right) \left( \frac{2}{\text{NTUP} + 2} \right)
\]

\[
= \text{___________ watts}
\]
11.3 Qvale Equation (Equation 4-105)
Do not have the information to calculate this.

12. Shuttle Conduction

12.1 High Pressure Engine (Equation 4-110)

\[ KG = \text{w/cm} \cdot \text{K at TR} = \text{K (From Table 4-9)} \]

\[ LT_1 = \text{cm} \quad @ \quad NU = \frac{\text{HZ}}{(LT \alpha \sqrt{NU})} \quad \text{(From Table 4-11)} \]

\[ LT_2 = \text{cm} \]

\[ K_1 = \text{w/cm} \cdot \text{K} \quad \text{From Figure 4-22} \]

\[ K_2 = \text{w/cm} \cdot \text{K} \]

\[ LB = 1 + \frac{KG}{2 \pi \cdot GR} \left( \frac{LT_1}{K_1} + \frac{LT_2}{K_2} \right) \]

\[ LB = 1 + \frac{1}{2 \pi} \left( \text{---} + \text{---} \right) \]

\[ LB = \text{---} \]

\[ QSH = \left( \frac{1 + LB}{1 + (LB)^2} \right) \frac{\pi}{8} \left( \frac{SD^2 (KG)(THM - TCM)(DCY)}{(GR)(LD)} \right) \]

\[ = \left( \frac{1 + LB}{1 + (LB)^2} \right) \frac{\pi}{8} \left( \frac{\text{---}^2 (\text{---})}{(\text{---})(\text{---})} \right) \]

\[ = \text{---} \quad \text{watts} \]

12.2 Low Pressure, Thin Walled Engine (Equation 4-111)

\[ RO_1 = \text{g/cm}^3 \text{ displacer wall density} \]

\[ RO_2 = \text{g/cm}^3 \text{ cylinder wall density} \]

\[ CP_1 = \text{j/g} \cdot \text{K displacer wall heat capacity} \]

\[ CP_2 = \text{j/g} \cdot \text{K cylinder wall heat capacity} \]

\[ \text{From Handbook} \]
OMG = $2\pi (NU) = 2\pi$ ( ) = $\frac{\text{rad}}{\text{sec}}$

$KG = \frac{\text{w/cm} \cdot \text{K at TR}}{(\text{Table 4-9})}$

$SGM = \frac{(KG)(SD)}{(GR)(OMG)} \left( \frac{1}{(RO1)(CP1)(WT1)} + \frac{1}{(RO2)(CP2)(WT2)} \right)$

$QSH = \left( \frac{1}{\left(1 + (SGM)^2 \right)^2} \right)^{\frac{\pi}{8}} \left( \frac{(SD)^2(KG)(THM - TCM)(DCY)}{(GR)(LD)} \right)$

$QSH = \frac{\text{watts}}{}$

13. Static Heat Conduction

13.1 Gas Conduction Inside Displacer or Hot Cap

$DID = DCY - 2(\text{GR}) - 2(\text{WT1}) = ( ) - 2( ) - 2( )$

$AHT = \frac{\pi}{4} (DID)^2 = \frac{\pi}{4} ( )^2 = \frac{\text{cm}^2}{\text{cm}}$

$KG = \frac{\text{w/cm} \cdot \text{K}}{\text{from 12}}$

$QC = \frac{KG (AHT)(THM - TCM)(DCY)}{(LD)}$

$QC = \frac{\text{watts}}{}$

13.2 Radiation Inside Displacer or Hot Cap

$\frac{\text{DID}}{\text{LD}} = \frac{\text{}}{\text{}} = \frac{\text{}}{\text{}}$
If:

\[ 0 < \frac{\text{DID}}{\text{LD}} < 0.2 \text{ then } \text{FA} = \frac{\text{DID}}{\text{LD}} \]

If:

\[ 0.2 < \frac{\text{DID}}{\text{LD}} < 7 \text{ then } \text{FA} = 0.50 + 0.20 \ln\left(\frac{\text{DID}}{\text{LD}}\right) \]

If:

\[ \frac{\text{DID}}{\text{LD}} > 7 \text{ then } \text{FA} = 1 \]

\[ \text{FE} = (\text{EH})(\text{EC}) = ( \quad )( \quad ) = \quad \]

\[ \text{FN} = \frac{1}{1 + \text{NRS}} = \frac{1}{1 + \quad} = \quad \]

\[ \text{AHT} = \quad \text{cm}^2 \text{ (from 13.1)} \]

\[ \text{QR} = (\text{FA})(\text{FE})(\text{FN})(\text{AHT})(5.67 \times 10^{-12}) \left( (\text{THM})^4 - (\text{TCM})^4 \right) \]
\[ = ( \quad )( \quad )( \quad )( \quad )(5.67 \times 10^{-12} \quad \left( ( \quad )^4 - ( \quad )^4 \right) \]
\[ = \quad \text{watts} \]

13.3 Displacer Cylinder Wall

\[ \text{KM} = \quad \text{w/cm K at TR} = \quad \text{K (Figure 4-22)} \]

\[ \text{AHT} = \frac{\pi}{4} \left( (\text{DCY} - 2\text{GR})^2 - (\text{DID})^2 \right) = \frac{\pi}{4} \left( ( \quad )^2 - 2( \quad )^2 \right) \]
\[ = \quad \text{cm}^2 \]

\[ \text{QC} = \frac{\text{KM}(\text{AHT})(\text{THM} - \text{TCM})}{\text{LD}} = ( \quad )( \quad )( \quad ) \]
\[ = \quad \text{watts} \]

13.4 Displacer Gap

\[ \text{KG} = \quad \text{w/cm C from 13.1} \]
\[ AHT = n(DCY)(GR) = n( ) ( ) = \text{_________ cm}^2 \]

\[ QC = \frac{KG(AHT)(THM - TCM)}{LD} = \left( \frac{ }{ } \right) \left( \frac{ }{ } \right) \left( \frac{ }{ } \right) \]

\[ = \text{_________ watts} \]

13.5 Cylinder Wall

Using the numbers in 2.5,

Let:

\[ 1 = \left( \frac{(LHB)4}{AHTB + AHT} \right) = \left( \frac{4( )}{( ) + ( )} \right) = \text{_________} \]

Then:

\[ R_1 = \frac{\text{___}}{KMH + KMB} = \left( \frac{ }{ } \right) + \left( \frac{ }{ } \right) = \text{_________} \]

\[ = \text{_________ K/watt} \]

Let:

\[ 2 = \frac{4(LBA)}{AHTB + AHT} = \left( \frac{4( )}{( ) + ( )} \right) = \text{_________} \]

Then:

\[ R_2 = \frac{\text{___}}{KMB + KMA} = \left( \frac{ }{ } \right) + \left( \frac{ }{ } \right) = \text{_________ K/watt} \]

Let:

\[ 3 = \frac{4(LAC)}{AHTB + AHT} = \left( \frac{4( )}{( ) + ( )} \right) = \text{_________} \]

Then:

\[ R_3 = \frac{\text{___}}{KMA + KMC} = \left( \frac{ }{ } \right) + \left( \frac{ }{ } \right) = \text{_________ K/watt} \]

\[ QC = \frac{THM - TCM}{R_1 + R_2 + R_3} = \left( \frac{ }{ } \right) + \left( \frac{ }{ } \right) + \left( \frac{ }{ } \right) \]

\[ = \text{_________ watts} \]

\[ TB = THM - R_1(QC) = ( ) - ( )( ) \]

\[ = \text{_________ K} \quad \text{---- Original TB estimate was _______ K} \]
\[ TA = TB - R2(QC) = (\quad) - (\quad)(\quad) = \quad \text{K} \quad \text{---- Original TA estimate was} \quad \text{K} \]

Now:

\[ KMB = \quad \text{w/cm} \cdot \text{K at TB} \quad \text{Figure 4-22} \]

\[ KMA = \quad \text{w/cm} \cdot \text{K at TA} \]

Now go around again.

\[ R1 = \frac{1}{KMH + KMB} = \left(\quad\right) + \left(\quad\right) = \quad \text{K/watt} \]

\[ R2 = \frac{1}{KMB + KMA} = \left(\quad\right) + \left(\quad\right) = \quad \text{K/watt} \]

\[ R3 = \frac{1}{KMA + KMC} = \left(\quad\right) + \left(\quad\right) = \quad \text{K/watt} \]

\[ QC = \frac{THM - TCM}{R1 + R2 + R3} = \left(\quad\right) + \left(\quad\right) - \left(\quad\right) + \left(\quad\right) = \quad \text{watts} \]

\[ TB = THM - R1(QC) = (\quad) - (\quad)(\quad) = \quad \text{K} \quad \text{---- Previous estimate was}\quad \text{K} \]

\[ TA = TB - R2(QC) = (\quad) - (\quad)(\quad) = \quad \text{K} \quad \text{---- Original TA estimate was} \quad \text{K} \]

Now:

\[ KMB = \quad \text{w/cm} \cdot \text{K at TB} \quad \text{Figure 4-22} \]

\[ KMA = \quad \text{w/cm} \cdot \text{K at TA} \]

Now go around again.

\[ R1 = \frac{1}{KMH + KMB} = \left(\quad\right) + \left(\quad\right) = \quad \text{K/watt} \]
R2 = \frac{\alpha_2}{K_{MB} + K_{MA}} = \left( \left( \right) + \left( \right) \right) \quad \text{K/watt}

R3 = \frac{\alpha_3}{K_{MA} + K_{MC}} = \left( \left( \right) + \left( \right) \right) \quad \text{K/watt}

Q_c = \frac{\theta_{TH} - \theta_{TC}}{R_1 + R_2 + R_3} = \left( \left( \right) + \left( \right) \right) \quad \text{watts}

TB = \theta_{TH} - R_1(Q_c) = \left( \right) - \left( \right) \quad \text{K}

TA = TB - R_2(Q_c) = \left( \right) - \left( \right) \quad \text{K}

Does the difference significantly change the thermal conductivities?

[ ] Yes \quad \text{Then go around again}

[ ] No \quad \text{Accept last Qc as accurate}

13.6 Regenerator Walls

One regenerator wall will be calculated and then multiplied

NR. Using the numbers from 2.4

Let (1) \quad \frac{4(LMB)}{\Theta_{TH} + \Theta_{TB}} = \left( \left( \right) + \left( \right) \right) \quad \text{K/watt}

Then R1 = \frac{\alpha_1}{K_{MB} + K_{MB}} = \left( \left( \right) + \left( \right) \right) \quad \text{K/watt}

Let (2) \quad \frac{4(LBA)}{\Theta_{TB} + \Theta_{TA}} = \left( \left( \right) + \left( \right) \right) \quad \text{K/watt}

Then R2 = \frac{\alpha_2}{K_{MB} + K_{MA}} = \left( \left( \right) + \left( \right) \right) \quad \text{K/watt}

Let (3) \quad \frac{4(LAC)}{\Theta_{TA} + \Theta_{TC}} = \left( \left( \right) + \left( \right) \right) \quad \text{K/watt}

Then R3 = \frac{\alpha_3}{K_{MA} + K_{MC}} = \left( \left( \right) + \left( \right) \right) \quad \text{K/watt}
\[ Q = \frac{\text{THM} - \text{TCM}}{R_1 + R_2 + R_3} = \left( \frac{1}{R_1} \right) + \left( \frac{1}{R_2} \right) + \left( \frac{1}{R_3} \right) = \text{_______ watts} \]

\[ TB = \text{THM} - R_1(Q) = \left( \frac{1}{R_1} \right) - \left( \frac{1}{R_2} \right)(...) = \text{_______ K} \text{ Previous estimate was } \text{_______ K} \]

\[ TA = TB - R_2(Q) = \left( \frac{1}{R_2} \right) - \left( \frac{1}{R_3} \right)(...) = \text{_______ K} \text{ Previous estimate was } \text{_______ K} \]

Now:

\[ K_{MB} = \text{_______ w/cm} \cdot \text{K at } TB \]

\[ K_{MA} = \text{_______ w/cm} \cdot \text{K' at } TA \]

Now go around again:

\[ R_1 = \frac{1}{K_{MH} + K_{MB}} = \left( \frac{1}{K_{MH}} \right) + \left( \frac{1}{K_{MB}} \right) = \text{_______ K/watt} \]

\[ R_2 = \frac{2}{K_{MB} + K_{MA}} = \left( \frac{1}{K_{MB}} \right) + \left( \frac{1}{K_{MA}} \right) = \text{_______ K/watt} \]

\[ R_3 = \frac{3}{K_{MA} + K_{MC}} = \left( \frac{1}{K_{MA}} \right) + \left( \frac{1}{K_{MC}} \right) = \text{_______ K/watt} \]

\[ Q = \frac{\text{THM} - \text{TCM}}{R_1 + R_2 + R_3} = \left( \frac{1}{R_1} \right) + \left( \frac{1}{R_2} \right) + \left( \frac{1}{R_3} \right) = \text{_______ watts} \]

\[ TB = \text{THM} - R_1(Q) = \left( \frac{1}{R_1} \right) - \left( \frac{1}{R_2} \right)(...) = \text{_______ K} \text{ Previous estimate was } \text{_______ K} \]

\[ TA = TB - R_2(Q) = \left( \frac{1}{R_2} \right) - \left( \frac{1}{R_3} \right)(...) = \text{_______ K} \text{ Previous estimate was } \text{_______ K} \]

Does the difference significantly change the thermal conductivities?

Yes [ ] then go around again

No [ ] then go on

\[ QC = Q(NR) = (\text{_______}) = \text{_______ watts} \]
13.7 Regenerator Matrix (from 2.2.1)

13.7.1 Screen Stack (Equation 4-170)

\[
\begin{align*}
 KM & = \text{w/cm} \cdot K \text{ at IR} \\
 KG & = \text{w/cm} \cdot K \text{ at IR}
\end{align*}
\]

Let \( T \) be \[
\begin{pmatrix}
 1 & -\left( \frac{\text{KM}}{\text{KG}} \right) \\
 1 & -\left( \frac{\text{KM}}{\text{KG}} \right)
\end{pmatrix}
\]

\[
\begin{align*}
\text{KMX} & = \left( \begin{pmatrix} T_{21} - E \end{pmatrix} \right) = \left( \begin{pmatrix} 1 & \{ \} \end{pmatrix} - \left( \begin{pmatrix} 1 & \{ \} \end{pmatrix} \right) \right) \\
\end{align*}
\]

\[ \text{w/cm} \cdot K \]

\[
\begin{align*}
\text{AHT} & = \frac{3}{4} \left( \text{DR} \right) \left( \frac{3}{4} \right) \quad \text{cm}^3 \\
\text{QC} & = \left( \text{NR} \right) \text{KMX} \left( \text{AHT} \right) \left( \text{THM} - \text{TCM} \right) \quad \text{watts}
\end{align*}
\]

13.7.2 Slots. Multiple Annuli (Equation 4-170a)

\[
\begin{align*}
\text{KM}, \text{KG} & \text{ see 3.2.1 (numbers from 2.2.2)} \\
\text{KMX} & = \frac{\text{KG} \left( 3IR \right) + \text{KMX} \left( \text{THI} \right)}{\text{GR} + \text{THI}} \quad \left( \begin{pmatrix} 1 & \{ \} \end{pmatrix} + \left( \begin{pmatrix} 1 & \{ \} \end{pmatrix} \right) \right) \\
\end{align*}
\]

\[ \text{w/cm} \cdot K \]

\[
\begin{align*}
\text{AHT} & = \frac{\text{GR} \left( 3IR \right) + \text{THI}}{\text{GR}} \quad \left( \begin{pmatrix} 1 & \{ \} \end{pmatrix} \right) \\
\text{QC} & = \left( \text{KMX} \left( \text{AHT} \right) \left( \text{THM} - \text{TCM} \right) \right) \quad \text{watts}
\end{align*}
\]

\[ \text{watts} \]
13.8 Summary of Static Heat Conduction

Section
13.1 Gas Cond. Inside Displ. \( QC = \) __________ watts
13.2 Radiation Inside Displ. \( QR = \) __________
13.3 Displ. Wall \( QC = \) __________
13.4 Displacer Gap \( QC = \) __________
13.5 Cylinder Wall \( QC = \) __________
13.6 Regenerator Wall \( QC = \) __________
13.7 Regenerator Matrix \( QC = \) __________

