ENGINEERING DESCRIPTION OF THE
ASCENT/DESCENT BET PRODUCT

August 1986

Prepared for

CONTRACT NAS9-17554
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LYNDON B. JOHNSON SPACE CENTER
HOUSTON, TEXAS

Prepared by

A. W. Seacord, II

(NASA-CR-171953) ENGINEERING DESCRIPTION OF
THE ASCENT/DESCENT BET PRODUCT (TRW Defense
Systems Group) 127 p CSCL 12A

System Development Division
TRW Defense Systems Group
Houston, Texas
ENGINEERING DESCRIPTION OF THE
ASCENT/DESCENT BET PRODUCT

August 1986

Prepared for

CONTRACT NAS9-17554
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LYNDON B. JOHNSON SPACE CENTER
HOUSTON, TEXAS

Prepared by

A. W. Seacord, II

System Development Division
TRW Defense Systems Group
Houston, Texas
ENGINEERING DESCRIPTION
OF THE
ASCENT/DESCENT BET PRODUCT

August 1986

Prepared for
Contract NAS9-17554
National Aeronautics and Space Administration
Lyndon B. Johnson Space Center
Houston, Texas

Prepared by
A. W. Seacord, II
Navigation Analysis Section

Approved by
S. M. Kindall, Head
Navigation Analysis Section

Approved by
D. K. Phillips, Manager
Systems Engineering and Analysis Department

TrW Defense Systems Group
Houston, Texas
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 INTRODUCTION AND SCOPE OF DOCUMENT</td>
<td>1</td>
</tr>
<tr>
<td>1.0.1 ASCENT/DESCENT OUTPUT OVERVIEW</td>
<td>1</td>
</tr>
<tr>
<td>1.0.2 SCOPE OF THIS DOCUMENT</td>
<td>1</td>
</tr>
<tr>
<td>2.0 NOMENCLATURE</td>
<td>3</td>
</tr>
<tr>
<td>2.0.1 VECTORS</td>
<td>3</td>
</tr>
<tr>
<td>2.0.2 MATRICES</td>
<td>4</td>
</tr>
<tr>
<td>2.0.3 SPECIFIC LOCATIONS</td>
<td>5</td>
</tr>
<tr>
<td>2.0.4 COORDINATE SYSTEMS</td>
<td>6</td>
</tr>
<tr>
<td>3.0 APPLICABLE DOCUMENTS</td>
<td>8</td>
</tr>
<tr>
<td>4.0 THE OUTPUT PRODUCTS</td>
<td>9</td>
</tr>
<tr>
<td>4.0.1 THE BETDATA FILE</td>
<td>9</td>
</tr>
<tr>
<td>4.0.1.1 OPIP Calculations</td>
<td>9</td>
</tr>
<tr>
<td>4.0.1.2 Subroutines Called by OPIP</td>
<td>64</td>
</tr>
<tr>
<td>4.0.2 THE NAVBLK FILE</td>
<td>112</td>
</tr>
<tr>
<td>4.0.2.1 Definition of Terms</td>
<td>112</td>
</tr>
<tr>
<td>4.0.2.2 Computation of Terms in DELTA and RSUBO</td>
<td>114</td>
</tr>
<tr>
<td>4.0.2.3 NAVBLK File Format</td>
<td>116</td>
</tr>
<tr>
<td>ATTACHMENT</td>
<td></td>
</tr>
<tr>
<td>A TOPODETIC EULER ANGLE RATES</td>
<td>A-1</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1</td>
<td>Definition of Terms</td>
<td>113</td>
</tr>
<tr>
<td>Table 2</td>
<td>NAVBLK File Format</td>
<td>116</td>
</tr>
</tbody>
</table>
1.0 INTRODUCTION AND SCOPE OF DOCUMENT

1.1 ASCENT/DESCENT OUTPUT OVERVIEW

The Ascent/Descent output product is produced in the OPIP routine from three files which constitute its input. One of these, OPIP.IN, contains mission specific parameters. Meteorological data, such as atmospheric wind velocities, temperatures, and density, are obtained from the second file, the Corrected METeorological DATA file (METDATA). The third file is the TRJATTDATA file which contains the time-tagged state vectors that combine trajectory information from the Best Estimate of Trajectory (BET) filter, LRRET5 (Applicable Document 4, Section 3.0), and Best Estimate of Attitude (REA) derived from IMU telemetry.

Ascent/Descent output products are provided with two files. The major product contains the BET, which is in the file BETDATA. The other product file contains the system parameters which were used to determine the BET. This is The Navigation Block, or NAVBLK, file.

Each of the product files is delivered on three magnetic computer tapes. Two of these are in binary format, one of which is formatted to be read by a CDC Cyber machine, and the other is formatted to be read by a Sperry Univac machine. The third tape is written in ASCII format and is used to produce a microfiche display of the output.

1.2 SCOPE OF DOCUMENT

This engineering description defines each term in the two output data files. The description of the BETDATA file includes an outline of the algorithm used to calculate each term. Most computations are performed in the program OPIP or in simple subroutines called by OPIP. Some subroutines are extensive, however, and their algorithms are described separately in a section following that for OPIP.
To facilitate describing the algorithms, a nomenclature is defined in Sections 2.1 through 2.4. The description of the nomenclature includes a definition of the coordinate systems used.

The NAVBLK file contains navigation input parameters. Each term in NAVBLK is defined, and its source (e.g., user input) is listed. The production of NAVBLK requires only two computational algorithms. These two algorithms, which compute the terms DELTA and RSUBO, are described in Section 4.2.2 following the definition of NAVBLK terms. Finally, the distribution of data in the NAVBLK records is listed.

The English system of units is used throughout this document. Unless specified otherwise, lengths, velocities, and accelerations are output in feet, feet/sec, and feet/sec², respectively. Mass is output in slugs, force (and weight) in pounds, temperature in degrees Rankine, and angular measure (e.g., latitude, longitude, Euler angles, angle of attack and sideslip angle) are in degrees. Also, unless specified otherwise, time is output in seconds.
2.1 VECTORS

- \( \hat{\mathbf{r}}_A^B[F] \) will represent a three-element position vector from point A to point B; i.e., the position of B with respect to A. The components of the vector are with respect to the reference frame F whose orthogonal basis set is \([\hat{i}_F, \hat{j}_F, \hat{k}_F]\); \( \hat{i}_F, \hat{j}_F, \) and \( \hat{k}_F \) are unit vectors. Thus,

\[
\hat{\mathbf{r}}_A^B[F] = x_B \hat{i}_F + y_B \hat{j}_F + z_B \hat{k}_F.
\]

- \( \hat{\mathbf{r}}_A^B[F] \) is the unit vector in the direction of \( \hat{\mathbf{r}}_A^B[F] \);

\[
\hat{\mathbf{r}}_A^B[F] = \frac{\hat{\mathbf{r}}_A^B[F]}{|| \hat{\mathbf{r}}_A^B[F] ||}.
\]

- \( \dot{\mathbf{r}}_A^B[F] \) which is a velocity if F is an inertial frame.

\[
\dot{\mathbf{r}}_A^B[F] = \frac{d}{dt} \hat{\mathbf{r}}_A^B[F].
\]

- \( \ddot{\mathbf{r}}_A^B[F] \) which is an acceleration if F is an inertial frame.

\[
\ddot{\mathbf{r}}_A^B[F] = \frac{d^2}{dt^2} \hat{\mathbf{r}}_A^B[F].
\]

- \( \mathbf{v}_wind[F] \) If a relative velocity (say, wind relative to some point, P) is indicated or if it is not useful to express a velocity in terms of a time derivative, then the velocity may be expressed as \( \mathbf{v}_wind[F] \).

where, again, F is the coordinate frame in which the vector
components are expressed.

- \( \mathbf{\hat{a}}_{A}^{P}[F] \) Likewise, the acceleration of a point \( P \) with respect to a point \( A \) as expressed in frame \( F \) is as shown. In this document, the term "contact acceleration" (which is the term most often used in the software listings) is equivalent to sensed acceleration.

- \( \mathbf{\dot{S}}[M50] \) is the M50 state vector.

- \( \mathbf{\hat{\sigma}}(\mathbf{\hat{a}}) \) will represent the standard deviation of the components of the vector \( \mathbf{\hat{a}} \). The term "uncertainty" will be equivalent to the standard deviation. If \( \mathbf{\hat{a}} \) has three components, then so will \( \mathbf{\hat{\sigma}}(\mathbf{\hat{a}}) \). That is, if \( \mathbf{\hat{a}} = w_x \hat{i} + w_y \hat{j} + w_z \hat{k} \), then \( \mathbf{\hat{\sigma}}(\mathbf{\hat{a}}) = [\sigma(w_x), \sigma(w_y), \sigma(w_z)]^T \).

- \( \mathbf{\hat{\omega}}[F] \) is the angular velocity expressed in the \( F \)-frame.

- \( \mathbf{\hat{\omega}}[F] \) is the angular acceleration expressed in the \( F \)-frame.

2.2 MATRICES

- \([F1 \rightarrow F2]\) represents a transformation matrix which transforms a vector
from frame $F_1$ to frame $F_2$. Thus,

$$\hat{\mathbf{r}}_A[F_2] = [F_1 + F_2] \hat{\mathbf{r}}_A[F_1]$$

In terms of elements, this matrix may be expressed as

$$[F_1 + F_2] = \mathbf{M} = [m_{ij}]$$

where $i$ represents the row and $j$ represents the column of element $m_{ij}$.

- $[F_1 + F_2]^T$ represents the transpose of the above matrix. Thus,
  since

  $$[F_1 + F_2]^T = [F_2 + F_1],$$

  then

  $$\hat{\mathbf{r}}_A[F_1] = [F_1 + F_2]^T \hat{\mathbf{r}}_A[F_2].$$

- $\mathbf{\xi}(\mathbf{\hat{w}})$ represents the covariance matrix of the vector $\mathbf{\hat{w}}$. If $\mathbf{\hat{w}}$ has $n$ elements, then $\mathbf{\xi}(\mathbf{\hat{w}})$ will be an $n \times n$ matrix.

2.3 SPECIFIC LOCATIONS

The following convention will be used to express the following points, or locations.

- CM = the center of mass (usually equivalent to the center of gravity).
• NB = the Navigation Base (= Nav Base).

• Θ = the center of the Earth.

• L = origin of Landing Field (Runway) coordinate system.

• S = origin of either Launch Site for Ascent analysis or Landing Site Runway threshold for Descent analysis.

• P = a general, specified point.

2.4 COORDINATE SYSTEMS

The following abbreviations will be used for the indicated coordinate systems. These coordinate systems are discussed in Applicable Document 1 listed in Section 3.0. For all of them, the Y-axis forms a right hand orthogonal triad with the X and Z axes.

• M50 - The Mean of 1950 is the Earth-centered inertial system whose X-axis is in the direction of the equinox at the beginning of the Besselian year 1950, and the X-Y plane is in the mean equator of that epoch. The Z-axis lies along the mean Earth's rotation axis of that epoch.

• TOD - The True Of Date is the Earth-centered inertial system whose X-axis is in the direction of the Vernal Equinox of the midnight prior to launch. The XY plane is in the equitorial plane of that epoch, and the Z-axis lies along the Earth's rotation axis of that epoch.

• ECI - The Earth-Centered Inertial system has its X-axis in the mean equator of the epoch (midnight prior to launch) and remains (i.e., is inertial) in the direction of the Greenwich meridian at that time. The Z-axis lies along the Earth's rotation axis of that epoch.

• GEO - The GEOgraphic frame is Earth-centered and Earth-fixed with the X-axis lying in the equator of date and passing through the Greenwich
Note that $h_1$, $h_2$, and $h_3$ are not all mutually orthogonal. By linear transformation of coordinates, each $h_i$ can be given in M50 by $H_i$ according to

$$H_1 = F_3,$$

$$H_2 = (-\sin \psi) F_1 + (\cos \psi) F_2,$$

$$H_3 = (\cos \psi \cos \theta) F_1 + (\sin \psi \cos \theta) F_2 + (-\sin \theta) F_3.$$

In accordance with foregoing arguments, the topodetic Euler angle rates are given by the projections of the body angular velocity $\mathbf{U}$ as seen in the topodetic frame onto the axes $H_1$, $H_2$, and $H_3$ used for the Euler angle rotations $\psi$, $\theta$, and $\phi$:

$$\dot{\psi} = U \cdot H_1,$$

$$\dot{\theta} = U \cdot H_2,$$ and

$$\dot{\phi} = U \cdot H_3.$$

4. REFERENCE

meridian. Its Z-axis lies along the Earth's rotation axis of that date.

- **TOP** - The TOPodetic system is an inertial system whose origin is on the Navigation Base with the X-axis pointing northward along the local meridian. The Z-axis points downward in the direction normal to the reference ellipsoid (presently, the Fischer Ellipsoid of 1960). Note that the topodetic frame is non-rotating; a new frame is redefined at each time point.

- **BOD** - The vehicle BODy system has its origin centered at the Navigation Base (not at the Center of Mass). The X-axis points toward the Orbiter's nose, and the Z-axis points down through the Orbiter's bottom.

- **PLM** - The PLuMbline coordinate system is Earth-centered and inertial. Its axes are parallel to a right handed orthogonal system centered at the launch site and fixed inertially at the time of SRB ignition (i.e., GET=0). Define \( \hat{n} \) as the unit vector normal to the tangent plane at the launch site; its direction is along the gravity gradient at that point. Also define a unit vector \( \hat{k} \) at the launch site, in the tangent plane, and pointing in the direction of the launch azimuth. The unit vectors \( \hat{n} \) and \( \hat{k} \) are normal to each other and to the third unit vector, \( \hat{j} \), of the triad. Then the plumbline X, Y, and Z axes are parallel and positive in the direction of the unit vectors \( \hat{n} \), \( \hat{j} \), and \( \hat{k} \) at the instant of SRB ignition. The Plumbline system is used only during ascent.

- **LF** - The Landing Field (or Runway) coordinate system is a rotating coordinate system whose origin is at the intersection of the runway centerline and threshold. The X-axis lies along the centerline, and is positive in the direction of rollout. The Z-axis is normal to the reference (i.e., Fischer) ellipsoid at the origin and is positive toward the Earth's center.
3.0 APPLICABLE DOCUMENTS


4.0 THE OUTPUT PRODUCTS

The output product of the Ascent/Descent analysis program consists of two files, BETDATA and NAVBLK. Each is produced by a major program. The purpose of this section is to describe the methods by which these programs calculate the output product files.

4.1 THE BETDATA FILE

The BETDATA file consists of 239 double precision words calculated by the program OPIP and subroutines called by it. The following Section 4.1.1 describes the processing within OPIP, and Section 4.1.2 describes the calculations of subroutines called by OPIP.

4.1.1 OPIP Calculations

The contents of the BETDATA file are discussed on the following pages in the order in which the words are calculated. The OPIP program and its subroutines calculate terms, each of which is represented by a name and an algebraic symbol and may consist of more than one double precision word in the file. For example, the M50 position vector has the term name M50, is designated by the algebraic symbol \( \mathbf{r}_B[M50] \), and consists of three double precision words.

\[ \text{NB} \]

The following discussion provides the term name, algebraic symbol, the program, or subroutine in which the terms is computed, and the algorithm by which the term is computed.
BET OUTPUT PRODUCTS

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>GET</td>
<td>t_{GET}</td>
<td>OPIP</td>
<td>Read next output time, t_{NEXT}, from TRJATTDATA file</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OPIP</td>
<td>Read SRB ignition time, t_{SRBI}, from OPIP.IN file.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OPIP</td>
<td>t_{GET} = t_{NEXT} - t_{SRBI}</td>
</tr>
</tbody>
</table>

Time from SRB ignition (in seconds); i.e., Ground Elapsed Time (GET)

Ground time in day:hour:min:sec of UT since the start of the year, but where day is the day number. That is, ground time is the elapsed Astronomical UT since the start of the year + 1 day.

2 GMTDAY  
3 GMTHR  
4 GMTMIN  
5 GMTSEC

Time is maintained internally in seconds from the astronomical UT start of year. The program is initialized with a Launch time (year, day number, seconds from the the UT midnight day of launch), stepped forward in time, and updated with time-tagged observations. Data records are written with GMTSEC as double precision.
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Onboard (Shuttle) time in day:hour:min:sec since SRB ignition

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>SGMTDAY</td>
<td>t_{SNEXT}</td>
<td>Compute onboard time of next output, t_{SNEXT}</td>
</tr>
<tr>
<td>7</td>
<td>SGMTHR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>SGMTMIN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>SGMTSEC</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
t_{SNEXT} = \frac{t_{NEXT} - \varepsilon_t + \Delta t_{drift} \cdot t_{OS}}{1 - \Delta t_{drift}}
\]

\(\varepsilon_t\) = Difference (or error) between onboard and ground times.

\(\Delta t_{drift}\) = Drift rate of onboard clock.

\(t_{OS}\) = time, in seconds, from midnight prior to launch as measured by the onboard clock. A value of \(\varepsilon_t\) is assumed.

TELAPSE The output is computed in the same way as are words 2, 3, 4, and 5. The times are onboard times, however, rather than ground times (e.g., \(t_{SNEXT}\) replaces \(t_{NEXT}\)).

Year of present (Ground) time, UT

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>GMTYR</td>
<td></td>
<td>TELAPSE The present UT year is calculated along with words 2, 3, 4, and 5.</td>
</tr>
</tbody>
</table>
### BET OUTPUT PRODUCTS (continued)

**DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER**

<table>
<thead>
<tr>
<th>word</th>
<th>name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position of the Nav Base relative to the (inertial) M50 coordinate frame</td>
<td>11 M50(1)</td>
<td>( \dot{r}_{\text{M50}} )</td>
<td>OPIC</td>
<td>Extracted from the state vector:</td>
</tr>
<tr>
<td></td>
<td>12 (2)</td>
<td>( \dot{r}_{\text{M50}} )</td>
<td>OPIC</td>
<td>( \dot{r}_{\text{M50}} = \text{M50STAT}(1,2,3) ) read in NB</td>
</tr>
<tr>
<td></td>
<td>13 (3)</td>
<td>( \dot{r}_{\text{M50}} )</td>
<td>OPIC</td>
<td>from the input file TRJATTDATA.</td>
</tr>
</tbody>
</table>

| Velocity of the Nav Base relative to the (inertial) M50 coordinate frame | 14 DM50(1) | \( \ddot{r}_{\text{M50}} \) | OPIC | Extracted from the state vector: |
|               | 15 (2)   | \( \ddot{r}_{\text{M50}} \) | OPIC | \( \ddot{r}_{\text{M50}} = \text{M50STAT}(4,5,6) \) read in NB |
|               | 16 (3)   | \( \ddot{r}_{\text{M50}} \) | OPIC | from the input file TRJATTDATA. |

| Contact (or sensed) acceleration of the Nav Base relative to the (inertial) M50 coordinate frame | 17 DDM50(1) | \( \dddot{r}_{\text{M50}} \) | OPIC | Extracted from the state vector: |
|               | 18 (2)   | \( \dddot{r}_{\text{M50}} \) | OPIC | \( \dddot{r}_{\text{M50}} = \text{M50STAT}(7,8,9) \) read in NB |
|               | 19 (3)   | \( \dddot{r}_{\text{M50}} \) | OPIC | from the input file TRJATTDATA. |
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word</th>
<th>name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>M5OC(1)</td>
<td>OPIP</td>
<td>Read in $\hat{r}_{NB}$ from TRJATTDATA file.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Position of the Orbiter Center of Mass (CM) relative to the M50 coordinate frame</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>$\hat{r}_\theta$ [M50]</td>
<td>M3TX1</td>
<td>$\hat{r}<em>{NB}$ [M50] = $[M50 + BOD]^T \hat{r}</em>{NB}$ [BOD]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Velocity of the Orbiter CM relative to the M50 coordinate frame</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>$\hat{v}_\theta$ [M50]</td>
<td>VADD</td>
<td>$\hat{v}<em>{NB}$ [BOD] = $\omega$ [BOD] x $\hat{r}</em>{NB}$ [BOD]</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>DM5OC(1)</td>
<td>OPIP</td>
<td>Read in $\hat{x}$ [BOD] = M50STAT(13,14,15)</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>$\hat{\hat{r}}_\theta$ [M50]</td>
<td>CROSS</td>
<td>$\hat{v}<em>{NB}$ [BOD] = $[M50 + BOD]^T \hat{v}</em>{NB}$ [BOD]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{\hat{r}}<em>\theta$ [M50] = $\hat{r}</em>{\theta}$ [M50] + $\hat{v}_{NB}$ [M50]</td>
<td>M3TX1</td>
<td>$\hat{v}<em>{NB}$ [BOD] = $[M50 + BOD]^T \hat{v}</em>{NB}$ [BOD]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{r}<em>{\theta}$ [M50] = $\hat{r}</em>{\theta}$ [M50] + $\hat{v}_{NB}$ [M50]</td>
<td>VADD</td>
<td>$\hat{r}<em>{\theta}$ [M50] = $\hat{r}</em>{\theta}$ [M50] + $\hat{v}_{NB}$ [M50]</td>
<td></td>
</tr>
</tbody>
</table>
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Sensed Acceleration of the Orbiter Center of Mass relative to the M50 coordinate frame

