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Trimming an Aircraft Model for Flight Simulation

Richard E. McFarland, Ames Research Center, Moffett Field, California

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Ames Research Center
Moffett Field, California 94035

SUMMARY

Real-time piloted aircraft simulations with digital computers have been performed at Ames Research Center (ARC) for over two decades. For the simulation of conventional aircraft models, the establishment of initial vehicle and control orientations at various operational flight regimes has been adequately handled by either analog techniques or simple inversion processes. However, exotic helicopter configurations have been recently introduced that require more sophisticated techniques because of their expanded degrees of freedom and environmental vibration levels.

At ARC, these techniques are used for the backward solutions to real-time simulation models as required for the generation of trim points. These techniques are presented in this paper with examples from a blade-element helicopter simulation model.

INTRODUCTION

For research such as that done at ARC, flight simulation is initiated from a configuration of vehicle states and pilot controls that is conceptually a single point in the history of a real or imagined vehicle flight. This point, called the "initial flight condition," is the origin of a region of interest to simulation researchers. For many reasons the vehicle model itself cannot be "flown" to this point, but rather must "magically" appear in this configuration, often repetitiously, with perhaps a single parameter being perturbed at the beginning of each real-time session. For example, a landing scenario may be repeatedly reenacted with different values for the vehicle initial flap setting. A change in flap setting (or a change in almost any variable on an aircraft) produces changes in the vehicle translational and rotational accelerations.

In real-time flight simulation, both the vehicle translational and rotational accelerations must vanish (be nulled or trimmed) during simulation initiation. It is not merely that pilots prefer to begin operations in a nonaccelerated configuration; it is actually a critical requirement. For instance, large-scale motion devices demand the absence of initial transients.

Initial Conditions

The set of initial conditions specified by engineers and pilots are invariably incomplete. For example, between two consecutive experiments, the computer operator might receive the following instructions from the simulator cab: "Beam me up, Scotty, to 500 ft on a glide slope of 5°." In all probability, the appropriate vehicle attitude and control configuration for the accomplishment of this trimmed initial condition would not be known a priori. In fact, the phenomenon of "incomplete initial conditions" presents an unusual problem to the discipline of flight simulation programming.

To appreciate the problem of incomplete initial conditions, consider the fact that a discrete real-time model is coded in forward, sequential flow. Given all of the pilot controls and all of the vehicle states, then the model is designed (only) to produce all of the vehicle states at the next time-point (one cycle-time later). However, the problem of trimming implies the backward solution of the model. The question is: If all of the accelerations are required to be zero on succeeding cycles, what is the required orientation of the vehicle and what are the required values for all of the pilot controls?

Modes

The problem becomes slightly less complicated when two distinct "modes" are programmed into the discrete model because there is (1) an initial condition mode (IC mode) during which the independent variable time "t" does not advance after each consecutive pass through the model, and (2) an operate mode (OP mode) during which time advances each pass through the model in accordance with $t = kT$, where T is the cycle time. During the IC mode, special program branches are implemented so that accelerations are prevented from producing velocity changes and velocities are prevented from producing changes in the linear and angular positions.

Complex dynamics nonetheless occur in the IC mode. They are caused by unavoidable algebraic loops and occasional free-running integrators, i.e., integrators that function during the IC mode. These processes complicate the production of "effects" whenever known "causes" are given in sequential processes.

Cause and Effect

Despite the presence of dynamic behavior in IC mode, cause and effect can be established, as will be described. It will require an exercise in the calculus of variations to establish the causes when only the desired effects are known.

Given the existence of cause and effect, the "forward solution only" characteristic of a flight simulation model may be used in a novel procedure to produce an effective "backward solution." This procedure is referred to as "trimming a vehicle model." It involves the production of partial derivative matrices and the creation of pseudoinverses thereof. An iterative procedure is developed by using classical mathematical techniques. These techniques are known to have quadratic convergence features.

In flight simulation the backward-solution process is actually a third mode; it is called "Trim mode." Since a pilot iterates his many controls based upon his many observable states, Trim mode emulates a pilot in actual flight (or OP mode). However, the trim process in flight simulation is automatic and functions only in IC mode; the simulator pilot is necessarily removed from the procedure. For the subject of "trimming an aircraft," the following definitions are used.

1. The "control vector" consists of elements either normally set by a pilot or, from physical considerations, consists of elements such as the aircraft orientation and position. Any given "control" must influence at least one element in the "state vector."

2. The "state vector" consists of the vehicle linear and angular accelerations, usually expressed in the body-axis frame.

PARTIAL DERIVATIVE EVALUATION

In terms of a set of n controls $C_1, C_2, \ldots, C_n$ in two distinct configurations, a differential control vector $\mathbf{x}$ may be described by the difference,

$$\mathbf{x} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}_b - \begin{bmatrix} C_1 \\ C_2 \\ \\ C_n \end{bmatrix}_a \tag{1}$$

Assume that these controls at points a and b influence the model and produce two distinct sets of values for the states. In terms of m states, a differential state vector $\mathbf{y}$ may then be observed from the model in response to the previously described control configurations.

$$\mathbf{y} = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_m \end{bmatrix}_b - \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_m \end{bmatrix}_a \tag{2}$$

The observations of the states at points a and b are valid only if the states are sufficiently quiescent at both points. In practical terms this means that the two points must each be independently acquired after some cyclic count K during which cause and effect are well established, and filtration techniques may be required, particularly for helicopter models, for the generation of trends rather than the known periodic variations that such models produce.

Response Interval

For blade-element helicopter simulation models such as the Rotor Systems Research Aircraft (ref. 1) or the Black Hawk (ref. 2), both of which are referred to as GENHEL models, it has been determined that approximately three complete rotor revolutions are required to attain the "tracking blade" state. Therefore, in response to a control perturbation, a large number of cycles through the model must be made before the state vector becomes sufficiently quiescent for algorithmic use. At a cycle time of 20 msec, three complete rotor revolutions translate to about 40 cycles through the model. Additionally, even after stationary equilibrium is attained, the rotor system number of blades per revolution (N/rev) harmonics (refs. 3 and 4) continue to obscure the vehicle states, as discussed in the next section.

Rotor response to a step input is a relatively slow dynamic process. This is compounded in IC mode by a smoothing filter that is usually used to suppress the rotor system N/rev harmonics. The resultant responses are "slow" especially following a collective input, where a reasonable step change (for the purpose of evaluating a partial derivative) in the control during the IC mode takes well over one half second (in pseudoreal time) to produce a vertical-axis acceleration response that is in the neighborhood of its final value.

In contrast to conventional aircraft simulation models, helicopter models usually contain more algebraic loops and free-running integrators. The blades cannot be static in this mode, but rather, they must advance in azimuth just as they do in OP mode in order to produce the appropriate forces and moments. Although no transition of time exists in IC mode, the azimuth angle for each computer cycle during this mode is the product of the rotor rpm and the cycle time T, just as in OP mode. Also, rotor downwash terms as well as flapping and lagging integrations function just as they do in OP mode. For these reasons there is an approximate cycle count equivalency during IC mode which may be expressed in terms of the requisite time for approximate convergence to final values (tracking blades) during OP mode. If the time for convergence is roughly T_c in OP mode, the requisite convergence number in IC mode is given approximately by $K = T_c/T$. For the example simulation model used here, the required value of T_c is about 0.8 sec. Complete quiescence is never actually achieved, but certain statistical operations may be used to aid convergence. They are shown below to satisfactorily isolate the effective trends of the state vector.

For a conventional aircraft simulation with a minimal number of algebraic loops and no free-running integrators, the value of K may be as small as 2 or 3, but for a blade-element helicopter model it is more probably in the neighborhood of 40 or 50.

Smoothing the States

The determination of values for states is a more complicated process than merely waiting a long time for inputs to propagate through the system and produce quiescent responses. Even when a good delay count of K (or interval T_c) is determined for a helicopter model (by examining responses), any given evaluation of the states will nonetheless be contaminated by the N/rev harmonics produced by its rotor system. This periodic contamination of the trends of the state vector renders any specific evaluation point quite useless for the computation of partial derivatives. For this reason the following regression technique is used.

Statistical parameters are accumulated over a sufficient interval (K cycles) and the assumed functionality is then evaluated at the end point. The assumption of quadratic behavior works well, particularly where the derivative at the end point is forced to be zero (i.e., is in quiescence). This regression technique is a least-squares solution for the best quadratic-curve fit (ref. 5), with a constraint. A detailed description follows.