Total \( QS = \) __________

14. Pumping Loss (Equation 4-126)

\[
P_{\text{MAX}} = \text{___________ MPa from 6}
\]

\[
P_{\text{MIN}} = \text{___________ MPa from 6}
\]

\[
RM = \frac{R}{MW} = 8.314 = \text{___________ J/g·K}
\]

\[
MW = \begin{cases} 
2.02 & \text{for H}_2 \\
4.00 & \text{for He} \\
29 & \text{for air}
\end{cases}
\]

\[
Z_1 = \text{___________ (normally 1 except when gas temp. < 70 K)}
\]

\[
Q_{\text{PU}} = \left( \frac{n \ DCY}{KG} \right)^{0.6} \left[ \frac{2(LD)(\text{THM} - \text{TCM})}{1.5(Z1)} \right] \left( \frac{P_{\text{MAX}} - P_{\text{MIN}}N(U(CP))^{2.6}}{(\text{THM} - \text{TCM})R(M) \cdot (\text{THM} - \text{TCM})R(M)} \right)^{1.6} 
\]

\[
Q_{\text{PU}} = \left( \frac{n \ DCY}{KG} \right)^{0.6} \left[ \frac{2(LD)(\text{THM} - \text{TCM})}{1.5(Z1)} \right] \left( \frac{P_{\text{MAX}} - P_{\text{MIN}}N(U(CP))^{2.6}}{(\text{THM} - \text{TCM})R(M) \cdot (\text{THM} - \text{TCM})R(M)} \right)^{1.6} 
\]

\[
= \text{___________ watts}
\]
15. Temperature Swing Loss (Equation 4-128)

\[ FCT = \text{see 11} \]
\[ ROM = \text{g/cm}^3 \]
\[ CPM = \text{j/g. K} \quad \text{from Standard References} \]

For Screen Regenerators:

\[ \text{MMX} = \text{NR} \\frac{3}{4} (DR)^2 (LR)(FF)(ROM) \]
\[ = ( \frac{3}{4} ) ( \frac{3}{4} ) \quad \text{g} \]

For Slot Regenerators:

\[ \text{MMX} = (LR)(THF)(AF)/GR = ( \frac{1}{2} ) \quad \text{g} \]

\[ \text{DELMX} = \frac{\text{WHS}(CV)(FCT)(THM - TCH)}{\text{NU(MMX)(CPM)}} = ( \frac{1}{2} ) \quad \text{K} \]

\[ \text{QTS} = \text{(FCT)(WHS)(CV)(DELMX)}/2 \]
\[ = ( \frac{1}{2} ) \quad \text{watts} \]

16. Internal Temperature Swing Loss

\[ KN = \text{w/cm \cdot K from Figure 4-22} \]

For Screens:

\[ \text{LMX} = \frac{\text{THM}}{2} = \frac{1}{2} \quad \text{cm} \]
\[ C3 = 0.25 \]

For Slots:

\[ \text{LMX} = \frac{\text{THF}}{2} = \frac{1}{2} \quad \text{cm} \]
\[ C3 = 0.32 \]
\[
Q_{TS} = \frac{Q_T S(C_3)(R_0)(C_{PM})(R_{MX})^2(N_U)}{(K_M)(F_C T)} \\
= \left( \frac{Q_T S(C_3)(R_0)(C_{PM})(R_{MX})^2(N_U)}{(K_M)(F_C T)} \right) \times \left( \frac{Q_T S(C_3)(R_0)(C_{PM})(R_{MX})^2(N_U)}{(K_M)(F_C T)} \right) \\
= \text{__________ watts}
\]

17. Performance Summary

<table>
<thead>
<tr>
<th>Net Power, watts</th>
<th>1st Iteration</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP = basic power =</td>
<td>_________</td>
<td>_______</td>
<td>_______</td>
</tr>
<tr>
<td>(from 7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WP = windage power =</td>
<td>_________</td>
<td>_______</td>
<td>_______</td>
</tr>
<tr>
<td>(from 8.4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MFL = mechanical friction loss =</td>
<td>_________</td>
<td>_______</td>
<td>_______</td>
</tr>
<tr>
<td>(from 9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NP = BP - WP - MFL =</td>
<td>_________</td>
<td>_______</td>
<td>_______</td>
</tr>
<tr>
<td>= net power</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Net Heat Input (watts) | 1st It. | 2nd | 3rd
(Equation 4-132)

| BHI = basic heat input = | _________ | _______ | _______ |
| (see 10) | | | |
Net Net Input (watts) | 1st It. | 2nd | 3rd
--- | --- | --- | ---
+QRH = reheat loss (see 11) | | | |
+QSH = shuttle heat cond. (see 12) | | | |
+QS = static heat cond. (see 13.8) | | | |
+QPU = pumping loss (see 14) | | | |
+QTS = temp. swing loss (see 15) | | | |
+QITS = internal temp. swing loss (see 16) | | | |
-WPH = heater windage power (see 8.4) | - | | |
\[ \frac{WPR}{2} = \text{half of regenerator windage power} \] (see 8.4) | | | |
QN = net heat input | | | |

18. Heat Exchanger Duty

| | 1st It. | 2nd | 3rd |
--- | --- | --- | ---
Gas Heater | | | |
QGH = QN = | | | |
Gas Cooler | | | |
QGC = QN - NP = | | | |
\[ = (\quad) - (\quad) \]
19. Gas Heater

19.1 Tubular Type

\[ \text{RE} = \text{from 8.2.1} \]

\[ \frac{LH}{DIH} = \text{from Figure 4-19} \]

\[ \frac{H(\text{PR})}{CP(\text{G})} = 1 \]

\[ G = \text{g/cm}^2 \text{ sec -- from 8.2.1} \]

\[ CV = \text{J/g \cdot K -- from Table 4-8} \]

\[ CP = \text{J/g \cdot K -- from Table 4-8} \]

\[ PR = \text{from Table 4-9} \]

\[ H = \frac{1}{2} \left( \frac{CP(\text{G})}{(PR)^3} \right) = \text{w/cm}^2 \cdot \text{K} \]

\[ \text{AHT} = (\text{NTH})(\pi)(\text{DIH})(LHHT) = \pi \left( \frac{1}{\text{cm}^2} \right) \]

\[ \text{NTUH} = \frac{H(\text{AHT})}{2(FCT)(WHS)(CV)} = \frac{1}{2} \left( \frac{1}{\text{cm}^2} \right) \]
1st Iteration

\[ TH = THM - \Sigma (FC) WHS (CV) \left[ \exp (NTH) - 1 \right] \]

\[ = \frac{1}{2} \left( \begin{array}{c} \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \exp \left( \frac{1}{2} \right) - 1 \right) \\ \end{array} \right) \]

\[ = \frac{1}{2} \left( \begin{array}{c} \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \exp \left( \frac{1}{2} \right) - 1 \right) \\ \end{array} \right) \]

2nd Iteration

\[ TH = THM - \frac{QGH}{\exp(1)} = \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \]

3rd Iteration

\[ TH = THM - \frac{QGH}{\exp(1)} = \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \]

19.2 Annular Gap Type Heater (Equation 4.136) (from 2.1.2)

\[ KG = \text{w/cm} \cdot \text{K} \quad \text{from Table 4.9 at THM} = \text{K} \]

\[ H = \frac{4.86(KG)}{4(GH)} = \frac{1.22}{(1.22)} = \text{w/cm}^2 \cdot \text{K} \]

\[ AHT = n(\pi)(LHT) = n(\pi)(\text{cm}) = \text{cm}^2 \]

\[ NTH = \Sigma (FC) (WH) (CV) = \Sigma (FC) (WH) (CV) \left( \exp (NTH) - 1 \right) \]

1st Iteration

\[ TH = THM - \Sigma (FC) (WH) (CV) \left( \exp (NTH) - 1 \right) \]
= \frac{1}{2} \left( \frac{1}{(1)} \right) \left( \frac{1}{(1)} \right) \left( \frac{1}{(1)} \right) \left( \frac{1}{(1)} \right) = \frac{1}{1} = K

2nd Iteration

\text{TH} = \text{THM} - \frac{\text{QGH}}{1} = \left( \frac{1}{1} \right) - \left( \frac{1}{1} \right) = \frac{1}{1} = K

3rd Iteration

\text{TH} = \text{THM} - \frac{\text{QGH}}{1} = \left( \frac{1}{1} \right) - \left( \frac{1}{1} \right) = \frac{1}{1} = K

19.3 Isothermalizer Cone Type Heater (Equation 4-137)

\text{KG} = \frac{1}{1} \text{ w/cm} \cdot \text{K} -- \text{from Table 4-9 at THM} = \frac{1}{1} \text{ K}

\text{GTA} = \frac{1}{1} \text{ cm} -- \text{from 8.2.3}

\text{AHT} = \pi \text{(NC)}(\text{DC}) \sqrt{ \frac{(\text{DC})^2}{2} + (\text{LC})^2 }

= \pi \left( \frac{1}{1} \right) \left( \frac{1}{1} \right) \sqrt{ \frac{(\text{DC})^2}{2} + (\text{LC})^2 } = \frac{1}{1} \text{ cm}^2

1st Iteration

\text{TH} = \text{THM} - \frac{\text{QGH}}{6(\text{KG})(\text{AHT})} = \left( \frac{1}{1} \right) - \left( \frac{6(\text{KG})(\text{AHT})}{1} \right) = \frac{1}{1} = \frac{1}{1} \text{ K}

\text{TH} = \frac{1}{1} \text{ K}
2nd Iteration
\[ TH = THM - \frac{QGH}{10} = \left( \ldots \right) - \left( \ldots \right) \]
\[ = \frac{\ldots}{K} \]

3rd Iteration
\[ TH = THM - \frac{QGH}{10} = \left( \ldots \right) - \left( \ldots \right) \]
\[ = \frac{\ldots}{K} \]

20. Gas Cooler

20.1 Correction of effective cold metal temperature due to temperature rise in cooling water.
\[ \Delta T = \frac{QGC}{T_{CW}(4.1868)} = \left( \ldots \right) \cdot 4.1868 \]
\[ = \frac{\ldots}{K} \]
\[ TCM = TCWI + \Delta T/2 \]
\[ = \frac{\ldots}{K} + \left( \ldots \right) = \ldots K \]

20.2 Tubular Type

\[ RE = \ldots \text{ from 8.3.1} \]

From Figure 4-19:
\[ \frac{H(PR)^{2}}{CP(G)} = \frac{\cdot}{\cdot} \]
\[ G = \ldots \text{ g/sec cm}^2 \text{ -- from 8.3.1} \]
\[ WCS = \ldots \text{ g/sec -- from 8.3.1} \]
\[ CV = \ldots \text{ J/g} \cdot \text{K} \text{ From Table 4-8} \]
\[ CP = \ldots \text{ J/g} \cdot \text{K} \]
\[ PR = \ldots \text{ from Table 4-9} \]
H = \frac{1}{2} \left( \frac{CP}{2} \right) (G) \frac{2}{(PR)}^{\frac{2}{3}} = \text{w/cm}^2 \cdot \text{K}

AHT = (NTC)(\pi)(DIC)(LCHT) = (\pi)(\pi)(\pi)
= \text{cm}^2

NTUC = \frac{H(AHT)}{2(FCT)(WCS)(CV)} = \frac{2}{2}(\exp(NTUC) - 1)

First Iteration

TC = TCM + \frac{QGC}{2(FCT)(WCS)(CV)(\exp(NTUC) - 1)}

TC = (\text{TCM}) + \frac{2}{2}(\exp(NTUC) - 1)

2nd Iteration

TC = TCM + \frac{QGC}{1}

= \text{TCM}

3rd Iteration

TC = TCM + \frac{QGC}{1}

20.3 Annular Gap Type Cooler (Equation 4-136)

KG = \text{w/cm} \cdot \text{K} from Table 4-9 at TCM = \text{K}

H = \frac{1.22KG}{GC} = 1.22(\text{K}) = \text{w/cm}^2 \cdot \text{K}

AHT = (DCY)(LCHT) = \pi(\pi)(\pi) = \text{cm}^2
Calculate NTUC and TC in 20.1

20.4 Isothermalizer Cone Type Cooler (Equation 4-138)

\[ KG = \frac{W}{cm \cdot K} \] from Table 4-9 at TCM = _____ K

GTA = _____ cm from 8.3.3

From 2.3.3:

\[ AHT = \pi (NC) (DC1) \sqrt{\left(\frac{DC1}{2}\right)^2 + (LC1)^2} \]

\[ = \pi (\_) (\_) \sqrt{\left(\frac{\_}{2}\right)^2 + (\_)^2} \]

\[ = \frac{\_}{\_} \text{cm}^2 \]

1st Iteration

\[ TC = TCM + \frac{QGC}{6(KG)(AHT)} = (\_) + \left(\frac{\_}{6(KG)(AHT)}\right) \]

\[ TC = \_ \text{K} \]

2nd Iteration

\[ TC = TCM + \frac{QGC}{\_} = (\_) + \left(\frac{\_}{\_}\right) \]

\[ = \_ \text{K} \]

3rd Iteration

\[ TC = TCM + \frac{QGC}{\_} = (\_) + \left(\frac{\_}{\_}\right) \]

\[ = \_ \text{K} \]

21. Conclusion

Final Net Power = _______ watts
Final Indicated Power = _______________ watts

Final Net Heat Input = _______________ watts

Brake Efficiency = _______________%

Indicated Efficiency = _______________%

7.2 Sample Design Calculation

In Section 7.1 a design form was presented which would allow the designer to start with a set of dimensions and operating conditions for his machine and then calculate through a series of equations, all conveniently specified, to determine the basic power output and heat input as well as the power losses and heat losses. The end result is a computed net heat input and power output and a net efficiency. In this section this form is excerpted to show only those parts that need to be filled out to calculate the expected performance of the GPU-3 engine described in Section 3.3. In particular, Test Point 1 on Table 3-8 is calculated as a sample. The GPU-3 engine is classified as a Beta type engine with a single displacer and power piston pair and a rhombic drive.

1. Type of Piston Arrangement

1.2 Beta: Displacer-Piston Overlapping Strokes

DCY = diameter of engine cylinder 7.010 cm.

SD = stroke of displacer 3.068 cm.

SP = stroke of power piston 3.068 cm.

DDR = diameter of displacer drive rod 0.953 cm.

2. Dead Volumes and Heat Exchangers and Wall Thicknesses

2.1 Volumes and Heat Exchangers at Heat Source Temperature

VHDX = extra hot dead volume besides that in the gas heater 12.389 cm³.

2.1.1 Tubular Gas Heater

NTH = number of heater tubes per power unit 40.

LH = total length of each heater tube 24.229 cm.

LHHT = heated length of each heater tube 15.545 cm.
DIH = inside diameter of heater tubes 0.302 cm.
DOH = outside diameter of heater tubes 0.483 cm.

2.2 Volumes at Regenerator Temperature

2.2.1 Screen Regenerators

LR = length of regenerator 2.261 cm.
NR = number of regenerators/unit 8
DR. = diameter of each regenerator 2.261 cm.
NS = number of screen layers 308
MSH = mesh size 83.9 wires/cm.
THW = thickness of wire in screens 0.0041 cm.
FF = filler factor, fraction of regenerator volume filled with wires 0.286
Screen material Stainless Steel

2.2.3 Displacer or Hot Cap Gap Volume

LD = length of displacer 4.359 cm.
DCY = diameter of cylinder around displacer 7.010 cm.
GR = gap between displacer and cylinder wall 0.025 cm.
Displacer wall material Stainless Steel
Cylinder wall material Stainless Steel
WT1 = wall thickness of displacer, 0.178 cm.
WT2 = wall thickness of cylinder wall 0.406 cm.
EH = emissivity of hot inside of displacer or cylinder wall 0.6
EC = emissivity of cold inside of displacer or cylinder wall 0.5
NRS = number of radiation shields inside the displacer or hot cap.

2.3 Volumes and Heat Exchangers at Heat Sink Temperature

\[ VCDX = \text{extra cold dead volume besides that in the gas cooler} \]
\[ = 5.782 \text{ cm}^3, \left[ \frac{\pi}{4} (0.235)^2 (0.625 + 0.017) \right] 8(2.54)^3 \]

2.3.1 Tubular Gas Cooler

NTC = number of cooler tubes per power unit.