26 DDM50C(1) OPIP Read in \( \dot{\omega} = M50STAT(16,17,18) \)

\[
\begin{align*}
27 \quad (2) & \quad \ddot{r}_\Theta [M50] \\
& \quad \text{CROSS} \\
& \quad \begin{align*}
\dot{v}_{NB}[BOD] &= \dot{\omega}[BOD] \times \dot{r}_{NB}[BOD] \\
\dot{a}_{NB}[BOD] &= \ddot{\omega}[BOD] \times \dot{v}_{NB} \\
& \quad \text{VADD} \end{align*}
\end{align*}
\]

\[
\begin{align*}
28 \quad (3) & \quad \ddot{r}_\Theta [M50] \\
& \quad \text{CROSS} \\
& \quad \begin{align*}
\dot{v}_{NB}[BOD] &= \dot{\omega}[BOD] \times \dot{v}_{NB} \\
\dot{a}_{NB}[BOD] &= \ddot{\omega}[BOD] \times \dot{r}_{NB}[BOD] \\
& \quad \text{VADD} \end{align*}
\end{align*}
\]

\[
\begin{align*}
M3TX1 & \quad \ddot{\dot{a}}_{NB}[BOD] = [M50 + BOD] \dddot{\dot{a}}_{NB}[BOD] \\
& \quad \text{VADD} \quad \ddot{r}_\Theta [M50] = \ddot{r}_\Theta [M50] + \dddot{\dot{a}}_{NB}[M50]
\end{align*}
\]

Position of the Nav Base in the GEOgraphic coordinate system

29 GEO(1) M3X1 \( \ddot{r}_\Theta [ECI] = [M50 \rightarrow ECI] \ddot{r}_\Theta [M50] \)

\[
\begin{align*}
30 \quad (2) & \quad \ddot{r}_\Theta [GEO] \\
& \quad \text{NB} \\
& \quad \begin{align*}
[ECI \rightarrow GEO] &= \text{fctn}(\omega_\Theta, t_{NEXT}) \\
\dot{r}_\Theta [GEO] &= [ECI \rightarrow GEO] \ddot{r}_\Theta [ECI]
\end{align*}
\end{align*}
\]

31 (3) \( \ddot{r}_\Theta [GEO] = [ECI \rightarrow GEO] \ddot{r}_\Theta [ECI] \)

14
Velocity of the Nav Base in the GEOgraphics coordinate system

32 DGE0(1) M3X1 $\mathbf{\ddot{r}}_{\text{ECI}} = [\mathbf{M50} + \mathbf{ECI}] \mathbf{\ddot{r}}_{\text{M50}}$

33 (2) $\mathbf{\ddot{r}}_{\text{GEO}}$

34 (3) $\mathbf{\ddot{r}}_{\text{GEO}} = [\mathbf{ECI} + \mathbf{GEO}] \mathbf{\ddot{r}}_{\text{ECI}}$

- $\omega$ = Earth's rotation rate vector

Sensed Acceleration of the Nav Base in the GEOgraphic coordinate system

35 DDGE0(1) M3X1 $\mathbf{\ddot{a}}_{\text{ECI}} = [\mathbf{M50} + \mathbf{ECI}] \mathbf{\ddot{a}}_{\text{M50}}$

36 (2) $\mathbf{\ddot{a}}_{\text{GEO}}$

37 (3) $\mathbf{\ddot{a}}_{\text{GEO}} = [\mathbf{ECI} + \mathbf{GEO}] \mathbf{\ddot{a}}_{\text{ECI}}$
Total Acceleration of the Nav Base expressed in the GEographic Coordinate System

38  DDTGEO(1)

\[ \hat{\mathbf{a}}_{\text{NB}}^{\text{ECI}} = \text{fctn}(\dot{\mathbf{r}}_\Theta^{\text{ECI}}, t_{\text{NEXT}}) \]

39  \( \dot{\mathbf{r}}_\Theta^{\text{GEO}} \)

\[ \dot{\mathbf{r}}_\Theta^{\text{GEO}} = [\text{ECI} \rightarrow \text{GEO}] \dot{\mathbf{r}}_\Theta^{\text{ECI}} \]

\[ - \omega_\Theta \times (\omega_\Theta \times \dot{\mathbf{r}}_\Theta^{\text{ECI}}) \]

\[ - 2\omega_\Theta \times [\text{ECI} \rightarrow \text{GEO}] \dot{\mathbf{r}}_\Theta^{\text{ECI}} \]

Gravitational Acceleration of the Nav Base expressed in the GEographic coordinate system

41  DDGGE(1)

\[ \hat{\mathbf{g}}_{\text{NB}}^{\text{ECI}} = \text{fctn}(\dot{\mathbf{r}}_\Theta^{\text{ECI}}, t_{\text{NEXT}}) \]

42  \( \hat{\mathbf{g}}_{\text{NB}}^{\text{GEO}} \)

\[ \hat{\mathbf{g}}_{\text{NB}}^{\text{GEO}} = [\text{ECI} \rightarrow \text{GEO}] \hat{\mathbf{g}}_{\text{NB}}^{\text{ECI}} \]

43  \( \hat{\mathbf{g}}_{\text{NB}}^{\text{GEO}} \)

\[ \hat{\mathbf{g}}_{\text{NB}}^{\text{GEO}} = \text{M3X1} \hat{\mathbf{g}}_{\text{NB}}^{\text{ECI}} \]
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program algorithm</th>
</tr>
</thead>
</table>

Initial computations for all wind-relative velocities which follow

SELMETA (Ascent) Read the meteorological data (Ascent)
input file CMETDATA, interpolate
DESCMET (Descent) and generate the array METDATA for that
location. Define the following terms
from METDATA:

\( w_H \) = Horizontal wind speed

\( \sigma_S(w_H) \) = Systematic uncertainty in \( w_H \)

\( \sigma_N(w_H) \) = Noise uncertainty in \( w_H \)

\( \theta_{WH} \) = Direction of horizontal wind
\( (= 0^\circ \text{ North and is positive } \) clockwise from that)\n
\( \sigma_S(\theta_{WH}) \) = Systematic uncertainty
in \( \theta_{WH} \)

\( \sigma_N(\theta_{WH}) \) = Noise uncertainty in \( \theta_{WH} \)

\( \sigma_N(w_V) \) = Noise uncertainty in
vertical wind speed

The total uncertainty in horizontal wind
speed, \( \sigma_T(w_H) \), is
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word</th>
<th>name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

\[ \sigma_T(w_H) = \left[ \sigma_S(w_H)^2 + \sigma_N(w_H)^2 \right]^{1/2} \]

The total uncertainty in horizontal wind direction, \( \sigma_T(\theta_{WH}) \), is

\[ \sigma_T(\theta_{WH}) = \left[ \sigma_S(\theta_{WH})^2 + \sigma_N(\theta_{WH})^2 \right]^{1/2} \]

Wind-relative velocity of the Nav Base projected onto the GEOgraphic coordinate system

44 DGEOW(1) OPIP

Set conversion constant \( C_{DO/R} \) = degrees/radian

45 (2) \( \hat{v} \) [GEO] wind

Read \( R_E \), Earth's equitorial radius and \( R_P \), Earth's polar radius from input file OPIP.IN

46 (3) \( \dot{\theta} \) [GEO] is computed above in words (29-31)

NB

GEOGEOD Compute Geodetic latitude (\( \phi_{GEOD} \)), longitude (\( \lambda_{GEOD} \)), height (\( h_{GEOD} \)), and declination, \( \delta \). (See description of GEOGEOD in Section 4.1.2)

GEOTOP \([GEO + TOP] = fctn(\lambda_{GEOD}, \phi_{GEOD})\)

M3X1 \( \dot{\theta} \) [TOP] = \([GEO + TOP] \dot{\theta} \) [GEO] \( \dot{\theta} \) [GEO]
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>DGEOW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WINDTOP</td>
<td>$v_{\text{wind}[\text{TOP}]} = [-w_H \cos \theta_{WH}, -w_H \sin \theta_{WH}, 0]^T$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\psi(v_{\text{wind}[\text{TOP}]}) = \text{ftctn} (\sigma(w_H), \sigma(\theta_W), w_H, \sigma(w_H))$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(See the description of WINDTOP in Section 4.1.2 for the expressions for the matrix elements in $\psi(v_{\text{wind}[\text{TOP}]})$.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VSUB</td>
<td>$v_{\text{wind}[\text{TOP}]} = \dot{r}<em>\theta [\text{TOP}] - v</em>{\text{wind}[\text{TOP}]}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M3TX1</td>
<td>$v_{\text{wind}[\text{GEO}]} = [\text{GEO} + \text{TOP}]^T v_{\text{wind}[\text{TOP}]}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Position of the Center of Mass in the GEOgraphic coordinate system

47 GEOC(1) \{ 
48 \begin{align*}
    \dot{r}_\Theta [\text{GEO}] \\
    \text{OPIP} \quad \text{Read } [M50 + \text{ECI}] \text{ from OPIP.IN} \\
    \text{CM} \quad \text{OPIP} \quad \text{Read } \dot{r}_{\text{NB}}[\text{BOD}] \text{ and } [M50 + \text{BOD}] \\
    \text{CM} \quad \text{from TRJATTDATA} \\
\end{align*}
49 \{ 
48 \begin{align*}
    \text{ECIGEO} \\
    \dot{r}_\Theta [\text{GEO}] \text{ defined previously in words (29-31)} \\
\end{align*}
\}
\}
\{ 
48 \begin{align*}
    \text{M3TX1} \\
    \dot{r}_{\text{NB}}[M50] = [M50 + \text{BOD}]^T \dot{r}_{\text{NB}}[\text{BOD}] \\
    \text{CM} \\
\end{align*}
\}

19
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word</th>
<th>name</th>
<th>symbol</th>
<th>program algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOC continued</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \dot{r}_{NB}[ECI] = [M50 + ECI] \dot{r}_{NB}[M50] \]

\[ \dot{r}_{NB}[GEO] = [ECI + GEO] \dot{r}_{NB}[ECI] \]

\[ \dot{\theta}_i [GEO] \text{ is defined in words (29-31)} \]

\[ \dot{r}_{i[ECI]} = [M50 + ECI] \dot{r}_{NB}[M50] \]

Velocity of the Center of Mass in the GEOgraphic coordinate system

50  DGEOC(1)  

\( \dot{\omega}[BOD] = M50\text{STAT}(13,14,15) \)

51  (2)  

\( \dot{\omega}_i [GEO] \)

\( \dot{r}_{NB}[BOD] = \dot{\omega}[BOD] \times \dot{r}_{NB}[BOD] \)

52  (3)  

\( \dot{r}_{NB}[M50] = [M50 + BOD] \dot{r}_{NB}[BOD] \)

\( \dot{r}_{NB}[M50] = [M50 + ECI] \dot{r}_{NB}[M50] \)
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name symbol program algorithm

DGEOC continued

\[ \dot{\mathbf{r}}_{NB}[\text{GEO}] = \dot{\mathbf{r}}_{NB}[\text{ECI}] - \mathbf{\omega} \times \mathbf{r}_{NB}[\text{GEO}] \]

\[ \dot{\mathbf{r}}_{\theta}[\text{GEO}] = \dot{\mathbf{r}}_{\theta}[\text{GEO}] + \mathbf{r}_{NB}[\text{GEO}] \]

Sensed acceleration of the Center of Mass in the GEOgraphic coordinate system.

53  DDGEOC(1)  OPIP  Read in \( \dot{\omega}[^{\text{BOD}}] = \text{M50STAT}(16,17,18) \)

54  (2)  CROSS  \( \dot{\mathbf{a}}_{CM}[\text{GEO}] \)  sensed

55  (3)  CROSS  Centrifugal acceleration:
\[ \ddot{\mathbf{a}}_{\text{CEN}}[^{\text{BOD}}] = \ddot{\omega}[^{\text{BOD}}] \times \mathbf{r}_{NB}[^{\text{BOD}}] \]

CROSS  Rotational acceleration:
\[ \dot{\mathbf{a}}_{\text{ROT}}[^{\text{BOD}}] = \dot{\omega}[^{\text{BOD}}] \times \mathbf{r}_{NB}[^{\text{BOD}}] \]

VADD  \( \ddot{\mathbf{r}}_{NB}[^{\text{BOD}}] = \ddot{\mathbf{a}}_{\text{CEN}}[^{\text{BOD}}] + \ddot{\mathbf{a}}_{\text{ROT}}[^{\text{BOD}}] \)

MT3X1  \( \ddot{\mathbf{r}}_{NB}[^{\text{M50}}] = \left[^{\text{M50}} \rightarrow ^{\text{BOD}} \right]^{T} \ddot{\mathbf{r}}_{NB}[^{\text{BOD}}] \)
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

DDGEOC continued

\[
\begin{align*}
\text{M3X1} & : \quad \mathbf{r}_{\text{NB}[\text{ECI}]} = [\mathbf{M50} + \mathbf{ECI}] \mathbf{r}_{\text{NB}[\mathbf{M50}]} \\
\text{ECIGEO} & : \quad \mathbf{a}_{\text{CM}[\text{GEO}]} = [\mathbf{ECI} + \mathbf{GEO}] \mathbf{r}_{\text{NB}[\text{ECI}]}, \\
& \quad \text{sensed} \quad \mathbf{CM} \\
& \quad - \mathbf{\omega}_e \times (\mathbf{\omega}_e \times \mathbf{r}_{\text{NB}[\mathbf{ECI}]}) \\
& \quad - 2\mathbf{\omega}_e \times [\mathbf{ECI} + \mathbf{GEO}] \mathbf{r}_{\text{NB}[\mathbf{ECI}]}, \quad \mathbf{CM}
\end{align*}
\]

Wind-relative velocity of the Center of Mass projected into the Geographic system

\[
\begin{align*}
56 \quad \text{DGEOWC(1)} & : \quad \mathbf{\dot{v}}_{\text{GEO]} \text{computed previously, words (44-46)} \\
\quad \text{OPIP} & \quad \mathbf{\dot{v}}_{\text{GEO]} \text{computed previously, words (44-46)} \\
\quad \text{wind} & \quad \mathbf{\dot{v}}_{\text{GEO]} \text{computed previously, words (44-46)} \\
\quad \text{NB} & \quad \mathbf{\dot{v}}_{\text{GEO]} \text{computed previously, words (44-46)} \\
\quad \mathbf{\dot{r}}_{\text{NB][GEO]} \text{calculated previously, in order} \\
\quad \text{CM} & \quad \mathbf{\dot{r}}_{\text{NB][GEO]} \text{calculated previously, in order} \\
\quad \text{CM} & \quad \mathbf{\dot{r}}_{\text{NB][GEO]} \text{calculated previously, in order} \\
\quad \text{CM} & \quad \mathbf{\dot{r}}_{\text{NB][GEO]} \text{calculated previously, in order} \\
\text{57} & \quad \mathbf{\dot{r}}_{\text{NB}[\text{GEO}]} \text{calculated previously, in order} \\
\quad (2) & \quad \mathbf{\dot{r}}_{\text{NB}[\text{GEO}]} \text{calculated previously, in order} \\
\quad \text{wind} & \quad \mathbf{\dot{r}}_{\text{NB}[\text{GEO}]} \text{calculated previously, in order} \\
\quad \text{CM} & \quad \mathbf{\dot{r}}_{\text{NB}[\text{GEO}]} \text{calculated previously, in order} \\
\quad \text{CM} & \quad \mathbf{\dot{r}}_{\text{NB}[\text{GEO}]} \text{calculated previously, in order} \\
\quad \text{CM} & \quad \mathbf{\dot{r}}_{\text{NB}[\text{GEO}]} \text{calculated previously, in order} \\
\text{58} & \quad \mathbf{\dot{r}}_{\text{NB}[\text{GEO}]} \text{calculated previously, in order} \\
\quad (3) & \quad \mathbf{\dot{r}}_{\text{NB}[\text{GEO}]} \text{calculated previously, in order} \\
\quad \text{VADD} & \quad \mathbf{\dot{v}}_{\text{GEO]} \text{computed previously, words (44-46)} \\
\quad \text{wind} & \quad \mathbf{\dot{v}}_{\text{GEO]} \text{computed previously, words (44-46)} \\
\quad \text{wind} & \quad \mathbf{\dot{v}}_{\text{GEO]} \text{computed previously, words (44-46)} \\
\quad \text{CM} & \quad \mathbf{\dot{v}}_{\text{GEO]} \text{computed previously, words (44-46)} \\
\quad \text{CM} & \quad \mathbf{\dot{v}}_{\text{GEO]} \text{computed previously, words (44-46)} \\
\quad \text{CM} & \quad \mathbf{\dot{v}}_{\text{GEO]} \text{computed previously, words (44-46)}
\end{align*}
\]
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position of the Nav Base relative to the landing runway expressed in Landing Field coordinates</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

59 LF(1)  GEOLF  First Pass obtain \([\text{GEO} \rightarrow \text{LF}]\) and \(\hat{\mathbf{r}}_\text{GEO}\) from Landing site geophysical parameters.

60 (2)  \(\begin{align*} \hat{\mathbf{r}}_\text{L}^{\text{LF}} \end{align*}\)  NB  \(\hat{\mathbf{r}}_\text{L}^{\text{LF}} = \hat{\mathbf{r}}_\text{GEO} + \hat{\mathbf{r}}_\text{GEO} - \hat{\mathbf{r}}_\text{GEO} \)

61 (3)  OPIP  \(\hat{\mathbf{v}}_\text{GEO}\) computed previously, words (29-31)

Velocity of the Nav Base relative to the landing runway in Landing Field coordinates

62 DLF(1)  OPIP  \(\hat{\mathbf{r}}_\text{GEO}\) computed previously, words (32-34)

63 (2)  \(\begin{align*} \hat{\mathbf{v}}_\text{L}^{\text{LF}} \end{align*}\)  M3X1  \(\hat{\mathbf{v}}_\text{L}^{\text{LF}} = [\text{GEO} + \text{LF}] \hat{\mathbf{r}}_\text{GEO} \)

64 (3)
**DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER**

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensed acceleration of the Nav Base relative to landing runway in Landing Field coordinates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65 DDLF(1)</td>
<td>OPIP</td>
<td>$\hat{r}_{\Theta} \left[ \text{GEO} \right]$ computed previously, words (38-40)</td>
<td></td>
</tr>
<tr>
<td>$\hat{a}_L \left[ \text{LF} \right]$</td>
<td>M3X1</td>
<td>$\hat{a}<em>L \left[ \text{LF} \right] = \left[ \text{GEO} + \text{LF} \right] \hat{r}</em>{\Theta} \left[ \text{GEO} \right]$</td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Geodetic velocity of the Nav Base projected in the TOPodetic coordinate frame

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>68 DTOP(1)</td>
<td>GEOGEOD</td>
<td>Geodetic latitude and longitude, $\phi$ and $\lambda$, respectively, were</td>
<td></td>
</tr>
<tr>
<td>$\hat{v}_{\Theta} \left[ \text{TOP} \right]$</td>
<td>M3X1</td>
<td>computed previously (words 44-48) from the geophysical parameters and $\hat{r}_{\Theta} \left[ \text{GEO} \right]$ which was computed previously, words 29-31</td>
<td></td>
</tr>
</tbody>
</table>

$$\text{GEOTOP} \left[ \text{GEO} + \text{TOP} \right] = \text{fctn}(\lambda_{\text{GEOD}}, \phi_{\text{GEOD}})$$

$$\hat{v}_{\Theta} \left[ \text{TOP} \right] = \left[ \text{GEO} + \text{TOP} \right] \hat{r}_{\Theta} \left[ \text{GEO} \right]$$
Geodetic sensed acceleration projected in the TOPodetic coordinate frame

\[ \mathbf{a}_{\theta}[\text{TOP}] = [\text{GEO} + \text{TOP}] \mathbf{r}_{\theta}[\text{GEO}] \]

Uncertainty (one sigma) in \( \mathbf{v}_{\theta}[\text{TOP}] \)

\[ \sigma(\mathbf{v}_{\theta}[\text{TOP}]) = [\text{GEO} + \text{TOP}] \sigma(\mathbf{v}_{\theta}[\text{GEO}]) [\text{GEO} + \text{TOP}]^T \]

where \( [C_{ij}] = \sigma(\mathbf{v}_{\theta}[\text{TOP}]) \)
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty (one sigma) in ( \dot{a}_\theta ) [TOP]</td>
<td>( \sigma_{\dot{a}_\theta \text{ sensed}} )</td>
<td>OPIP</td>
<td>( \sigma(\hat{r}<em>\text{M5O}) ) is extracted from ( \sigma(\hat{S}</em>\text{M5O}) )</td>
</tr>
<tr>
<td>( \ddot{a}<em>\theta ) [TOP] ( \dddot{a}</em>\theta ) [TOP]</td>
<td>( \hat{M}_3 \times 3 \text{UN}_2 )</td>
<td>M3X3UN2</td>
<td>( \sigma(\hat{r}<em>\text{ECI}) = [M5O \times ECI] \sigma(\hat{r}</em>\text{M5O}) [M5O \times ECI]^T )</td>
</tr>
<tr>
<td>( \dddot{a}_\theta ) [TOP]</td>
<td>( \hat{M}_3 \times 3 \text{UN}_2 )</td>
<td>M3X3UN2</td>
<td>( \sigma(\hat{r}<em>\text{GEO}) = [EIO \times GEO] \sigma(\hat{r}</em>\text{ECI}) [EIO \times GEO]^T )</td>
</tr>
<tr>
<td>( \dddot{a}_\theta ) [TOP]</td>
<td>( \hat{M}_3 \times 3 \text{UN}_2 )</td>
<td>M3X3UN2</td>
<td>( \sigma(\hat{a}<em>\theta \text{ sensed}) = [GEO \times TOP] \sigma(\hat{r}</em>\text{GEO}) [GEO \times TOP]^T )</td>
</tr>
<tr>
<td>( \dot{a}_\theta ) [TOP]</td>
<td>( \hat{M}_3 \times 3 \text{UN}_2 )</td>
<td>M3X3UN2</td>
<td>( \sigma(\dot{a}<em>\theta \text{ sensed}) = [GEO \times TOP] \sigma(\dot{r}</em>\text{GEO}) [GEO \times TOP]^T )</td>
</tr>
<tr>
<td>( \dddot{a}<em>\theta ) [TOP] ( \dddot{a}</em>\theta ) [TOP]</td>
<td>( \hat{M}_3 \times 3 \text{UN}_2 )</td>
<td>M3X3UN2</td>
<td>( \sigma(\dddot{a}<em>\theta \text{ sensed}) = [GEO \times TOP] \sigma(\dddot{r}</em>\text{GEO}) [GEO \times TOP]^T )</td>
</tr>
<tr>
<td>SIGMAS</td>
<td>( \dddot{a}_\theta ) [TOP]</td>
<td>SIGMAS</td>
<td>( \sigma(\dddot{a}<em>\theta \text{ sensed}) = [C</em>{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}]^T )</td>
</tr>
</tbody>
</table>