Assume a quadratic history of a particular state S as a function of the cycle count k. This is a more reasonable functionality than time t (where, for a discrete model $t = kT$) because time does not advance in IC mode. Observations of a state S are made at each point k, with the assumed functionality

$$(S)_k = a_1 + a_2 k + a_3 k^2 \tag{3}$$

where $k = 1, 2, \ldots, K$. If k is assumed to be a continuous variable, the derivative of this function with respect to the count is given by

$$\frac{d(S)_k}{dk} = a_2 + 2a_3 k \tag{4}$$

When equation (4) is set equal to zero at the final time point ($k = K$), one of the unknowns is eliminated.

$$a_3 = -\frac{a_2}{2K} \tag{5}$$

The function is then expressed in terms of two unknowns a_1 and a_2.

$$(S)_k = a_1 + \frac{a_2 k(2K - k)}{2K} \tag{6}$$

By defining the operator σ as the summation from 1 to K, the method of least squares can be used to give the solutions in terms of the observations.

$$
\begin{aligned}
a_1 &= \frac{\sigma[k^2(2K - k)^2]\sigma[S_k] - \sigma[k(2K - k)]\sigma[S_k k(2K - k)]}{K\sigma[k^2(2K - k)^2] - \sigma[k[2K - k)]^2} \\
a_2 &= \frac{2K^2\sigma[S_k k(2K - k)] - 2K\sigma[k(2K - k)]\sigma[S_k]}{K\sigma[k^2(2K - k)]^2 - \sigma[k(2K - k)]^2}
\end{aligned}
\tag{7}
$$

The final value (or best estimate of S after K cycles) is then given by the value of the quadratic function at $k = K$.

$$
\begin{aligned}
(S)_k &= a_1 + \frac{1}{2} a_2 K \\
&= \frac{\sigma[S_k]\{\sigma[k^2(2K - k)^2] - K^2\sigma[k(2K - k)]\} + \sigma[S_k k(2K - k)]\{K^3 - \sigma[k(2K - k)]\}}{K\sigma[k^2(2K - k)^2] - \sigma[k(2K - k)]^2} \\
&= \frac{30\sigma[k(2K - k)S_k] - 6(K + 1)(2K + 1)\sigma[S_k]}{K(K + 1)(8K - 11)}
\end{aligned}
\tag{8}
$$

For K observation points (K cycles) after a control deflection is initiated, the two indicated running summations are computed for each state. This process produces the best estimate of each state under the assumptions.

Evaluation Sequence

Each control is independently incremented and the resultant states are evaluated using the regression technique just described. The sequence is as follows. (1) All of the states are computed for the initial control vector, here defined as the "primary point." (2) The first control is incremented by an amount to be later discussed. (3) Evaluation of all of the states is made after K cycles according to the regression techniques. (4) The first control is then returned to its initial (primary) point and the second control is incremented by a designated amount. (5) This iterative process of incrementing and evaluating continues until all of the n controls are completed. At the end of this sequence, the partials of all states with respect to control variations have been obtained by comparison with the "primary point" values, i.e., the H matrix has been computed.

$$
H = \begin{bmatrix}
\Delta S_1/\Delta C_1 & \Delta S_1/\Delta C_2 & \cdots & \Delta S_1/\Delta C_n \\
\Delta S_2/\Delta C_1 & \Delta S_2/\Delta C_2 & \cdots & \Delta S_2/\Delta C_n \\
\vdots & & & \\
\Delta S_m/\Delta C_1 & \Delta S_m/\Delta C_2 & \cdots & \Delta S_m/\Delta C_n
\end{bmatrix}
\tag{9}
$$

Note that this $\mathbf{H}$ matrix (m by n) relates the differential control vector $\mathbf{x}$ (n by 1) to the differential state vector $\mathbf{y}$ (m by 1) according to

$$\mathbf{y} = \mathbf{Hx} \tag{10}$$

for small changes in the controls. The elements of the $\mathbf{H}$ matrix are referred to as the "stability derivatives."

Range of Controls

The maximum and minimum control values must be specified. Each of the n controls then has a given total range.

$$\begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}_{max} - \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}_{min} \tag{11}$$

The maximum and minimum control deflections are used for more than one purpose. By limiting the allowable control excursion range they prevent the controls from entering areas of nonlinearities from which they may never recover. For example, given a large initial pitch or roll angle, certain aircraft may trim themselves upside down! At ARC we arbitrarily use 0.5% of this range as the partial evaluation size in the development of the $\mathbf{H}$ matrix:

$$\Delta C_i = 0.005\ R_i \qquad (i = 1, 2, \ldots n) \tag{12}$$

Changes in controls that are used for evaluation purposes must be significant, but must not cause states to transcend a reasonable region of linearity. Once the partial derivatives have been evaluated, a new "primary evaluation" may be performed. Equation (10) is the basis for this evaluation, as is outlined in the next section.

PRIMARY EVALUATIONS

The transition of a complete control vector from set a to set b as in equation (1) for the trimming problem is driven by the fact that the desired set b of states in equation (2) is the null vector. This null vector, within some small error criteria, is the definition of trim for aircraft simulation work. In terms of aircraft body linear and angular accelerations the state vector that is required from equation (2) is the null vector minus the values observed at the initial primary point a. The differential state vector $\mathbf{y}$ for one cycle of trimming is thus defined.

$$\mathbf{y} = -\begin{bmatrix} \dot{q} \\ \dot{w} \\ \dot{p} \\ \dot{v} \\ \dot{r} \\ \dot{u} \end{bmatrix} \tag{13}$$

The indicated ordering of the state vector is not arbitrary, as will be discussed under "Ground Trim."

System Definitions

Equation (10) is restated below with the appropriate dimensions included for clarification.

$$\mathbf{y}[m \cdot 1] = \mathbf{H}[m \cdot n]\mathbf{x}[n \cdot 1] \tag{14}$$

This system is overdetermined if there are more states than controls ($m > n$); it is determined if the number of states is equal to the number of controls ($m = n$); and it is underdetermined if the number of states is less than the number of controls ($m < n$). The least-squares method of solution to both the over- and underdetermined cases is outlined in the next two sections. The determined case, which is trivial, is a subset of either of these techniques.

The following mathematical techniques are not universally applicable. However, they are quite useful in the aircraft simulation field, where trim points are known to exist. It is helpful to keep in mind that this is an iterative procedure in which small changes are made. In the process of trimming an aircraft model, only small linear subspaces of the total region need be considered.

The Overdetermined Case

The overdetermined case means that there are more states than controls. For the overdetermined case, there is in general no exact solution for $\mathbf{x}$ because of probable inconsistencies in the system. However, a vector $\mathbf{x}$ which minimizes the error norm (a scalar) may be obtained. This scalar is created by forming the error norm in combination with an arbitrary, symmetric, positive-definite (m-by-m) weighting matrix $\mathbf{W}$.

For aircraft simulation work, the weighting matrix need not be quite so arbitrary. With the state vector $\mathbf{y}$ defined as in equation (13), the angular accelerations are given in rad/sec^2 and the linear accelerations are given in ft/sec^2. For approximate equivalency, this leads to the requirement that in terms of pure numbers (without units) the angular terms should have an order of magnitude more influence than the linear terms to satisfy motion-system acceptance criteria. For example, if linear accelerations during convergence are in the region of 0.01 ft/sec^2, the angular accelerations should be in the region of 0.001 rad/sec^2. According to the definition of equation (13), this produces the following arrangement of $\mathbf{W}$ when the number of states $m = 6$.

$$\mathbf{W} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix} \tag{15}$$

The overdetermined solution proceeds as follows. The scalar error norm is expressed

$$Q = (\mathbf{Hx} - \mathbf{y})'\mathbf{W}(\mathbf{Hx} - \mathbf{y}) \tag{16}$$

and differentiating this (ref. 6) with respect to the vector $\mathbf{x}$ giving

$$\frac{dQ}{d\mathbf{x}} = 2\mathbf{H}'\mathbf{WHx} - 2\mathbf{H}'\mathbf{Wy} \tag{17}$$

Setting this equal to zero (for extreme values) implies

$$\mathbf{H}'\mathbf{WHx} = \mathbf{H}'\mathbf{Wy} \tag{18}$$

The solution vector is thus given by

$$\mathbf{x} = (\mathbf{H}'\mathbf{WH})^{-1}\mathbf{H}'\mathbf{Wy} \tag{19}$$

The Underdetermined Case

The underdetermined case means that there are more controls than states. For the underdetermined case, there are infinitely many solutions for $\mathbf{x}$ that satisfy the equation exactly, but to make the selection unique, the condition may be imposed that the norm $\mathbf{x}'\mathbf{Vx}$ be minimized where $\mathbf{V}$ is an arbitrary symmetric positive-definite (n-by-n) weighting matrix. The Legrange method of multipliers calls for the formation of the sum

$$Q = \mathbf{x}'\mathbf{Vx} - 2\boldsymbol{\beta}'(\mathbf{Hx} - \mathbf{y}) \tag{20}$$

where $\boldsymbol{\beta}$ is a vector multiplier. Differentiating with respect to the vector $\mathbf{x}$ yields

$$\frac{dQ}{d\mathbf{x}} = 2\mathbf{V}\mathbf{x} - 2\mathbf{H}'\boldsymbol{\beta} \tag{21}$$

and setting this equal to zero gives

$$\mathbf{x} = \mathbf{V}^{-1}\mathbf{H}'\boldsymbol{\beta} \tag{22}$$

This produces the solution vector

$$\mathbf{x} = \mathbf{V}^{-1}\mathbf{H}'(\mathbf{H}\mathbf{V}^{-1}\mathbf{H}')^{-1}\mathbf{y} \tag{23}$$

According to the dimensions of $\mathbf{V}$, in the underdetermined case the controls rather than the states are weighted. This operation has redundancy with the control limiting of equation (11) in that the ranges adequately isolate the relative-magnitude importance of the controls. For this reason $\mathbf{V}$ is taken to be the identity matrix so that the final underdetermined solution becomes

$$\mathbf{x} = \mathbf{H}'(\mathbf{H}\mathbf{H}')^{-1}\mathbf{y} \tag{24}$$

Note that both pseudoinverses $(\mathbf{H}'\mathbf{W}\mathbf{H})^{-1}\mathbf{H}'\mathbf{W}$ and $\mathbf{V}^{-1}\mathbf{H}'(\mathbf{H}\mathbf{V}^{-1}\mathbf{H}')^{-1}$ reduce to $\mathbf{H}^{-1}$ (if it exists) for the determined case.