LC = total length of each cooler tube \(4.470\) cm.

LCHT = cooled length of each cooler tube \(3.480\) cm.

DIC = inside diameter of cooler tubes \(0.102\) cm.

DOC = outside diameter of cooler tubes \(0.152\) cm.

2.4 Regenerator Wall Dimensions (see Figure 4-23)

\[ \text{AHTH} = \text{heat conduction area at hot end} \quad 1.425 \text{ cm}^2. \]

\[ \text{AHTB} = \text{heat conduction area at level B} \quad 1.425 \text{ cm}^2. \]

\[ \text{AHTA} = \text{heat conduction area at level A} \quad 0.853 \text{ cm}^2. \]

\[ \text{AHTC} = \text{heat conduction area at cold end} \quad 1.425 \text{ cm}^2. \]

\[ \text{LHB} = \text{hot length regenerator wall} \quad 1.016 \text{ cm.} \]

\[ \text{LBA} = \text{middle length regenerator wall} \quad 1.194 \text{ cm.} \]

\[ \text{LAC} = \text{cold length regenerator wall} \quad 0.051 \text{ cm.} \]

Regenerator wall material: Stainless Steel.

<table>
<thead>
<tr>
<th>Temp K</th>
<th>Thermal Conductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>THM = 978</td>
<td>KMH = 0.25 w/cm·K−1</td>
</tr>
<tr>
<td>TB = 800 (est.)</td>
<td>KMB = 0.225</td>
</tr>
<tr>
<td>TA = 350 (est.)</td>
<td>KMA = 0.16</td>
</tr>
<tr>
<td>TCM = 294</td>
<td>KMC = 0.15</td>
</tr>
</tbody>
</table>
2.5 Cylinder Wall Dimensions (see Figure 4-24)

\[
\begin{align*}
\text{AHTH} &= \text{heat conduction area at hot end} \quad 15.915 \text{ cm}^2. \\
\text{AHTB} &= \text{heat conduction area at level B} \quad 10.726 \text{ cm}^2. \\
\text{AHTA} &= \text{heat conduction area at level A} \quad 9.469 \text{ cm}^2. \\
\text{AHTC} &= \text{heat conduction area at cold end} \quad 15.915 \text{ cm}^2. \\
\text{LHB} &= \text{hot length regenerator wall} \quad 2.858 \text{ cm.} \\
\text{LBA} &= \text{middle length regenerator wall} \quad 1.016 \text{ cm.} \\
\text{LAC} &= \text{cold length regenerator wall} \quad 1.245 \text{ cm.}
\end{align*}
\]

Cylinder wall material: Stainless Steel

<table>
<thead>
<tr>
<th>Level Temp K</th>
<th>Thermal Conductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>THM = 978</td>
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<td>TB = 800 (est)</td>
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<td>KMA = 0.16</td>
</tr>
<tr>
<td>TCM = 294</td>
<td>KMC = 0.15</td>
</tr>
</tbody>
</table>

3. Drives

\[N = \text{Number of power units/engine} \quad 1\]

3.3 Beta-Rhombic Drive

\[\text{LCR} = \text{connecting rod length} \quad 4.602 \text{ cm.} \]
\[\text{RC} = \text{crank radius} \quad 1.397 \text{ cm.} \]
\[\text{ECC} = \text{crank eccentricity} \text{ (see Figure 3-30)} \quad 2.065 \text{ cm.}\]

4. Given Operating Conditions

\[\text{TC} = \text{effective temperature in cold, compression space} \quad 300 \text{ K.}\]
\[K = C + 273.1 \quad \text{Make initial estimate}\]
\[K = (F + 460)/1.8\]
\[\text{TH} = \text{effective temperature in hot, expansion space} \quad 978 \text{ K.}\]
FCW = cooling water flow $ \frac{379}{\sec} $ g/sec.

TCWI = Temperature of cooling water into engine 294.4 K.

NU = engine frequency 25 Hz.

(P = RPM/60)

PMAX = maximum engine pressure Not Spec. MPa.

MPa = 0.006894 (psia)

= 0.1013 (atm)

PAVG = time averaged mean pressure 2.068 MPa.

PMAH = heat source metal temperature 978 K.

TCM = heat sink metal temperature 295 K.

Working Gas Hydrogen.

5. Computation of Engine Volumes

5.1 Live Volumes

Note: Live volumes for crank and rhombic drives need not be defined.

5.2 Dead Volumes

VHD = hot dead volume, cm$^3$

for tubular gas heater

= $VHD + \frac{n}{4} (DIH)^2 (LH) (NTH)$

= $12,389 + \frac{n}{4} (0.302)^2 (24.229)(40) = 81,811$ cm$^3$.

VRD = regenerator dead volume, cm$^3$

for screen regenerators, short calculation

VRD = $NR \left(\frac{n}{4}\right) (DR)^2 (LR) (1 - FF)$

= $(8) \frac{n}{4} (2.261)^2 (2.261) (1 - 0.286) \approx 51,354$ cm$^3$. 
VCD = cold dead volume
for tubular gas coolers
\[ \text{VCD} = \text{VCD}_X + \frac{\pi}{4}(\text{DIC})^2(\text{LC})(\text{NTC}) \]
\[ = 5.782 + \frac{\pi}{4}(0.102)^2(4.470)(312) = 17.178 \text{ cm}^3. \]

6. General Intermediate Parameters

\[ TR = \frac{(\text{THM} - \text{TCM})}{\ln(\text{THM}/\text{TCM})} \]
\[ = \frac{(978 - 295)}{\ln(978/295)} = 570 \text{ K} \]

7.4 Rhombic Drive Philips Engine (See Figure 4-15 and 4-16)

\[ A = \sqrt{(\text{LCR})^2 - (\text{ECC} - \text{RC})^2} \]
\[ = \sqrt{(4.602)^2 - (2.065 - 1.397)^2} = 4.553 \text{ cm} \]

\[ B = \sqrt{(\text{LCR})^2 - (\text{ECC} + \text{RC})^2} \]
\[ = \sqrt{(4.602)^2 - (2.065 + 1.397)^2} = 3.034 \text{ cm} \]

For a Check:

\[ \text{VCL} = (A - B)\frac{\pi}{2}(\text{DCY})^2 - (\text{DDR})^2 \]
\[ = (4.553 - 3.034)\frac{\pi}{2}(7.010)^2 - (0.953)^2 = 115.06 \text{ cm}^3. \]

\[ \text{VCL}_X = \frac{\pi}{2}(\text{DCY})^2 - (\text{DDR})^2 \left\{ A - \sqrt{(\text{LCR})^2 - (\text{ECC} - \text{RC} \cos(\text{PHI}))^2} \right\} \]
\[ = \frac{75.762}{21.178} \left( 4.553 - \sqrt{21.178 - (2.065 - 1.395 \cos(\text{PHI}))^2} \right) \]

\[ \text{VC} = \text{VCD} + \text{VCL}_X = \text{VCL}_X + 17.178 \]

\[ \text{VC} = 17.178 + 75.762 \left( 4.553 - \sqrt{21.178 - (2.065 - 1.395 \cos(\text{PHI}))^2} \right) \]
VC is calculated for PHI = 0, 30, 60, ..., 360°
(See next page)

\[ D_1 = \sqrt{(L_C + R_C)^2 - (L_{CC})^2} \]
\[ = \sqrt{(4.60^2 + 1.397)^2 - (2.066)^2} = 5.632 \text{ cm.} \]

\[ C = \sqrt{(L_C - R_C)^2 - (E_{CC})^2} \]
\[ = \sqrt{(4.60^2 - 1.397)^2 - (2.066)^2} = 2.451 \text{ cm.} \]

\[ V_{HL} = \frac{(D_1 - C)}{4} (D_C)^2 \]
\[ = \frac{(5.632 - 2.451)}{4} (7.010)^2 = 122.784 \text{ cm}^3. \]

\[ V_H = V_{HL} + V_{HD} = \frac{n}{4} (D_C)^2 \left\{ \sqrt{(L_C)^2 - \left( E_{CC} - (R_C \cos(\Phi_H)) \right)^2} \right. \]
\[ + \left. R_C \sin(\Phi_H) - C \right\} + V_{HD} \]
\[ V_H = \frac{n}{4} (7.010)^2 \left\{ \sqrt{(4.602)^2 - \left( 2.066 - 1.367 \cos(\Phi_H) \right)^2} \right. \]
\[ + \left. 1.397 \sin(\Phi_H) - 2.451 \right\} + 81.811 \]

\[ V_H = 38.595 \left[ \sqrt{21.178 - \left( 2.066 - 1.397 \cos(\Phi_H) \right)^2} + 1.397 \sin(\Phi_H) \right. \]
\[ - \left. 2.451 \right] + 81.811 \]

\[ V_H \text{ is calculated for } \Phi_H = 0 \rightarrow 360 \text{ (See next page)} \]

\[ V_T = V_H + V_C + V_{RD} = V_H + V_C + 51.854 \]

\[ V_T \text{ is calculated for } \Phi_H = 0 \rightarrow 360 \text{ (See next page)} \]

\[ P = \frac{1}{\frac{V_H}{V_I} + \frac{V_{RD}}{V_{IR}} + \frac{V_C}{V_{IC}}} = \frac{V_H}{978} + \frac{1}{570} + \frac{V_{IC}}{300} \]

\[ P \text{ is calculated for } \Phi_H = 0 \text{ to 360} \text{ (See next page)} \]
### COMPUTATION OF BETA PHILIPS BASIC POWER

<table>
<thead>
<tr>
<th>PHI</th>
<th>VH</th>
<th>VC</th>
<th>VT</th>
<th>P</th>
<th>PC</th>
<th>DELW</th>
<th>TH</th>
<th>FC</th>
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PM = 2.436

\[
M(R) = \frac{P_{AVG}}{PM} = \frac{2.068}{2.436} = 0.8490 \text{ J/K}
\]

More accurate calculation by programmable calculator gives 94.218
Note: Do Not Add in P at PHI = 0

\[ PM = \sum_{\text{PHI} = 30}^{\text{PHI} = 360} \frac{P}{12} \]

\[ PM = 2.436 \]

(see previous page)

\[ M(R) = \frac{P_{\text{AVG}}}{PM} = \frac{2.068}{2.436} = 0.8490 \text{ J/K} \]

\[ PC = \frac{P_{\text{AVG}}}{PM} = P \times 0.8490 \]

PC is calculated for PHI = 0, 30, ..., 360 on previous page.

PC is integrated verses VT using the trapezoidal rule, i.e.

\[ \text{DELW} = \frac{P_{C1} + P_{C2}}{2} (V_{T2} - V_{T1}) \]

For the first increment the subscript 1 stands for PHI = 0° and the subscript 2 stands for PHI = 30°. For the second increment the subscript 1 stands for PHI = 30° and the subscript 2 stands for PHI = 60°. DELW is summed for all 12 increments.

\[ BP = \text{NU}(1.045) \sum_{i=1}^{12} \text{DELW} \]

\[ = 25 \times (1.045) \times 94.218 = 2461 \text{ watts.} \]

The mass flow rates are now computed:

\[ \text{FH} = \frac{P_{(VH)}}{M(R)(\text{Tin})} = \frac{P_{(VH)}}{(0.8490)(978)} \]

\[ M(R) = 0.8490 \text{ J/K} \]

\[ M = \frac{M(R)}{8.314} = \frac{0.8490}{8.314} \]

\[ M = 0.1021 \text{ g mol} \]

FH is calculated for PHI = 30 to 360 (results on previous page) and is graphed on Figure 7-1.

\[ \text{FHMAX} = 0.560 \quad \text{FHMIN} = 0.155 \]
Figure 7-1. Mass Flow Approximation.
\[ FC11 = \frac{0.325}{2} \quad FC2 = \frac{0.325}{2} \]

\[ FC_{TH} = \frac{FC11 + FC2}{2} = \frac{0.325 + 0.325}{2} = 0.325 \]

\[ WHS = \frac{(F_{MAX} - F_{MIN})M(MW)}{FC_{TH}/NU} \]

\[ = \frac{(0.560 - 0.155)(0.1021)(2.02)}{(0.325/25)} = 6.425 \text{ g/sec} \]

\[ FC = \frac{P(VC)}{M(R)(IC)} = \frac{0.8490}{300} \]

FC is calculated for PHI = 30 to 360° (see previous table) and is graphed on Figure 7-1.

\[ FC_{MAX} = 0.695 \quad FC_{MIN} = 0.175 \]

\[ FC3 = 0.303 \quad FC4 = 0.325 \]

\[ FC_{TC} = \frac{FC3 + FC4}{2} = \frac{0.303 + 0.325}{2} = 0.314 \]

\[ WC_{S} = \frac{(FC_{MAX} - FC_{MIN})M(MW)}{FC_{TC}/NU} \]

\[ = \frac{(0.695 - 0.175)(0.1021)(2.02)}{(0.314/25)} = 8.538 \text{ g/sec} \]

For regenerator:

\[ FC = \frac{FC_{TH} + FC_{TC}}{2} = \frac{0.325 + 0.314}{2} = 0.320 \]

\[ WRS = \frac{WHS + WC_{S}}{2} = \frac{6.425 + 8.538}{2} = 7.482 \text{ g/sec} \]
8. Fluid Friction loss

8.1 Regenerator Windage

8.1.1 Screens -- VHL = \frac{122.784}{\text{cm}^3} \text{ (from 5.1 or 7.4)}

TR = 570 \text{ K (from 6)}

PAVG = 2.068 \text{ MPa (from 4)}

for hydrogen:

\[ \mu = 88.73 \times 10^{-6} + 0.2 \times 10^{-6} (\text{TR} - 293) + 0.118 \times 10^{-6} (\text{PAVG}) \]

\[ = \frac{570}{2.068} \]

\[ = 144.4 \times 10^{-6} \text{ g/cm sec} \]

\[ \rho = \frac{\text{MW}}{22.414 \times 0.1013 \times 273} \text{TR} = 0.1202 \frac{\text{MW(PAVG)}}{\text{TR}} \]

\[ = 0.1202(2.02)(2.068) = 8.81 \times 10^{-4} \text{ g/cm}^3 \]

For numerical Schmidt analysis (from 7.4)

\[ WRS = 7.482 \text{ g/sec} \]

\[ AC = \frac{VRD}{LR} = \frac{51.854}{2.261} = 22.93 \text{ cm}^2 \]

\[ G = \frac{WRS}{AC} = \frac{7.482}{22.93} = 0.3262 \text{ g/sec cm}^2 \]

\[ \text{AHT} = \frac{(\pi)^2}{2} (\text{MSH})(\text{THW})(\text{DR})^2(\text{NR})(\text{NS}) \]

\[ = \frac{(\pi)^2}{2} (83.9)(.0041)(2.261)^2(8)(308) \]

\[ = 21382 \text{ cm}^2 \]
\[ \text{RH} = \frac{AC(LR)}{(AI/IT)} = \frac{(22.93)(2.261)}{21382} = 2.425 \times 10^{-3} \text{ cm} \]

\[ \text{RE} = \frac{4(RH)G}{\mu} = \frac{4(2.425 \times 10^{-3})(0.3262)}{(1.44 \times 10^{-4})} = 21.97 \]

If \( \text{RE} < 60 \):
\[ \log F = 1.73 - 0.93 \log (\text{RE}) \]
\[ 3.035 \quad 21.97 \]

If \( 60 < \text{RE} < 1000 \):
\[ \log F = 0.714 - 0.365 \log (\text{RE}) \]

If \( \text{RE} > 1000 \):
\[ \log F = 0.015 - 0.125 \log (\text{RE}) \]

\[ \text{DELP} = \frac{F(G)^2(LR)}{2(10^4)(\text{RH})(\text{RHOM})} \]
\[ \text{DELP} = \frac{(3.035)(0.3262)^2(2.261)}{(2 \times 10^4)(2.42 \times 10^{-3})(8.81 \times 10^{-4})} = 0.0172 \text{ MPa} \]