\( \dot{v}_\text{wind} \) [TOP] = \([-w_H \cos \theta_{WH}, -w_H \sin \theta_{WH}, 0]^T \)

Wind-relative velocity of the Nav Base projected onto the TOPodetic frame

\( \dot{v}_\text{wind} \) [TOP] = \( fctn(\sigma(w_H), \sigma(u_{WH}), w_H, \sigma(w_v)) \)

(See description of WINDTOP in Section 4.1.2 for the expressions for the matrix elements)
Velocity of the Nav Base in the GEOgraphic coordinate system, but projected into the vehicle BODY axis coordinate system

\[
\begin{align*}
83 & \quad \text{DBOD(1)} \quad \text{ECIGEO} \quad \dot{r}_\theta [\text{GEO}] \text{ computed previously, words (32-34)} \\
84 & \quad \text{(2)} \quad \dot{v}_\theta [\text{BOD}] \quad M3X3T \quad [\text{ECI + BOD}] = [\text{M50 + BOD}] [\text{M50 + ECI}]^T \\
85 & \quad \text{(3)} \quad M3X3T \quad [\text{GEO + BOD}] = [\text{ECI + BOD}] [\text{ECI + GEO}]^T \\
\end{align*}
\]

Sensed acceleration of the Nav Base projected onto the vehicle BODY system

\[
\begin{align*}
86 & \quad \text{DDBOD(1)} \quad \text{OPIP} \quad \ddot{r}_\theta [\text{M50}] \text{ and } [\text{M50 + BOD}] \\
87 & \quad \text{(2)} \quad \ddot{a}_\theta [\text{BOD}] \\
88 & \quad \text{(3)} \quad M3X1 \quad \ddot{a}_\theta [\text{BOD}] = [\text{M50 + BOD}] \ddot{r}_\theta [\text{M50}] \\
\end{align*}
\]
Wind vector projected into the vehicle BODY coordinate system

89  WBODY(1)  WINDTOP  \( \mathbf{v}_{\text{wind}}^{\text{TOP}} = [-w_H \cos \theta_{\text{wind}}, -w_H \sin \theta_{\text{wind}}, 0]^T \)

90  (2)  \( \mathbf{v}_{\text{wind}}^{\text{BOD}} = [\mathbf{v}_{\text{TOP}} + \mathbf{v}_{\text{BOD}}] \)

91  (3)

Wind-relative velocity of the Nav Base expressed in vehicle BODY coordinates

92  DBODW(1)  OPiP  \( \mathbf{v}_{\text{TOP}}^{\text{NB}} \) computed previously, words (68-70)

93  (2)  \( \mathbf{v}_{\text{wind}}^{\text{BOD}} = [\mathbf{v}_{\text{TOP}} + \mathbf{v}_{\text{BOD}}] \)

94  (3)  VSUB  \( \mathbf{v}_{\text{TOP}}^{\text{NB}} = \mathbf{v}_{\text{TOP}}^{\text{NB}} - \mathbf{v}_{\text{TOP}}^{\text{wind}} \)

M3X1  \( \mathbf{v}_{\text{TOP}}^{\text{NB}} = [\mathbf{v}_{\text{TOP}} + \mathbf{v}_{\text{BOD}}] \mathbf{v}_{\text{TOP}}^{\text{NB}} \)
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Uncertainty of velocity of Nav Base in GEOgraphics coordinates projected into BOD system

95  DBODU(1)  OPIP  \( \frac{\dot{r}}{\dot{\theta}} [M50] \) extracted from \( \frac{\dot{s}}{s} [M50] \)

96  (2)  \( \frac{\dot{\phi}}{\dot{\theta}} [BOD] \)  OPIP  \( \phi, \theta, \psi \) extracted from \( \frac{\dot{\phi}}{\dot{\theta}} [S[M50]] \); \( \phi = \) roll, \( \theta = \) pitch, \( \psi = \) yaw

97  M3X3UN2  \( \frac{\dot{r}}{\dot{\theta}} [ECI] \) =

\[
[M50 + ECI] \frac{\dot{s}}{s} [M50] [M50 + ECI]^T
\]

ECIGEO  \{ [ECI + GEO] computed previously, for words (19-31)

M3X3UN2  \( \frac{\dot{r}}{\dot{\theta}} [GEO] \) =

\[
[ECI + GEO] \frac{\dot{s}}{s} [ECI] [ECI + GEO]^T
\]

M3X3T  \{ [ECI + BOD] = [M50 + BOD] [M50 + ECI]^T

M3X3T  \{ [GEO + BOD] = [ECI + BOD] [ECI + GEO]^T
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>DBODU continued</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M3XUN2</td>
<td>$\phi^\prime(\dot{\mathbf{v}}_\Theta [BOD]) = $</td>
<td>NB</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$[\text{GEO} + \text{BOD}] \phi(\ddot{r}_\Theta [\text{GEO}]) [\text{GEO} + \text{BOD}]^\top$</td>
<td>NB</td>
<td></td>
</tr>
</tbody>
</table>

| SBCALC | $\phi(\mathbf{v}_\Theta [BOD]) = \phi(\mathbf{v}_\Theta [BOD]) + \{\text{effects of body } \hat{\mathbf{v}}\}$ | NB | |
| | (See the description of SBCALC in Section 4.1.2) | | |

| SIGMAS | $\mathbf{\sigma}(\mathbf{v}_\Theta [BOD]) = [C_{11}^{\frac{1}{2}}, C_{22}^{\frac{1}{2}}, C_{33}^{\frac{1}{2}}]^\top$ | NB | |
| | where $[C_{ij}] = \phi(\mathbf{v}_\Theta [BOD])$ | NB | |

Uncertainty of sensed acceleration of Nav Base projected onto the BODy system

98 DDBODU(1)

| OPIP | $\phi(\ddot{r}_\Theta [M50]) \& \phi(\phi, \Theta, \Psi)$ from $\phi(\mathbf{s}[M50])$ | NB | |

99 (2)

| OPIP | $\mathbf{\sigma}(\dot{\mathbf{a}}_\Theta [BOD])$ | NB | [M50 + BOD] from TRJATTDATA |

100 (3)

| M3X3UN2 | $\phi(\dot{\mathbf{a}}_\Theta [BOD]) = $ | NB | |
| | $[\text{M50} + \text{BOD}] \phi(\ddot{r}_\Theta [\text{M50}]) [\text{M50} + \text{BOD}]^\top$ | NB | |
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word</th>
<th>name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>DDBODU</td>
<td>continued</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**SPBCALC**

\[ \ddot{a}_\theta [\text{BOD}] = \dot{a}_\theta [\text{BOD}] + \text{effects of body axis errors} \]

(See the description of SPBCALC in Section 4.1.2)

**SIGMAS**

\[ \ddot{a}_\theta [\text{BOD}] = [C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}]^T \]

where \([C_{ij}] = \ddot{a}_\theta [\text{BOD}]\)

Uncertainty in wind vector projected onto BODY axis coordinates

101  **WBODVU(1)**

OPIP  \( \phi(\phi, \theta, \psi) \) extracted from \( \ddot{a} [\text{M50}] \)

102  (2)

\( \ddot{\psi}_\theta [\text{BOD}] \) OPIP  \([\text{M50} + \text{BOD}] \) read from the TRJATTDATA file

103  (3)

GPIP  \([\text{M50} + \text{ECI}] \) read from the OPIP.IN file

**M3X3T**

\([E\text{CI} + \text{BOD}] = [\text{M50} + \text{BOD}] [\text{M50} + \text{ECI}]^T \)

**GEOTOP**

\([\text{GEO} + \text{TOP}] = \text{fctn}(\phi_{\text{GEOD}}, \lambda_{\text{GEOD}}) \)

**ECIGEO**

\([\text{ECI} + \text{GEO}] = \text{fctn}(\omega_{\theta}, \text{t}, \dot{r}_\theta [\text{ECI}], \dot{r}_\theta [\text{ECI}], \dot{r}_\theta [\text{ECI}]) \)

**M3X3**

\([\text{ECI} + \text{TOP}] = [\text{GEO} + \text{TOP}] [\text{ECI} + \text{GEO}] \)

**M3X3T**

\([\text{TOP} + \text{BOD}] = [\text{ECI} + \text{BOD}] [\text{ECI} + \text{TOP}]^T \)
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>WBODVU</td>
<td>continued</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**WINDTOP**

\[ \zeta(\vec{v} \text{ [TOP]} ) = \text{fctn}(\sigma(w_H), \sigma(\theta_{WH}), w_H, \sigma(w_v)) \]

(See description of WINDTOP in Section 4.1.2 for expressions for the matrix elements)

**M3X3UN2**

\[ \zeta'(\vec{v} \text{ [BOD]} ) = \text{wind} \]

\[ [\text{TOP + BOD}] \zeta(\vec{v} \text{ [TOP]} ) [\text{TOP + BOD}]^T \text{wind} \]

(See description of SPBCALC in Section 4.1.2)

**SIGMAS**

\[ \sigma(\vec{v} \theta \text{ [BOD]} ) = [C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}] \text{wind} \]

where \[ [C_{ij}] = \zeta(\vec{v} \theta \text{ [BOD]} ) \text{wind} \]

Uncertainty in the wind-relative velocity of the Nav Base expressed in BODY coordinates

104 **DBODWU(1)**

OPIP

\( \zeta(\phi, \theta, \psi) \) extracted from \( \zeta([M50]) \)

105 **(2)**

\[ \zeta(\vec{v} \text{ [BOD]} ) \text{wind} \]

OPIP

[\text{TOP + BOD}] \text{computed previously, words (101-103)}

106 **(3)**

\[ \zeta(\vec{v} \theta \text{ [TOP]} ) \text{computed previously, NB words (74-76)} \]
### DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>Word Name</th>
<th>Symbol</th>
<th>Program</th>
<th>Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>DBODWU continued</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### WINDTOP

\[
\vec{v}_{\text{TOP}} = [-w_H \cos \theta_{wH}, -w_H \sin \theta_{wH}, 0]^T_{\text{wind}}
\]

#### WINDTOP

\[
\phi(\vec{v}_{\text{TOP}}) = \text{fctn}(\sigma(w_H), \theta_{wH}, w_H, \sigma(w_v))_{\text{wind}}
\]

(See description of WINDTOP in Section 4.1.2 for the expressions for the matrix elements)

#### VSUB

\[
\vec{v}_{\text{TOP}} = \vec{v}_{\text{TOP}} - \vec{v}_{\text{TOP}}_{\text{NB}}
\]

#### OPIP

\[
\phi(\vec{v}_{\text{TOP}})_{\text{NB}} = \phi(\vec{v}_{\text{TOP}})_{\text{NB}} + \phi(\vec{v}_{\text{TOP}})_{\text{wind}}
\]

#### M3X3UN2

\[
\phi'(\vec{v}_{\text{BOD}})_{\text{wind}} = \left[\text{TOP + BOD}\right] \phi(\vec{v}_{\text{TOP}})_{\text{wind}}
\]

#### SPBCALC

\[
\phi(\vec{v}_{\text{BOD}})_{\text{wind}} = \phi'(\vec{v}_{\text{BOD}})_{\text{wind}} + \{ \text{effects of body axis errors} \}_{\text{NB}}
\]

(See description of SPBCALC in Section 4.1.2)
SIGMAS \( \tilde{\mathbf{v}} \) (wind \[BOD\]) = \([C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}]^T \)

where \([C_{ij}] = \psi(\mathbf{v} \text{ [BOD]})\)

Quaternion for transforming from M50 coordinates of BODy coordinates

107 Q50(1) \quad \text{OPIP} \quad \text{Read directly from the input file TRJATTDATA}
108 (2) \quad \tilde{q} \text{ [M50 + BOD]} \quad \text{and equivalenced to the output buffer array elements}
109 (3) \quad \text{110 (4)}

Index of selected IMU

111 PIMU \quad \text{OPIP} \quad \text{Read directly from the input file TRJATTDATA}
\quad \text{and equivalenced to the output buffer array element}

Transformation from the ECI to launch-site-located Plumbline coordinate system

\[ \text{ECIPLM [ECI + PLM]} \] is calculated on the first pass for ascent (only) from geophysical parameters, launch site parameters, and the time of SRB ignition
Position of the Nav Base relative to the Earth's center expressed in Plumbline coordinates (Ascent only)

112 PLM(1) OPIP Read \([\text{M50} \rightarrow \text{ECI}]\) from input file OPIP.IN

113 (2) \(\hat{\mathbf{r}}_{\theta} \left[ \text{PLM} \right]\) OPIP Read \(\hat{\mathbf{r}}_{\theta} \left[ \text{M50} \right]\) from the input file TRJATTDATA

\[ \text{NB NB} \]

114 (3) M3X1 \(\hat{\mathbf{r}}_{\theta} \left[ \text{ECI} \right] = [\text{M50} \rightarrow \text{ECI}] \hat{\mathbf{r}}_{\theta} \left[ \text{M50} \right]\)

\[ \text{NB NB} \]

\[ \text{NB NB} \]

Velocity of the Nav Base relative to the Earth's center expressed in Plumbline coordinates (Ascent only)

115 DPLM(1) OPIP Read \(\dot{\hat{\mathbf{r}}}_{\theta} \left[ \text{M50} \right]\) from TRJATTDATA

\[ \text{NB} \]

116 (2) \(\dot{\hat{\mathbf{r}}}_{\theta} \left[ \text{PLM} \right]\) M3X1 \(\dot{\hat{\mathbf{r}}}_{\theta} \left[ \text{ECI} \right] = [\text{M50} \rightarrow \text{ECI}] \dot{\hat{\mathbf{r}}}_{\theta} \left[ \text{M50} \right]\)

\[ \text{NB NB} \]

117 (3) M3X1 \(\dot{\hat{\mathbf{r}}}_{\theta} \left[ \text{PLM} \right] = [\text{ECI} \rightarrow \text{PLM}] \dot{\hat{\mathbf{r}}}_{\theta} \left[ \text{ECI} \right]\)

\[ \text{NB NB} \]

\[ \text{NB NB} \]
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name  symbol  program  algorithm

Sensed acceleration of the Nav Base relative to Earth's center expressed in Plumpline coordinates (Ascent only)

118 DDPLM(1)  OPIP  Read $\hat{r}_\Theta \,[M50]$ from TRJATTDATA

119 (2) $\hat{r}_\Theta \,[PLM]$  $\hat{r}_\Theta \,[ECI] = [M50 + ECI] \hat{r}_\Theta \,[M50]$

120 (3) $\hat{r}_\Theta \,[PLM] = [ECI + PLM] \hat{r}_\Theta \,[ECI]$

Uncertainty in the position of the Nav Base relative to the Earth's center expressed in Plumpline coordinates (Ascent only)

121 PLMU(1)  OPIP  $\Phi(\hat{r}_\Theta \,[M50])$ is extracted from $\Phi(\hat{r}_\Theta \,[M50])$

122 (2) $\Phi(\hat{r}_\Theta \,[PLM])$  M3X3UN2  $\Phi(\hat{r}_\Theta \,[ECI]) = [M50 + ECI] \Phi(\hat{r}_\Theta \,[M50]) [M50 + ECI]^T$

123 (3) M3X3UN2  $\Phi(\hat{r}_\Theta \,[PLM]) = [ECI + PLM] \Phi(\hat{r}_\Theta \,[ECI]) [ECI + PLM]^T$

36
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

PLMU continued

\[ \sigma(r_{\oplus}[\text{PLM}]) = \begin{bmatrix} C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2} \end{bmatrix}^T \]

where \([C_{ij}] = \Phi(r_{\oplus}[\text{PLM}])\)

Uncertainty in the velocity of the Nav Base relative to the Earth's center expressed in Plumline coordinates (Ascent only)

124 DPLMU(1) OPIP \( \dot{\Psi}(r_{\oplus}[M50]) \) is extracted from \( \dot{\Psi}(\hat{\mathbf{S}}[M50]) \)

125 M3X3UN2 \( \dot{\Psi}(r_{\oplus}[ECI]) = \)

\[ [M50 + ECI] \dot{\Psi}(r_{\oplus}[M50]) [M50 + ECI]^T \]

126 M3X3UN2 \( \dot{\Psi}(r_{\oplus}[PLM]) = [ECI + PLM] \dot{\Psi}(r_{\oplus}[ECI]) [ECI + PLM]^T \)

SIGMAS \( \dot{\sigma}(r_{\oplus}[\text{PLM}]) = \begin{bmatrix} C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2} \end{bmatrix}^T \)

where \([C_{ij}] = \Phi(r_{\oplus}[\text{PLM}])\)
Uncertainty in the sensed acceleration of the Nav Base relative to the Earth's center expressed in Plumbline coordinates (Ascent only)

127 DDPLMU(1)

OPIP $\mathcal{O}(r_{\oplus}[M50])$ is extracted from $\mathcal{O}(s_{M50})$

128 (2)

$\mathcal{O}(r_{\oplus}[PLM]) =$

$\mathcal{O}(r_{\oplus}[ECI]) =$

$\mathcal{O}(r_{\oplus}[M50]) [M50 + ECI]^T$

129 (3)

$\mathcal{O}(r_{\oplus}[M50]) =$

$\mathcal{O}(r_{\oplus}[ECI]) [ECI + PLM]^T$

SIGMAS $\mathcal{O}(r_{\oplus}[PLM]) = [C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}]^T$

where $[C_{ij}] = \mathcal{O}(r_{\oplus}[PLM])$
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word</th>
<th>name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Wind-relative velocity of the Nav Base but projected onto Plumbline coordinates (Ascent only)

130 PPLMW(1)  VSUB  \( \hat{\mathbf{v}} \) [TOP] was computed previously for wind NB words (104-106)

131 (2)  \( \hat{\mathbf{v}} \) [PLM]  M3X3T  \([\mathbf{ECI} + \mathbf{TOP}]\) was computed previously for words (101-103)

132 (3)  ECIPLM  \([\mathbf{ECI} + \mathbf{PLM}]\) was computed previously just before words (112-114)

\[
\begin{align*}
\mathbf{M3TX1} & \quad \hat{\mathbf{v}} \ [\mathbf{ECI}] = [\mathbf{ECI} + \mathbf{TOP}]^T \hat{\mathbf{v}} \ [\mathbf{TOP}] \\
& \quad \text{wind} \quad \text{wind} \\
& \quad \text{NB} \quad \text{NB}
\end{align*}
\]

\[
\begin{align*}
\mathbf{M3X1} & \quad \hat{\mathbf{v}} \ [\mathbf{PLM}] = [\mathbf{ECI} + \mathbf{PLM}] \hat{\mathbf{v}} \ [\mathbf{ECI}] \\
& \quad \text{wind} \quad \text{wind} \\
& \quad \text{NB} \quad \text{NB}
\end{align*}
\]
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Uncertainty in the wind-relative velocity of the Nav Base projected onto the Plumbline coordinate system (Ascent only)

133 DPLMWU(1)  WINDTOP \( \mathcal{C}(\mathbf{v}_{\text{top}}) \) computed previously for wind words (80-82)

134 (2)  M3X3UN2 \( \mathcal{C}(\mathbf{v}_{\oplus \text{top}}) \) computed previously for NB words (74-76)

135 (3)  OPIP \( \mathcal{C}(\mathbf{v}_{\text{top}}) = \mathcal{C}(\mathbf{v}_{\oplus \text{top}}) + \mathcal{C}(\mathbf{v}_{\text{top}}) \)

M3X3UN1 \( \mathcal{C}(\mathbf{v}_{\text{eci}}) = \)
\( [\text{eci} + \text{top}]^T \mathcal{C}(\mathbf{v}_{\text{top}}) [\text{eci} + \text{top}] \)

M3X3UN2 \( \mathcal{C}(\mathbf{v}_{\text{plm}}) = \)
\( [\text{eci} + \text{plm}] \mathcal{C}(\mathbf{v}_{\text{eci}}) [\text{eci} + \text{plm}]^T \)