Mathematical Rigor

The linear techniques just described are not global-search algorithms. When larger solution spaces are considered, mathematical exceptions to these techniques occur. For instance, the process is easily thwarted by an "inflection point." However, for the aircraft trimming problem, aberrant points are few and far between, especially if the initial control vector is in the approximate region of the trim point upon process initiation.

A linearly independent set of controls must be selected in order to avoid indeterminate pseudoinverses. No two controls may produce proportional state-vector changes.

The direct use of either equations (19) or (24) in a sequence of primary evaluations may lead to the discrete "limit cycle" phenomenon. For this reason, a scalar gain $G = 0.5$ is nominally associated with these equations. Optionally, this gain may be scheduled in the interval (0, 1).

COMMUNICATION PARAMETERS

At ARC the trimming procedures are implemented in a stand-alone, generalized FORTRAN subroutine called STILL which is part of our BASIC structure (ref. 7). This routine communicates with an aircraft simulation model by using two different media. All kinematic variables (such as the state vector) and discrete switches use our standard BASIC COMMON, but the subroutine calling sequence is used to communicate the number of applicable controls, their identity, and their extreme values. The technique of using a calling sequence for these quantities was selected because it permits flight simulation programmers to incorporate alternate trimming procedures within the same program. For example, a conventional trim procedure using aircraft pitch attitude as one of the controls in a horizontal flight configuration may be alternated with a procedure in which pitch attitude may be frozen in a process of determining the requisite flightpath angle or power level.

Boundary Control

The maximum and minimum control-deflection values of equation (11) are used to maintain the control vector within a region of interest. A check of each control is made at the end of each primary evaluation. If any given control violates its boundary, it is arbitrarily set back from the boundary by an amount equal to 5% of its range. This procedure is occasionally useful during the initial debug of a simulation model, during which phase the utilized initial control values tend to be quite removed from their requisite trim values.

Ground Trim

A discrete (e.g., a switch) exists within the BASIC COMMON subroutine structure that informs various programs when at least one gear is compressed (ref. 8). In this case, the calling sequence of subroutine STILL is ignored, and alternate control and state definitions are automatically employed. A takeoff configuration is thus easily sequenced into a simulation effort. In this case the selected differential state-vector is a subset of equation (13):

$$\mathbf{y} = -\begin{bmatrix} \dot{q} \\ \dot{w} \\ \dot{p} \\ \dot{v} \end{bmatrix} \tag{25}$$

For the vector in equation (25), m = 4. Note that by using the same sequence as in equation (13), subroutine STILL maintains the same notation for all options.

The ground-trim control vector was selected because of its capacity to trim the state vector given the complex interactions of a helicopter model. It consists of the vehicle pitch attitude, its height, and its roll angle.

$$C = \begin{bmatrix} \theta \\ h \\ \phi \end{bmatrix} \tag{26}$$

In equation (25), no attempt is made to null the longitudinal-acceleration component because motion simulators are normally capable of accommodating this variable by a static offset called "residual tilt." Any yaw acceleration during takeoff may be accommodated by either a (more) symmetric thrust distribution or by the foot pedals (or even differential braking action). Yaw acceleration has not been a problem during ground trim except at extremely low velocities where gear braking and side-force models had inadequate breakout limits.

The determination of the maximum and minimum control deflections is automatic during ground trim. The extreme values are computed from geometrical relationships involving gear locations with respect to the vehicle c.g. These locations are also part of the BASIC COMMON structure used by STILL.

Airborne Trim

For airborne trim, the calling sequence to STILL specifies the order of the controls in terms of their sequential partial evaluations. The calling sequence is padded to the tenth value but any integer value for the numer of controls N from one to ten is permissible. The calling sequence is given by:

```
CALL STILL (N, C1, CMIN1, CMAX1, C2,  CMIN2,  CMAX2,
            C3, CMIN3, CMAX3, C4,  CMIN4,  CMAX4,
            C5, CMIN5, CMAX5, C6,  CMIN6,  CMAX6,
            C7, CMIN7, CMAX7, C8,  CMIN8,  CMAX8,
            C9, CMIN9, CMAX9, C10, CMIN10, CMAX10)
```

If the inelegance of this structure seems objectionable, then consider the fact that STILL functions on a variety of computers including Xerox Sigmas, IBM PCs, VAXs, and CDC 7600s.

Observation of Progress

For convenience of observation, subroutine STILL provides a discrete that informs the user exactly when a primary evaluation is completed. This discrete, called IEVAL, is set for only one cycle when both the controls and states are absolutely compatible, i.e., the display or print commands slaved to this discrete show the true trim progress independent of intermediate partial evaluations. This discrete is necessarily set at the end of the interval during which the value of IPART is zero (the primary evaluation interval).

EXAMPLES

A typical blade-element rotor system is used in order to display some of the features of STILL. This system illustrates two different control configurations. These configurations are selected to show the convergence features for the over- and underdetermined cases and to exemplify some of the challenges associated with a blade-element helicopter model. Neither the vehicle flight regime nor the particular units associated with the selected controls or states are germane to this presentation.

The first configuration, that of an overdetermined case, is also used to identify the separate features of the partial evaluation sequence by use of a subset of the trim history.

The control list for the overdetermined case includes (1) a collective-proportional control "THETAO," (2) a longitudinal cyclic control "B1S," (3) a lateral cyclic control "A1S," (4) a tail rotor control "PEDAL," and (5) the roll attitude of the vehicle "PHI."

Partial Evaluation Sequence

The initial 12 sec of trimming data are given in figures 1 and 2. The controls are shown in figures 1(a) through 1(e), and a partial-evaluation-sequence identifier (a control variable subscript) called "IPART" is shown in figure 1(f). If only the initial portion of the trim process is used, the separate partial evaluations are easily identified, although trim is not accomplished in this short interval. For this particular model the cycle time used was $T = 20$ msec and the selected time for convergence (for each evaluation) $T_c = 0.8$ sec. Hence, for 40 cycles ($K = 40$), or 0.8 sec of pseudoreal time, each partial evaluation is made, and this is repeated by cycling through the controls in sequence.

In figure 1(a) we see that the first control, THETAO, is incremented for 0.8 sec (at $t = 0.8$ sec, after the initial primary evaluation) and then returned to its original value at $t = 1.6$ sec. The second control, B1S, is then incremented for a like duration. This process continues until all five controls are incremented, as is shown in figure 1(f). The quantity "IPART" shows which control is being incremented. When IPART = 0 a "primary evaluation" is being made. After this occurs, the entire process repeats itself. Hence, $(N + 1)T_c = 4.8$ sec are required for each complete primary evaluation when $N = 5$.

The influence of the above procedure upon the states is given in figures 2(a) through 2(f) for the angular and linear states. In figure 2(b) we see that QBD, the pitch acceleration, responds to the first partial THETAO of figure 1(a) beginning at $t = 0.8$ sec with a transient behavior, and that the transient is really not completed even after $t = 1.6$ sec. However, at this time the first partial column of equation (9) is evaluated. This is one example of how the final-value regression technique of equation (8) is useful in isolating the trend of the state history. In figure 2(f), this same control is shown to cause transient behavior in the vertical acceleration WBD. Note that when the first partial is returned to its initial value (at $t = 1.6$ sec) and the second partial is incremented, a similar transient behavior occurs.

This process of incrementing and observing (statistically) continues until all five partials are evaluated at $t = 4.8$ sec (five controls plus one primary evaluation times 0.8 sec). At this point another primary evaluation is initiated. All of the controls are changed at this time (fig. 1), and all of the states respond (fig. 2). The statistical evaluation of the state response to this vector change is completed at $t = 5.6$ sec, at which time equation (19) produces a new primary evaluation point. Each state indicates a quadratic-type transient with superimposed noise characteristics during this primary interval, as is shown in all traces of figure 2 beginning at $t = 4.8$ sec and ending at $t = 5.6$ sec. This is another reason for using the final-value regression technique, here evaluated at the final point given by $t = 5.6$ sec.

From simple observation of the first 12 sec of the trimming procedure, no clear trends of the controls can be determined. This is because 12 sec of data corresponds to only two-and-a-half primary evaluations, as shown by IPART in figure 1(f). The complete trimming interval is examined in the next section.

Overdetermined Convergence

Figures 3 and 4 show the complete convergence process for the overdetermined example. Less than one minute was required for 12 primary evaluations, which caused all states to have negligible values (after the last primary evaluation). The trimming algorithm terminates automatically when this occurs.

For the overdetermined example, the value of vehicle yaw angle psi = 0.0°, and the vehicle pitch angle THETA = 1.26°; these quantities are not part of the control list in this case. The five controls that were used, their ranges, and their initial and final values are given in table 1.