For numerical Schmidt analysis
\[ \text{WPR} = \frac{\text{DELP}(\text{WRS})(2)(\text{FCT})}{(\text{RHOM})} \]
\[ = \frac{(0.0172)(7.482)(2)(0.320)}{(8.81 \times 10^{-4})} = 93 \text{ watts} \]
8.2 Gas Heater Windage

8.2.1 Tubular Heater

For Hydrogen:

\[ \mu = 10^{-6} \left\{ 88.73 + 0.2(\text{THM} - 293) + 0.118(\text{PAVG}) \right\} \]

\[ = \frac{978}{2.068} \]

\[ = 2.26 \times 10^{-4} \text{ g/cm}\cdot\text{sec} \]

MW = 2.02 H₂ / 4.00 air

RHOM = 0.1202 \( \frac{\text{MN(PAVG)}}{\text{(THM)}} \)

\[ = 0.1202 \left( \frac{2.02}{2.068} \right) = \frac{978}{2.068} = 5.13 \times 10^{-4} \text{ g/cm}^3 \]

For numerical Schmidt analysis (from 7.4)

\[ \text{WHS} = 6.425 \text{ g/sec} \]

\[ \text{AC} = (\text{NTH})(\text{DIH})^2 \frac{\pi}{4} = (40)(0.302)^2 \frac{\pi}{4} = 2.87 \text{ cm}^2 \]

\[ G = \frac{\text{WHS}}{\text{AC}} = \frac{6.425}{2.87} = 2.239 \text{ g/cm}^2\cdot\text{sec} \]

\[ \text{RE} = \frac{(\text{DIH})G}{\mu} = \frac{(0.302)(2.239)}{2.26 \times 10^{-4}} = 2991 \]

If \( \text{RE} < 2000 \), \( F = 16/\text{RE} = 16/2991 \)

\( \checkmark \) If \( \text{RE} > 2000 \), \( \log F = -1.34 - 0.20 \log (\text{RE}) \)

\[ = 9.22 \times 10^{-3} \]

For numerical Schmidt analysis

\[ \text{DELP} = \frac{2(F)(G)^2(LH)}{10^7(\text{DIH})(\text{RHOM})} \]

\[ = \frac{2(0.22 \times 10^{-3})(2.239)^2(24.23)}{10^7(0.302)(5.13 \times 10^{-4})} = 0.00145 \text{ MPa} \]
WPH = \frac{DELP(WHS)2(FCTH)}{RHOM} = \left(0.00145\right)\left(6.425\right)^2\left(0.325\right) = \left(5.13 \times 10^{-4}\right) 11.77 \text{ watts}

8.3 Gas Cooler Windage

8.3.1 Tubular Cooler

for Hydrogen:

\[MU = 10^{-6} \left\{ 88.73 + 0.2(TCM - 293) + 0.118(PAVG) \right\} \]

\[\frac{295}{2.068} \]

\[= 8.94 \times 10^{-5} \text{ g/cm sec} \]

\[\text{MW} = \begin{array}{c} \text{H}_2 \text{ He} \text{ air} \\ 2.02 \ 4.00, 29 \end{array} \]

\[\text{RHOM} = 0.1202(\text{MW})(PAVG)/TCM \]

\[= 0.1202(2.02)(2.068)/(295) = 1.70 \times 10^{-3} \text{ g/cm}^3 \]

For numerical Schmidt analysis (from 7.4)

\[\text{WCS} = 8.538 \text{ g/sec} \]

\[\text{AC} = (NTC)(DIC)^2 \frac{\pi}{4} = \left(312\right)(0.102)^2 \frac{\pi}{4} = 2.55 \text{ cm}^2 \]

\[G = \frac{\text{WCS}}{\text{AC}} = \frac{8.538}{2.55} = 3.348 \text{ g/sec cm}^2 \]

\[\text{RE} = \frac{(DIC)G}{MU} = \frac{(0.102)(3.348)}{8.94 \times 10^{-5}} = 3820 \]

If \(\text{RE} < 2000\), \(F = 16/\text{RE} = 16/\) \(\)

\(\checkmark\) If \(\text{RE} > 2000\), \(\log F = -1.34 - 0.20 \log(\text{RE})\)

\[8.78 \times 10^{-3} \quad 3820\]
For numerical Schmidt analysis:

\[
\text{DELP} = \frac{2(F)(G)^2(\text{LC})}{10^7(\text{DIC})(\text{RHOM})} = \frac{2(6.78 \times 10^{-3})(3.348)^2(4.47)}{10^7(0.102)(1.70 \times 10^{-3})} = 5.075 \times 10^{-4} \text{ MPa}
\]

\[
\text{WPC} = \frac{\text{DELP}(\text{WCS})^2(\text{FCTC})}{\text{RHOM}} = \frac{(5.075 \times 10^{-4})(8.538)^2(0.314)}{(1.70 \times 10^{-3})} = 1.60 \text{ watts}
\]

8.4 Fluid Friction Loss Summary

Gas Heater, WPH = 12 watts
Regenerator, WPR = 93 watts
Gas Cooler, WPC = 2 watts
Total, WP = 107 watts

9. Mechanical Friction Loss

MFL = based upon experimental measurement

or

MFL = 0.2 BP = 0.2(2461) = 492 watts

10. Basic Heat Input

\[
\text{BHI} = \frac{\text{BP}}{1 - \frac{\text{TC}}{\text{TH}}} = \frac{2461}{1 - \frac{300}{978}} = 3550 \text{ watts}
\]

11. Reheat Loss

11.1 Constant Volume Assumption (Equation 4-97)

FCT = 0.32 (usually 1/3) (from 7.4)

WRS = 7.482 g/sec (from 8.1)
CV = \(10.42\) for TR = \(570\) K (see Table 4-8)

CP = \(14.55\) for TR = \(570\) K (see Table 4-8)

AHT = \(21382\) cm² (from 8.1)

G = \(0.3262\) g/cm² sec (from 8.1)

PR = \(0.753\) for TR = \(570\) °K (see Table 4-9)

RE = \(21.97\) (from 8.1)

\[
\log \left( \frac{H}{G \cdot CP} \left( \frac{PR}{3} \right)^{\frac{2}{3}} \right) = -0.13 - 0.412 \log (RE)
\]

\[
\log \left( \left( \frac{H}{0.3262 \cdot 14.55} \right)^{\frac{2}{3}} \left( \frac{0.753}{2} \right) \right) = -0.13 - 0.412 \log (21.97)
\]

\(H = \frac{1.190}{w/cm² \cdot K}\)

\[
NTUV = \frac{H \cdot AHT}{(WRS) \cdot (CV)} = \frac{(1.190) \cdot (21382)}{(7.482) \cdot (10.42)} = 326.4
\]

\[
QRH = \frac{(FCT) \cdot (WRS) \cdot (CV) \cdot (THM - TCM)}{NTUV + 2} \times \left( \frac{2}{NTUV + 2} \right)
\]

\[
= \frac{(0.32) \cdot (7.482) \cdot (10.42) \cdot (978 - 295)}{326.4 + 2}
\]

\(= 103.76\) watts

12. Shuttle Conduction

12.1 High Pressure Engine (Equation 4-110)

\(KG = \frac{28.06 \times 10^{-4}}{w/cm \cdot K} \) at TR = \(570\) K (From Table 4-9)

\(LT1 = \frac{0.152}{cm} \) @ NU = \(25\) HZ (From Table 4-11)

\(LT2 = \frac{0.152}{cm} \) (LT α \(\sqrt{NU}\))

\(K1 = \frac{0.19}{w/cm \cdot K} \) From Figure 4-23

\(K2 = \frac{0.19}{w/cm \cdot K} \)
\[ \text{LB} = 1 + \frac{\text{KG}}{2} \frac{(\text{LT1} + \text{LT2})}{\text{GR}} \]

\[ \text{LB} = 1 + \frac{28.06 \times 10^{-4}}{2} \frac{(0.152 + 0.152)}{0.19} \]

\[ \text{LB} = \frac{0.129}{0.129} \]

\[ \text{QSH} = \left( \frac{1 + \text{LB}}{1 + (\text{LB})^2} \right)^\frac{\pi}{8} \frac{(\text{SD})^2 \text{KG}(\text{THM} - \text{TCM})(\text{DCY})}{(\text{GR})(\text{LD})} \]

\[ \text{QSH} = \left( \frac{1 + 1.029}{1 + (1.029)^2} \right)^\frac{\pi}{8} \frac{(3.068)^2 (28.06 \times 10^{-4})(978 - 295)(7.010)}{(0.025)(4.359)} \]

\[ \text{QSH} \approx 449 \text{ watts} \]

13. Static Heat Conduction

13.1 Gas Conduction Inside Displacer or Hot Cap

\[ \text{DID} = \text{DCY} - 2(\text{GR}) - 2(\text{WT1}) = 6.604 \text{ cm} \]

(Direct from engine dimensions)

\[ \text{AHT} = \frac{\pi}{4} (\text{DID})^2 = \frac{\pi}{4} (6.604)^2 = 34.25 \text{ cm}^2 \]

\[ \text{KG} = 28.06 \times 10^{-4} \text{ w/cm} \cdot \text{C from 12} \]

\[ \text{QC} = \frac{\text{KG}(\text{AHT})(\text{THM} - \text{TCM})}{(\text{LD})} = \frac{(28.06 \times 10^{-4})(34.25)(978 - 295)}{(4.359)} \]

\[ \text{QC} = 15 \text{ watts.} \]

13.2 Radiation Inside Displacer or Hot Cap

\[ \frac{\text{DID}}{\text{LD}} = 6.604 \quad 4.359 = 1.515 \]

If:

\[ 0 < \frac{\text{DID}}{\text{LD}} < 0.2 \text{ then } \text{FA} = \frac{\text{DID}}{\text{LD}} \]
If: 
\[ 0.2 < \frac{\text{DID}}{\text{LD}} < 7 \] then \( FA = 0.50 + 0.20 \ln \left( \frac{\text{DID}}{\text{LD}} \right) \) \( FA = 0.503 \)

If:
\[ \frac{\text{DID}}{\text{LD}} > 7 \] then \( FA = 1 \)

\( FE = (EH)(EC) = (0.6)(0.5) = 0.3 \)

\( FN = \frac{1}{1 + \text{NRS}} = \frac{1}{1 + 2} = \frac{1}{3} \)

\( \text{AHT} = 34.25 \text{ cm}^2 \) (from 13.1)

\[ \text{QR} = (FA)(FE)(FN)(\text{AHT})(5.67 \times 10^{-12})(\text{THM})^4 - (\text{TCM})^4 \]
\[ = (0.503)(0.3)(1/3)(34.25)5.67 \times 10^{-12} \left( 978 \right)^4 - 295^4 \]
\[ = 10 \text{ watts} \]

13.3 Displacer Cylinder Wall

\( \text{KM} = 0.19 \text{ w/cm K at TR} = \frac{570}{6.96} \text{ K (Figure 4-22)} \)

\[ \text{AHT} = \frac{\pi}{4} \left( (\text{DCY} - 2\text{GR})^2 - (\text{DID})^2 \right) = \frac{\pi}{4} \left( \left( \frac{7.01}{2} \right) - (6.604)^2 \right) \]
\[ = 3.788 \text{ cm}^2 \]

\[ \text{QC} = \frac{\text{KM(THM - TCM)}}{\text{LD}} = (0.19)(3.788)(978 - 295) \]
\[ (4.359) \]
\[ = 113 \text{ watts} \]

13.4 Displacer Gap

\( \text{KG} = 28.06 \times 10^{-4} \text{ w/cm} \cdot \text{C from 13.1} \)

\[ \text{AHT} = \pi(\text{DCY})(\text{GR}) = \pi(7.01)(0.025) = 0.551 \text{ cm}^2 \]
\[ QC = \frac{KG(AHT)(THM - TCM)}{LU} = \left(3.8 \times 10^{-4}\right)\left(0.861\right)\left(0.978 - 295\right) \]
\[ = \frac{0.24}{4.369} \text{ watts} \]

13.5 Cylinder Wall

Using the numbers in 2.5,

Let:

\[ 1 = \left(\frac{(LHB)4}{AHTB + AHTB}\right) = \frac{4(2.858)}{(15.915) + (10.726)} = 0.429 \]

Then:

\[ R1 = \frac{1}{KMB + KMA} = \frac{0.429}{(0.25) + (0.225)} = 0.903 \text{ K/watt} \]

Let:

\[ 2 = \frac{4(LBA)}{AHTB + AHTA} = \frac{4(1.016)}{(10.726) + (9.469)} = 0.201 \]

Then:

\[ R2 = \frac{2}{KMB + KMA} = \frac{(0.201)}{(0.225) + (0.16)} = 0.523 \text{ K/watt} \]

Let:

\[ 3 = \frac{4(LAC)}{AHTA + AHTC} = \frac{4(1.245)}{(9.469) + (15.915)} = 0.196 \]

Then:

\[ R3 = \frac{3}{KMA + KMC} = \frac{(0.196)}{(0.16) + (0.15)} = 0.633 \text{ K/watt} \]

\[ QC = \frac{THM - TCM}{R1 + R2 + R3} = \frac{(0.978 - 295)}{(0.903 + 0.523 + 0.633)} \]
\[ = \frac{332}{\text{ watts}} \]

\[ TB = THM - R1(QC) = (0.978) - (0.903)(332) \]
\[ = 678 \text{ K} \text{ --- Original TB estimate was 800 K} \]
TA = TB - R2(QC) = (678) - (0.515)(334) = 504 K ---- Original TA estimate was 350 K

Now:

KMB = 0.21 w/cm K at TB
KMA = 0.18 w/cm K at TA

Figure 4-23

Now go around again.

R1 = \frac{1}{KMB + KMB} = \frac{(0.429)}{(0.25) + (0.21)} = 0.933 K/watt

R2 = \frac{2}{KMB + KMA} = \frac{(0.201)}{(0.21) + (0.18)} = 0.515 K/watt

R3 = \frac{3}{KMA + KMC} = \frac{(0.196)}{(0.18) + (0.15)} = 0.594 K/watt

QC = \frac{THM - TCM}{R1 + R2 + R3} = \frac{(978) - (95)}{(0.933) + (0.515) + (0.594)} = 334 watts

TB = THM - R1(QC) = (978) - (0.933)(334) = 666 K ---- Previous estimate was 678 K

= TB - R2(QC) = (666) - (0.515)(334) = 494 K ---- Previous estimate was 504 K

Does the difference significantly change the thermal conductivities?

☐ Yes ------ Then go around again

☑️ No ------ Accept last QC as accurate QC = 334

13.6 Regenerator Walls

One regenerator wall will be calculated and then multiplied

NR. Using the numbers from 2.4
Let \( \frac{4(LHB)}{AHTH + AHTB} = \frac{4(1.016)}{1.425 + (1.425)} = 1.426 \)

Then \( R1 = \frac{1}{KMH + KMB} = \frac{(1.426)}{(.25) + (.225)} = 3.002 \) K/watt

Let \( \frac{4(LBA)}{AHTB + AHTA} = \frac{4(1.194)}{1.425 + (0.853)} = 2.097 \)

Then \( R2 = \frac{2}{KMB + KMA} = \frac{(2.097)}{(0.225) + (.16)} = 5.446 \) K/watt

Let \( \frac{4(LAC)}{AHTA + AHTC} = \frac{4(0.051)}{0.853 + (1.425)} = 0.090 \)

Then \( R3 = \frac{3}{KMA + KMC} = \frac{(0.090)}{(0.16) + (.15)} = 0.289 \) K/watt

\[
Q = \frac{THM - TCM}{R1 + R2 + R3} = \frac{(0.978) - (0.295)}{(3.002) + (5.446) + (0.289)} = 78 \text{ watts.}
\]

\[
TB = THM - R1(Q) = (0.978) - (3.002) = 743 \text{ K ---- Original estimate was 800 K}
\]

\[
TA = TB - R2(Q) = (743) - (5.446) = 317 \text{ K ---- Original estimate was 350 K}
\]

Now:

\[
KMB = \frac{0.21}{w/cm \cdot K \text{ at } TB}
\]

\[
KMA = \frac{0.155}{w/cm \cdot K \text{ at } TA}
\]

Now go around again:

\[
R1 = \frac{1}{KMH + KMB} = \frac{(1.426)}{(.25) + (.21)} = 3.100 \text{ K/watt}
\]

\[
R2 = \frac{2}{KMB + KMA} = \frac{(2.097)}{(0.21) + (.155)} = 5.744 \text{ K/watt}
\]

\[
R3 = \frac{3}{KMA + KMC} = \frac{(0.090)}{(0.155) + (.150)} = 0.294 \text{ K/watt}
\]

\[
Q = \frac{THM - TCM}{R1 + R2 + R3} = \frac{(0.978) - (0.295)}{(3.100) + (5.744) + (0.294)} = 74.75 \text{ watts.}
\]
TB = THM - R1(Q) = (9/8) - (3.100)(74.75) = \frac{746}{746} K --- Previous estimate was \frac{743}{743} K

TA = TB - R2(Q) = \frac{746}{746} - (5.744)(74.75) = \frac{317}{317} K --- Previous estimate was \frac{317}{317} K

Does the difference significantly change the thermal conductivities?