SIGMAS \( \mathbf{\sigma}(\mathbf{v}_{\text{plm}}) = [c_{11}^{1/2}, c_{22}^{1/2}, c_{33}^{1/2}]^T \)
where \( [c_{ij}] = \mathcal{C}(\mathbf{v}_{\text{plm}}) \)
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLMC(1)</td>
<td>r^[PLM]</td>
<td>OPIP</td>
<td>read r^[NB][BOD] from the input file TRJATTDATA</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ECIPLM</td>
<td>ECI + PLM computed previously before words (112-114)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M3X1</td>
<td>( \mathbf{M50} \rightarrow \mathbf{BOD} ) ( \mathbf{T} ) r^[NB][BOD]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M3X1</td>
<td>r^[NB][ECI] = ( \mathbf{M50} \rightarrow \mathbf{ECI} ) r^[NB][M50]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M3X1</td>
<td>r^[NB][PLM] = ( \mathbf{ECI} \rightarrow \mathbf{PLM} ) r^[NB][ECI]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OPIP</td>
<td>r^[PLM] is word set (112-114)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>VADD</td>
<td>r^[PLM] = r^[PLM] + r^[NB][PLM]</td>
</tr>
</tbody>
</table>

Position of the vehicle Center of Mass relative to Earth's center expressed in Plumbline coordinates (Ascent only)
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Velocity of the vehicle Center of Mass relative to the Earth's center expressed in Plumbline coordinates (Ascent only)

139 DPLMC(1)

\[ \mathbf{v}_{\text{NB}}(\text{BOD}) = \mathbf{\dot{\omega}}(\text{BOD}) \times \mathbf{r}_{\text{NB}}(\text{BOD}) \]  

140 (2)

\[ \mathbf{r}_{\text{NB}}(\text{M50}) = (\text{M50} + \text{BOD})^T \mathbf{v}_{\text{NB}}(\text{BOD}) \]

141 (3)

\[ \mathbf{r}_{\text{NB}}(\text{ECI}) = (\text{M50} + \text{ECI}) \mathbf{r}_{\text{NB}}(\text{M50}) \]

\[ \mathbf{r}_{\text{NB}}(\text{PLM}) = (\text{ECI} + \text{PLM}) \mathbf{r}_{\text{NB}}(\text{ECI}) \]

142 DDPLMC(1)

\[ \mathbf{\dot{\omega}}_{\text{NB}}(\text{BOD}) = \mathbf{\dot{\omega}}(\text{BOD}) \times \mathbf{\dot{v}}_{\text{NB}}(\text{BOD}) \]

143 (2)

\[ \mathbf{r}_{\text{NB}}(\text{M50}) = [\text{M50} + \text{BOD}]^T \mathbf{\ddot{a}}_{\text{NB}}(\text{BOD}) \]

Sensed acceleration of the vehicle Center of Mass expressed in Plumbline coordinates (Ascent only)

142 DDPLMC(1)

\[ \mathbf{\dot{\omega}}_{\text{NB}}(\text{BOD}) = \mathbf{\dot{\omega}}(\text{BOD}) \times \mathbf{\dot{v}}_{\text{NB}}(\text{BOD}) \]

143 (2)

\[ \mathbf{r}_{\text{NB}}(\text{M50}) = [\text{M50} + \text{BOD}]^T \mathbf{\ddot{a}}_{\text{NB}}(\text{BOD}) \]
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>DDPLMC</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \mathbf{M3X1} \]

\[ \mathbf{\ddot{r}_{NB}[ECI]} = [\mathbf{M50} \rightarrow \mathbf{ECI}] \mathbf{\ddot{r}_{NB}[M50]} \]

\[ \Rightarrow \mathbf{CM} \]

\[ \mathbf{M3X1} \]

\[ \mathbf{\ddot{r}_{NB}[PLM]} = [\mathbf{ECI} \rightarrow \mathbf{PLM}] \mathbf{\ddot{r}_{NB}[ECI]} \]

\[ \Rightarrow \mathbf{CM} \]

\[ \Rightarrow \mathbf{OPIP} \]

\[ \mathbf{\ddot{r}_{\theta}[PLM]} \text{ is word set (118-120)} \]

\[ \Rightarrow \mathbf{NB} \]

\[ \Rightarrow \mathbf{VADD} \]

\[ \mathbf{\ddot{r}_{\theta}[PLM]} = \mathbf{\ddot{r}_{\theta}[PLM]} + \mathbf{\ddot{r}_{NB}[PLM]} \]

\[ \Rightarrow \mathbf{CM} \quad \mathbf{NB} \quad \mathbf{CM} \]

Wind-relative velocity of the vehicle Center of Mass expressed in Plumbline coordinates (Ascent only)

145 DPLMCW(1)

146 \( \ddot{v} \quad [\mathbf{PLM}] \)

\[ \Rightarrow \mathbf{wind} \quad \mathbf{CM} \]

147 \( \mathbf{M3X1} \)

\[ \mathbf{\ddot{r}_{NB}[PLM]} \text{ computed previously for } \mathbf{CM} \]

\[ \Rightarrow \text{words (139-141)} \]

148 \( \mathbf{M3X3} \)

\[ [\mathbf{ECI} \rightarrow \mathbf{TOP}] \text{ computed previously for words (101-103)} \]
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECIPLM</td>
<td>$\dot{\mathbf{v}}$ $[\text{ECI} \times \text{PLM}]$</td>
<td>computed previously for words (112-114)</td>
<td></td>
</tr>
<tr>
<td>M3TX1</td>
<td>$\dot{\mathbf{v}}$ $[\text{ECI}] = [\text{ECI} \times \text{TOP}]^T \dot{\mathbf{v}} [\text{TOP}]$</td>
<td>$\dot{\mathbf{v}}$ $[\text{TOP}]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{wind}$ $\text{wind}$</td>
<td>$\text{wind}$ $\text{wind}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{NB}$ $\text{NB}$</td>
<td>$\text{NB}$ $\text{NB}$</td>
<td></td>
</tr>
<tr>
<td>M3X1</td>
<td>$\dot{\mathbf{v}}$ $[\text{PLM}] = [\text{ECI} \times \text{PLM}] \dot{\mathbf{v}} [\text{ECI}]$</td>
<td>$\dot{\mathbf{v}}$ $[\text{ECI}]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{wind}$ $\text{wind}$</td>
<td>$\text{wind}$ $\text{wind}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{NB}$ $\text{NB}$</td>
<td>$\text{NB}$ $\text{NB}$</td>
<td></td>
</tr>
<tr>
<td>VADD</td>
<td>$\dot{\mathbf{v}}$ $[\text{PLM}] = \dot{\mathbf{v}} [\text{PLM}] + \dot{r}_{\text{NB}[\text{PLM}]}$</td>
<td>$\dot{\mathbf{v}} [\text{PLM}]$ $\dot{r}_{\text{NB}[\text{PLM}]}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{wind}$ $\text{wind}$</td>
<td>$\text{wind}$ $\text{CM}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{CM}$ $\text{NB}$</td>
<td>$\text{CM}$ $\text{NB}$</td>
<td></td>
</tr>
</tbody>
</table>

Yaw, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

148 YAW $\psi$ EULANG Given in the description of EULANG in Section 4.1.2

149 YAWU $\sigma(\psi)$

Pitch, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

150 PITCH $\theta$ EULANG Given in the description of EULANG in Section 4.1.2

151 PITCHU $\sigma(\theta)$
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Roll, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

152 ROLL \( \phi \) EULANG Given in the description of EULANG in Section 4.1.2

153 ROLLU \( \sigma(\phi) \)

Yaw Rate, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

154 YAWD \( \dot{\psi} \) EULANG Given in the description of EULANG in Section 4.1.2

155 YAWDU \( \sigma(\dot{\psi}) \)

Pitch Rate, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

156 PITCHD \( \dot{\theta} \) EULANG Given in the description of EULANG in Section 4.1.2

157 PITCHDU \( \sigma(\dot{\theta}) \)

Roll Rate, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

158 ROLLD \( \dot{\phi} \) EULANG Given in the description of EULANG in Section 4.1.2

159 ROLLDU \( \sigma(\dot{\phi}) \)
### DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>160 BA</td>
<td>$\phi$</td>
<td>BANK</td>
<td>Given in the description of BANK in Section 4.1.2</td>
</tr>
</tbody>
</table>

#### Longitude of the Nav Base

| 161 GEOLONG | $\lambda_{\text{GEOD}}$ | GEOGEOD | Given in the description of GEOGEOD in Section 4.1.2 |

#### Geodetic Latitude of the Nav Base

| 162 GEODLAT | $\phi_{\text{GEOD}}$ | GEOGEOD | Given in the description of GEOGEOD in Section 4.1.2 |

#### Geodetic altitude (height) above the (Fischer) reference ellipsoid of the Nav Base

| 163 GEODALT | $h_{\text{GEOD}}$ | GEOGEOD | Given in the description of GEOGEOD in Section 4.1.2 |

#### Uncertainty in the geodetic altitude (height) of the Nav Base

| 164 GEOHU | $\sigma(h_{\text{GEOD}})$ | OPIP | Let the local vertical unit vector, $\hat{n}$, which is normal to the reference ellipsoid be $\hat{n} = [n_1, n_2, n_3]^T$ where $n_1 = \cos\phi_{\text{GEOD}} \cos\lambda_{\text{GEOD}}$, $n_2 = \cos\phi_{\text{GEOD}} \sin\lambda_{\text{GEOD}}$, $n_3 = \sin\phi_{\text{GEOD}}$ |

46
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOHU</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

OPIP Read M50COV from input file TRJATTDATA

\[ \Omega(\hat{r}_\theta [M50]) = \text{MVCOV}(i,j); i = 1,3; j = 1,3 \]

\[ \text{OPIP} \ (f, [M50]) = MVCOV(i,j); i = 1,3; j = 1,3 \]

\[ \text{NB} \]

\[ M3X3UN2 \ (f, [ECI]) = \]

\[ \text{NB} \]

\[ [M50 + ECI] \Omega(\hat{r}_\theta [M50]) [M50 + ECI]^T \]

\[ \text{NB} \]

\[ M3X3UN2 \ (f, [GEO]) = \]

\[ \text{NB} \]

\[ [ECI + GEO] \Omega(\hat{r}_\theta [ECI]) [ECI + GEO]^T \]

\[ \text{NB} \]

\[ \text{OPIP} \quad \sigma(h_{GEOD})^2 = \hat{n} \Omega(\hat{r}_\theta [GEO]) \hat{n}^T \]

\[ \text{NB} \]

\[ \text{OPIP} \quad \sigma(h_{GEOD}) = [\sigma(h_{GEOD})^2]^{1/2} \]

Geodetic altitude rate of the Nav Base

165 DH \[ \dot{h}_{GEOD} \]

\[ \text{OPIP} \quad \dot{\hat{r}}_\theta [GEO] \] computed previously, words (32-34)

\[ \text{NB} \]

\[ \dot{h}_{GEOD} = \hat{n} \cdot \dot{\hat{r}}_\theta [GEO] \]

\[ \text{NB} \]
Uncertainty in geodetic altitude rate of the Nav Base

\[
\sigma(h_{GEOD})^2 = \hat{n} \cdot \dot{\sigma}(\hat{r}_\theta [M50]) \cdot \hat{n}^T
\]

\[
\sigma(h_{GEOD}) = \left[\sigma(h_{GEOD})^2\right]^{1/2}
\]

Declination of the Nav Base

\[
\delta_{NB} = \arctan \frac{z_g}{(x_g^2 + y_g^2)^{1/2}}
\]
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Magnitude of the geocentric position vector of the Nav Base

168 RM $\mathbf{R}_\theta$ FPANG2

$\mathbf{R}_\theta = |\mathbf{r}_\theta^{\text{[M50]}}|$ NB NB NB

Magnitude of inertial velocity of the Nav Base

169 VM $\mathbf{V}_{NB}$ FPANG2

$\mathbf{V}_{NB} = |\mathbf{\dot{r}}_\theta^{\text{[M50]}}|$ NB

Flight path angle as observed in the ECI coordinate frame

170 FPM $\alpha$ FPANG2

Given in the description of FPANG2 in Section 4.1.2

Azimuth of the velocity observed in the ECI coordinate frame

171 AZM $A_z$ FPANG2

Given in the description of FPANG2 of Section 4.1.2
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slant range of the Nav Base measured from the launch site (Ascent) or Landing Site (Runway threshold, Descent)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

172 RS $R_s$ OPIP Read geodetic position parameters $[\phi (s), \lambda (s), h (s)]$ for $s =$ Origin of Launch Site (Ascent) or $s =$ Origin of Landing Site (Descent), from input file OPIP.IN

GEODGEO $r_s^\theta [GEO] = fctn(\phi (s), \lambda (s), h (s))$ for $s =$ Origin of Launch Site (Ascent) or $s =$ Origin of Landing Site (Descent), (Algorithm given in the description of GEODGEO in Section 4.1.2)

VSUB $r_s^\theta [GEO] = r_s^\theta [GEO] - r_s^\theta [GEO]$ for $s =$ Origin of Launch Site (Ascent) or $s =$ Origin of Landing Site (Descent)

OPIP $R_s = |r_s^\theta [GEO]|$
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Transformation matrix, M50 frame to vehicle BODY frame

\[
M_{BOD}(\begin{array}{ccc}
173 & M_{50BOD}(1,1) & OPIP \\
174 & (1,2) & \text{Read from the input file TRJATTDATA} \\
175 & (1,3) & \text{and loaded directly into the output} \\
176 & (2,1) & \text{array.} \\
177 & (2,2) & \text{[M50 + BOD]} \\
178 & (2,3) & \\
179 & (3,1) & \\
180 & (3,2) & \\
181 & (3,3) & 
\end{array})
\]

Wind-relative velocity magnitude of the vehicle Center of Mass

\[
V_{\text{wind}}^{\text{CM}} \quad \text{FPANG} \quad \text{Given in the description of FPANG in Section 4.1.2}
\]

Uncertainty in the wind-relative velocity magnitude of the vehicle Center of Mass

\[
\sigma(V_{\text{wind}}^{\text{CM}}) \quad \text{FPANG} \quad \text{Given in the description of FPANG in Section 4.1.2}
\]

Wind-relative flight path angle of the vehicle Center of Mass

\[
\gamma_{\text{wind}}^{\text{CM}} \quad \text{FPANG} \quad \text{Given in the description of FPANG in Section 4.1.2}
\]
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty in the wind-relative flight path angle of the vehicle Center of Mass</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>185 GAMTOPU</td>
<td>$\sigma(\gamma_{\text{wind}})$</td>
<td>FPANG</td>
<td>Given in the description of FPANG in Section 4.1.2</td>
</tr>
<tr>
<td>Wind-relative azimuth of the vehicle Center of Mass</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>186 PSITOP</td>
<td>$\psi_{\text{wind}}$</td>
<td>FPANG</td>
<td>Given in the description of FPANG in Section 4.1.2</td>
</tr>
<tr>
<td>Uncertainty in the wind-relative azimuth of the vehicle Center of Mass</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>187 PSITOPU</td>
<td>$\sigma(\psi_{\text{wind}})$</td>
<td>FPANG</td>
<td>Given in the description of FPANG in Section 4.1.2</td>
</tr>
<tr>
<td>Earth surface range from subvehicle (Nav Base) point to $s$, where $s$ is the Launch Site origin (for Ascent analysis) or Runway threshold (for Descent analysis)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>188 S</td>
<td>$R_{NBS}$</td>
<td>OPIP</td>
<td>Read geodetic position parameters, $[\phi(s), \lambda(s), h(s)]$</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>GEOD</td>
<td>GEOD</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>for $s =$ Origin of Launch Site (Ascent) or $s =$ Origin of Landing Site (Descent), from input file OPIP.IN</td>
<td></td>
</tr>
</tbody>
</table>
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word</th>
<th>name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>S continued</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**GEODGEO**

\[ \hat{r}_@^{[\text{GEO}]} = \text{fctn}(\phi(s), \lambda(s), h(s)) \]

(Algorithm given in the description of GEODGEO in Section 4.1.2)

Let \( \hat{r}_@^{[\text{GEO}]} = [x_s, y_s, z_s]^T \)

and \( \hat{r}_@^{[\text{GEO}]} = [x_{NB}, y_{NB}, z_{NB}]^T \)

Then

\[ R_@^2 = x_s^2 + y_s^2 + z_s^2 \]

\[ R_@^2 = x_{NB}^2 + y_{NB}^2 + z_{NB}^2 \]

Then,

**OPIP**

\[ R_{@S} = R_@^2 \]

\[ R_{@S} = R_{@S} \beta \]

\( \beta \) is the angle, in radians, between the geocentric vectors to \( s \) and to the Nav Base and is calculated from

\[ R_{@S}^2 - (R_@ R_@ \cos \beta)^2 \]

**OPIP**

\[ \beta = \text{arc tan} \left( \frac{R_@ R_@ \cos \beta}{s_{NB} s_{NB}} \right) \]

where

\[ R_@ R_@ \cos \beta = \hat{r}_@^{[\text{GEO}]} \cdot \hat{r}_@^{[\text{GEO}]} \]

\[ s_{NB} s_{NB} s_{NB} \]
**BET OUTPUT PRODUCTS (continued)**

**DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER**

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Wind-relative angle of attack for the vehicle Center of Mass

189 ALPHA \( \alpha_w \) AERODYN Given in the description of AERODYN in Section 4.1.2

Uncertainty in the wind-relative angle of attack for the vehicle Center of Mass

190 ALPHAU \( \sigma(\alpha_w) \) AERODYN Given in the description of AERODYN in Section 4.1.2

Wind-relative guidance sideslip angle for the vehicle Center of Mass

191 BETAP \( \beta_{WG} \) AERODYN Given in the description of AERODYN in Section 4.1.2

Uncertainty in the wind-relative guidance sideslip angle for the Center of Mass

192 BETAPU \( \sigma(\beta_{WG}) \) AERODYN Given in the description of AERODYN in Section 4.1.2

Wind-relative dynamic pressure for the total Center of Mass velocity, in lb/ft²

193 QAERO \( q \) AERODYN Given in the description of AERODYN in Section 4.1.2
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty in the dynamic pressure for the velocity of the Center of Mass</td>
<td>194 QAEROU</td>
<td>σ(q)</td>
<td>AERODYN Given in the description of AERODYN in Section 4.1.2</td>
</tr>
<tr>
<td>Wind-relative pitch dynamic pressure for the Center of Mass, in lb-deg/ft²</td>
<td>195 QAALPHA</td>
<td>qₐ</td>
<td>AERODYN Given in the description of AERODYN in Section 4.1.2</td>
</tr>
<tr>
<td>Wind-relative yaw dynamic pressure for the Center of Mass, in lb-deg/ft²</td>
<td>196 QBETA</td>
<td>qₜ</td>
<td>AERODYN Given in the description of AERODYN in Section 4.1.2</td>
</tr>
<tr>
<td>Wind-relative Mach number at the Center of Mass</td>
<td>197 MACH</td>
<td>M</td>
<td>AERODYN Given in the description of AERODYN in Section 4.1.2</td>
</tr>
<tr>
<td>Uncertainty in the wind-relative Mach number at the Center of Mass</td>
<td>198 MACHU</td>
<td>σ(M)</td>
<td>AERODYN Given in the description of AERODYN in Section 4.1.2</td>
</tr>
</tbody>
</table>
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word</th>
<th>name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind-relative viscous parameter affecting the motion of the Center of Mass</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

199 VOO $v$ AERODYN Given in the description of AERODYN in Section 4.1.2

Uncertainty in the wind-relative viscous parameter for the Center of Mass

200 VOOU $\sigma(v)$ AERODYN Given in the description of AERODYN in Section 4.1.2

Ambient atmospheric temperature in °R

201 METDATA(3) $T_A$ OPPIP Placed in output buffer directly

Uncertainty in ambient atmospheric temperature in °R

202 METDATA(7) $\sigma(T_A)$ OPPIP Placed in output buffer directly

Ambient atmospheric pressure in lb/ft$^2$

203 METDATA(4) $P_A$ OPPIP Placed in output buffer directly
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Uncertainty in ambient atmospheric pressure in lb/ft²

204 METDATA(8) \( \sigma(P_A) \) OPIP Placed in output buffer directly

Atmospheric mass density in slugs/ft²

205 METDATA(5) \( \rho_{AM} \) OPIP Placed in output buffer directly

Wind-relative equivalent air speed at the Center of Mass

206 EAS \( V_{EA} \) CM AERODYN Given in the description of AERODYN in Section 4.1.2

Uncertainty in the wind-relative equivalent air speed at the Center of Mass

207 EASU \( \sigma(V_{EA}) \) CM AERODYN Given in the description of AERODYN in Section 4.1.2

Load factor at the Center of Mass, in g's [ft/sec²]

208 L \( n \) AERODYN Given in the description of AERODYN in Section 4.1.2
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Wind-relative drag over mass for the Center of Mass

209 DOM  \( F_{D/M} \)  AERODYN  Given in the description of AERODYN in Section 4.1.2

Wind-relative lift over drag for the Center of Mass

210 LOD  \( L_D \)  AERODYN  Given in the description of AERODYN in Section 4.1.2

Wind-relative aerodynamic side slip angle for the Center of Mass

211 BETA  \( \beta_{WA} \)  AERODYN  Given in the description of AERODYN in Section 4.1.2

Uncertainty in the wind-relative aerodynamic sideslip angle

212 BETAU  \( \sigma(\beta_{WA}) \)  AERODYN  Given in the description of AERODYN in Section 4.1.2

Euler angles, yaw (\( \psi_T \)), pitch (\( \theta_T \)), and roll (\( \phi_T \)), expressed in TOPodetic coordinates