The quadratic-convergence features of the trimming algorithm are easily seen from figures 3 and 4, even though the intermediate partial evaluations tend to obscure the traces.

TABLE 1.- OVERDETERMINED PARAMETERS

Control number	Name	Minimum value	Maximum value	Initial value	Final trim
1	THETAO	10	18	15.5	15.8427
2	B1S	0	2	1.1	1.3632
3	A1S	-2	0	-0.5	-0.8312
4	PEDAL	0	2	0.8	1.0441
5	PHI	-2	2	0.5	0.5265

Underdetermined Convergence

For this example the control list is expanded to include the yaw angle PSI and the pitch angle THETA along with the original five controls. The list then includes seven controls; one greater than the number of states. The seven controls used, their ranges, and their initial and final values are given in table 2.

The final trim point is only approximately equal to that of the overdetermined case. This occurs because two additional degrees of freedom have been introduced.

TABLE 2.- UNDERDETERMINED PARAMETERS

Control number	Name	Minimum value	Maximum value	Initial value	Final trim
1	THETAO	10	18	15.5	15.8311
2	B1S	0	2	1.1	1.3629
3	A1S	-2	0	-0.5	-0.8256
4	PEDAL	0	2	0.8	1.0491
5	PHI	-2	2	0.5	0.5482
6	PSI	-2	2	0.0	-0.1517
7	THETA	0	4	1.26	1.3193

Trim completion also needs 12 primary evaluations for the underdetermined case; this requires a longer total time interval because there are two more controls to be evaluated per primary point. The control history is presented in figure 5 and the state history is presented in figure 6. For convenience the partial-evaluation-sequence identifier IPART is presented both in figure 5(f) and 5(i).

CONCLUSIONS

The regression techniques given here are sufficient to trim a wide variety of simulation models. The curve-fit feature for raw input data expands the capacity of the algorithms to handle models with algebraic loops, free-running integrators, and noisy environments such as those of helicopter models.

The procedures presented in this paper are represented by a stand-alone FORTRAN subroutine called STILL. This program, which performs all the required mathematical operations including matrix inversion, has many useful features. By use of over- and underdetermined control lists, it can create trim maps over complete flight envelopes. These data are important for simulation debug and systematic checkout. Additionally, real-time operations invariably require the STILL capability to determine trim controls at arbitrary initial-flight conditions.

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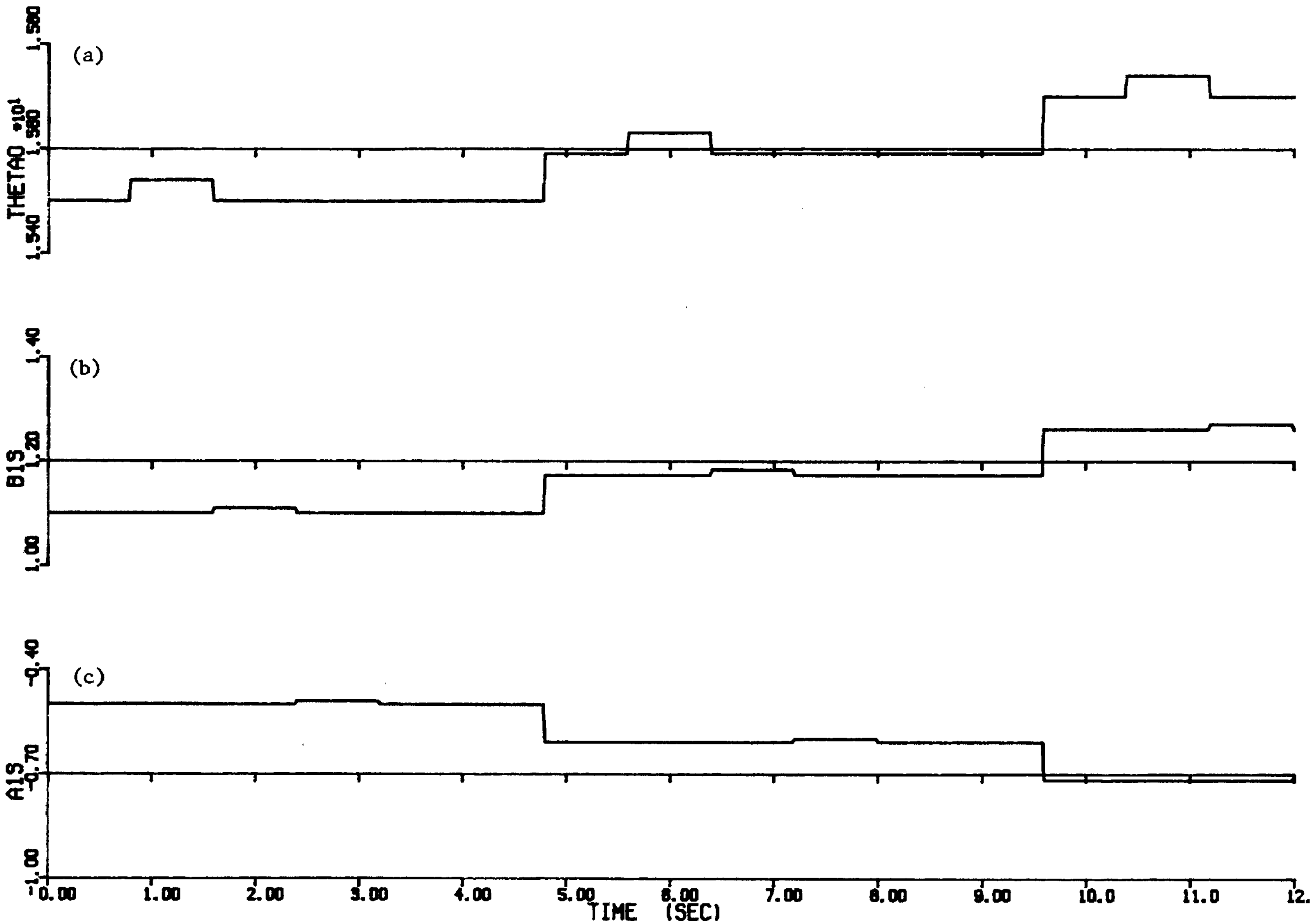

Figure 1.- Overdetermined controls, short sequence. (a) Collective THETAO. (b) Longitudinal cyclic B1S. (c) Lateral cyclic A1S.

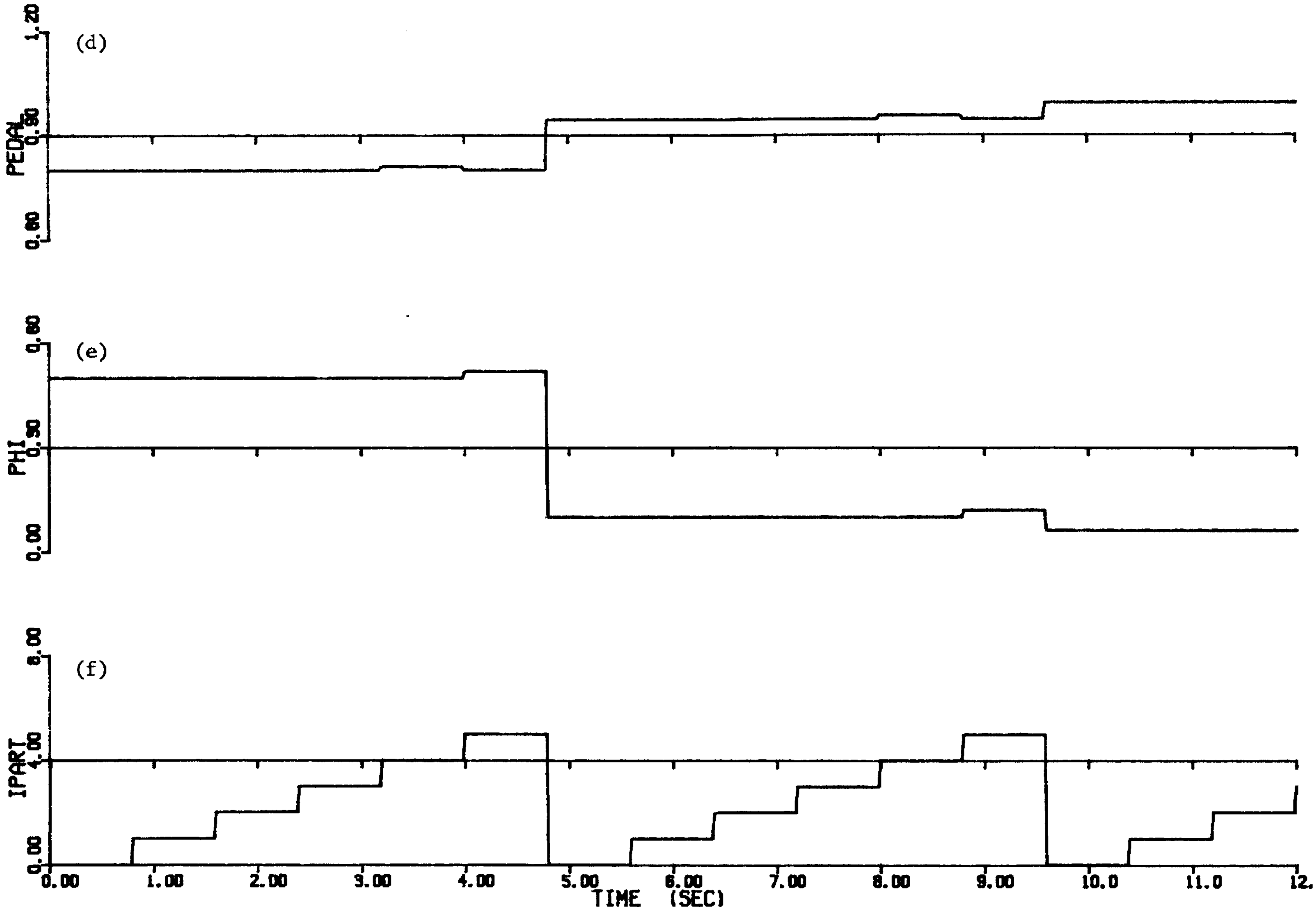

Figure 1.- Concluded. (d) Yaw control PEDAL. (e) Roll angle PHI. (f) Evaluation sequence identifier IPART.