Yes [ ] then go around again

No [X] then go on

QC = Q(NR) = (74.75)(8) = \frac{598}{598} \text{ watts}

13.7 Regenerator Matrix (From 2.2.1)

13.7.1 Screen Stack (Equation 4-120)

\[ KM = \frac{0.19}{w/cm \cdot K \text{ at TR} = \frac{570}{570} K \text{ (Fig. 4-22)}} \]

\[ KG = \frac{28.06 \times 10^{-4}}{w/cm \cdot K \text{ at TR} = \frac{570}{570} K \text{ (Table 4-9)}} \]

Let \( \Omega = \left( \frac{1 + \frac{KM}{KG}}{1 - \frac{KM}{KG}} \right) = \left( \frac{1 + \frac{0.19}{28.06 \times 10^{-4}}}{1 - \frac{0.19}{28.06 \times 10^{-4}}} \right) = \frac{-1.03}{1} \]

\[ KMX = KG \left( \frac{\Omega - FF}{\Omega + FF} \right) = (28.06 \times 10^{-4}) \left( \frac{-1.03}{-1.03} - \frac{0.296}{0.296} \right) \]

\[ = \frac{4.96 \times 10^{-3}}{w/cm \cdot K} \]

\[ AHT = \frac{n}{4} \cdot (DR)^2 = \frac{n}{4} \cdot (2.261)^2 = \frac{4.02}{4.02} \text{ cm}^2 \]

\[ QC = \frac{(NR)KMX(AHT)(THM - TCM)}{LR} = (8)(4.96 \times 10^{-3})(4.02)(\frac{978}{978} - \frac{705}{705}) \]

\[ = \frac{48}{48} \text{ watts} \]
### 13.8 Summary of Static Heat Conduction

Section

<table>
<thead>
<tr>
<th>Description</th>
<th>QC Value (watts)</th>
</tr>
</thead>
<tbody>
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<td>13.1 Gas Cond. Inside Displ.</td>
<td>15</td>
</tr>
<tr>
<td>13.2 Radiation Inside Displ.</td>
<td>10</td>
</tr>
<tr>
<td>13.3 Displ. Wall</td>
<td>113</td>
</tr>
<tr>
<td>13.4 Displacer Gap</td>
<td>0</td>
</tr>
<tr>
<td>13.5 Cylinder Wall</td>
<td>334</td>
</tr>
<tr>
<td>13.6 Regenerator Wall</td>
<td>598</td>
</tr>
<tr>
<td>13.7 Regenerator Matrix</td>
<td>48</td>
</tr>
</tbody>
</table>

**Total QS = 1118 watts**

### 14. Pumping Loss (Equation 4-126)

- \( P_{\text{MAX}} = 2.880 \) MPa from 6
- \( P_{\text{MIN}} = 1.337 \) MPa from 6

\[
RM = \frac{R}{MW} = \frac{9.314}{2.02} = 4.616 \text{ J/kg} \cdot \text{K}
\]

\[
MW = \begin{cases} 2.02 & \text{for H}_2 \\ 4.00 & \text{for He} \\ 29 & \text{for air} \end{cases}
\]

\[
Z_1 = 1 \quad \text{(normally 1 except when gas temp. < 70 K)}
\]

\[
QPU = \left( \frac{\pi DCY}{KG} \right)^{0.6} \cdot \frac{2(LD)(THM - TCM)}{1.5(21)} \cdot \left( \frac{(P_{\text{MAX}} - P_{\text{MIN}})NU(CP)}{(THM - TCM)RM} \right)^{1.6} \cdot GR^{2.6}
\]

\[
QPU = \left( \frac{\pi (7.01)}{(28 \times 10^{-4})} \right)^{0.6} \cdot \frac{2(4.359)(978 - 295)}{1.5(1)} \cdot \left( \frac{(2.88 - 1.337)(25)(14.55)}{(978 - 295)(4.116)} \right)^{1.6} \cdot (0.025)^{2.6}
\]

\[
= 13.6 \text{ watts}
\]
15. Temperature Swing Loss (Equation 4-128)

\[
FCT = 0.32 \text{ see 11}
\]
\[
ROM = 7.5 \text{ g/cm}^3\]
\[
CPM = 1.05 \text{ j/g} \cdot \text{K}
\]

For Screen Regenerators:

\[
MMX = NR \pi \left(\frac{D}{4}\right)^2 (LR)(FF)(ROM)
\]
\[
= (8 \pi \left(\frac{2.261}{4}\right)^2 (2.261)(0.286)(7.5)
\]
\[
= 155.78 \text{ g}
\]

\[
DELTMX = \frac{\text{WRS} (CV) (FCT) (THM - TCM)}{\text{NU} (MMX) (CPM)}
\]
\[
= \frac{(7.482)(10.42)(0.32)(978 - 295)}{(25)(155.78)(1.05)}
\]
\[
= 4.167 \text{ K}
\]

\[
QTS = (FCT) \text{WRS} (CV) (DELTMX) / 2
\]
\[
= (0.32)(7.482)(10.42)(4.167) / 2
\]
\[
= 51.98 \text{ watts}
\]

16. Internal Temperature Swing Loss

\[
KM = 0.19 \text{ w/cm} \cdot \text{K from Figure 4-22}
\]

For Screens:

\[
LMX = THW / 2 = (0.0041) / 2 = 0.00205 \text{ cm}
\]
\[
C3 = 0.25
\]

\[
QITS = \frac{QTS (C3)(ROM)(CPM)(LMX)^2 (NU)}{(KM)(FCT)}
\]
\[
= \frac{(51.98)(0.25)(7.5)(1.05)(0.00205)^2(25)}{(0.19)(0.32)}
\]
\[
= 0.177 \text{ watts}
\]
17. Performance Summary

<table>
<thead>
<tr>
<th>Net Power, watts</th>
<th>1st Iteration</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP = basic power =</td>
<td>2461</td>
<td>2287</td>
<td></td>
</tr>
<tr>
<td>(from 7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WP = windage power =</td>
<td>107</td>
<td>107</td>
<td></td>
</tr>
<tr>
<td>(from 8.4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MFL = mechanical friction loss =</td>
<td>492</td>
<td>457</td>
<td></td>
</tr>
<tr>
<td>(from 9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NP = BP - WP - MFL =</td>
<td>1862</td>
<td>1723</td>
<td></td>
</tr>
</tbody>
</table>

= net power

Net Heat Input (watts)

(Equation 4-132)

<table>
<thead>
<tr>
<th></th>
<th>1st It.</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>BHI = basic heat input =</td>
<td>3550</td>
<td>3415</td>
<td></td>
</tr>
<tr>
<td>(see 10)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+QRH = reheat loss =</td>
<td>104</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(see 11)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+QSH = shuttle heat cond. =</td>
<td>449</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(see 12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+QS = static heat cond. =</td>
<td>1118</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(see 13.8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+QPU = pumping loss =</td>
<td>14</td>
<td></td>
<td>1687</td>
</tr>
<tr>
<td>(see 14)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+QTS = temp. swing loss =</td>
<td>52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(see 15)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+QITS = internal temp. swing loss =</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(see 16)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-WPH = heater windage power =</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(see 8.4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[\frac{WPH}{2}] = half of regenerator windage power =</td>
<td>-47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(see 8.4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QN = net heat input =</td>
<td>5237</td>
<td>5102</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
18. Heat Exchanger Duty

<table>
<thead>
<tr>
<th></th>
<th>1st It.</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas Heater</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QGH = QN</td>
<td>5237</td>
<td>5102</td>
<td></td>
</tr>
<tr>
<td>Gas Cooler</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QGC = QN - NP</td>
<td>3375</td>
<td>3379</td>
<td></td>
</tr>
</tbody>
</table>

\[ QGC = (5237) - (1862) \]

19. Gas Heater

Not needed because in GPU-3 the effective gas temperature is assumed to be measured with a thermocouple. \( TH = 978 \) K.

20. Gas Cooler —

20.1 Correction of effective cold metal temperature due to temperature rise in cooling water.

\[ \Delta T = \frac{QGC}{FCW(4.1868)} = \frac{(3375)}{(379)(4.1868)} \]

\[ = 2.13 \text{ K} \]

\[ TCM = TCWI + \frac{\Delta T}{2} \]

\[ = 294.4 + \frac{(2.13)}{2} = 295.5 \text{ K} \]

20.2 Tubular Type

\[ RE = \frac{3820}{3.48} \text{ from 8.3.1} \]

\[ \frac{LCHT}{DID} = \frac{3.48}{0.102} = 34.1 \]

From Figure 4-19:

\[ \frac{H(PR)}{CP(G)} = 0.0031 \]

\[ G = 3.348 \text{ g/sec cm}^2 \text{ from 8.3.1} \]

\[ WCS = 8.538 \text{ g/sec -- from 8.3.1} \]
CV = \frac{10.18}{\text{j/g\cdot K}} \quad \text{From Table 4-8} \quad \text{for TCM = 295 K}

CP = \frac{14.31}{\text{j/g\cdot K}} \quad \text{from Table 4-9}

PR = 0.72

\begin{align*}
H &= \frac{\Omega(CP)(G)}{3} = \frac{(0.0031)(14.31)(3.348)}{(0.72)^3} = 0.1848 \text{ w/cm}^2 \cdot \text{K} \\
AHT &= (NTC)(\pi)(DIC)(LCHT) = (312)(\pi)(0.102)(3.48) = 347.92 \text{ cm}^2 \\
NTUC &= \frac{H(AHT)}{2(FCT)(WC)(CV)} = \frac{(0.1848)(347.92)}{2(0.314)(8.538)(10.18)} = 1.178
\end{align*}

First Iteration:

\begin{align*}
TC &= TCM + \frac{QGC}{2(FCT)(WC)(CV)(\exp(NTUC) - 1)} \\
TC &= (295.5) + \frac{3375}{2(0.314)(8.538)(10.18)(\exp(1.178) - 1)} \\
&= 323.00 \text{ K}
\end{align*}
An additional iteration was now made by programmable calculator.

\[ \text{TH} = 978.00 \quad \text{TC} = 323.00 \quad \text{M(R)} = 0.8258 \]

\begin{align*}
\text{PHI} & \quad \text{PC} & \quad \text{FH} & \quad \text{FC} \\
0 & 2.653 & -- & -- \\
30 & 2.394 & 0.561 & 0.175 \\
60 & 2.118 & 0.585 & 0.231 \\
90 & 1.827 & 0.453 & 0.345 \\
120 & 1.562 & 0.341 & 0.486 \\
150 & 1.388 & 0.239 & 0.607 \\
180 & 1.361 & 0.176 & 0.673 \\
210 & 1.530 & 0.161 & 0.670 \\
240 & 1.908 & 0.195 & 0.594 \\
270 & 2.418 & 0.276 & 0.457 \\
300 & 2.807 & 0.384 & 0.305 \\
330 & 2.851 & 0.477 & 0.208 \\
360 & 2.653 & 0.537 & 0.170 \\
\end{align*}

\[ \sum_{i=1}^{12} \text{DELW} = 87.538 \]

\[ \text{BP} = 2287 \quad \text{BHI} = 3415 \]

FH and FC are plotted in Figure 7-1. The second iteration is seen to offset the results and does not change the flow rates or times. BP and BHI are then used in 17 to calculate NP and QN as the second iteration. Then these are used to calculate QGH and QGC in 18. Finally, in 20 a new TC is calculated:

2nd Iteration

\[ \text{TC} = \text{TCM} + \frac{QGC}{1} = (295.5) + \frac{(3379)}{(122.73)} \]

\[ = 323.03 \text{ K} \quad \text{Third iteration not necessary.} \]

21. Conclusion

Final Net Power = 1723 watts
Final Indicated Power = 2180 watts
Final Net Heat Input = 5102 watts
Brake Efficiency = 33.8 %
Indicated Efficiency = 42.7 %
8. REFERENCES

The references given in this section (See Table 8-1) have been accumulated from previous bibliographies particularly Walker (Ref. 73) and United Stirling of Sweden, compiled by Karin Adler. They also have been obtained from the authors own files and publications and from the references listed in these papers and reports that relate directly to Stirling engines. For the recent material the following computer based literature files have been searched:

- Compendex 1970 -- (Engineering Index)
- ISMEC 1973 -- Information service in Mechanical Engineering (INSPEC)
- NTIS 1964 --
- NASA Literature Search No. 35884

The references have been organized by year of publication. Within each year each reference has been given a letter designation. The reference list has been indexed by personal author (See Table 8-2) and by corporate author if applicable (See Table 8-3).

Not every publication listed in the reference list has been obtained by the authors. In most cases theses were not sought because the main results are given in subsequent journal articles.

Patents are included if they were referenced in publications or were in the authors file. The author is indebted to Ted Finkelstein who graciously allowed his file of Stirling engine patents to be copied. No independent search was made of the patent literature since this search would need to be done by specialists at the patent office.

The author intends to maintain this file of Stirling engine references. He would appreciate receiving copies of publications that are not now included. The author has a copy of the paper on file if an asterisk (*) appears at the end of the reference.

Besides indexing this reference list by author and corporate author, this publication discusses the different aspects of Stirling engines and refers to this reference list by number and sometimes by chief author name. In the 1900's the 19 is omitted for brevity. Also each article has been classified by subject using the classification scheme given in Table 8-4. The kind of Stirling engine is classified by type of heat input, arrangement of parts and intended use. Design considerations and experimental results is also divided into a number of categories. Table 8-5 gives the paper numbers that relate to each classification from Table 8-4. These classifications have been determined by a perusal of the publication if it were available, otherwise the classification was determined from the title or possibly the abstract. This classification index has been found useful in preparing this publication. It is hoped that the readers will find it useful.
Table 8-1
Stirling Engine References
Organized by Year of Publication


1845 a Poingdestre, W. W., Proceedings ICE, 1845
1845 b Stirling, R., Proceedings ICE, 1845.


1869 b Eckerth, "Technische Blatter," 1, Jahrgang, Prague, 1869.


1875 b "Air Engines," Editorial, Engineering, Vol. 19, Part 1 - March 12, 1875, pp. 200-201; Part 2 - March 26, 1875, pp. 241-242; Part 3 - April 9, 1875, pp. 287-289; Part 4 - April 30, 1875, pp. 355-356; Part 5 - May 21, 1875, pp. 417-418; Part 6 - June 18, 1875, pp. 504-505. AO. 

1876 a Ericsson, J., Contributions to the Centennial Exhibition, 1876. 


1885 a Babcock, G. H., "Substitutes for Steam," Trans ASME, pp. 680-741, 1885. AO. 


1899 a Appleton Cyclopaedia of Applied Mechanics, New York, 1899. AO.

1899 b Knoke, J. O., "Die Kraftmaschine Des Kleingewerbes," Berlin, 1899. AO. *


1911 a Donkin, B., "A Text Book on Gas, Oil and Air Engines," London, 1911. AO.


1948 g Bohr, E., "Den Moderna Varmluftsmotorn," Teknisk Tidskrift, No. 18, pp. 595-599, 1948. AO.


<table>
<thead>
<tr>
<th>Year</th>
<th>Source</th>
<th>Details</th>
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         A4, B3, B4, B6, B8, D2. *


B1, B6, B8, C3, E3.