213 TOPEUL(1)  \( \psi_T \)  OPIP  Let \( [\text{TOP} \rightarrow \text{BOD}] = [m_{ij}] \) which was calculated previously for words (101-103)
BET OUTPRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>
| 214 TOPEUL(2) | θ_T | OPIP | (1) Pitch not vertical  
ψ = arctan(m_{12}/m_{11})  
θ = - arctan(m_{13}) |
| 215 TOPEUL(3) | φ_T | OPIP | (2) Pitch vertical  
ψ = 0  
θ = ± 90° = - sign(90°, m_{13})  
ϕ = arctan(m_{23}/m_{33}) |
| 215 TOPEUL(3) | φ_T | OPIP | (3) For Ascent and if ϕ < 0°,  
ϕ = ϕ + 360° |

Euler angle rates relative to the TOPodetic frame

| 216 TOPRATE(1) | \( \hat{\omega}^{[\text{TOP}]} \) | OPIP | [ECI + TOP] was computed previously for words (101-103) |
| 217 (2) | \( \hat{\omega}^{[\text{TOP}]} \) | OPIP | [M50 + ECI] was read from input file OPIP.IN |
| 218 (3) | \( \hat{\omega}^{[\text{TOP}]} \) | M3X3 | [M50 + TOP] = [ECI + TOP] [M50 + ECI] |
| 218 | \( \hat{\omega}^{[\text{TOP}]} \) | EULRATE | \( \hat{\omega}^{[\text{TOP}]} = [\dot{\psi}_T, \dot{\phi}_T, \dot{\phi}_T]^T \) |
| | | | calculated by algorithm given in the description of EULRATE in Section 4.1.2 |
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name          symbol    program algorithm

Inertial body angular rates around the BODY axes

\[
\begin{align*}
\omega_{1B} &= S_{13} \cdot C_{D/R} \\
\omega_{2B} &= S_{14} \cdot C_{D/R} \\
\omega_{3B} &= S_{15} \cdot C_{D/R}
\end{align*}
\]

Uncertainties inertial body angular rates around the BODY axes

\[
\begin{align*}
\mathbf{\omega}^{\circ}[\text{BOD}] &= [\omega_{1B}, \omega_{2B}, \omega_{3B}]^T
\end{align*}
\]
BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBANGA(1)</td>
<td></td>
<td>OPIP</td>
<td></td>
</tr>
<tr>
<td>IBANGA(2)</td>
<td></td>
<td>OPIP</td>
<td></td>
</tr>
<tr>
<td>IBANGA(3)</td>
<td></td>
<td>OPIP</td>
<td></td>
</tr>
</tbody>
</table>

Inertial body angular accelerations around the BODY axes

\[ \sigma(\omega_{xB}) = \left( C_{13,13} \right)^{1/2} \times C_{D/R} \]
\[ \sigma(\omega_{yB}) = \left( C_{14,14} \right)^{1/2} \times C_{D/R} \]
\[ \sigma(\omega_{zB}) = \left( C_{15,15} \right)^{1/2} \times C_{D/R} \]

\[ \mathbf{\hat{\omega}}(\mathbf{[BOD]}) = [\sigma(\omega_{xB}), \sigma(\omega_{yB}), \sigma(\omega_{zB})]^T \]

225 IBANGA(1) --- OPIP Read M50 state vector from input file

\[ \mathbf{\hat{\omega}} = [S_{16} \ldots S_{18}]^T \]

convert from radians to degrees

226 IBANGA(2) --- OPIP

\[ \omega_{xB} = S_{16} \times C_{D/R} \]
\[ \omega_{yB} = S_{17} \times C_{D/R} \]
\[ \omega_{zB} = S_{18} \times C_{D/R} \]

\[ \mathbf{\omega} = [\omega_{xB}, \omega_{yB}, \omega_{zB}]^T \]

61
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Uncertainties in the inertial body accelerations around the BODY axes

228 IBANGAU(1)  
229 (2) \( \hat{\sigma}(\hat{\omega}) \)  
230 (3) \[
\begin{align*}
\hat{\sigma}(\omega_{xB}) &= (C_{16,16})^{1/2} \cdot C_{D/R} \\
\hat{\sigma}(\omega_{yB}) &= (C_{17,17})^{1/2} \cdot C_{D/R} \\
\hat{\sigma}(\omega_{zB}) &= (C_{18,18})^{1/2} \cdot C_{D/R}
\end{align*}
\]

228 IBANGAU(1)  
229 (2) \( \hat{\sigma}(\hat{\omega}) \)  
230 (3) \[
\hat{\sigma}(\omega) = [\omega_{xB}, \omega_{yB}, \omega_{zB}]^T
\]

Sensed acceleration of the vehicle Center of Mass expressed in BODY axes coordinates

231 DDBODC(1)  
232 (2) \( \ddot{r}_{\Theta}[\text{BOD}] \)  
233 (3) \[
\begin{align*}
\dot{r}_{\Theta}[\text{BOD}] &= \dot{r}_{\Theta}[\text{M50}] + \dot{r}_{\Theta}[\text{M50}] \\
\dot{r}_{\Theta}[\text{BOD}] &= \dot{r}_{\Theta}[\text{BOD}] + \dot{r}_{\Theta}[\text{BOD}] \\
\dot{r}_{\Theta}[\text{BOD}] &= \dot{r}_{\Theta}[\text{BOD}] + \dot{r}_{\Theta}[\text{BOD}]
\end{align*}
\]
DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

<table>
<thead>
<tr>
<th>word name</th>
<th>symbol</th>
<th>program</th>
<th>algorithm</th>
</tr>
</thead>
</table>

Uncertainty in the sensed acceleration of the vehicle Center of Mass in BODY coordinates

234 \text{DDBODCU(1)} \begin{align*}
\text{OPIP} & \quad \text{Read } \hat{s}[M50], \hat{r}_{NB}[BOD], [M50 \rightarrow BOD] \\
\text{CM} & \quad \text{and } \zeta(\hat{s}[M50]) \text{ from TRJATTDATA}
\end{align*}

235 \begin{align*}
\text{OPIP} & \quad \text{Read } \hat{\sigma}(\hat{r}_{NB}[BOD])^2 \text{ from OPIP.IN} \\
\text{CM} & \quad \text{CM}
\end{align*}

\text{DDBC} \quad \zeta(\hat{r}_{NB}[BOD]) = fctn(\hat{s}[M50], \hat{r}_{NB}[BOD], \\
\text{CM} & \quad [M50 \rightarrow BOD], \zeta(\hat{s}[M50]), \sigma(\hat{r}_{NB}[BOD])^2) \\
\text{CM}

\text{OPIP} \quad \hat{\sigma}(\hat{r}_{NB}[BOD]) = [C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}]^T \\
\text{CM} & \quad \text{CM}

\text{where } [C_{ij}] = \zeta(\hat{r}_{NB}[BOD])

Position of the vehicle Center of Mass relative to the Nav Base in BODY coordinates

237 \text{BODCG(1)} \quad \text{OPIP} \quad \text{Read from the input file TRJATTDATA and placed directly into the output buffer array.}

238 \begin{align*}
\text{OPIP} & \quad \hat{r}_{NB}[BOD] \\
\text{CM} & \quad \text{CM}
\end{align*}

239 \begin{align*}
\text{OPIP} & \quad \text{Read from the input file TRJATTDATA and placed directly into the output buffer array.}
\end{align*}
4.1.2 Subroutines Called by OPIP

This section contains the description of subroutines called by OPIP and which contain significant algorithms needed for computing of Ascent/Descent output products. The subsection for each subroutine starts with a list of terms in the subroutine calling argument. Each output term is indicated in the text by the symbol *>>* next to the left-hand margin.

The list of subroutines documented in this section is as follows:

<table>
<thead>
<tr>
<th>SUBROUTINE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>AERODYN</td>
<td>65</td>
</tr>
<tr>
<td>BANK</td>
<td>76</td>
</tr>
<tr>
<td>EULANG</td>
<td>79</td>
</tr>
<tr>
<td>EULRATE</td>
<td>94</td>
</tr>
<tr>
<td>FPANG</td>
<td>96</td>
</tr>
<tr>
<td>FPANG2</td>
<td>99</td>
</tr>
<tr>
<td>GEODEGEO</td>
<td>103</td>
</tr>
<tr>
<td>GEOGEOD</td>
<td>105</td>
</tr>
<tr>
<td>SPBCALC</td>
<td>108</td>
</tr>
<tr>
<td>WINDTOP</td>
<td>110</td>
</tr>
</tbody>
</table>
Calling Argument List (in list order)

Input

$L_1$  
First-pass flag.

$C_{D/R}$  
Conversion constant equal to degrees per radian.

$\hat{v}_{\text{wind}}^{\text{CM}}$  
Velocity of vehicle Center of Mass with respect to the wind. This vector is expressed in vehicle BODy coordinates.

$\mathcal{C}(\hat{v}_{\text{wind}}^{\text{CM}})$  
Covariance matrix of $\hat{v}_{\text{wind}}^{\text{CM}}$.

$\hat{a}_{\text{CM}}^{\text{BOD}}$  
Sensed acceleration of the Center of Mass expressed in BODy coordinates.

$\rho_{\text{AM}}$  
Atmospheric mass density (slugs/ft$^3$).

$\sigma(\rho_{\text{AM}})$  
Uncertainty in atmospheric mass density.

$T_{\text{AR}}$  
Atmospheric temperature in degrees Rankine.

$\sigma(T_{\text{AR}})$  
Uncertainty in the atmospheric temperature, $T_{\text{AR}}$.

Output

$\alpha_W$  
Wind-relative angle of attack (degrees).

$\sigma(\alpha_W)$  
Uncertainty in angle of attack.

$\beta_{WG}$  
Wind-relative guidance sideslip angle (degrees).
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\beta_{WG})$</td>
<td>Uncertainty in $\beta_{WG}$.</td>
</tr>
<tr>
<td>$\beta_{WA}$</td>
<td>Wind-relative aerodynamic sideslip angle.</td>
</tr>
<tr>
<td>$\sigma(\beta_{WA})$</td>
<td>Uncertainty in $\beta_{WA}$.</td>
</tr>
<tr>
<td>$q$</td>
<td>Total dynamic pressure.</td>
</tr>
<tr>
<td>$\sigma(q)$</td>
<td>Uncertainty in the total dynamic pressure.</td>
</tr>
<tr>
<td>$q_\alpha$</td>
<td>Pitch dynamic pressure.</td>
</tr>
<tr>
<td>$q_\beta$</td>
<td>Yaw dynamic pressure.</td>
</tr>
<tr>
<td>$M$</td>
<td>Wind-relative Mach number.</td>
</tr>
<tr>
<td>$\sigma(M)$</td>
<td>Uncertainty in $M$.</td>
</tr>
<tr>
<td>$V_{EA}$</td>
<td>Equivalent air speed of the Center of Mass (knots).</td>
</tr>
<tr>
<td>$\sigma(V_{EA})$</td>
<td>Uncertainty in $V_{EA}$.</td>
</tr>
<tr>
<td>$n$</td>
<td>Wind-relative load factor (in units of g, the sea-level acceleration of gravity).</td>
</tr>
<tr>
<td>$\hat{a}_{WD}$</td>
<td>Wind-relative drag acceleration (magnitude).</td>
</tr>
<tr>
<td>$L_D$</td>
<td>Wind-relative lift over drag.</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Wind-relative viscous parameter.</td>
</tr>
<tr>
<td>$\sigma(\nu)$</td>
<td>Uncertainty in the wind-relative viscous parameter.</td>
</tr>
</tbody>
</table>
The Algorithms

Constants (set on first pass through the subroutine)

\[ g = 32.174 \] Earth-surface gravitational acceleration.

\[ k_{ew} = 12.1527 \] Constant for computing the equivalent air speed, \( V_{EA} \).

\[ k_M = 4289.05 \] Constant for computing the Mach number, \( M \).

\[ \hat{y} = [0, 1, 0]^T \] The (column) unit vector used to compute lift over drag, \( L_D \).

Preliminary computations: Magnitude of wind-relative velocity and its variance

Let the magnitude of \( \dot{\mathbf{v}} \) [BOD] be \( V \);

\[ V = \left( v_x^2 + v_y^2 + v_z^2 \right)^{1/2}. \]

To compute the variance of \( V \), first \( \left[ \frac{\partial V}{\partial v_x}, \frac{\partial V}{\partial v_y}, \frac{\partial V}{\partial v_z} \right]^T \) is computed.

\[ \frac{\partial V}{\partial v_x} = \frac{\partial}{\partial v_x} \left( v_x^2 + v_y^2 + v_z^2 \right)^{1/2} = \frac{v_x V}{V}. \]

Likewise, \( \frac{\partial V}{\partial v_y} = \frac{v_y V}{V} \) and \( \frac{\partial V}{\partial v_z} = \frac{v_z V}{V} \).

The variance of \( V \) is computed from this and the covariance matrix:

\[ \sigma(V)^2 = \left[ \frac{\partial V}{\partial v_x}, \frac{\partial V}{\partial v_y}, \frac{\partial V}{\partial v_z} \right] \sigma(v \text{ [BOD]}) \left[ \frac{\partial V}{\partial v_x}, \frac{\partial V}{\partial v_y}, \frac{\partial V}{\partial v_z} \right]^T. \]

The uncertainty, which will be used later, used here is the standard
deviation given by

\[ \sigma(V) = \left[ \sigma(V)^2 \right]^{1/2}. \]

Wind-relative angle of attack, \( \alpha_W \)

Let \( V_{xz} = (v_x^2 + v_z^2)^{1/2} \).

Then \( \sin \alpha_W = \frac{v_z}{V_{xz}} \text{ and } \cos \alpha_W = \frac{v_x}{V_{xz}} \).

So,

\[ \alpha_W = \arcsin\left(\frac{v_z}{V_{xz}}\right) \]

and \( \alpha_W \) is converted to degrees and adjusted so that \( 0 \leq \alpha_W \leq 180^0 \).

Uncertainty in \( \alpha_W \)

First, the partial derivative matrix is computed:

\[
\begin{bmatrix}
\frac{\partial \alpha_W}{\partial v_x} \\
\frac{\partial \alpha_W}{\partial v_y} \\
\frac{\partial \alpha_W}{\partial v_z}
\end{bmatrix}_{\text{wind}}^{\text{CM}} = \begin{bmatrix}
\frac{\partial \alpha_W}{\partial v_x}, \frac{\partial \alpha_W}{\partial v_y}, \frac{\partial \alpha_W}{\partial v_z}
\end{bmatrix}_T = \begin{bmatrix}
-\frac{\sin \alpha_W}{V_{xz}}, 0, \frac{\cos \alpha_W}{V_{xz}}
\end{bmatrix}_T.
\]

Using this and the covariance matrix, the variance is calculated from

\[ \sigma(\alpha_W)^2 = \frac{\partial \alpha_W}{\partial v_x}, \frac{\partial \alpha_W}{\partial v_y}, \frac{\partial \alpha_W}{\partial v_z} \begin{bmatrix} f(v) & [\text{BOD}] \end{bmatrix}_{\text{wind}}^{\text{CM}} \begin{bmatrix}
\frac{\partial \alpha_W}{\partial v_x}, \frac{\partial \alpha_W}{\partial v_y}, \frac{\partial \alpha_W}{\partial v_z}
\end{bmatrix}^T.
\]

The uncertainty, or standard deviation, in \( \alpha_W \) is then

\[ \sigma(\alpha_W) = \left[ \sigma(\alpha_W)^2 \right]^{1/2}. \]
Wind-relative aerodynamic side slip angle, $\beta_{WA}$

As before,
\[
V = |\mathbf{v}_{[\text{BOD}]}| = \left(v_x^2 + v_y^2 + v_z^2\right)^{1/2}.
\]

Then,
\[
\sin \beta_{WA} = \frac{v_y}{V} \quad \text{and} \quad \cos \beta_{WA} = \frac{(v_x + v_z)^{1/2}}{V}.
\]

So,
\[
\beta_{WA} = \arcsin\left(\frac{v_y}{V}\right)
\]
and $\beta_{WA}$ is converted to degrees and adjusted so that $0 \leq \beta_{WA} \leq 180^\circ$.

Uncertainty in $\beta_{WA}$

First, the partial derivative matrix is computed:
\[
\left[\frac{\partial \beta_{WA}}{\partial v_{[\text{BOD}]}}\right]_{\text{wind CM}} = \left[\frac{\partial \beta_{WA}}{\partial v_x}, \frac{\partial \beta_{WA}}{\partial v_y}, \frac{\partial \beta_{WA}}{\partial v_z}\right]^T
\]

\[
= \left[\frac{-\cos \alpha_w \sin \beta_{WA}}{V}, \frac{\cos \beta_{WA}}{V}, \frac{-\sin \alpha_w \sin \beta_{WA}}{V}\right]^T.
\]

Using this and the covariance matrix, the variance is calculated from
\[
\sigma^2(\beta_{WA}) = \frac{\left[\frac{\partial \beta_{WA}}{\partial v_x}, \frac{\partial \beta_{WA}}{\partial v_y}, \frac{\partial \beta_{WA}}{\partial v_z}\right]^T v_{[\text{BOD}]} \left[\frac{\partial \beta_{WA}}{\partial v_x}, \frac{\partial \beta_{WA}}{\partial v_y}, \frac{\partial \beta_{WA}}{\partial v_z}\right]}{V}
\]

The uncertainty, or standard deviation, in $\beta_{WA}$ is then
\[
\sigma(\beta_{WA}) = \left[\sigma(\beta_{WA})^2\right]^{1/2}.
\]
Wind-relative guidance side slip angle, $\beta_{WG}$

Let $V_{xy} = (v_x^2 + v_y^2)^{1/2}$.

Then,
$$\sin \beta_{WG} = \frac{v_y}{V_{xy}} \text{ and } \cos \beta_{WG} = \frac{v_x}{V_{xy}}.$$ 

So,
$$\beta_{WG} = \arcsin \left( \frac{v_y}{V_{xy}} \right)$$

and $\beta_{WG}$ is converted to degrees and adjusted so that $0 \leq \beta_{WG} \leq 180^\circ$.

Uncertainty in $\beta_{WG}$

First compute the partial derivative matrix

$$\frac{\partial \beta_{WG}}{\partial v} = \begin{bmatrix} \frac{\partial \beta_{WG}}{\partial v_x} & \frac{\partial \beta_{WG}}{\partial v_y} & \frac{\partial \beta_{WG}}{\partial v_z} \end{bmatrix} = \begin{bmatrix} \frac{-\sin \beta_{WG}}{V_{xy}} & \frac{\cos \beta_{WG}}{V_{xy}} & 0 \end{bmatrix}. $$

Using this and the covariance matrix, the variance is calculated from

$$\sigma^2(\beta_{WG}) = \left[ \frac{\partial \beta_{WG}}{\partial v} \right]^T \Sigma \left[ \frac{\partial \beta_{WG}}{\partial v} \right].$$

The uncertainty, or standard deviation, in $\beta_{WG}$ is then

$$\alpha(\beta_{WG}) = \left[ \sigma(\beta_{WG})^2 \right]^{1/2}. $$

Dynamic pressures and the uncertainty in $q$

The (total) dynamic pressure, $q$, is calculated by

$$q = \frac{1}{2} \rho_{AM} V.$$
where \( \rho_{AM} \) is the atmospheric mass density.

The uncertainty in \( q \) is calculated from

\[
\sigma(q) = \left[ \rho_{AM}^2 V \sigma(V)^2 + \frac{1}{4} V^2 \sigma(\rho_{AM})^2 \right]^{1/2}
\]

where \( \sigma(\rho_{AM}) \) is the uncertainty in \( \rho_{AM} \). Note that both \( \rho_{AM} \) and \( \sigma(\rho_{AM}) \) are brought in through the argument list.