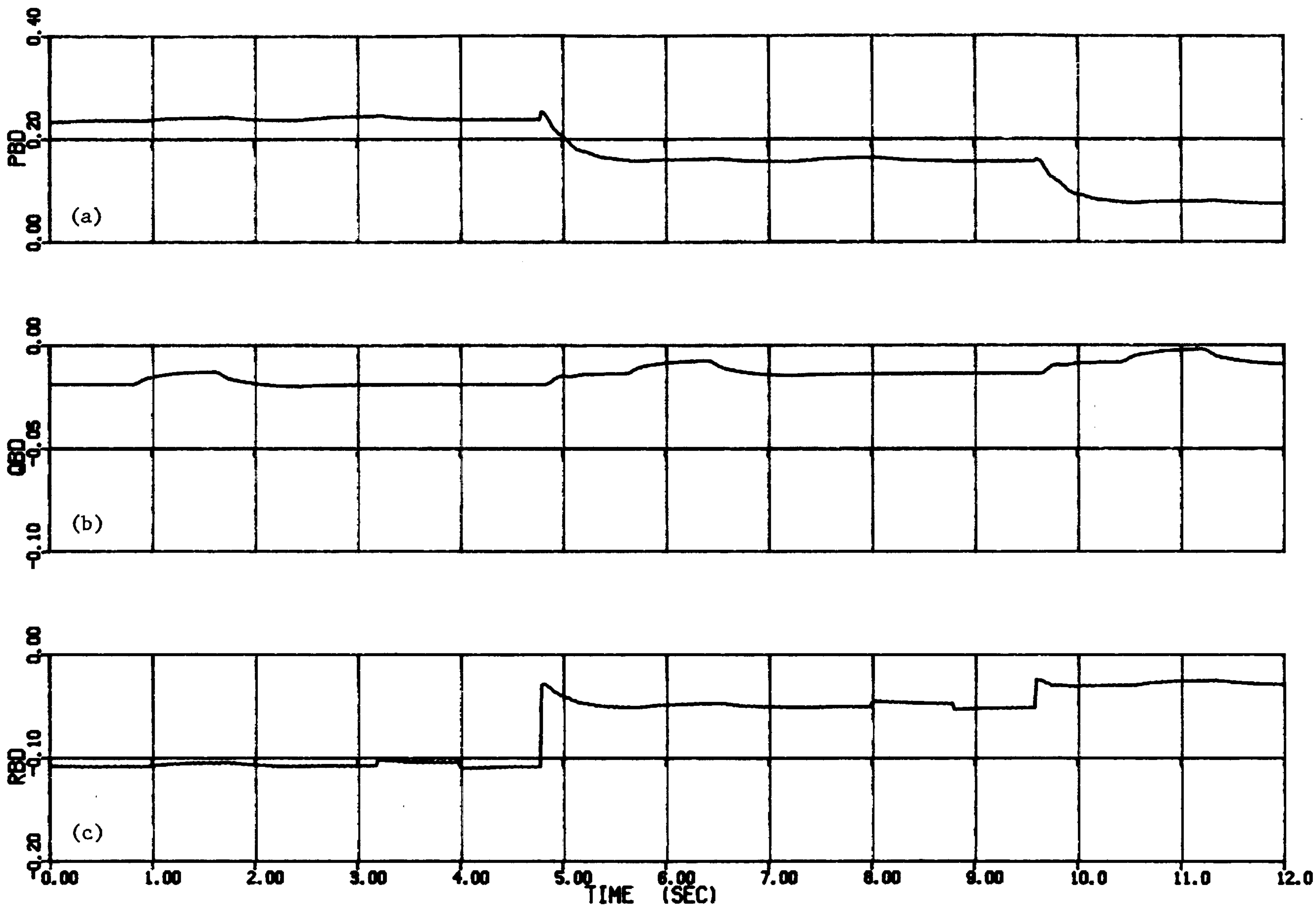

Figure 2.- Overdetermined states, short sequence. (a) Roll acceleration PBD.
(b) Pitch acceleration QBD. (c) Yaw acceleration RBD.

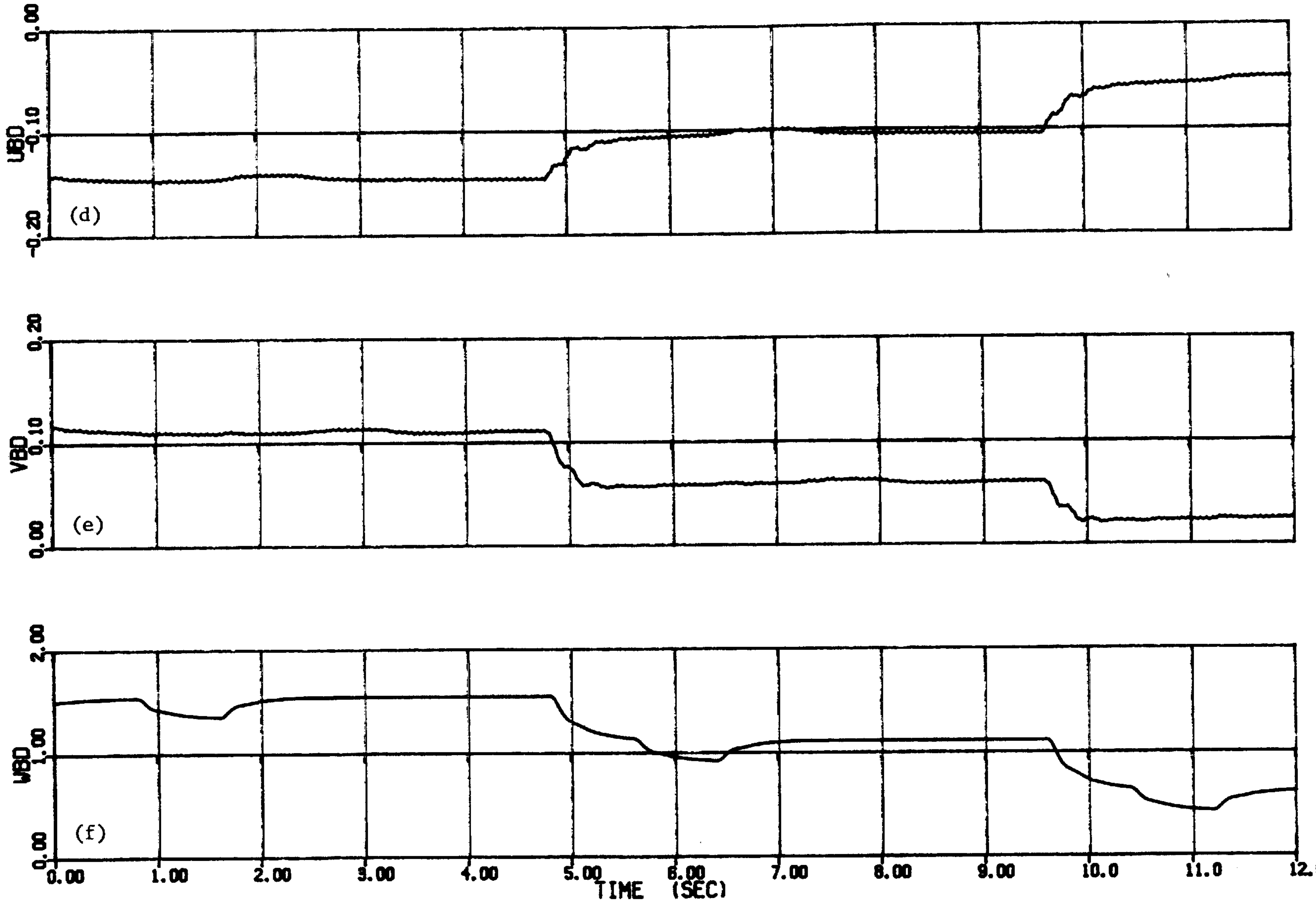

Figure 2.- Concluded. (d) Forward acceleration UBD. (e) Side acceleration VBD. (f) Downward acceleration WBD.