1971 aw Vacant

1971 ax Vacant


1972 ab "Ford Signs Licensing Pact to Develop Stirling Engine," AMM/MN, August 14, 1972. Cl. *

1972 ac "Ford Will Develop 'Hot Air' Engines With Dutch Partner," L. A. Times, August 9, 1972. Cl. *


1974 be sec 00000e


1975 ao Carrett, K., "Stirling Engine is a Hot Contender for Car Powerplants," 
Engineer, Vol. 240, No. 6208, March, 1975. A1, A2, A6, B3, B6, B8, 
C1, D8, D18. *

1975 ap Mortimer, J., "Low Cost, Long Life (Stirling) Engine May be Ideal for 
B5, B6, B8. *

1975 aq Wilson, S. S., "Possible Developments in Transportation," Aspects of 
Energy Convers., Proc. of a Summer School, Lincoln Coll., Oxford, 

No. 12, pp. 18-23, 57, Dec. 1975. C1. *

1975 as Grigorenko, N. M., Savchenko, V. I., and Prusman, Yu. O., "Results of 

1975 at Vacant

1975 au Andrus, S., Faeser, R. J., Moise, J., Hoffman, L. C., and Rudnicki, M. E., 
"Development and Evaluation of a Stirling Powered Cardiac Assist 
System," Aerojet Liquid Rocket Co., Annual Rept. No. N01-HV-3-2930, 

1975 av Biryukov, V. I., and Sergeev, P. V., "Synthesis of Three-Parameter 
Mechanisms of the Rhombic Drive of a Stirling Engine," Izv Vyssh 
Uchebn Zaved Mashinostr., No. 11, pp. 70-76, 1975. B6, B8


1975 ay Balas, C., Jr., "Design and Fabrication of a Rhombic Drive Stirling 
Cycle Cryogenic Refrigerator," Philips Labs, Final Rept. No. DAAK02-
72-C-0224, April, 1975. B3, B6, B8, C3, E3.

1975 az Carlqvist, S. G., Lia, T., and Lundholm, G. S. K., "Stirling Engines: 
Their Potential Use in Commercial Vehicles and Their Impact on Fuel 

1975 ba Smith, L., Sandquist, G., Olsen, D. B., Arnett, G., Gentry, S., and 
Kolff, W. J., "Power Requirements for the A.E.C., Artificial Heart," 


1976 at Vacant


0000 b "Applications of Cryogenic Equipment in Hydrocarbon Processing and Transport," Philips Corp. C3. *

0000 c Zacharias, F. A., "Advanced Development on Stirling Engines at MWM." 2nd NATO-CCMS Symposium, Dusseldorf, West Germany, Nov. 4, 1974.

0000 d "Cryogenic Equipment," Philips Corp. C3, C4. *
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<th>Air Product &amp; Chemicals, Inc.</th>
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<td>71 j, 71 be</td>
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<th>American Gas Association</th>
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<th>Arthur D. Little, Inc.</th>
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<td>59 j, 60 d, 61 h</td>
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<td>72-e, 73 ax</td>
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Zacharias, F. A., 00 c, 71 m, 71 w, 71 au, 73 y
Zanzig, J., 65 q
Zapf, Horst, 70 i, 70 l
Zarinchang, J., 75 d
Zeuner, G., 1887 a
Zimmerman, F. J., 71 be
Zimmerman, M. D., 71 c, 71 x
Zindler, G. F., 69 uj
Zykov, V. M., 74 am
ERG, Inc. 73 p, 77 a, 77 u
Fairchild Space & Electronics Co. 76 ag
Ford Motor Co. 73 h, 76 ah, 77 k, 77 l, 77 aq, 77 bk
General Electric 76 j, 77 w, 77 aj, 77 bn
General Motors Corporation 42 b, 60 a, 64 g, 65 t, 68 p, 68 v, 69 f, 69 k, 69 v, 69 ao, 69 ap
Goddard Space Flight Center 69 aa
HEW 69 d
Hughes Aircraft 68 x, 72 t, 75 a
IIT Research Institute 65 i, 65 j
Institute of Gas Technology 67 f, 75 e
Jet Propulsion Laboratory 75 t, 77 ac, 77 ae
Johns Hopkins University 77 ax
Joint Center for Graduate Study 75 ag, 76 c, 76 ay, 77 h, 77 x, 77 aa, 77 ao
Josam Manufacturing Co. 76 p
Kaiser Engineers 60 m
Kings College, London 61 a, 61 k, 62 e, 62 f, 63 f, 63 g, 76 h, 77 z, 77 ba
Laboratoriet for Energiteknik 74 ab, 76 w
Lafayette College 71 be
M.A.N.-MWN 71 m, 71 w, 71 au, 72 aq, 74 u
McDonnell Douglas Astronautics 68 c, 68 l, 68 u, 69 a, 69 x, 69 al, 70 v, 70 x, 71 i, 71 ao, 71 ba, 72 b, 72 d, 72 m, 72 ak, 72 an, 73 q, 73 w, 74 o, 74 n, 74 av, 75 r, 75 be, 76 r, 76 v, 76 as, 76 ay, 77 x
Mechanical Technology Inc. 72 v, 76 az, 77 b, 77 m, 77 s
Medtronic, Inc. 73 w
M.I.T. 68 r, 69 n, 69 o, 71 an, 73 ay
NASA-Lewis 61 i, 71 am, 77 p, 77 ab, 77 ao, 77 as, 77 at, 77 au, 77 av
National Bureau of Standards 77 ad
National Heart and Lung Institute
59 a, 70 x, 71 b, 71 i,
71 j, 71 ba, 72 d, 72 ak,
72 an, 73 an, 74 av, 75 be,
76 as

National Research Council
61 j

Ohio University
69 h, 71 g, 73 t

Penn State College
58 g, 69 b

Philips, Eindhoven
43 b, 46 c, 48 j, 48 k, 49 d,
49 e, 49 f, 49 g, 49 h, 49 i,
49 j, 50 b, 50 c, 50 d, 51 g,
51 h, 51 i, 51 j, 51 k, 51 l,
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52 p, 52 q, 52 r, 52 s, 52 d,
53 f, 53 g, 53 h, 53 i, 53 j,
54 d, 60 c, 60 e, 63 e, 64 i,
65 b, 65 g, 65 h, 65 x, 66 k,
68 d, 69 e, 69 r, 70 d, 70 u,
71 e, 71 f, 71 m, 72 a, 72 ah,
73 d, 73 h, 74 c, 74 d, 74 u,
75 f, 75 h, 75 m, 76 f, 77 bb,
00 a

Philips, North American
57 g, 57 k, 58 c, 58 h, 58 i,
59 d, 59 h, 59 l, 59 m, 65 v,
66 l, 67 c, 70 h, 71 l, 71 p,
71 v, 73 x, 74 b, 74 w, 75 b,
75 m, 76 e, 76 am, 77 f, 77 v,
77 y, 77 ax

Purdue University
68 r, 71 aj, 71 ak

RCA
72 af

Reactor Centrum Nederland
66 d

Rocketdyne
64 c, 65 c, 67 c, 67 d

Roesel Lab
74 s

Stanford University
50 a, 76 ak

Stone and Webster Engineering Corp.
71 ak

Sunpower
75 n, 75 s

Syracuse University
64 d, 65 d

TCA Stirling Engine Research
and Development Company
70 g, 72 u

Thermo Electron Corporation
71 b, 72 d, 74 ba, 76 bc

United Stirling
70 o, 71 m, 71 ah, 73 s, 74 z,
75 j, 77 i, 77 j, 77 al, 77 am,
77 bj

University of Calgary
68 n, 69 q, 70 g, 71 n,
72 j, 73 j, 73 m, 73 u,
73 v, 76 ax

University of Florida
65 o

University of London
67 f

University of Michigan
68 b
University of Tokyo
61 m

University of Wisconsin
60 j, 61 b

University of Witwatersrand
75 w, 76 i, 76 x, 76 y,
77 c, 77 d, 77 e, 77 g

Washington State University,
Medical College
77 x

Wayne State University
72 r

Westinghouse
73 ax, 76 am, 76 ao, 76 ap

Table 8-4
Classification of Stirling Engine References

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<td>A1</td>
<td>Liquid or Gaseous Fuel</td>
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<tr>
<td>A2</td>
<td>Nuclear Reactor</td>
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<tr>
<td>A3</td>
<td>Radioisotope</td>
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<tr>
<td>A4</td>
<td>Solar Heat</td>
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<tr>
<td>A5</td>
<td>Solid Fuel</td>
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<tr>
<td>A6</td>
<td>Stored Thermal Energy</td>
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<tr>
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<td>B3</td>
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B4 offset
B5 Dual Piston Engines
B6 Crank Operated Displacer
B7 Free Displacer (not crank)
B8 Crank Operated Power Piston
B9 Free Power Piston (not crank)

\underline{Intended or Actual Useful Output}

Mechanical Power
C1 Vehicle
C2 Other

Pumping
C3 Heat (refrigeration)
C4 Liquid
C5 Gas
C6 Other
C7 Electric Generation

\underline{Design Considerations}

D1 System Studies

Engine
Thermodynamic
D2 1st Order
D3 2nd Order
D4 3rd Order

Mechanical
D5 Efficiency
D6 Sealing
D7 Engine Starting
D8 Engine Control
D9 Power Takeoff
D10 Gas Transport
D11 Materials of Construction
D12 Other

Heat Transfer and Fluid Flow
D13 Air Preheater
D14 Working Gas Heater
D15 Regenerator
D16 Working Gas Cooler
D17 Heat Rejection
D18 Working Fluid Selection

Experimental Results

Full Systems
E1 Vehicles
E2 Other

Engines
Components

E3
E4 Seals
E5 Mechanical Power Train
E6 Control Mechanism
E7 Air Preheater
E8 Working Gas Heater
Table 8-5

Paper Numbers Related to Each
Stirling Engine Subject Classification

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<th>AO</th>
<th>General Stirling Engine News</th>
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A2 Nuclear Reactor

- 1975 ao-
- 1977 a, f

A3 Radioisotope

- 1964 j
- 1965 y
- 1966 d, f, n
- 1967 i
- 1968 h, j, k, l, s, x, y, ab
- 1969 i, aa, aj, ak
- 1970 r, t, v
- 1971 a, b, i, j, az, ab
- 1972 c, d, e, f, h, j, k, l, m, ak, an
- 1973 c, d, j, am, at, ax
- 1974 au, av, ax, ay, az, ba
- 1975 l, p, r, y, ab, ai, au, ax, ba, be
- 1976 j, r, s, v, ag, aj, am, aq, au, ay

A4 Solar Heat

- 1960 b
- 1961 e
- 1962 n
- 1965 o, u
- 1966 c
- 1967 b
- 1968 x
- 1970 q
- 1971 g, h
- 1973 j
- 1974 u, bb
- 1975 n, q, ac
- 1976 e, k, m, o, af, aw
- 1977 u, ac

A5 Solid Fuel

- 1816 a
- 1853 a
- 1871 b
- 1875 a
- 1889 a
- 1973 j
- 1975 n
- 1976 f, o
- 1977 u

A6 Stored Thermal Energy

- 1962 g
- 1968 h, l, v
- 1969 f, t, z, ap
- 1970 o, t, x
- 1971 i, ac
- 1972 ak, an
- 1973 c, d, j
- 1974 e, av
- 1975 ae, ao, be
- 1976 c, o, p, ae, am, ay

A7 None of Above

- 1961 d, h
- 1968 e, l
- 1969 f, g
- 1970 d
- 1973 c, d, x
- 1974 u, af
- 1975 f, h
- 1976 ak, ao, ap
- 1977 f, ae
Type of Engine

Displacers Only Engines

B1 VM Cycle

1918 a
1928 c
1961 h
1988 x
1969 a
1970 g, h, ac
1971 i; am, bf
1972 t, z, af
1974 i, y
1975 a, b, ac
1976 l

B2 Thermocompressor

1949 a
1957 f
1962 c
1988 e, h, l, u, ak
1969 a, i, r, x
1970 f, r, v
1971 a, i, j, az
1972 b, k, ak, al
1973 r, at
1974 m, x, au, aw, bb
1975 p, v, ap, au, ba
1976 t, u, aj, al, aq, ay, bc
1977 c, h

Displacer - Piston Engines

B3 Inline

1816 a
1827 a
1840 a
1853 a
1880 b
1888 b, c
1897 a
1903 a
1906 a
1913 a
1914 a

B4 Offset

1879 a
1890 a
1899 a
1906 a
1947 c
1951 f
B4 Continued

1959 c
1960 j
1961 e
1966 c, o
1968 k
1970 p
1973 j
1974 bd
1975 ai, bg
1976 c, n, p, r, v, ax
1977 e, k

B5 Dual Piston Engines

1833 a
1853 b
1854 b
1871 b
1881 a
1905 a
1919 a
1938 b
1946 a
1947 b, c
1948 k
1949 c.
1952 s
1954 e
1957 k
1958 a
1959 c.
1961 d
1962 l
1963 e, h
1965 b, f, l
1966 k, m
1967 c
1968 g, r, w
1969 f
1970 d
1971 c, d, w, x, ag
1972 c, ar
1973 j, y, ay
1974 ae
1975 af, ap, bd, bf
1976 d, g, k, m, o, q
1977 c, e, g, h, i, j, ae, aq, bb, bj

B6 Crank Operated Displacer

1816 a
1853 a
1880 b
1888 b, c
1897 a
1903 a
1906 a
1913 a
1932 b
1946 a
1947 c
1949 b, c
1951 f
1952 b, f, g
1954 f
1955 c
1956 d, e
1957 i
1958 i
1959 c, d, f, h, n
1960 e, j, t
1961 c, d, e
1962 i, l, n
1963 f, i, r
1964 f
1965 f, l, m, u
1966 g, m, o
1967 f
1968 a, f, k, ad
1969 a, e, f, r, v, ab
1970 a, d, h, j, o, p, v
1971 c, d, e, f, l, v, w, x, ab, ac, ag
1972 f, j, m, q, an
1973 c, d, h, j
1974 m, ax, ay, az, bc, bd
1975 ab, af, ai, an, ao, ap, av, ay, bd, ag
1976 a, b, c, e, j, n, p, ae, am, ax, az
1977 c, h, ab, aj, ar, ax
0000 c

B7 Free Displacer (not crank)

1968 h, i, z
1969 a, h, ak
1970 p, r
Continued

1971 g, i, ao, aq, ay, az
1972 b, j, k, m, x, y
1973 b, j, p, t, u, al, as, at
1974 f, g, j, o, p, q, r, ac, ad, au
1975 n, o, q, r, s, v, x, z, ad, au, be
1976 k, r, v, z, ak, aq
1977 a, b, f, j, m, r, s, t, u, ac

Crank Operated Power Piston

1816 a
1853 a
1871 b
1880 b
1881 a
1888 b, c
1889 a
1897 a
1903 a
1906 b
1913 a
1932 b
1946 a
1947 c
1949 b, c
1951 f
1952 c, d, f, g
1954 e, f
1955 c
1956 d, e
1957 i
1958 a, h, i
1959 c, d, f, h, i, j, n
1960 e, j, t
1961 c, d, e
1962 i, l, n
1963 f, h, i, r, s
1965 b, f, i, u
1966 c, g, j, k, m, o
1967 c, f

Continued

1968 a, f, k, l, w, y, ad
1969 a, e, f, v, ab
1970 a, d, h, j, o, p, s, v
1971 c, d, e, f, l, t, v, w, x, ab, ac
1972 c, f, j, m, q, af, ar
1973 c, d, h, j, a1, ay
1974 m, r, ae, ax, ay, az, bc, bd
1975 t, ab, af, an, ao, ap, av, ay, bd, bf, bg
1976 a, b, c, d, e, g, j, n, p, q, r, v, z, ae, am, ax, az, ba
1977 c, h, k, q, ab, aj, aq, ar, ax, bb, bg
0000 c

Free Power Piston (not crank)

1968 i, z
1969 a, f, h, ac
1970 d
1971 c, d, g, k, ag, ao, aq, ay
1972 j, m, x, y, ad
1973 b, j, p, t, u
1974 f, g, j, o, p, q, ac, ad
1975 n, o, q, r, s, x, z, ad, af
1976 k, m, ak, hb
1977 a, b, m, r, s, t, u, ac
Intended Or Actual Useful Output