The pitch dynamic pressure, \( q_\alpha \), is calculated from

\[
q_\alpha = \alpha \rho q.
\]

The yaw dynamic pressure, \( q_\beta \), is calculated from

\[
q_\beta = \beta \rho q.
\]

The equivalent air speed, \( V_{EA} \), and its uncertainty, \( \sigma(V_{EA}) \)

\[
V_{EA} = k_{ew} \rho_{AM}^{1/2} V.
\]

Its uncertainty is calculated from

\[
\sigma(V_{EA}) = k_{ew} \rho_{AM} V + \frac{1}{4} V \sigma(\rho_{AM})^2 \right]^{1/2}
\]

The load factor, \( n \)

The load factor is computed by dividing the sensed acceleration of the Center of Mass, \( \ddot{z}_{CM} \) brought in through the calling argument, by the Earth's (surface) gravitational acceleration, \( g \). Thus,
The wind-relative drag acceleration, $a_{WD}$

First, the unit vector of the wind-relative velocity of the Center of Mass is calculated; let it be $\hat{v}_W$. Then

$$\hat{v}_{[\text{BOD}]}^{\text{wind}} = \frac{CM}{v} \hat{v}_W.$$ 

And then the magnitude of the drag acceleration is computed from

$$a_{WD} = -\hat{v}_W \cdot \hat{a}_{[\text{BOD}]}^{\text{CM}}.$$ 

The wind-relative lift-over-drag, $L_D$

Lift is computed as follows. $\hat{y} = [0,1,0]^T$ was set up during the first pass through the program along with other constants. The direction, $\hat{L}$, of the lift force is determined from the cross product

$$\hat{L} = \hat{y} \times \hat{v}_W.$$ 

The unit vector in this direction is

$$\hat{L} = \frac{\hat{L}}{|\hat{L}|}.$$ 

The lift acceleration, $a_L$, is the component of $\hat{a}_{[\text{BOD}]}^{\text{CM}}$ along $\hat{L}$; thus,

$$a_L = \hat{a}_{[\text{BOD}]}^{\text{CM}} \cdot \hat{L}.$$ 

The lift-over-drag is the ratio of $a_L$ to $a_{WD}$ if $a_{WD} \neq 0$. Thus,

$$L_D = \begin{cases} 
\frac{a_L}{a_{WD}} & \text{if } a_{WD} \neq 0 \\
0 & \text{if } a_{WD} = 0
\end{cases}.$$
Mach number, \( M \), viscous parameter, \( \nu \), and their uncertainties

First, the ambient atmospheric temperature and its uncertainty is converted from degrees Rankine to degrees Kelvin:

\[
T_{AK} = \frac{5}{9} T_{AR} = T_A \\
\sigma(T_{AK}) = \frac{5}{9} \sigma(T_{AR}) = \sigma(T_A).
\]

Next, the stagnation temperature, \( T_S \), and its uncertainty are computed using \( V \), the magnitude of the wind-relative Center of Mass velocity. The algorithm is

\[
T_S = 726.97 + 0.468 T_A + 3.4098447 \times 10^{-6} V^2.
\]

Its uncertainty is computed from

\[
\sigma(T_S) = \left[ (0.468 \sigma(T_A))^2 + (4.650816351 \times 10^{-11} V \sigma(V))^2 \right]^{1/2}.
\]

Next, the "intermediate coefficient of viscosity", \( C_0 \), and its uncertainty, \( \sigma(C_0) \), are calculated using the following algorithms:

\[
C_0 = \frac{T_S^{1/2} \left[ T_A + 122.1 \left( 10^{-5}/T_A \right) \right]}{\left[ T_S + 122.1 \left( 10^{-5}/T_S \right) \right]}
\]

and its uncertainty,

\[
\sigma(C_0) = \left[ \frac{\sigma(T_S)^2}{4T_S T_A^3} + \frac{T_S \sigma(T_A)^2}{4T_A^3} \right]^{1/2}.
\]

Next, the coefficient of dynamic viscosity, \( \mu \), and its uncertainty, \( \sigma(\mu) \), are calculated using the following algorithms based on the U. S. Standard Atmosphere of 1976:
\[ \mu = 3.0449939 \times 10^{-8} \left( \frac{T_A^{3/2}}{T_A + 110.4} \right) \]

and its uncertainty,

\[ \sigma(\mu) = 3.0449939 \times 10^{-8} \left( T_A^{1/2} \sigma(T_A) \right) \left( \frac{T_A + 165.6}{T_A + 110.4} \right)^2. \]

Finally, the wind-relative Mach number and its uncertainty are computed using the following algorithms:

\[ M = V(k_M T_A)^{-1/2} \]

and its uncertainty,

\[ \sigma(M) = \left[ \frac{\sigma(V)^2}{4289.05 \ T_A} + \frac{V^2 \sigma(T_A)^2}{17156.2 \ T_A^3} \right]^{1/2}. \]

where, as before, \( V \) is the magnitude of \( \vec{v} \) and \( \sigma(V) \) is its uncertainty.

The constant \( k_M \) is set during the first pass through the routine.

The last output terms to be computed are the wind-relative "viscous parameter", \( \nu \), and its uncertainty, \( \sigma(\nu) \). They are computed with the following algorithms based on the U. S. Standard Atmosphere of 1976:

\[ \nu = M \left[ \frac{C_0 \mu}{107.5 \ V \ \rho_{AM}} \right]^{1/2}. \]

Its uncertainty is computed from the following series.

These terms are computed:

\[ M_{pss} = \frac{\sigma(M)^2 \ C_0 \mu}{107.5 \ V \ \rho_{AM}} \]
With these, the uncertainty in the "viscous parameter" is

\[ \sigma(v) = \left( M_{pss} + C_{pss} + \mu_{pss} + V_{pss} + \rho_{pss} \right)^{1/2} \].
Calling Argument List (in list order)

Input

\( C_{D/R} \)  
Conversion constant equal to degrees per radian.

\( L_{ASC} \)  
Flag which is .TRUE. if ascent analysis is being done.

\([\text{TOP} \rightarrow \text{BOD}]\)  
Transformation matrix, TOPodetic to BODY frame.

\( \dot{v} [\text{TOP}]_{wind}^{\text{CM}} \)  
Wind-relative velocity of the Center of Mass expressed in the topodetic frame.

Output

\( \phi_W \)  
Wind-relative bank angle in degrees.

The Algorithm

First, the unit vector

\[
\hat{v}_W = \frac{\dot{v} [\text{TOP}]_{wind}^{\text{CM}}}{|| \dot{v} [\text{TOP}]_{wind}^{\text{CM}} ||}
\]

is calculated. Next, the body pitch and roll axes are extracted from the transformation matrix. Let
\[
\begin{bmatrix}
\text{TOP + BOD} = \\
\begin{bmatrix}
  r_x & r_y & r_z \\
  p_x & p_y & p_z \\
  y_x & y_y & y_z
\end{bmatrix}
= \\
\begin{bmatrix}
  \hat{r}_T \\
  \hat{p}_T \\
  \hat{y}_T
\end{bmatrix}
\end{bmatrix}
\]

This means that each row of the TOP to BOD matrix is the row unit vector representation of a rotation axis expressed in the topodetic frame. Expressed as the column vectors, they are

\[\hat{r} = \text{roll body axis expressed in the topodetic frame};\]
\[\hat{p} = \text{pitch body axis expressed in the topodetic frame};\text{ and}\]
\[\hat{y} = \text{yaw body axis expressed in the topodetic frame}.\]

Let \(\hat{n}\) be the unit vector normal to both \(\hat{r}\) and \(\hat{v}_W\). The normal vector, \(\hat{n}\) is found from the cross product,

\[\hat{n} = \hat{r} \times \hat{v}_W.\]

Its unit vector, which has the components \(n_1, n_2, n_3\), is

\[\hat{n} = \frac{\hat{n}}{|\hat{n}|} = [n_1, n_2, n_3]^T.\]

The angle, \(\alpha\), between \(\hat{r}\) and \(\hat{v}_W\) is found from

\[\alpha = \arccos(\hat{r} \cdot \hat{v}_W).\]

Then the rotation matrix, \([R]\) which rotates \(\hat{r}\) about the axis \(\hat{n}\) through the angle \(\alpha\) is:

\[
[R] =
\begin{bmatrix}
[n_1^2 + (1-n_1^2) \cos \alpha] & [n_1n_2(1-\cos \alpha) - n_3 \sin \alpha] & [n_1n_3(1-\cos \alpha) + n_2 \sin \alpha] \\
[n_1n_2(1-\cos \alpha) + n_3 \sin \alpha] & [n_2^2 + (1-n_2^2) \cos \alpha] & [n_2n_3(1-\cos \alpha) - n_1 \sin \alpha] \\
[n_1n_3(1-\cos \alpha) - n_2 \sin \alpha] & [n_2n_3(1-\cos \alpha) + n_1 \sin \alpha] & [n_3^2 + (1-n_3^2) \cos \alpha]
\end{bmatrix}
\]
Next, determine the new pitch axis, $\hat{p}$, after the rotation. It is found by the following logic:

If $\hat{v}$ [TOP] does NOT lie along $\hat{r}$; $\hat{p} = [R] \hat{p}$
wind
CM

If $\hat{v}$ [TOP] DOES lie along $\hat{r}$; $\hat{p} = \hat{p}$
wind
CM

Define $\hat{w}$ as the unit vector in the topodetic x-y plane which is also normal to $\hat{v}_w$. If $\hat{v}_w$ is expressed in terms of its components,

$\hat{v}_w = [v_x, v_y, v_z]^T$,
then
$\hat{w} = [-v_y, v_x, 0]^T$

and its unit vector is

$\hat{w} = \frac{\hat{w}}{|\hat{w}|}$

Finally, the wind-relative bank angle, $\phi_w$, is calculated by

$$
\phi_w = \arccos(\hat{w} \cdot \hat{p})
$$

This, being in radians, must be converted to degrees.

Finally, adjustments must be made. Given that $\hat{p} = [p_x, p_y, p_z]^T$, then

if $p_z < 0$, change the sign of $\phi_w$: $\phi_w = -\phi_w$

Also, if this is an ascent analysis ($L_{ASC}$ = .TRUE.) and $\phi_w < 0$, then add $360^\circ$ to $\phi_w$.
EULANG

Calling Argument List (in list order)

Input

$C_{D/R}$ Conversion constant equal to degrees per radian.

$L_{ASC}$ Flag which is .TRUE. if ascent analysis is being done.

$L_1$ Flag which is .TRUE. for the first pass through the routine.

$N_{err}$ Error counter (This can be incremented within the routine, so it also can become an output).

$\xi$ The 15-element state vector $= [S_1 \ldots S_{15}]^T$.

$[S_1, S_2, S_3]^T = \hat{\mathbf{r}}_{\mathbf{BOD}}$ Geocentric position vector of the Nav Base in body coordinates.

$[S_4, S_5, S_6]^T = \mathbf{
abla}_{\mathbf{BOD}}$ Wind-relative velocity of the Nav Base in body coordinates.

$[S_7, S_8, S_9]^T = \mathbf{\ddot{a}}_{\mathbf{BOD}}$ Wind-relative acceleration of the Nav Base in body coordinates.

$S_{10}, S_{11}, S_{12}$ Not used.

$[S_{13}, S_{14}, S_{15}]^T = \mathbf{\omega}_B$ Inertial (ECI) angular rates of the body axes.

$\mathbf{\Omega}(S)$ Covariance matrix of the state vector, $\xi$. 79
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{\hat{w}}_{[\text{BOD}]}$</td>
<td>Wind velocity expressed in body axis coordinates.</td>
</tr>
<tr>
<td>$\Phi(\mathbf{\hat{w}}_{[\text{BOD}]})$</td>
<td>Covariance matrix of $\mathbf{\hat{w}}_{[\text{BOD}]}$.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_w$</td>
<td>Wind-relative yaw angle in degrees.</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Wind-relative pitch angle in degrees.</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>Wind-relative roll angle in degrees.</td>
</tr>
<tr>
<td>$\sigma(\psi_w)$</td>
<td>Uncertainty (standard deviation) in $\psi_w$.</td>
</tr>
<tr>
<td>$\sigma(\theta_w)$</td>
<td>Uncertainty (standard deviation) in $\theta_w$.</td>
</tr>
<tr>
<td>$\sigma(\phi_w)$</td>
<td>Uncertainty (standard deviation) in $\phi_w$.</td>
</tr>
<tr>
<td>$\dot{\psi}_w$</td>
<td>Wind-relative yaw rate in degrees/sec.</td>
</tr>
<tr>
<td>$\dot{\theta}_w$</td>
<td>Wind-relative pitch rate in degrees/sec.</td>
</tr>
<tr>
<td>$\dot{\phi}_w$</td>
<td>Wind-relative roll rate in degrees/sec.</td>
</tr>
<tr>
<td>$\sigma(\dot{\psi}_w)$</td>
<td>Uncertainty (standard deviation) in $\dot{\psi}_w$.</td>
</tr>
<tr>
<td>$\sigma(\dot{\theta}_w)$</td>
<td>Uncertainty (standard deviation) in $\dot{\theta}_w$.</td>
</tr>
<tr>
<td>$\sigma(\dot{\phi}_w)$</td>
<td>Uncertainty (standard deviation) in $\dot{\phi}_w$.</td>
</tr>
</tbody>
</table>


Algorithms

Determination of a wind-related coordinate system

First, obtain the relative velocity $\mathbf{v}_{R} = \mathbf{v}_{R_{\text{wind}}} - \mathbf{v}_{B_{\text{wind}}}$ which is

$$\mathbf{v}_{R} = \mathbf{v}_{B} - \mathbf{v}_{B_{\text{wind}}} = [v_{R_{x}}, v_{R_{y}}, v_{R_{z}}]^{T}$$

Then, define a coordinate system $\hat{x}, \hat{y}, \hat{z}$ such that

- $\hat{y}_{W}$ is normal to $\mathbf{v}_{R}$ and $\mathbf{r}_{\theta_{B_{\text{wind}}}}$: the "right" vector;
- $\hat{z}_{W}$ is opposite in direction to $\mathbf{r}_{\theta_{B_{\text{wind}}}}$: the "down" vector;
- $\hat{x}_{W}$ is normal to $\hat{y}_{W}$ and $\hat{z}_{W}$: the "forward" vector (which completes the triad).

For notational simplicity, let $\mathbf{r}_{\theta_{B_{\text{wind}}}}$ be represented by $\mathbf{r}_{B} = [x_{B}, y_{B}, z_{B}]^{T}$.

Since uncertainties of the wind-relative Euler angles are to be calculated, it is necessary to calculate partial derivative matrices. They are set up in subroutines P3UV and PCROSS. The following notation will be used.

If $\mathbf{a} = [a_{1}, a_{2}, a_{3}]^{T}$ and $\mathbf{b} = [b_{1}, b_{2}, b_{3}]^{T}$ are two vectors, then the partial derivative matrices will be indicated by

$$\left[ \frac{\partial \mathbf{a}}{\partial \mathbf{b}} \right] = \begin{bmatrix}
\frac{\partial a_{1}}{\partial b_{1}} & \frac{\partial a_{1}}{\partial b_{2}} & \frac{\partial a_{1}}{\partial b_{3}} \\
\frac{\partial a_{2}}{\partial b_{1}} & \frac{\partial a_{2}}{\partial b_{2}} & \frac{\partial a_{2}}{\partial b_{3}} \\
\frac{\partial a_{3}}{\partial b_{1}} & \frac{\partial a_{3}}{\partial b_{2}} & \frac{\partial a_{3}}{\partial b_{3}}
\end{bmatrix}$$
Compute the "right" unit vector, \( \hat{y} \), and its partial derivative matrices

The subroutine PCROSS is called to compute the cross product

\[
\hat{y} = \hat{v}_R \times \hat{r}_B = \hat{v} [BOD] \times \hat{r}_B [BOD].
\]

Then it computes the skew-symmetric partial derivative matrices of \( \hat{y} \) with respect to the two input vectors, \( \hat{v}_R \) and \( \hat{r}_B \), of which \( \hat{y} \) is the cross product,

\[
\begin{bmatrix}
\frac{\partial \hat{y}}{\partial \hat{v}_R} \\
\frac{\partial \hat{y}}{\partial \hat{r}_B}
\end{bmatrix}
\]

Another subroutine, P3UV, is called which computes the unit vector of \( \hat{y} \)

\[
\hat{y} = \frac{\hat{y}}{|\hat{y}|} = [y_1, y_2, y_3]^T
\]

and the partial derivative matrix of \( \hat{y} \) with respect to \( \hat{y} \):

\[
\frac{\partial \hat{y}}{\partial \hat{y}}.
\]

Then, the following partial derivative matrices are computed:

\[
[P(\hat{y})] = \begin{bmatrix}
\frac{\partial \hat{y}}{\partial \hat{y}} & \frac{\partial \hat{y}}{\partial \hat{r}_B}
\end{bmatrix}
\]

and

\[
[P(\hat{y})] = \begin{bmatrix}
\frac{\partial \hat{y}}{\partial \hat{v}_R} & \frac{\partial \hat{y}}{\partial \hat{r}_B}
\end{bmatrix}.
\]

Compute the "down" unit vector, \( \hat{z} \), and its partial derivative matrix

The subroutine P3UV is called to calculate

\[
\hat{r}_B = \frac{\hat{r}_B}{|\hat{r}_B|} \text{ and } \frac{\partial \hat{r}_B}{\partial \hat{r}_B}.
\]
Since the "down" direction is opposite to that of \( \hat{r}_B \),

\[
\hat{z} = -\hat{r}_B = [z_1, z_2, z_3]^T
\]

and its partial derivative matrix is

\[
\left[ P(\frac{\hat{z}}{r_B}) \right] = -\left[ \frac{\partial \hat{r}_B}{\partial r_B} \right].
\]

Compute the "forward" unit vector, \( \hat{x} \), and its partial derivative matrices

Since the "forward" vector \( \hat{x} \) completes the triad, the subroutine PCROSS is called to calculate

\[
\hat{x} = y \hat{x} z = [x_1, x_2, x_3]^T,
\]

\[
\left[ P(\frac{\hat{x}}{y}) \right] = \left[ \frac{\partial \hat{x}}{\partial y} \right]
\]

and

\[
\left[ P(\frac{\hat{x}}{z}) \right] = \left[ \frac{\partial \hat{x}}{\partial z} \right].
\]

Compute the wind-relative pitch angle, \( \theta_W \), and its partial derivatives

\[
\theta_W = -C_{D/R} \arcsin \left( \frac{z_1}{\left( \frac{2}{x_1 + z_1} \right)^{2.1/2}} \right).
\]

\( \theta_W \) is constrained so that \(-180 \leq \theta_W \leq 180\).

The partial derivative matrices are expressed as row matrices:

\[
\begin{bmatrix}
\frac{\partial \theta_W}{\partial x} \\
\frac{\partial \theta_W}{\partial z_1} \\
\frac{\partial \theta_W}{\partial z_2} \\
\frac{\partial \theta_W}{\partial z_3}
\end{bmatrix} = \begin{bmatrix}
\frac{z_1}{2} \\
\frac{2.1/2}{x_1 + z_1} \\
0 \\
0
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\frac{\partial \theta_W}{\partial z} \\
\frac{\partial \theta_W}{\partial z_1} \\
\frac{\partial \theta_W}{\partial z_2} \\
\frac{\partial \theta_W}{\partial z_3}
\end{bmatrix} = \begin{bmatrix}
\frac{-x_1}{2} \\
\frac{2.1/2}{x_1 + z_1} \\
0 \\
0
\end{bmatrix}
\]
These will be used to determine the angle rates and uncertainties.

Compute the wind-relative yaw angle, $\psi_W$, and its partial derivatives

$$\psi_W = C_{D/R} \arcsin (y_1)$$

Its partial derivatives with respect to $y$ are

$$\left[\frac{\partial \psi_W}{\partial y} \right] = \left[ \frac{\partial \psi_W}{\partial y_1}, \frac{\partial \psi_W}{\partial y_2}, \frac{\partial \psi_W}{\partial y_3} \right] = \left[ \frac{1}{(y_2^2 + y_3^2)^{1/2}}, 0, 0 \right]$$

Compute the wind-relative roll angle, $\phi_W$, and its partial derivatives

$$\phi_W = -C_{D/R} \arcsin \left( \frac{y_3}{\left(y_2^2 + y_3^2\right)^{1/2}} \right)$$

$\phi_W$ is constrained so that $-180^\circ \leq \phi_W \leq 180^\circ$.

For ascent and $\phi_W < 0$, $\phi_W$ is made positive: $\phi_W = \phi_W + 360^\circ$.

Its partial derivatives are

$$\left[\frac{\partial \phi_W}{\partial y} \right] = \left[ \frac{\partial \phi_W}{\partial y_1}, \frac{\partial \phi_W}{\partial y_2}, \frac{\partial \phi_W}{\partial y_3} \right] = \left[ 0, \frac{-y_2}{y_2^2 + y_3^2}, \frac{y_3}{y_2^2 + y_3^2} \right].$$

Compute partial derivative matrices used to compute angle rates and their uncertainties

The matrix of the partial derivatives of the wind-relative Euler angles, $E_{W_i}$, with respect to the unit vectors $\hat{y}$ and $\hat{z}$ is calculated to be
The individual derivatives, or elements, of this matrix are as follows:

\[
\begin{bmatrix}
\frac{\partial \psi_W}{\partial y_1} & \frac{\partial \psi_W}{\partial y_2} & \frac{\partial \psi_W}{\partial y_3} & \frac{\partial \psi_W}{\partial z_1} & \frac{\partial \psi_W}{\partial z_2} & \frac{\partial \psi_W}{\partial z_3} \\
\frac{\partial \theta_W}{\partial y_1} & \frac{\partial \theta_W}{\partial y_2} & \frac{\partial \theta_W}{\partial y_3} & \frac{\partial \theta_W}{\partial z_1} & \frac{\partial \theta_W}{\partial z_2} & \frac{\partial \theta_W}{\partial z_3} \\
\frac{\partial \phi_W}{\partial y_1} & \frac{\partial \phi_W}{\partial y_2} & \frac{\partial \phi_W}{\partial y_3} & \frac{\partial \phi_W}{\partial z_1} & \frac{\partial \phi_W}{\partial z_2} & \frac{\partial \phi_W}{\partial z_3}
\end{bmatrix}
\]

The matrices of partial derivatives of the state vector position, \( \hat{r}_B \), and wind-relative velocity, \( \hat{v}_R \), with respect to the wind-relative Euler angles, \( \hat{E}_{W_i} \), are as follows. With \( \hat{r}_B = [x_B, y_B, z_B]^T \),
With $v_R = [v_{Rx}, v_{Ry}, v_{Rz}]^T$, another matrix that is used for the determination of Euler angle rates is that containing the partial derivatives of $\gamma$ and $\dot{z}$ with respect to $r_B$, $\dot{v}_B$, $\psi_W$, $\theta_W$, $\phi_W$. With $r_B = [x_B, y_B, z_B]^T$ and $\dot{v} = [v_{Rx}, v_{Ry}, v_{Rz}]^T$,

$$\begin{bmatrix}
\frac{\partial v_{Rx}}{\partial \psi_W} & \frac{\partial v_{Rx}}{\partial \theta_W} & \frac{\partial v_{Rx}}{\partial \phi_W} \\
\frac{\partial v_{Ry}}{\partial \psi_W} & \frac{\partial v_{Ry}}{\partial \theta_W} & \frac{\partial v_{Ry}}{\partial \phi_W} \\
\frac{\partial v_{Rz}}{\partial \psi_W} & \frac{\partial v_{Rz}}{\partial \theta_W} & \frac{\partial v_{Rz}}{\partial \phi_W}
\end{bmatrix} = \begin{bmatrix}
0 & -z_B & y_B \\
z_B & 0 & -x_B \\
-y_B & x_B & 0
\end{bmatrix}.$$
The elements $p_{ij}$, $i = 1 - 6$ and $j = 7, 8, 9$ are computed as follows. Let $E_{W_1} = \psi_W$, $E_{W_2} = \theta_W$, and $E_{W_3} = \phi$, then for $i = 1, 2, 3$ and $j = 7, 8, 9$

$$p_{ij} = \sum_{k=1}^{3} \left( \frac{\delta y_i}{\delta r_k} \frac{\delta E_{W_k}}{\delta r_j} + \frac{\delta y_i}{\delta v_k} \frac{\delta E_{W_k}}{\delta v_j} \right)$$

and for $i = 4, 5, 6$ with $j = 7, 8, 9$
\[ P_{ij} = \sum_k \left( \frac{aE_{W_k}}{ar_k} \right) \left( \frac{aE_{W_k}}{ar_j} \right). \]

The index \( k \) is used as follows: for \( k=1 \), \( r_k = x_B \) and \( v_k = v_{Rx} \); for \( k=2 \), \( r_k = y_B \) and \( v_k = v_{Ry} \); and for \( k=3 \), \( r_k = z_B \) and \( v_k = v_{Rz} \).