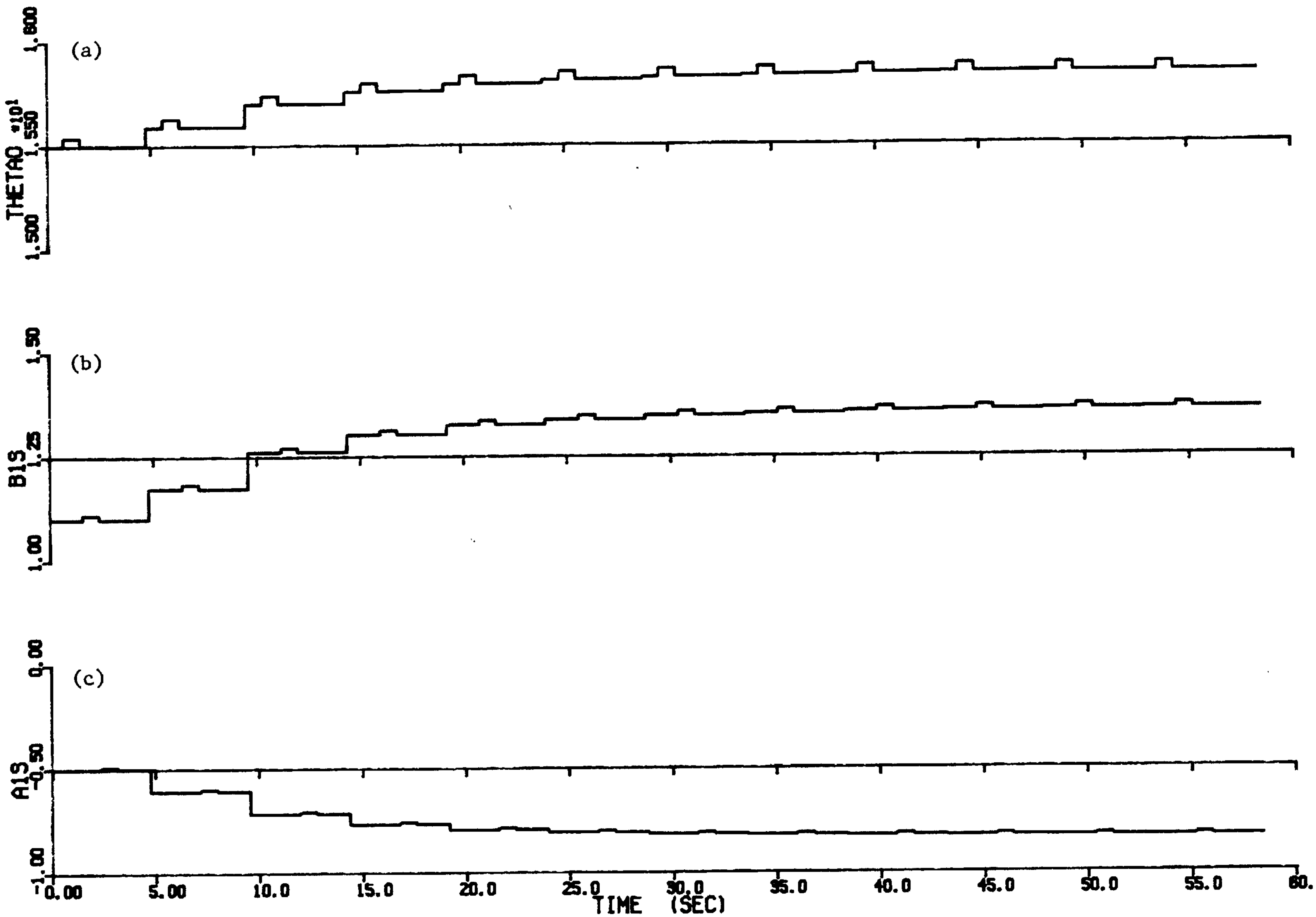

Figure 3.- Overdetermined controls, complete trim. (a) Collective THETAO. (b) Longitudinal cyclic B1S. (c) Lateral cyclic A1S.

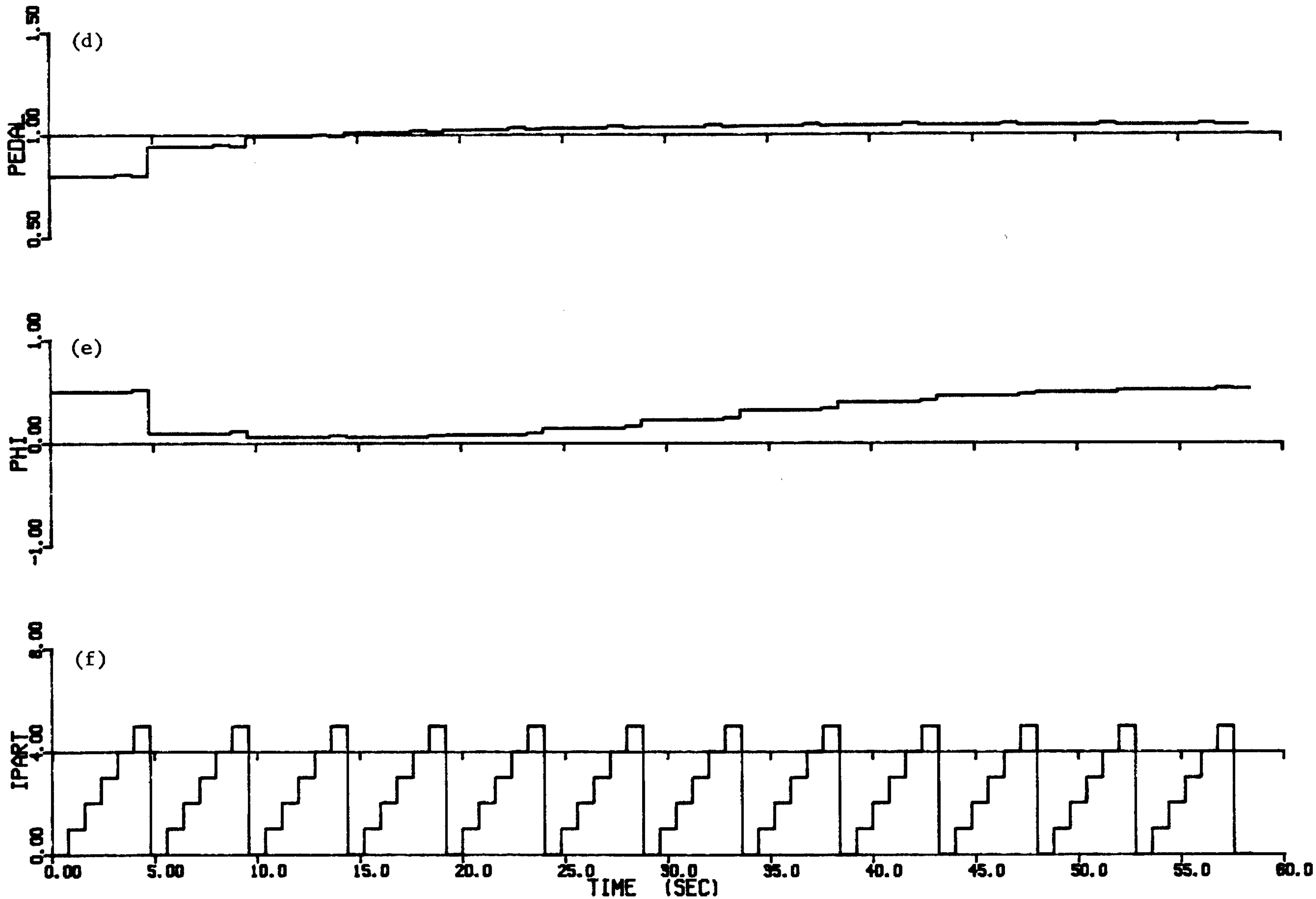

Figure 3.- Concluded. (d) Yaw control PEDAL. (e) Roll angle PHI.
(f) Evaluation sequence identifier IPART.