Mechanical Power

C1 Vehicle

1946 b
1960 i, n
1965 y
1967 f
1968 a, f, aa
1969 c, d, f, j, w, y, z
1970 a, c, d, j, k, i
1971 c, d, m, u, w, x, ab, ac, ad, ag, a1, as, au, aw, bc, ba
1972 a, m, o, v, w, aa, ab, ac, aj
1973 c, d, g, h, j, n, y, ac, af
1974 a, d, e, t, ag, ah, an, aq, ar, bc
1975 m, s, t, u, ae, am, ao, aq, ar, bd
1976 a, c, d, g, aa, ad, ae, ai, an
1977 f, j, n, ab, ah, ap, aq, bk, bm
0000 c

C2 Other

1946 d
1952 n
1957 d, e
1961 m
1966 g
1969 m
1970 p
1973 au
1976 b
1977 ah

Pumping

C3 Heat (refrigeration)

1850 a
1874 a
1938 b
1942 b
1946 d
1949 c
1951 f
1952 c, e, g, n
1953 c
1954 b, e, f
1955 b, c, e, f, g
1956 d, e, f
1957 f, h, i, j
1958 f
1959 d, h, j, n
1960 c, d, h
1961 c, h, l
1962 b, f
1963 c, h, n, o, q
1964 d, f, i, k
1965 b, d, f, j, l, n, v, x, z
1966 i, k, l
1967 f
1968 d, e, g, r, w, x, ad
1969 l, aa, ai, al, an
1970 h, v, x, ac, ax, bn
1971 b, l, p, s, v, ak, ba, bf
1972 d, e, f, t, y, z, ae, af, am, an
1973 j, o, p, aa, ae, ap, aq, at, au, ax
1974 l, o, y, ak, am, ao, as, av
1975 b, e, q, x, ac, af, ag, aj, ak, as, ay, be
1976 l, ab, ar, as, ax
1977 f, h, n, r, t, w, ad, ax, bn
0000 c, d
### C4 Liquid

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### C7 Electric Generation

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### Design Considerations

#### D1 System Studies

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D6 Sealing

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1948 j
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1962 n
1965 b, f, h, u
1967 e, g
1968 e, f
1969 e
1973 b
1974 a, av, bc
1975 b, be
1976 c, l, ae, ba, bb
1977 b, v

D8 Continued

1971 b, j, ac, ba
1973 b, q, af
1974 c, av
1975 ao
1976 c, al, ba
1977 q, v
0000 c

D9 Power Takeoff

1920 a
1951 h, i, m
1952 j
1957 c
1959 l
1960 e, o, p, q
1962 f
1969 e, q
1972 af
1973 b, p, q, w, y
1974 c, v
1975 b, bc, be
1976 ba
1977 b

D7 Engine Starting

1962 n
1966 c
1973 b, q
1974 ag
1975 f
1976 c
1977 q

D10 Gas Transport

1920 a
1952 n
1965 a, m
1971 l, s, ap
1973 b
1977 b

D8 Engine Control

1943 b
1949 e
1950 b
1951 n, p
1952 h, q, r
1953 h
1955 d
1956 b
1957 g
1959 m
1960 s
1962 n
1967 f
1969 al
1970 s

D11 Materials of Construction

1953 g
1957 h
1971 l
1973 ax, ay
1974 h
1975 t
1976 ao, bb
1977 b, v, y
### D12 Other

**Heat Transfer and Fluid Flow**

#### D13 Air Preheater
- 1947 b
- 1951 j
- 1968 p
- 1971 q
- 1974 bc
- 1976 ae, bb
- 1977 l, o, p, ab
- 0000 c

#### D14 Working Gas Heater
- 1926 a
- 1943 a
- 1946 c
- 1948 b
- 1949 d, h, i, j
- 1950 b, c, d
- 1951 g, k, o, p
- 1952 d, f, h, k, l, m, o
- 1953 d, i, j
- 1954 d
- 1957 d
- 1959 a
- 1960 p, r
- 1962 c
- 1965 a, u
- 1966 b, p
- 1967 m
- 1969 aa
- 1971 af
- 1973 s, y, ab, aj
- 1974 s, u, z, aa
- 1975 f, i, j, k, ac
- 1976 ao, av
- 1977 b, z

### D15 Regenerator
- 1917 b, d
- 1927 a
- 1928 a
- 1929 a, b, c
- 1930 a
- 1931 b
- 1932 a
- 1934 a
- 1938 a
- 1940 a
- 1942 a
- 1943 a
- 1947 a, b
- 1948 c, d, e
- 1949 d, f, g
- 1950 a, c, d
- 1951 a, b, c, d, e, q
- 1952 a, c, m
- 1953 a, e, g
- 1964 c, f
- 1956 a
- 1957 a, h, i
- 1958 i
- 1959 b, g, i
- 1960 g
- 1961 a, f, g, i, j, k, n
- 1962 a, b, c, f, j
- 1963 b, d, f, g
- 1964 a, e, g, h
- 1965 a, j, r
- 1966 h, k
- 1967 c, l
- 1968 e, r, ac
- 1969 n, p, aa, ah
- 1970 m, u, y, z
- 1971 l, o, s, aj, ak, an
- 1972 i
- 1973 j, l, ax
- 1974 m, aa, bc
- 1975 d, i, ac, aw
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- 1977 a, p, ab
- 0000 a
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9. DIRECTORY

A relatively small number of individuals and organizations are now actively doing anything with Stirling engines. The author hopes that this manual will encourage other individuals and possibly companies to become active in Stirling engine development. To aid in this process, the following directory is offered. Organizations are listed by their company name and the name of the head is given. Individuals and groups associated with universities are listed by the name of the head individual. The organizations and individuals will now be listed in alphabetical order.

Aerojet Liquid Rocket Company
P.O. Box 13222
Sacramento, California 95813

Contacts: John Moise
Larry Hoffman

Aerojet has been working since 1967 on a small thermocompressor type Stirling engine to power an artificial heart. As the free displacer oscillates, check valves pump helium through the engine. It produces 5 watts of pneumatic power at 18% overall efficiency.

About 10 people are employed on the project.

AGA Navigation Aids Limited
77 High Street
Brentford
Middlesex TW8 0AB
England

Contacts: Mr. K. C. Sutton-Jones
Mr. N. Spottiswoode

This Company, a Subsidiary of Swedish AGA, concentrates on marine navigational aids, and has a large part of the World market. The Company is sponsoring development and application of the Harwell Thermo Mechanical Generator, and is contracted to install a 60-watt version of this generator in a major lighthouse on the Irish Coast.

Amtech Incorporated
141 California Street
Newton, MA 02158

Contact: Dr. Larry C. Hoagland, Director of Research
Amtech, Incorporated Continued


Energy Research and Generation, Inc.
Lowell and 57th Streets
Oakland, California 94608

Contact: G. M. Benson, Director R, D & E

ERG has been developing for over ten years resonant free-piston Stirling-type machines (Thermoscillators) including hydrostatic drives, linear alternators, heat pumps, cryogenic refrigerators and gas compressors. In addition, development has continued on a cruciform variable displacement crank-type Stirling engine having a Rinia arrangement. ERG is performing R & D on heat exchangers, heat pipes, isothermalizers, regenerators, gas springs, gas bearings, seals materials (including silicon nitride and silicon carbide), and computer modeling as well as on linear motors and alternators, hydraulic drive components and external heat exchangers and heat sources (including combustors and solar collectors). ERG has built and tested several test engines and presently has separate electromechanical, hydraulic, engine and heat exchanger test cells. ERG sells heat exchangers, regenerators, linear motor/alternators, linear motoring dynamometer test stands, gas springs/bearings and dynamic seals. ERG plans to sell soon an oil-free isothermal compressor with linear motor drive and small Thermoscillators and laboratory demonstrations. The current status on ERG Stirling engines is given in references 77 a and u.

Dr. Benson reports that 5 people are now working solely on corporately funded Stirling engine programs.

Entwicklungsgruppe Stirling Motor M.A.N.-MWM
M.A.N. Werk, Augsburg
D8900 Augsburg 1
Stadtbachstrasse 1
West Germany

Contact: Dr-Ing. F. A. Zacharias

M.A.N.-MWM is a licensee of Philips and cooperate with Philips on some few projects. M.A.N.-MWM is following an independent course drawing on their experience as makers of Diesel engines. The 1973 information from JPL who visited there in 1974 (75 t) indicates that they are using double-acting, crank-operated pistons in an in-line or in a Vee arrangement. The heater tubes are investment cast, some with fins. The tubes are arranged in a line instead of a ring as employed by United Stirling. They use a straight accordion folded counter-flow air preheater arranged in parallel.
Entwicklungsgruppe Stirling Motor M.A.N.-MWM Continued

To the straight heater tube banks. Apart from the usual pressure level control by working gas compressor they have followed up a unique form of power control called intermittent short circuit control. For part of the time during compression and also expansion a valve is opened connecting the working space to the buffer space. The fraction of the time determines the power. "Very quick engine response, moderate efficiency under partial load and moderate construction costs are expected from this form of control." (00 c)

Together with Battelle Institute, Frankfurt, they have a concept of a hydrostatic drive mechanism with a stroke regulating control. Best partial load efficiencies are expected.

Due to another source it is known that M.A.N. in cooperation with Philips have realized a Stirling engine heating system by Li-SF₆ combustion via heat pipes.

At present about 40 people are employed in the group.

Fairchild Industries
Germantown, Maryland

Contact: Al Schock

Fairchild has a contract with DOE to perform technical services. One important thing has been that Al Schock has developed a 3rd order computer program to analyze the Beale free-piston engine that Sunpower is furnishing to MTI. Nothing has so far been published.

About 2 people are working on Stirling engines.

FFV Industrial Products
Sweden

See: Stirling Power Systems Corp.

FFV is a Swedish government-owned industrial group who is 50% owner of United Stirling. Independent of United Stirling, but using United Stirling technology, they have built a medium performance engine generator set which operates very quietly for commercial and military applications. The first technical paper on this product will be released in late 1978 and the engine-generator will be for sale to the general public in 1979. The product will be sold in the United States by Stirling Power Systems Corp.

FFV now employs 50 people on the Stirling engine project.
Ford Motor Company
Powertrain Research Office
Dearborn, Michigan

Contact: Norman D. Postma

Since 1971, the Ford Motor Company has been working with N.V. Philips Company of Holland evaluating Stirling engines for automobile propulsion. The promise of this development is given in reference 73. The present status is presented in Section 3.1. Ford is a licensee of Philips. The program at both Dearborn and Eindhoven was designed to put two Stirling engines into Ford Torino automobiles for evaluation of vehicle performance, economy, emissions, etc. Ford is now working on a $161 million cost sharing contract with DOE to go from 1 Oct 1977 to 30 Sept 1985. The goal of this program is to determine whether or not the Stirling can be an attractive automobile engine.

Currently Ford has about 50 employees working directly on the first task of the DOE program which requires a high confidence assessment of the Stirling engine fuel economy potential.

General Electric, Advanced Energy Programs
Valley Forge Space Division
Valley Forge, Pennsylvania 19481

Contact: Mr. J. A. Bledsoe

GE is developing a Stirling Radioisotope Power System (SIPS). North American Philips is doing the engine (76). Plutonium-238 oxide is the fuel and 1000 w(e) is the power.

Contact: Mr. L. Dutram

Also on a separate program GE is continuing a development program started at Sunpower of a gas heated free piston, free-displacer Stirling engine operating a Freon compressor (77 w). They plan to build and test a three ton cooling capacity prototype by 1979. The project is sponsored by the American Gas Association, the Department of Energy and General Electric.

Forty people are working on Stirling engines at General Electric.

John I. Griffin
Solar Engines
2937 West Indian School Road
Phoenix, Arizona 85017

Mr. Griffin is building and selling model Stirling engines. He has one on the market now and plans to market 6 more models over the next two years. Three thousand of the first model have already been sold as of 31 Jan 1978.
Harwell Laboratory
Instrumentation & Applied Physics Division
AEA
Harwell
Oxfordshire OX11 ORA
England

Contact: Mr. E. H. Cooke-Yarborough

Work at Harwell is concentrated on Stirling-cycle devices which do not use rotation or sliding surfaces. The primary object of this work has been to generate electricity from heat. The machine which has been evolved has a springmounted displacer oscillating at a frequency in the region of 100Hz in helium, and a metal diaphragm to translate the resultant gas pressure changes into a mechanical movement having an amplitude of the order of 1mm. The oscillating hub of the diaphragm is coupled to a special alternator in which a permanent magnet vibrates between two pole pieces carrying windings on which the alternating output voltage is generated.

A number of thermo-mechanical generators giving an alternating power output in the region of 30W have been built. One of these has been in service since the summer of 1975, to provide the power for the UK National Data Buoy. Seven machines have been built, one with a heat-to-electricity efficiency of 16.9%.

Development sponsored by AGA Navigation Aids Limited has resulted in the up-rating of this design to 60W. During 1978 AGA will install a 60-watt TMG as the main power source in a major lighthouse off the Irish Coast.

A further development, which stemmed from the work on the TMG, is the Fluidyne, which uses an oscillating column of liquid as a displacer, and another liquid column, oscillating under the influence of the gas pressure changes, to provide the power output. This system is well adapted to the pumping of liquids. Construction and operation are relatively simple, and this has potentialities for water-pumping in developing countries. An experimental machine has pumped at a rate in excess of 60 gallons per hour.

The current program is aimed at improving the power output and efficiency of the TMG to meet specific requirements of users. It is also aimed at adapting the Fluidyne to meet specific user requirements.

The equivalent of two people are now working on Stirling engines at Harwell.
Hughes Aircraft Company Continued

Hughes Aircraft Company has been developing Stirling type cryogenic refrigerators since 1960 and a family of these units have been developed for ground, air and space applications. Currently, emphasis is being placed upon Stirling and Vuilleumier refrigerators including special modified versions of each to meet the specific needs of various applications.

About 45 people are involved in Stirling type cryo cooler development.

Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, California 91103

Contact: Frank W. Hoehn

The Jet Propulsion Laboratory is currently working on a program to develop a Stirling Laboratory Research Engine which can eventually be produced commercially and be made available to researchers in academic, industrial, and government laboratories. A first generation 10 Kw engine has been designed, fabricated, and assembled. The preprototype engine is classified as a horizontally-opposed, two-piston, single-acting machine with a dual crankshaft drive mechanism. The test engine, which is designed for maximum modularity, is coupled to a universal dynamometer. Individual component and engine performance data will be obtained in support of a wide range of analytical modeling activities.

The laboratory is also sponsoring work on a 1 Kw, solar heated free-piston Stirling linear electric generator.

JPL did the study (75 t) which influenced DOE to concentrate their efforts on the Stirling engine and the Gas Turbine for future automobile engines.

Approximately 3-6 people support the Stirling engine-related activities at JPL.

Laboratoriet for Energiteknik
Danmarks Tekniske Hojskole
Bygning 403 · DK-2800 Lyngby
Denmark

Contact: Professor Bjorn Qvale
Professor Niels Elmo Anderson

Due to Professor Qvale's thesis at MIT on Stirling engines and his subsequent interest in this field there have been a number of papers from this laboratory on Stirling engines. A recent letter states that almost no work is now going on in this field.
Professor W. R. Martini  
Joint Center for Graduate Study  
100 Sprout Road  
Richland, Washington 99352

Professor Martini heads a group varying from 1 to 5 working on various basic aspects of Stirling engines. He is the author of the Stirling engine design manual. He also sends out the Stirling Engine Research Institute Newsletter. He is interested in determining the usefulness of isothermalizers in Stirling engine designs and in building a heat operated heat pump using two Stirling cycles. He started the artificial heart power source program at McDonnell Douglas and continues an interest in this program.

Mechanical Technology Inc.  
968 Albany-Shaker Road  
Latham, New York 12110

Contact: Bruce Goldwater.

In conjunction with Sunpower MTI is developing a Free-Piston Stirling Linear Alternator power conversion system. They are aggressively expanding their own Stirling engine capability. They are working with United Stirling to demonstrate Stirling engines for automobiles as part of a major DOE funded study.

MTI currently has 20 people working on Stirling engines.