The next step is to calculate the partial derivative matrix

\[
\left[ \frac{a(\psi_W, \theta_W, \phi_W)}{a(r_B, \psi_B, \theta_B, \phi_B)} \right] = \left[ \frac{a(y, z)}{a(r, \psi_B, \theta_B, \phi_B)} \right]
\]

where

\[
P_{11} = \frac{aE_{W_1}}{ay} \frac{ay_1}{ar_R} \frac{aE_{W_1}}{az} \frac{az_1}{ar_R};
\]

\[
P_{12} = \frac{aE_{W_1}}{ay} \frac{ay_2}{ar_B} \frac{aE_{W_1}}{az} \frac{az_2}{ar_B};
\]

\[
P_{13} = \frac{aE_{W_1}}{ay} \frac{ay_3}{ar_B} \frac{aE_{W_1}}{az} \frac{az_3}{ar_B};
\]

\[
P_{14} = \frac{aE_{W_1}}{ay} \frac{ay_1}{ar_R} \frac{aE_{W_1}}{az} \frac{az_1}{ar_R};
\]

\[
P_{15} = \frac{aE_{W_1}}{ay} \frac{ay_2}{ar_R} \frac{aE_{W_1}}{az} \frac{az_2}{ar_R};
\]

\[
P_{16} = \frac{aE_{W_1}}{ay} \frac{ay_3}{ar_R} \frac{aE_{W_1}}{az} \frac{az_B}{ar_R};
\]

The remaining elements are taken directly from \([P_{ij}]\); that is,

\[ P_{ij} = p_{ij} \quad \text{for } i = 1, 2, 3 \text{ and } j = 7, 8, 9. \]
The angle rates, $\dot{\psi}_W$, $\dot{\theta}_W$, and $\dot{\phi}_W$, are calculated from

$$\mathbf{\dot{\omega}}_{[BOD]} = \begin{bmatrix} \dot{\psi}_W, \dot{\theta}_W, \dot{\phi}_W \end{bmatrix}^T$$

$$= \frac{a(\psi_W, \theta_W, \phi_W)}{a(\mathbf{r}_B, \mathbf{v}_R, \psi_W, \theta_W, \phi_W)} \begin{bmatrix} v_x, v_y, v_z, \dot{v}_x, \dot{v}_y, \dot{v}_z, \dot{\psi}_I, \dot{\theta}_I, \dot{\phi}_I \end{bmatrix}^T$$

Note the right hand matrix is a portion of the state vector, $\mathbf{\xi}$, which includes words 4 through 9 and 13 through 15.

Specifically in terms of the elements $P_{ij}$

$$[P_{ij}] = \frac{a(\psi_W, \theta_W, \phi_W)}{a(\mathbf{r}_B, \mathbf{v}_R, \psi_W, \theta_W, \phi_W)}.$$  

the angle rates are

\[ \psi_W = P_{11}v_x + P_{12}v_y + P_{13}v_z + P_{14}v_x + P_{15}v_y + P_{16}v_z + P_{17}\dot{\psi}_I + P_{18}\theta_I + P_{19}\phi_I \]

\[ \theta_W = P_{21}v_x + P_{22}v_y + P_{23}v_z + P_{24}v_x + P_{25}v_y + P_{26}v_z + P_{27}\dot{\psi}_I + P_{28}\theta_I + P_{29}\phi_I \]

\[ \phi_W = P_{31}v_x + P_{32}v_y + P_{33}v_z + P_{34}v_x + P_{35}v_y + P_{36}v_z + P_{37}\dot{\psi}_I + P_{38}\theta_I + P_{39}\phi_I \]

These are converted from radians per sec to degrees per sec.

Compute the wind-relative Euler angle rate uncertainties

Consider the $[P]$ matrix as a three-element (column) matrix whose elements are nine-element row vectors; that is,
\[
\begin{bmatrix}
\dot{\psi}_w, \dot{\theta}_w, \dot{\phi}_w \\
\dot{r}_B, \dot{\psi}_w, \theta_w, \phi_w
\end{bmatrix} = [\dot{P}_1, \dot{P}_2, \dot{P}_3]^T
\]

where

\[
\dot{P}_i = [P_{i1}, \ldots P_{i9}].
\]

The state vector \(\dot{\mathbf{x}}\), is dimensioned as a 15-element array, and, therefore, its covariance matrix, \(\Phi(\mathbf{S})\), is dimensioned 15 x 15. However, elements 10, 11, and 12 are not used, so, for the Euler angle and angle rate uncertainties, terms of \(\dot{\mathbf{x}}\) and \(\Phi(\dot{\mathbf{x}})\) involving position, \(\dot{r}_B\), are not used. The elements of \(\dot{\mathbf{x}}\) and \(\Phi(\dot{\mathbf{x}})\) are accessed in such a way as to effectively make \(\dot{\mathbf{x}}\) a nine element vector and \(\Phi(\dot{\mathbf{x}})\) a 9 x 9 matrix.

Then consider \(\Phi(\dot{\mathbf{x}})\) as a nine element column matrix of nine element row vectors,

\[
\Phi(\dot{\mathbf{x}}) = [C_{1j}] = [C_1, C_2, \ldots C_9]^T = [\dot{C}_j],
\]

where

\[
\dot{C}_j = [C_{j1}, \ldots C_{j9}].
\]

Now, define \(R_{ij}\) as the dot product

\[
R_{ij} = \dot{C}_j \cdot \dot{P}_i^T
\]

Nine of these, \(1 \leq j \leq 9\), are arranged to form a vector, \(\dot{R}_i\).

The variance, \(\sigma(E_{W1})^2\), of the \(i\)th wind-relative Euler angle rate is computed from

\[
\sigma(E_{W1})^2 = \dot{P}_i \cdot \dot{R}_i; \quad i = 1, 2, 3.
\]

In other words,

\[
\sigma(\psi_W)^2 = \dot{P}_1 \cdot \dot{R}_1; \quad \sigma(\theta_W)^2 = \dot{P}_2 \cdot \dot{R}_2; \quad \sigma(\phi_W)^2 = \dot{P}_3 \cdot \dot{R}_3.
\]
All three variances are tested for being positive. Any which is found to be negative is set to zero, and a message is written to an output file.

Finally, the uncertainties (standard deviations) of the wind-relative Euler angle rates are computed from

\[ \sigma(\psi_W) = \left[ \sigma(\psi_W)^2 \right]^{1/2}; \]
\[ \sigma(\theta_W) = \left[ \sigma(\theta_W)^2 \right]^{1/2}; \]
\[ \sigma(\phi_W) = \left[ \sigma(\phi_W)^2 \right]^{1/2}. \]

These are all converted from radian/sec to degrees sec.

**Compute the wind-relative Euler angle uncertainties**

Again, considering \( \Phi(\mathbf{\hat{s}}) \) as a 9 x 9 matrix, the wind velocity covariance matrix, \( \Phi(\mathbf{\hat{v}} [\text{BOD}]) \), which is a 3 x 3 matrix, is added to it. If wind

\[ \Phi(\mathbf{\hat{v}} [\text{BOD}]) = [v_{ij}], \]

the sum, \( \Phi(\mathbf{\hat{w}}) \), a 9 x 9 matrix, is formed as follows:

\[ \Phi(\mathbf{\hat{w}}) = \Phi(\mathbf{\hat{s}}) + \Phi(\mathbf{\hat{v}} [\text{BOD}]). \]
\( \Psi(\tilde{\mathbf{w}}) = \)

\[
\begin{bmatrix}
C_{11} & . & . & & & & & & C_{34} & C_{35} & C_{36} \\
. & & & & & & & & \cdots & C_{44} + v_{11} & C_{45} + v_{12} & C_{46} + v_{13} & \cdots \\
. & & & & & & & & \cdots & C_{54} + v_{21} & C_{55} + v_{22} & C_{56} + v_{23} & \cdots \\
. & & & & & & & & \cdots & C_{64} + v_{31} & C_{65} + v_{32} & C_{66} + v_{33} & \cdots \\
. & & & & & & & & \cdots & C_{74} & \cdots & \cdots & \cdots & C_{76} \\
& & & & & & & & . & . & . & . \\
& & & & & & & & . & . & . & . \\
& & & & & & & & . & . & . & . \\
& & & & & & & & . & . & . & . \\
& & & & & & & & . & . & . & . \\
C_{99} & . & . & . & . & . & & & & & & \end{bmatrix}
\]

\( \Psi(\tilde{\mathbf{w}}) \) is considered as a column vector with each of the nine elements being a nine-element vector. Let

\[
\Psi(\tilde{\mathbf{w}}) = [\tilde{\mathbf{w}}_1, \tilde{\mathbf{w}}_2, \ldots, \tilde{\mathbf{w}}_9]^T = [\tilde{\mathbf{w}}_j]
\]
where each $\mathbf{\hat{w}}_j$ is

$$\mathbf{\hat{w}}_j = [w_{j1}, \ldots, w_{j9}].$$

Then, as before, using

$$\mathbf{R}_{ij} = \begin{bmatrix} \alpha(\psi_W, \theta_W, \phi_W) \\ \alpha(\hat{\psi}_B, \hat{\theta}_B, \hat{\phi}_B) \end{bmatrix} = [\hat{\psi}_1, \hat{\psi}_2, \hat{\psi}_3]^T = [\hat{\psi}_i],$$

the dot product $R_{ij}$ is computed from

$$R_{ij} = \mathbf{\hat{w}}_j \cdot \hat{\psi}_i.$$  

There are nine $1 \leq j \leq 9$ elements, $R_{ij}$, which form a vector $\hat{\mathbf{R}}_i$. The variance, $\sigma(E_{Wi})^2$, of the $i^{th}$ wind-relative Euler angle is computed from

$$\sigma(E_{Wi})^2 = \hat{\psi}_i \cdot \hat{\mathbf{R}}_i; \ i = 1,2,3.$$  

All three variances are tested for being positive. Any which is found to be negative is set to zero, and a message is written to the output file.

Finally, the uncertainties (standard deviations) of the wind-relative Euler angles are computed from

$$\sigma(\psi_W) = [\sigma(\psi_W)^2]^{1/2};$$

$$\sigma(\theta_W) = [\sigma(\theta_W)^2]^{1/2}; \text{ and }$$

$$\sigma(\phi_W) = [\sigma(\phi_W)^2]^{1/2}.$$  

These are all converted from radians to degrees.
Calling Argument List (in list order)

Input

\[ \mathbf{\hat{r}}_{\text{NB}}^{[\text{M50}]} \]  
Geocentric position of the Navigation Base in M50 coordinates.

\[ \mathbf{\hat{r}}_{\text{NB}}^{[\text{M50}]} \]  
Geocentric velocity of the Navigation Base in M50 coordinates.

\[ \mathbf{\hat{\omega}}^{[\text{BOD}]} \]  
Orbiter angular velocity in body axis coordinates.

\[ [\text{M50} + \text{BOD}] \]  
Transformation matrix from M50 to BODy axis coordinates (The Navigation Base is the origin of the BODy system.).

\[ [\text{M50} + \text{TOP}] \]  
Transformation matrix from M50 to TOPodetic coordinates.

\[ [\text{TOP} + \text{BOD}] \]  
Transformation matrix from TOPodetic to BODy coordinates.

Output

\[ \mathbf{\hat{\omega}}^{[\text{TOP}]} \]  
Topodetic Euler angle rates for yaw, pitch, and roll:  
\[ \mathbf{\hat{\omega}}^{[\text{TOP}]} = [\psi_T, \theta_T, \phi_T]^T \]

Input

\[ C_{\text{DR}} \]  
Conversion constant equal to degrees per radian.
Algorithm

The algorithm is described in TRW IOC 83:W482.4-28, dated 25 March 1983 and written by Darwin H. Poritz. This document is included in the Attachment.
Calling Argument List (in list order)

Input

\[ C_{D/R} \]
Conversion constant equal to degrees per radian.

\[ \vec{v} \ [\text{TOP}] \]
Wind-relative velocity of the Center of Mass in the TOPodetic frame.

\[ \sigma(\vec{v} \ [\text{TOP}]) \]
Uncertainty in \[ \vec{v} \ [\text{TOP}] \].

Output

\[ V_{WR} \]
Magnitude of \[ \vec{v} \ [\text{TOP}] \].

\[ \sigma(V_{WR}) \]
Uncertainty in \[ V_{WR} \].

\[ \gamma_W \]
Wind-relative ("local") flight path angle in degrees.

\[ \sigma(\gamma_W) \]
Uncertainty in \[ \gamma_W \].

\[ \psi_W \]
Wind-relative ("local") azimuth, or heading angle, in degrees.

\[ \sigma(\psi_W) \]
Uncertainty in \[ \psi_W \].
Compute the magnitude, $V_{WR}$, and its uncertainty, $\sigma(V_{WR})$

Let $\hat{\mathbf{v}}_{[\text{TOP}]} = [v_{Rx}, v_{Ry}, v_{Rz}]^T$

Then, the magnitude of this velocity is

$$V_{WR} = (v_{Rx}^2 + v_{Ry}^2 + v_{Rz}^2)^{1/2}.$$ 

Let $\sigma(\hat{\mathbf{v}}_{[\text{TOP}]}) = [\sigma_x(\hat{v}_{WR}), \sigma_y(\hat{v}_{WR}), \sigma_z(\hat{v}_{WR})]^T$

Then, the uncertainty, or standard deviation, in $V_{WR}$, the magnitude of the wind-relative velocity, is

$$\sigma(V_{WR}) = \sqrt{\frac{\sigma_x(\hat{v}_{WR})^2 + \sigma_y(\hat{v}_{WR})^2 + \sigma_z(\hat{v}_{WR})^2}{V_{WR}}}.$$ 

Compute the wind-relative flight path angle and its uncertainty

The wind-relative flight path angle is computed from

$$\gamma_W = \arcsin\left(-\frac{v_{Rx}}{V_{WR}}\right).$$

which is converted from radians to degrees.

Its uncertainty, or standard deviation, is computed from

$$\sigma(\gamma_W) =$$

$$\frac{\left[\frac{\sigma_x(\hat{v}_{WR})^2 v_{Rx}^2 + \sigma_y(\hat{v}_{WR})^2 v_{Ry}^2 + \sigma_z(\hat{v}_{WR})^2 (v_{Rx}^2 + v_{Ry}^2)^2}{V_{WR}^4 (v_{Rx}^2 + v_{Ry}^2)}\right]}{1/2}.$$
Compute the aximuth, or heading, and its uncertainty

The wind-relative aximuth angle is computed from

\[ \psi_W = \arctan\left(\frac{v_{Ry}}{v_{Rx}}\right) \]

which is converted from radians to degrees. Its uncertainty, or standard deviation, is computed with

\[ \sigma(\psi_W) = \sqrt{\frac{\sigma_x(\tilde{v}_{WR})^2 v_{Ry}^2 + \sigma_y(\tilde{v}_{WR})^2 v_{Rx}^2}{v_{Rx}^2 + v_{Ry}^2}} \]
Calling Argument List (in list order)

**Input**

- \(C_{D/R}\)  
  Conversion constant equal to degrees per radian.

- \(\mathbf{r}_{\theta}[M50]\)  
  Geocentric position of the Navigation Base in the M50 frame.

- \(\mathbf{v}_{\theta}[M50]\)  
  Geocentric velocity of the Navigation Base in the M50 frame.

**Output**

- \(R_{NB}\)  
  Magnitude of \(\mathbf{r}_{\theta}[M50]\).

- \(V_{NB}\)  
  Magnitude of \(\mathbf{v}_{\theta}[M50]\).

- \(\gamma_I\)  
  Inertial flight path angle measured, in degrees, in horizon plane.

- \(\psi_I\)  
  Inertial azimuth, or heading angle, measured in degrees, from the North.

**Algorithms**

Computation of the magnitudes of position and velocity

Let \(\mathbf{r}_{\theta}[M50] = [x_I, y_I, z_I]^T\) and \(\mathbf{v}_{\theta}[M50] = [v_x, v_y, v_z]^T\).
Then,

\[ R_{NB} = (x_I^2 + y_I^2 + z_I^2)^{1/2} \]

and

\[ v_{NB} = (v_x^2 + v_y^2 + v_z^2)^{1/2} \]

**Computation of the inertial flight path angle, \( \gamma_I \)**

First, the unit vectors \( \hat{r}_{NB} \) and \( \hat{v}_{NB} \) for the position and velocity vectors, respectively, are computed:

\[ \hat{r}_{NB} = \left[ \frac{x_I}{R_{NB}}, \frac{y_I}{R_{NB}}, \frac{z_I}{R_{NB}} \right]^T \]

and

\[ \hat{v}_{NB} = \left[ \frac{v_x}{v_{NB}}, \frac{v_y}{v_{NB}}, \frac{v_z}{v_{NB}} \right]^T. \]

The inertial flight path angle is computed from

\[ \gamma_I = \arcsin (\hat{r}_{NB} \cdot \hat{v}_{NB}) \]

which is converted from radians to degrees.

**Computation of the inertial azimuth or heading, \( \psi_I \)**

First, the cross product of velocity and position is calculated;

\[ \mathcal{L} = \hat{r}_{[M50]} \times \hat{r}_{[M50]} \]

Then, the horizontal velocity, \( \mathcal{V}_H \), is calculated from

\[ \mathcal{V}_H = \hat{r}_{[M50]} \times \mathcal{L} \]
and its unit vector, \( \hat{v}_H \), is determined;

\[
\hat{v}_H = \frac{\hat{v}_H}{|\hat{v}_H|}
\]

The vector, \( \hat{N} \), pointing North is computed from

\[
\hat{N} = r_{\theta}^* \begin{bmatrix} M50 \end{bmatrix} \times (\hat{k} \times r_{\theta}^* \begin{bmatrix} M50 \end{bmatrix})
\]

where \( \hat{k} \) is the (unit) vector \([0,0,1]^T\).

Then,

\[
\hat{N} = (x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}) \times (\hat{k} \times (x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}))
\]

\[
= -zx \hat{i} - zy \hat{j} + (x^2 + y^2) \hat{k}
\]

\[
= [-zx, -zy, (x^2 + y^2)]^T.
\]

The unit vector is calculated from

\[
\hat{N} = \frac{\hat{N}}{|\hat{N}|}
\]

Next, the unit vector, \( \hat{E} \), pointing East is computed from

\[
\hat{E} = \hat{N} \times r_{NB}.
\]

Then,

\[
\cos \psi_I = \hat{N} \cdot \hat{v}_H \quad \text{and} \quad \sin \psi_I = \hat{E} \cdot \hat{v}_H.
\]

To avoid a singularity when computing \( \psi_I \), \( \cos \psi_I \) is checked.

If \( \cos \psi_I = 0 \) and \( \sin \psi_I > 0 \), set \( \psi_I = 0^0 \)
If \( \cos \psi_I = 0 \) and \( \sin \psi_I \geq 0 \), set \( \psi_I = 270^0 \)

If \( \cos \psi_I \neq 0 \), then \( \psi \) is computed from

\[
\psi_I = \arctan \left( \frac{\sin \psi_I}{\cos \psi_I} \right)
\]

which is converted from radians to degrees.
Calling Argument List (in list order)

Input

$C_{D/R}$  Conversion constant equal to degrees per radian.

$PE$  Array of Earth geophysical parameters. Those used here are as follows:

$R_{\theta E} = $ Earth's equatorial radius = $PE(1)$

$R_{\theta P} = $ Earth's polar radius = $PE(6)$

$\phi_D$  Geodetic latitude, in degrees, of the desired location, $P$.

$\lambda_D$  Geodetic longitude, in degrees, of the desired location, $P$.

$h_D$  Geodetic height, in feet, of the desired location, $P$.

Output

$\mathbf{r}_p^{[GEO]}$  Geocentric position vector of the desired location, $P$, in the GEOgraphic ("Earth-Fixed-Greenwich") coordinate system.