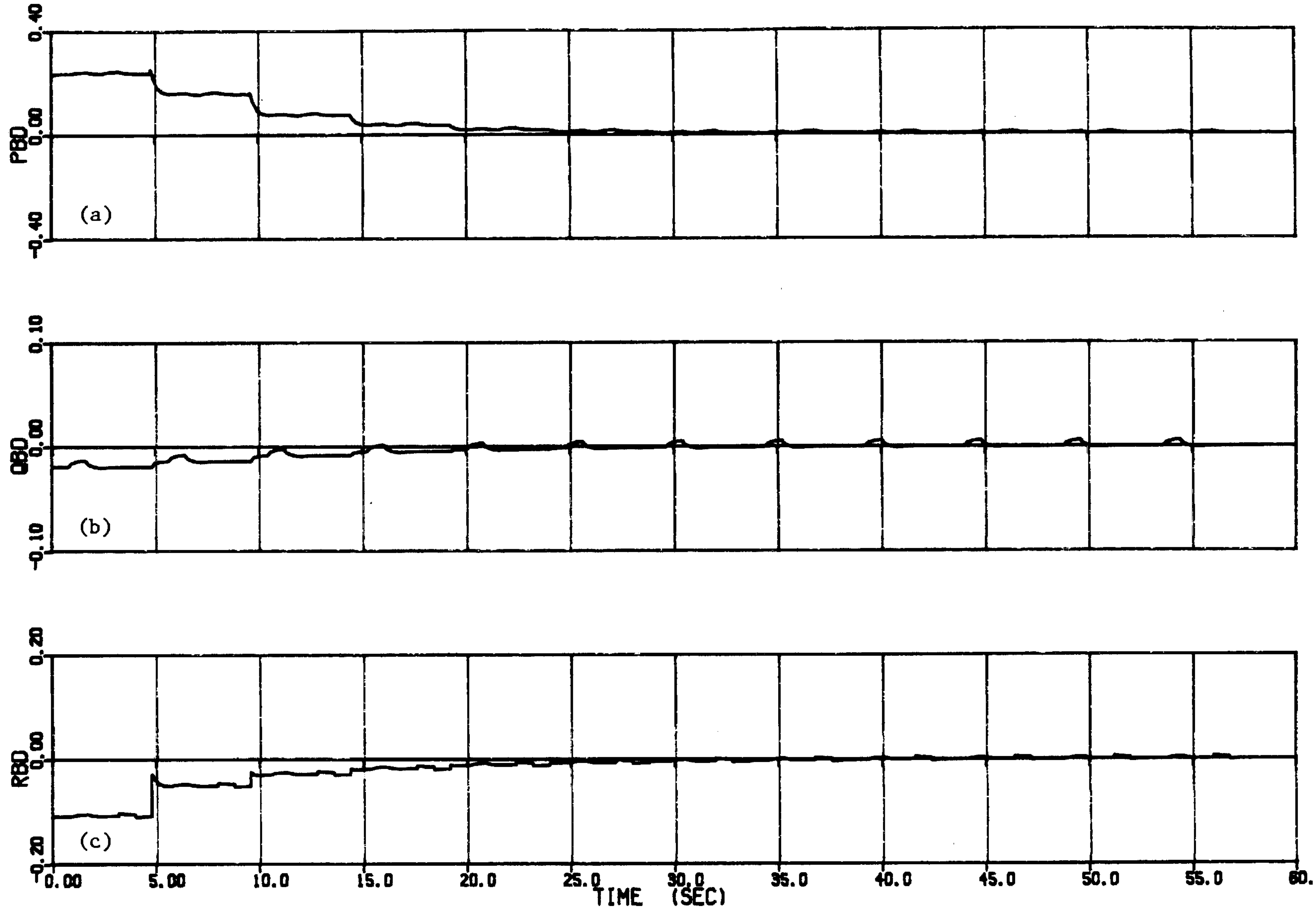

Figure 4.- Overdetermined states, complete trim. (a) Roll acceleration PBD. (b) Pitch acceleration QBD. (c) Yaw acceleration RBD.

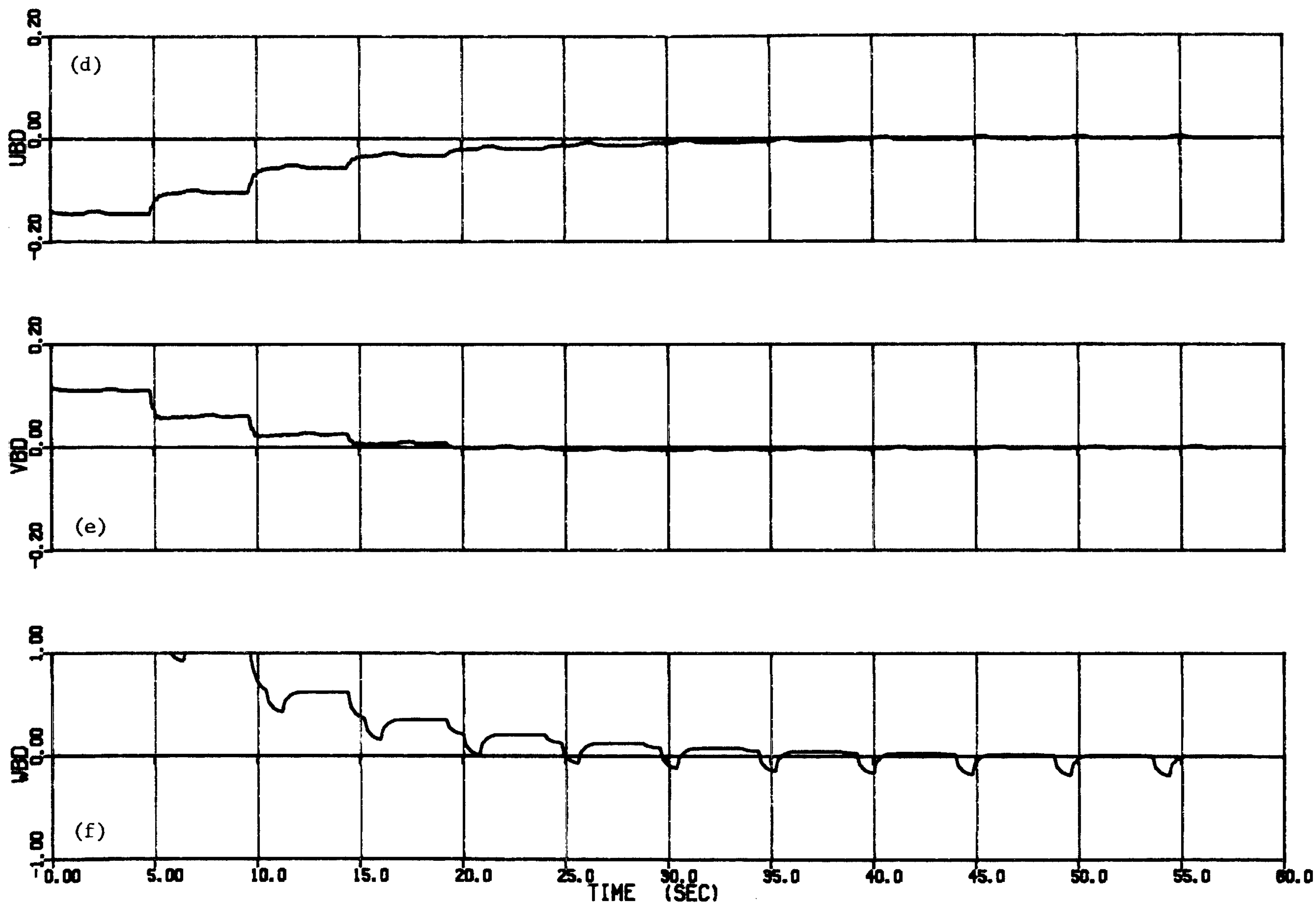

Figure 4.- Concluded. (d) Forward acceleration UBD. (e) Side acceleration VBD. (f) Downward acceleration WBD.

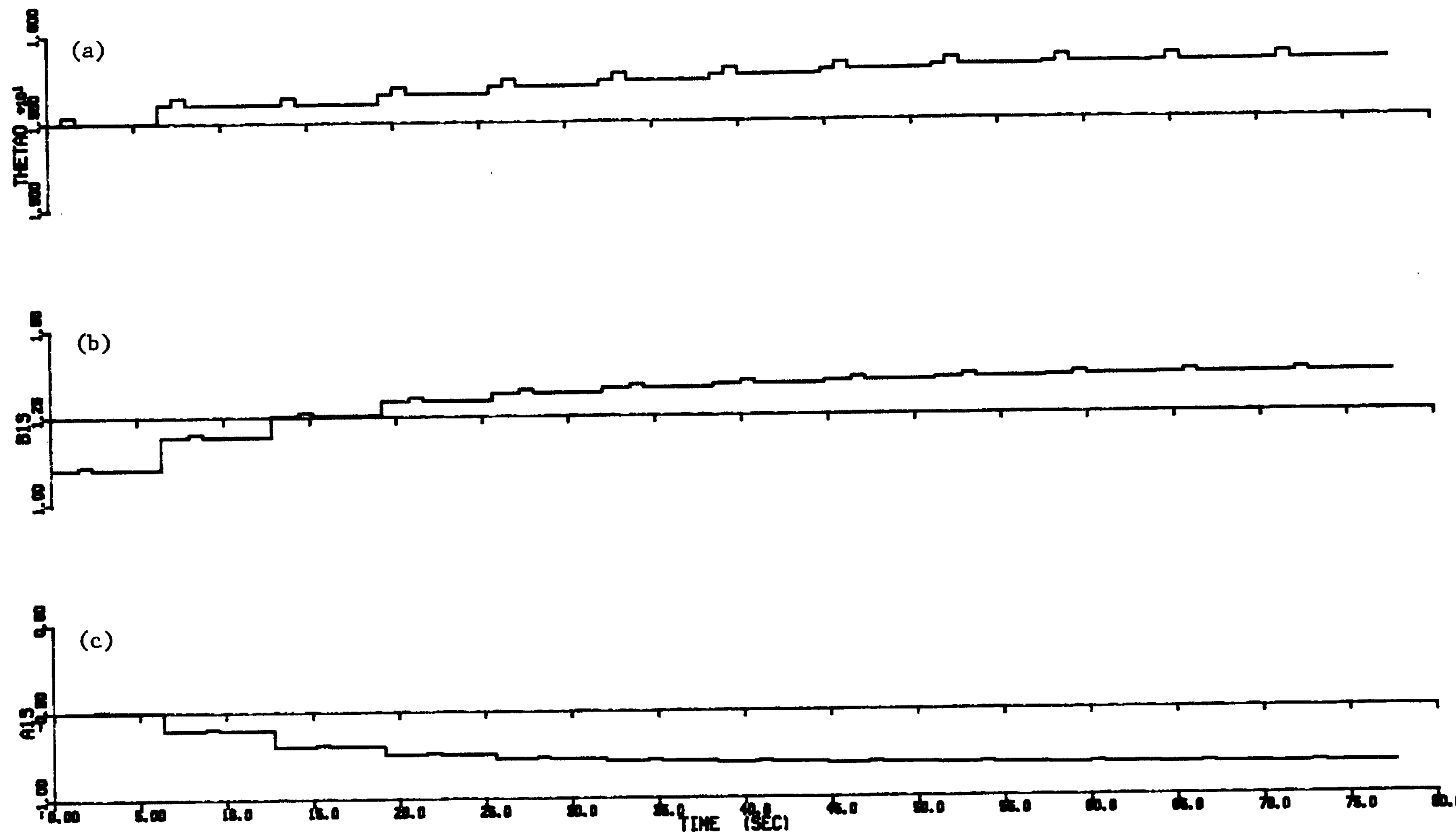

Figure 5.- Underdetermined controls, complete trim. (a) Collective THETAO. (b) Longitudinal cyclic B1S. (c) Lateral cyclic A1S.