McDonnell Douglas Corp.  
Richland Energy Laboratory  
100 Sprout Road  
Richland, Washington 99352

Contact: R. P. Johnston

Since 1967 this group has been developing miniature Stirling engines to power an artificial heart. The engine produces about 5 watts of hydraulic power from heat at about 18% efficiency. It employs a free displacer engine which applies pulsating gas pressure through a diaphragm to a freely oscillating oil pump. The engine is self starting and is controlled by a single valve adjustment.

Presently 11 people are employed on this project.
NASA-Lewis has been given the project management responsibilities by DOE to produce improved Stirling engine propulsion systems during the next decade. They have obtained two GPU-3 Stirling engines and are testing one to obtain some publicly available information on Stirling engine performance.

They have negotiated the $161 million contract with Ford Motor Co./Philips for development of Stirling and will negotiate a contract with MTI/United Stirling/American Motors for the same thing. They have sponsored the production of this Stirling engine design manual. They have a contract with Boeing and University of Toledo for evaluation of reciprocating seals. They are doing work on materials technology for both a metal and a ceramic Stirling engine. They have signed a contract with Illinois Institute of Technology Research Institute to measure hydrogen permeability in metals and ceramics. Near-term future plans are aimed at establishing a balanced, integrated mix of activities on improved engine development, advanced system definition studies and supporting research and technology.

Currently the equivalent of 20 people are working on Stirling engines at NASA-Lewis.

Mechanical Systems Section
Building Environment Division
Center for Building Technology
National Bureau of Standards
Washington D.C. 20234

Contact: Dr. David A. Didion

The initial work at NBS focused on the laboratory evaluation of a Philips 1-98 engine driving a Rankine cycle heat pump which used recovered engine heat to supplement its own capacity. The system was tested as a function of outdoor temperature, engine speed, and coolant temperature, and many of the results are presented in reference 77 ad. A subsequent analytical study was conducted on the performance of a variety of total energy configurations when powered by Stirling engines. This work will hopefully be presented at a future IECEC meeting. We are currently involved in developing a test and rating procedure for engine-driven heat pumps which will include Stirling engines.

One person is currently working on this project.
Dr. Allan J. Organ  
Department of Mechanical Engineering  
University of London King's College  
Strand London, England  
WC2R 2LS

Dr. Organ is a regular contributor since 1970 to the literature on Stirling engines. The recent ones have been highly mathematical. Those who have talked with him state that he is as much concerned with the mechanical part of the engine as he is with the heat transfer and fluid flow part. He has a grant from the United Kingdom Science Research Council for an experimental program. His department intends to offer starting October 1978 a course-unit option (one engineering degree subject credit) in the thermodynamics and computer modeling of Stirling cycle machines. This course will be for final year engineering students.

N. V. Philips  
Eindhoven  
Netherlands

dr. ir. C. L. Spigt is in charge of the Eindhoven Stirling engine work — about 100 people.

Dr. R. J. Meijer  
439 Huntington Place  
Ann Arbor, Michigan 48104

Dr. A. P. J. Michels  
1828 Mershon  
Ann Arbor, Michigan 48104

Science advisors to Ford Motor Company on Stirling engines.

The Philips Company was the first to recognize that the Stirling engine would be a useful prime mover if it were modernized. Philips has been publishing on Stirling engines since 1943. Almost all the companies developing Stirling engines for sale are licensees to Philips. These include United Stirling, FFV, M.A.N.-MWM, and the Ford Motor Company.

Philips has developed the rhombic drive Stirling and the 4 cylinder swashplate Stirling. They have perfected the oil backed roll sock seal. They have demonstrated very long life and very high efficiency in their machines. They have built engines to replace the automobile engine in size and power density. Besides developing engines that efficiently employ liquid fuel, they have demonstrated machines that use coal, Li-SF$_6$, and stored thermal energy. Their Stirling engine cryogenic refrigerators are a commercial success.
Philips Laboratories, a division of
North American Philips Corporation
345 Scharborough Road
Briarcliff Manor, New York 10510

Contact: Alexander Daniels

Philips Laboratories have close association with the N.V. Philips Company
of Holland, and have a number of programs.

They are working on the SIPS program with the General Electric Company
(76a) and they are doing a study for DOE on a total energy system
using Stirling engines (77f).

Fourteen people are working on Stirling engines at North American Philips.

Norman E. Polster
Argenta, B.C.
Canada

Mr. Polster has invented a self-starting, intrinsically controlled Stirling
engine. In a demonstration model made at the Ontario Science Centre, a
manually operated torque control level provides an instantaneous continously
controllable accelerating or decelerating torque, including zero torque
for any shaft position, any shaft speed and direction including a stationary
condition. Mr. Polster is joined with JOSAM Manufacturing Company, Michigan
City, Indiana, in further developments. Some work has been done at the
Joint Center for Graduate Study, Richland, Washington. (76c)

Professor C. J. Rallis
School of Mechanical Engineering
University of Witwatersrand
1 Jan Smuts Avenue
Johannesburg 2001 South Africa

Professor Rallis currently leads a team of 6 graduate and undergraduate
students in developing 2nd and 3rd order computations procedures and
checking them with experiments. They are also doing basic work on
pressure drop, time lag effects, and heat transfer in periodic flow heat
exchangers. They are also experimenting with fluidyne machines and are
consulting with Harwell on the analysis of the thermo mechanical
converter

Professor Graham Rice
Dept. of Engineering & Cybernetics
The University of Reading
Whiteknights, Reading RG 6ZAY, England

Professor Rice has been involved in a number of interesting Stirling engine
Professor Graham Rice Continued

experiments at the University of Reading (75 k). He is involved in a proposed consortium to design and build a Stirling engine in the United Kingdom.

Ross Enterprises
37 West Broad Street
Suite #630
Columbus, Ohio 43215

Contact: M. Andrew Ross

Mr. Ross is a practicing attorney who is also a model engineer. He designs and builds his own machines in his own shop. He is also the author of a number of popular articles on Stirling engines (76 a, 76 b), and has an impressive collection of antique Stirling engines.

Professor J. Senft
Division of Science and Mathematics
Minot State College
Minot, North Dakota 58701.

Professor Senft teaches mathematics, does research on Stirling and other heat engines, builds miniature engines in his own shop, and writes on engineering subjects. He has authored several articles on miniature Stirling engines in model engineering journals, and has also worked as analyst with the Sunpower group.

Professor J. L. Smith Jr.
Dept. of Mechanical Engineering
Massachusetts Institute of Technology
77 Massachusetts Avenue
Cambridge, Massachusetts 02139

Professor Smith is developing a valved hot gas engine which may have high torque at low speed like a Stirling engine but without the severe restrictions on heat exchanger dead volume inherent in the Stirling engine.

At this time, one graduate student is working with a test engine, concentrating on the periodic heat transfer between the working gas and the cylinder walls of the compressor and expander.
Stirling Power Systems Corp.
7101 Jackson Road
Ann Arbor, Michigan 48103

Contact: Mr. Lennart Johansson.

Stirling Power Systems is a marketing organization owned 80.5% by FFV a Swedish government owned industrial group and 19.5% by Thetford Company, a recreational vehicle-equipment supply firm of Ann Arbor, Michigan. They will market the FFV engine-generator.

Sunpower Inc.
W. T. Beale, President
48 West Union Street
Athens, Ohio 45701

Sunpower is an out-growth of Professor William T. Beale's work at Ohio University on free-piston Stirling engines. Sunpower is working with MTI of Latham, New York on a DOE sponsored 2KW(e) space power plant using a free-displacer, free-power piston Stirling engine driving a linear alternator. Sunpower has also built a free-piston engine for NASA-Lewis.

A new small solar-electric engine designed by Sunpower is being tested. The target for this system is overall conversion efficiency from solar energy to usable electric power approximately twice as high as that of a silicon solar cell using concentrated sunlight.

Sunpower currently employs 12 people working on Stirling engines.

Trans Computer Associates
Dr. T. Finkelstein, President
P.O. Box 643
Beverly Hills, California 90213

Dr. Finkelstein has worked on Stirling engines for a number of companies. He is now an independent consultant and has his 3rd order computer code available for use. He conducts a short course on Stirling engines every year at UCLA. He is the authority on the history of Stirling engines.

United Stirling (Sweden) AB & Co.
Fack
20110 Malmb 1, Sweden

Contact: Bengt Hallare, Corporate Planning and Marketing

United Stirling's program is well described in Section 3.2 of this manual. Briefly, they have designed a 40, a 75 and a 150 KW engine to be used in
United Stirling (Sweden) Continued

vehicles. Although a licensee of Philips, they have developed their own mechanical seal and their own engine designs, a crank operated Rinia arrangement.

United Stirling is identified along with Mechanical Technology Inc. and AM General as the "second team" to be funded by DOE to develop Stirling engines for automobiles.

United Stirling employs about 100 people working on Stirling engines.

Dr. Israel Urieli
Ormat Turbines
P.O. Box 68
Yavne, Israel

Dr. Urieli recently received his doctorate from the University of Witwatersrand on the subject of a third order analysis of a Stirling engine. He is lecturing part time at the University of Bersheva and is continuing his research on Stirling engines.

Professor G. Walker
Dept. of Mechanical Engineering
University of Calgary
Alberta, Canada

Professor Walker is measuring characteristics of reversing flow regenerators. He teaches courses on Stirling engines and is the author of an important book on Stirling engines. (73 j)

Westinghouse Electric Company
Advanced Energy Systems-Division
P.O. Box 10864
Pittsburgh, Pennsylvania 15236

Contact: W. D. Pouchot

Westinghouse has been doing the system work and North American Philips has been doing the engine work on a DOE sponsored artificial heart program (76 am). The Stirling engine work was phased out in U.S. Governments Fiscal Year 1977.
APPENDIX A

DERIVATION OF EQUATIONS FOR HEAT FLOW FROM VOLUMES WHICH ARE HEATED UNIFORMLY

During expansion and compression of a gas, the gas temperature of the entire volume changes uniformly before thermal conductivity makes a difference. Laser heating also approximates this. For the purpose of evaluation the equations will be derived for heat flow from a slab and from a cylinder.

From a Slab

In Figure A1 the gas is being cooled from both sides. The heat flow at $x$ is:

$$Q_x = Q_w \frac{x}{(s/2)} = -k_G A \frac{dT}{dx} \quad (A1)$$

where

- $Q_w =$ heat flow at wall, watts
- $x =$ distance from centerline, m
- $k_G =$ thermal conductivity of gas, w/m°C
- $A =$ area for heat flow, m²
- $T =$ gas temperature, °C

Integrating,

$$\frac{2Q_w}{s} \int_{0}^{s/2} x \, dx = -k_G A \int_{T_L}^{T_M} dT \quad (A2)$$

So

$$Q_w = \frac{4k_G A}{s} (T_L - T_M) \quad (A3)$$

However, we need to know the average gas temperature, $T_A$, instead of the centerline temperature, $T_L$. $T_A$ is defined by the equation
Figure A1. Assumed Gas Conduction in a Slab.

Figure A2. Assumed Gas Conduction in a Cylinder.
\[(T_A - T_e)\frac{s}{2} = \int_0^s (T - T_e)dx \tag{A4}\]

Using different limits on Equation A2:

\[\frac{2Q_w}{s} \int_0^x x dx = -k_GA \int_{T_e}^T dT \tag{A5}\]

\[\frac{2Q_w}{s} \frac{x^2}{2} = -k_GA(T - T_e) \tag{A6}\]

\[T - T_e = -\frac{Q_wx^2}{sk_GA} \tag{A7}\]

So

\[T_A - T_e = \int_0^{s/2} -\frac{Q_w}{sk_GA} x^2 dx = -\frac{Q_w s}{12k_GA} \tag{A8}\]

From Equation A3

\[T_L - T_M = \frac{Q_w s}{4k_GA} \tag{A9}\]

Therefore,

\[T_A - T_M = T_A - T_e + (T_e - T_M) \tag{A10}\]

\[= -\frac{Q_w s}{12k_GA} + \frac{Q_w s}{4k_GA} = \frac{Q_w s}{6k_GA} \]

or

\[Q_w = \frac{6k_GA}{s} (T_A - T_M) \tag{A11}\]

Also, from the standpoint of heat capacity,

\[Q_w = -\rho V C_v \frac{dT_A}{d_0} \]

where \(\rho = \text{gas density, kg/m}^3\)
\[ V = \text{gas volume, m}^3 \]
\[ C_v = \text{heat capacity at constant volume, j/kg } ^\circ \text{C} \]
\[ T_A = \text{average gas temperature, } ^\circ \text{C} \]
\[ t = \text{time, seconds} \]

**From a Cylinder**

In Figure A2 the gas is being cooled from the cylindrical surface. The heat flow at \( r \) is:

\[ q_r = q_w \frac{\pi r^2}{4D^2} = -k_G (2\pi r \xi) (\frac{dT}{dr}) \]

where \( \xi = \text{length of cylinder} \).

Integrating:

\[ \int_0^{D/2} q_w \frac{\pi r^2}{4D^2} \, dr = - \int_{T_L}^{T_M} dT \]

\[ \frac{2q_w}{\pi k_G D^2} \left[ \frac{r^2}{2} \right]_0^{D/2} = - \left[ T \right]_{T_L}^{T_M} \]

\[ \frac{Q_w D^2}{\pi k_G D^2} = T_L - T_M \]

\[ \frac{Q_w}{4\pi k_G} = T_L - T_M \]

However, what is needed is the integrated average gas temperature, \( T_A \). By definition,

\[ (T_A - T_L) \frac{n}{4} D^2 = \int_0^{D/2} (T - T_L) 2\pi r \, dr \]
Using different limits for Equation A13,
\[
\frac{2Q_w}{\pi \varepsilon k_G D^2} \int_0^r rdr = -\int_{T_L}^T dT \tag{A16}
\]

or
\[
\frac{Q_w r^2}{\pi \varepsilon k_G D^2} = T_L - T \tag{A17}
\]

Substituting into Equation A15,
\[
(T_A - T_L) \frac{\pi}{4} D^2 = \int_0^{D/2} -\frac{Q_w r^2}{\pi \varepsilon k_G D^2} 2\pi r dr = -\frac{2Q_w}{\varepsilon k_G D^2 \pi} \int_0^r r^3 dr \tag{A18}
\]

\[
T_A - T_L = -\frac{2Q_w (4)}{\varepsilon k_G D^2 \pi} \left[ \frac{r^4}{4} \right]_0^{D/2}
\]

\[
= -\frac{2Q_w (4) D^4}{\varepsilon k_G D^4 \pi (4)} = -\frac{Q_w}{8\varepsilon k_G \pi} \tag{A19}
\]

Therefore:
\[
T_A - T_M = T_A - T_L + T_L - T_M \tag{A20}
\]

Substituting Equation A14 and A19 in Equation A20:
\[
T_A - T_M = -\frac{Q_w}{2\pi \varepsilon k_G} + \frac{Q_w}{4\pi \varepsilon k_G} = \frac{Q_w}{8\pi \varepsilon k_G} \tag{A21}
\]

or
\[
Q_w = \frac{8\pi \varepsilon k_G (T_A - T_M)}{T} \tag{A22}
\]
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   Joint Center for Graduate Study
   100 Sprout Rd.
   Richland, WA 99352

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   Cleveland, Ohio 44135.

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   Project Manager, Harold H. Valentine, Systems Analysis and Assessment Office, NASA Lewis
   Research Center, Cleveland, Ohio 44135.

16. Abstract  
   This manual is intended to serve both as an introduction to Stirling engine analysis methods and
   as a key to the open literature on Stirling engines. Over 800 references are listed and these are
   cross referenced by date of publication, author and subject. Engine analysis is treated starting
   from elementary principles and working through cycle analysis. Analysis methodologies are
   classified as first, second or third order depending upon degree of complexity and probable
   application; first order for preliminary engine studies, second order for performance prediction
   and engine optimization, and third order for detailed hardware evaluation and engine research.
   A few comparisons between theory and experiment are made. A second order design procedure
   is documented step by step with calculation sheets and a worked out example to follow. Current
   high power engines are briefly described and a directory of companies and individuals who are
   active in Stirling engine development is included. Much remains to be done. Some of the more
   complicated and potentially very useful design procedures are now only referred to. Future
   support will enable a more thorough job of comparing all available design procedures against
   experimental data which should soon be available.

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