Algorithm

First $\phi_D$ and $\lambda_D$ are converted from degrees to radians. Then

$$d_{xy} = \frac{R_{\theta E}}{[1 + (\frac{R_{\theta P}}{R_{\theta E}} \tan \phi_E)^2]^{1/2}}$$

and
Then, the geocentric position vector is

\[ d_z = \left( \frac{R_{OP}}{R_{OE}} \right)^2 d_{xy} \tan \phi_D. \]

Then, the geocentric position vector is

\[
\mathbf{r}_G = \begin{bmatrix}
x_G \\
y_G \\
z_G
\end{bmatrix} = \begin{bmatrix}
(d_{xy} + h_D \cos \phi_D) \cos \lambda_D \\
(d_{xy} + h_D \sin \phi_D) \sin \lambda_D \\
\left( d_z + h_D \sin \lambda_D \right)
\end{bmatrix}.
\]
Calling Argument List (in list order)

Input

\texttt{C_{D/R}} \quad \text{Conversion constant equal to degrees per radian.}

\texttt{R_{OE}} \quad \text{Equitorial radius of the Earth.}

\texttt{R_{OP}} \quad \text{Polar radius of the Earth.}

\texttt{r_{GEO}[P]} \quad \text{Position vector of a point, P, (eg., for the vehicle, the Center of Mass, or the Navigation Base) expressed in GEOgraphic coordinates.}

Outputs

\texttt{\phi_{D}} \quad \text{Geodetic latitude, in degrees, of point P.}

\texttt{\lambda_{D}} \quad \text{Geodetic longitude, in degrees, of point P.}

\texttt{h_{D}} \quad \text{Geodetic height, in feet, of point P.}

\texttt{\delta} \quad \text{Declination, in degrees, of point P.}

Algorithms

Preliminary calculations

First, the Earth's flattening, \( f \), is calculated from its definition:
\[
f = \frac{R_{OE} - R_{OP}}{R_{OE}}
\]

The components of the geographic position vector are \(x_G, y_G, z_G\); that is,

\[
\begin{bmatrix}
  x_G \\
  y_G \\
  z_G
\end{bmatrix} = \mathbf{r}_{GEO}.
\]

The following parameters are calculated from \(f\) and the geocentric position:

\[
A = x_G^2 + y_G^2
\]

and

\[
D = (1 - f)^2 z_G^2.
\]

Now, a parameter, \(B_{p1}\), is calculated by means of a five-pass iteration. The iteration is initialized by setting

\[
B_0 = 0.0067
\]

Then, for \(i = 0, 1, 2, 3,\) and 4

\[\text{(1)}\quad B_{p1} = B_i + 1\]

\[\text{(2)}\quad C = \frac{A}{B_{p1}^2} \]

\[\text{(3)}\quad B_i = \frac{f(2 - f) R_{OE}}{(C + D)^{1/2}}\]

Steps 1 through 3 are repeated until \(i = 4\).

Then,

\[
B_{p1} = B_4 + 1
\]
Calculate the geodetic latitude, $\phi_D$, and longitude, $\lambda_D$

The latitude is calculated from

$$\phi_D = \arctan\left(\frac{z_G}{(A/B_{p1})^{1/2}}\right)$$

which is converted from radians to degrees.

The longitude is

$$\lambda_D = \arctan\left(\frac{y_G}{x_G}\right)$$

which is converted from radians to degrees.

Calculate the geodetic height, $h_D$

The height is calculated from

$$h_D = \frac{B_4(1 - f)^2}{\left[1 - \frac{f(2 - f)}{\left(-\frac{A}{B_{p1}^2} + z_G\right)^{1/2}}\right]}$$

Calculate the declination, $\delta$

The declination is calculated from

$$\delta = \arctan\left(\frac{z_G}{(x_G^2 + y_G^2)^{1/2}}\right)$$

which is converted from radians to degrees.
SPBCALC

Calling Argument List (in list order)

Input

\( \Omega(\hat{\varepsilon}[BOD]) \)

Covariance matrix of errors in body axis orientation which are errors in the body axis Euler angles in radians. 
\( \hat{\varepsilon}[BOD] = [\varepsilon_{\psi}, \varepsilon_{\theta}, \varepsilon_{\phi}]^T \) where \( \psi = \text{yaw}, \theta = \text{pitch}, \) and \( \phi = \text{roll}. \)

\( \hat{u} \)

Vector for which the covariance matrix is to be corrected for body axis orientation errors. The vector has the elements \( \hat{u} = [u_1, u_2, u_3]^T. \)

Input and Output

\( \Omega(\hat{u}) \)

Covariance matrix of the vector \( \hat{u}. \) It is input without the effect of body axis errors and output with these effects added to it.

Algorithm

First, the partial derivative matrix is constructed as follows:

\[
\begin{bmatrix}
\frac{\partial u_1}{\partial \varepsilon_j} \\
\frac{\partial u_2}{\partial \varepsilon_j} \\
\frac{\partial u_3}{\partial \varepsilon_j}
\end{bmatrix} = \begin{bmatrix}
0 & -u_3 & u_2 \\
u_3 & 0 & -u_1 \\
-u_2 & u_1 & 0
\end{bmatrix}.
\]

Then, the contribution of the axis uncertainty is calculated by

\[\Omega(\hat{\varepsilon}[\hat{u}]) = \left[\frac{\partial u_i}{\partial \varepsilon_j}\right] \Omega(\hat{\varepsilon}[BOD]) \left[\frac{\partial u_i}{\partial \varepsilon_j}\right]^T.\]

The axis uncertainty matrix is then added to \( \Omega(\hat{u}) \) to obtain the output;
*>> \( \zeta(\dot{u}) = \zeta(\dot{u})_{IN} + \zeta(\varepsilon[\dot{u}]) \)
WINDTOP

Calling Argument List

Input

\( C_{D/R} \) Conversion constant equal to degrees per radian.

\( W_H \) Horizontal wind speed.

\( \theta_H \) Direction angle of horizontal wind velocity in degrees.

\( \sigma(W_H) \) Uncertainty in \( W_H \).

\( \sigma(\theta_H) \) Uncertainty in \( \theta_H \) (\( \theta_H \) in degrees).

\( \sigma(W_V) \) Uncertainty in vertical wind speed.

Output

\( \hat{\mathbf{v}} \) [TOP] Wind velocity expressed in TOPodetic coordinates.

\( \mathbf{C}(\hat{\mathbf{v}} \) [TOP]) Covariance matrix of \( \hat{\mathbf{v}} \) [TOP].

Algorithm

Calculation of wind velocity expressed in topodetic coordinates

First, \( \theta_H \) and \( \sigma(\theta_H) \) are converted to radians by dividing by \( C_{D/R} \). Then

\[
\begin{bmatrix}
-W_H \cos \theta_H \\
-W_H \sin \theta_H \\
0
\end{bmatrix}
\]
Calculation of the covariance matrix of $\mathbf{\hat{v}}_{\text{TOP}}^{\text{wind}}$

The matrix elements are set as follows:

\[
\begin{bmatrix}
\{[\sigma(W_h) \cos \theta_h] + [W_h \sigma(\theta_h) \sin \theta_h] \} \cos \theta_h \sin \theta_h \{[\sigma(W_h) \cos \theta_h] + [W_h \sigma(\theta_h) \sin \theta_h] \}, 0 \\
\{[\cos \theta_h \sin \theta_h \{[\sigma(W_h) \cos \theta_h] + [W_h \sigma(\theta_h) \sin \theta_h] \} [\cos \theta_h \sin \theta_h \{[\sigma(W_h) \cos \theta_h] + [W_h \sigma(\theta_h) \sin \theta_h] \}, 0 \\
0 & 0 & \sigma(W_v)
\end{bmatrix}
\]
4.2 THE NAVBLK FILE

The purpose of the NAVBLK file is to provide the user with a magnetic tape file which contains a list of input parameter values used by the trajectory estimation (BET) system. This file is produced by the program IZIIPC. The contents of the NAVBLK file are gathered by the program INPUT whose inputs are mainly user inputs. The only calculated parameters are the declination (DELTA) and geocentric distance (RSUBO) of the Nav Base as the vehicle sits on the launch pad before launch. These two terms, DELTA and RSUBO, are calculated in INPUT; the algorithms for them are given in Section 4.2.2 of this document.

4.2.1 Definition of Terms

The NAVBLK file contains constants required for a BET solution. The terms in this file are defined in Table 1 on the next page. Section 4.2.3 shows how the contents of the NAVBLK are arranged in the file records.
<table>
<thead>
<tr>
<th>Term Name</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>IHEADR [180]</td>
<td>NAVBLock header from MET tape</td>
<td>See CMET[20]</td>
</tr>
<tr>
<td>CGT [200, 4]</td>
<td>Center of Mass (CG) history timeline</td>
<td>User input via terminal editor</td>
</tr>
<tr>
<td>DPSET [20, 200]</td>
<td>Special Event Timeline with description</td>
<td>User input via terminal editor</td>
</tr>
<tr>
<td>REF [3, 3, 3]</td>
<td>REFerence to Stable Member transformation MATrix (REFSMMAT)</td>
<td>User input via terminal editor</td>
</tr>
<tr>
<td>TRAC [6, 120]</td>
<td>Tracking Station data</td>
<td>Stored on file RADAR</td>
</tr>
<tr>
<td>CMET [20]</td>
<td>Corrected (units conversion) METeorological data</td>
<td>Entered by means of MET (data) tapes Ascent from NOAA station at KSC Descent from MSC</td>
</tr>
<tr>
<td>KAPPA</td>
<td>Earth-fixed launch azimuth</td>
<td>User input via terminal editor</td>
</tr>
<tr>
<td>PHI0</td>
<td>Geodetic latitude of launch site</td>
<td>User input via terminal editor</td>
</tr>
<tr>
<td>DELTA</td>
<td>Declination of Nav Base on launch pad</td>
<td>Computed in program INPUT</td>
</tr>
<tr>
<td>LAMO</td>
<td>Geodetic longitude of launch site</td>
<td>User input via terminal editor</td>
</tr>
<tr>
<td>RSUBO</td>
<td>Geocentric distance of Nav Base on launch pad</td>
<td>Computed in program INPUT</td>
</tr>
<tr>
<td>XSUBO</td>
<td>Geodetic height of Nav Base above Fischer ellipsoid</td>
<td>User input via terminal editor</td>
</tr>
<tr>
<td>NAVO [3]</td>
<td>Position of Nav Base with respect to structure frame</td>
<td>User input via terminal editor</td>
</tr>
</tbody>
</table>
4.2.2 Computation of the Terms DELTA and RSUBO

These terms are computed for ascent analysis in the program INPUT. RSUBO is the geocentric distance of the Navigation Base as the Shuttle sits on the launch pad. DELTA is the declination (or geocentric latitude) of the Navigation Base before launch. The following symbols will be used:

\[ R_{OE} \]  
Equatorial radius of the Earth.

\[ R_{OP} \]  
Polar radius of the Earth.

\[ \phi_0 \]  
Geodetic latitude of the launch site, in degrees.

\[ X_0 \]  
Geodetic height of the Nav Base, at the launch site, above the Fischer ellipsoid.

\[ r_0 \]  
RSUBO

\[ \delta \]  
DELTA

The latitude is converted to radians within the algorithm. The algorithm is as follows. Define

\[ S_{xy} = \frac{R_{OE}}{[1 + (\frac{R_{OP}}{R_{OE}} \tan \phi_0)^2]^{1/2}} \]

and

\[ S_z = \frac{R_{OP} \tan \phi_0}{[1 + (\frac{R_{OP}}{R_{OE}} \tan \phi_0)^2]^{1/2}} \]

Also define

\[ V_{xy} = \frac{S_{xy}}{R_{OE}^2} = \frac{1}{R_{OE}[1 + (\frac{R_{OP}}{R_{OE}} \tan \phi_0)^2]^{1/2}} \]

and
\[ V_z = \frac{S_z}{R_\oplus} = \frac{\tan \phi_0}{R_\oplus[1 + \left(\frac{R_\oplus}{R_\oplus \tan \phi_0}\right)^2]^{1/2}}. \]

Let
\[ V = (V_{xy}^2 + V_z^2)^{1/2}. \]

Then, let
\[ u_{xy} = \frac{V_{xy}}{V} \quad \text{and} \quad u_z = \frac{V_z}{V}. \]

Then, the declination is
\[ \delta = \arctan \left( \frac{S_z + u_z x_0}{S_{xy} + u_{xy} x_0} \right). \]

The geocentric distance of the Nav Base is
\[ r_0 = \left[ (S_z + u_z x_0)^2 + (S_{xy} + u_{xy} x_0)^2 \right]^{1/2}. \]
4.2.3 NAVBLK File Format

Each record written by the program I2IIPC is 1000 words long. The file format is shown in the following table.

Table 2. NAVBLK File Format

<table>
<thead>
<tr>
<th>TERM</th>
<th>RECORD</th>
<th>WORD</th>
<th>RECORD</th>
<th>WORD</th>
</tr>
</thead>
<tbody>
<tr>
<td>IHEADR</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>180</td>
</tr>
<tr>
<td>GRR</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>CGT</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>804</td>
</tr>
<tr>
<td>DPSET</td>
<td>2</td>
<td>805</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>REF</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>31</td>
</tr>
<tr>
<td>TRAC</td>
<td>6</td>
<td>32</td>
<td>6</td>
<td>631</td>
</tr>
<tr>
<td>CMET</td>
<td>6</td>
<td>632</td>
<td>26</td>
<td>631</td>
</tr>
<tr>
<td>KAPPA</td>
<td>6</td>
<td>632</td>
<td>6</td>
<td>632</td>
</tr>
<tr>
<td>PHIO</td>
<td>6</td>
<td>633</td>
<td>6</td>
<td>633</td>
</tr>
<tr>
<td>DELTA</td>
<td>6</td>
<td>634</td>
<td>6</td>
<td>634</td>
</tr>
<tr>
<td>LAMO</td>
<td>6</td>
<td>635</td>
<td>6</td>
<td>635</td>
</tr>
<tr>
<td>RSUBO</td>
<td>6</td>
<td>636</td>
<td>6</td>
<td>636</td>
</tr>
<tr>
<td>XSUBO</td>
<td>6</td>
<td>637</td>
<td>6</td>
<td>637</td>
</tr>
<tr>
<td>NAVO</td>
<td>6</td>
<td>638</td>
<td>6</td>
<td>640</td>
</tr>
</tbody>
</table>
APPENDIX A

TOPODETFIC EULER ANGLE RATES

(Applicable Document 5, Section 3.0)
1. INTRODUCTION

The Ascent/Descent Best Estimated Trajectory (BET) which is produced by the OPIP program provides space in words 216, 217, and 218 of each record for rates of Orbiter Euler angles relative to the instantaneous topodetic axes. This memorandum proposes a method for computing these angular rates and presents this method in Section 3 below. The definition of the Euler angles is given in Section 2.

2. TOPODETIC EULER ANGLES: YAW, PITCH, AND ROLL

The topodetic coordinate system is defined and displayed in a figure in Appendix A of Reference 1. The origin of the system is the point of interest, e.g., the Orbiter's center of gravity or navigation base. The unit vector \( \mathbf{f}_3 \) representing the topodetic z axis points down (in the direction of Earth's gravity) from the origin along the local perpendicular to the Fischer-ellipsoid model of the Earth's surface. The unit vector \( \mathbf{f}_1 \) representing the topodetic x axis is perpendicular to \( \mathbf{f}_3 \) and points northward in the origin's meridian plane. The unit vector \( \mathbf{f}_2 \) representing the topodetic y axis completes the right-handed, orthogonal system.

The Orbiter's body-axis coordinate system is also defined and displayed in a figure in Appendix A of Reference 1. The origin of the system is the Orbiter's center of gravity. The unit vector \( \mathbf{e}_1 \) representing the body x axis is parallel to the Orbiter structural body \( X_0 \) axis and positive toward the Orbiter's nose. The unit vector \( \mathbf{e}_3 \) representing the body z axis is parallel to the Orbiter's plane of symmetry, is perpendicular to \( \mathbf{e}_1 \), and points downward with respect to the Orbiter's fuselage. The unit vector \( \mathbf{e}_2 \) representing the body y axis completes the right-handed, orthogonal system.

The topodetic Euler angles of yaw \( \psi \), pitch \( \theta \), and roll \( \phi \) are defined by fictitiously aligning the Orbiter's body axes \( x \), \( y \), and \( z \) with the topodetic unit vectors \( \mathbf{f}_1 \), \( \mathbf{f}_2 \), and \( \mathbf{f}_3 \), respectively, and then rotating the Orbiter in a particular way until the body axes \( x \), \( y \), and \( z \) are aligned with the body-axis unit vectors \( \mathbf{e}_1 \), \( \mathbf{e}_2 \), and \( \mathbf{e}_3 \), respectively. The first rotation is about the fictitiously aligned body \( z \) axis through the angle \( \psi \). The second rotation is through the angle \( \theta \) about the fictitiously aligned by \( y \) axis after the first rotation through
The third rotation is through the angle $\psi$ about the fictitiously aligned $x$ axis after the rotations through $\psi$ and $\theta$.

Let $I(\psi)$, $J(\theta)$, and $K(\phi)$ denote the orthogonal transformations for the successive rotations of yaw, pitch, and roll by $\psi$, $\theta$, and $\phi$ radians respectively. From simple figure drawings, it can be shown that

$$
I(\psi) = \begin{bmatrix} 
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1 
\end{bmatrix},
$$

$$
J(\theta) = \begin{bmatrix} 
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta 
\end{bmatrix}, \text{ and}
$$

$$
K(\phi) = \begin{bmatrix} 
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi 
\end{bmatrix}.
$$

The complete rotation $L(\psi, \theta, \phi)$ is given by the product

$$
L(\psi, \theta, \phi) = K(\phi) J(\theta) I(\psi).
$$

3. TOPODETIC EULER ANGLE RATES

It is convenient to calculate the Euler angle rates $\dot{\psi}$, $\dot{\theta}$, and $\dot{\phi}$ from the perspective of an inertial coordinate system. The mean-of-1950 (M50) inertial coordinate system is defined and displayed in a figure in Appendix A of Reference 1. Let $g_1$, $g_2$, and $g_3$ be the unit vectors in the directions of the M50 $x$, $y$, and $z$ axes, respectively. Let
be the orthogonal transformation from M50 to body-axis coordinates. It follows that the $E_j$'s are the body axes $e_1$, $e_2$, and $e_3$ expressed in M50 coordinates. Let

$$B = (f_i \cdot q_j) = (F_1, F_2, F_3)^t$$

be the orthogonal transformation from M50 to topodetic coordinates. The $F_j$'s are the topodetic axes $f_1$, $f_2$, and $f_3$ expressed in M50 coordinates. The matrices $A$ and $B$ are already computed within the OPIP program.

The body angular velocity $\omega$ expressed in body-axis coordinates is supplied as an input to the OPIP program. Since the body axes are fixed in the Orbiter, the angular velocity of the body axes relative to an inertial frame is the same as the angular velocity of the body. Let

$$\omega = A^t \omega$$

be the body angular velocity expressed in M50 coordinates.

Define $\Omega$ as the angular velocity of the topodetic axes as seen from the M50 inertial frame and as expressed in M50 coordinates. The body angular velocity as seen in the topodetic frame is found by removing the rotational motion of the topodetic frame from the inertial body angular velocity. Therefore

$$\Omega = \omega - \Omega$$

is the body angular velocity as seen in the topodetic frame and as expressed in M50 coordinates. The next step is to compute $\Omega$.

$\Omega$ can be decomposed into rotational motion about $f_2$ due to northward motion of the Orbiter and into rotational motions about $f_1$ and $f_3$ due to eastward motion of the Orbiter. Let $R$ and $V$ be respectively the inertial
position and velocity of the Orbiter in M50 coordinates. An approximate value for $Q$ is

$$Q = \frac{(F_2 \times R) \cdot V}{|F_2 \times R| |R|} F_2 + \frac{(F_1 \times R) \cdot V}{|F_1 \times R| |R|} F_1 + \frac{(F_3 \times R) \cdot V}{|F_3 \times R| |R|} F_3$$

The first term above is approximate because it neglects the additional rotation about $f_2$ due to the increase in flattening of the Fischer ellipsoid with motion toward the pole. This effect is believed to be negligible.

All rotational motion of the Orbiter as seen in the topodetic frame is due entirely to the Orbiter's angular velocity as seen in the topodetic frame. Therefore, the topodetic Euler angle rates $\dot{\psi}$, $\dot{\theta}$, and $\dot{\phi}$ are the components of body angular velocity along the instantaneous axes of rotation used for the topodetic Euler angles $\psi$, $\theta$, and $\phi$, respectively. The next step is to compute the axes of rotation which will be denoted by $h_1$, $h_2$, and $h_3$.

The yaw is about $f_3$ and thus

$$h_1 = -f_3.$$  

The pitch is about $f_2$ after rotation about $h_1$ by $\psi$. Since the columns of $I^t(\psi)$ are the rotated axes in topodetic coordinates, the second column of $I^t(\psi)$ gives $h_2$, the rotated $f_2$, in topodetic coordinates:

$$h_2 = (-\sin \psi) f_1 + (\cos \psi) f_2.$$  

The roll is about $f_1$ after rotation about $h_1$ by $\psi$ followed by rotation about $h_2$ by $\theta$. By a similar argument, the first column of $[J(\theta) I(\psi)]^t$ gives $h_3$, the rotated $f_1$, in topodetic coordinates:

$$h_3 = (\cos \psi \cos \theta) f_1 + (\sin \psi \cos \theta) f_2 + (-\sin \theta) f_3.$$