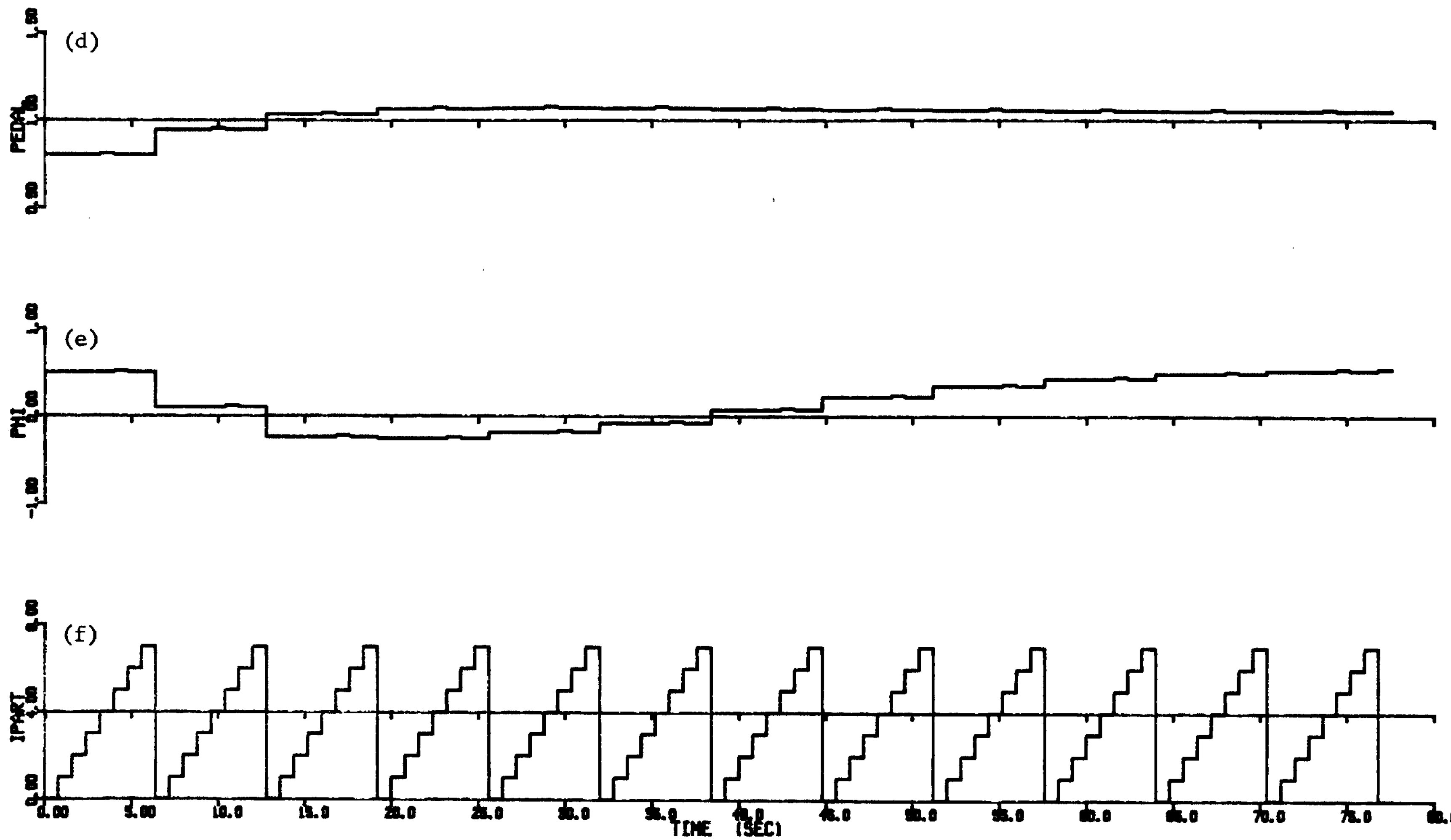

Figure 5.- Continued. (d) Yaw control PEDAL. (e) Roll angle PHI.
(f) Evaluation sequence identifier IPART.

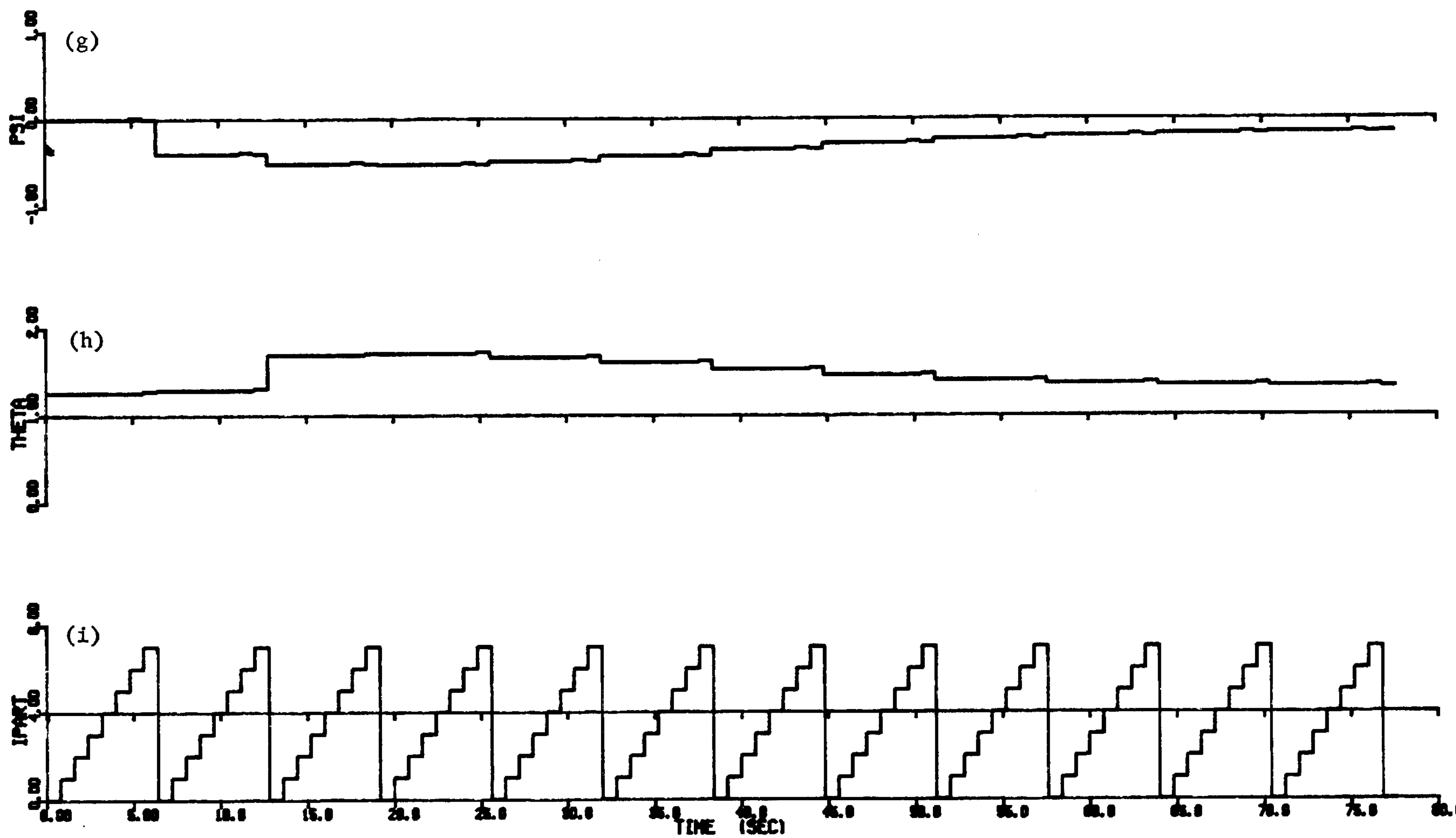

Figure 5.- Concluded. (g) Yaw angle PSI. (h) Pitch angle THETA.
(i) Evaluation sequence identifier IPART.

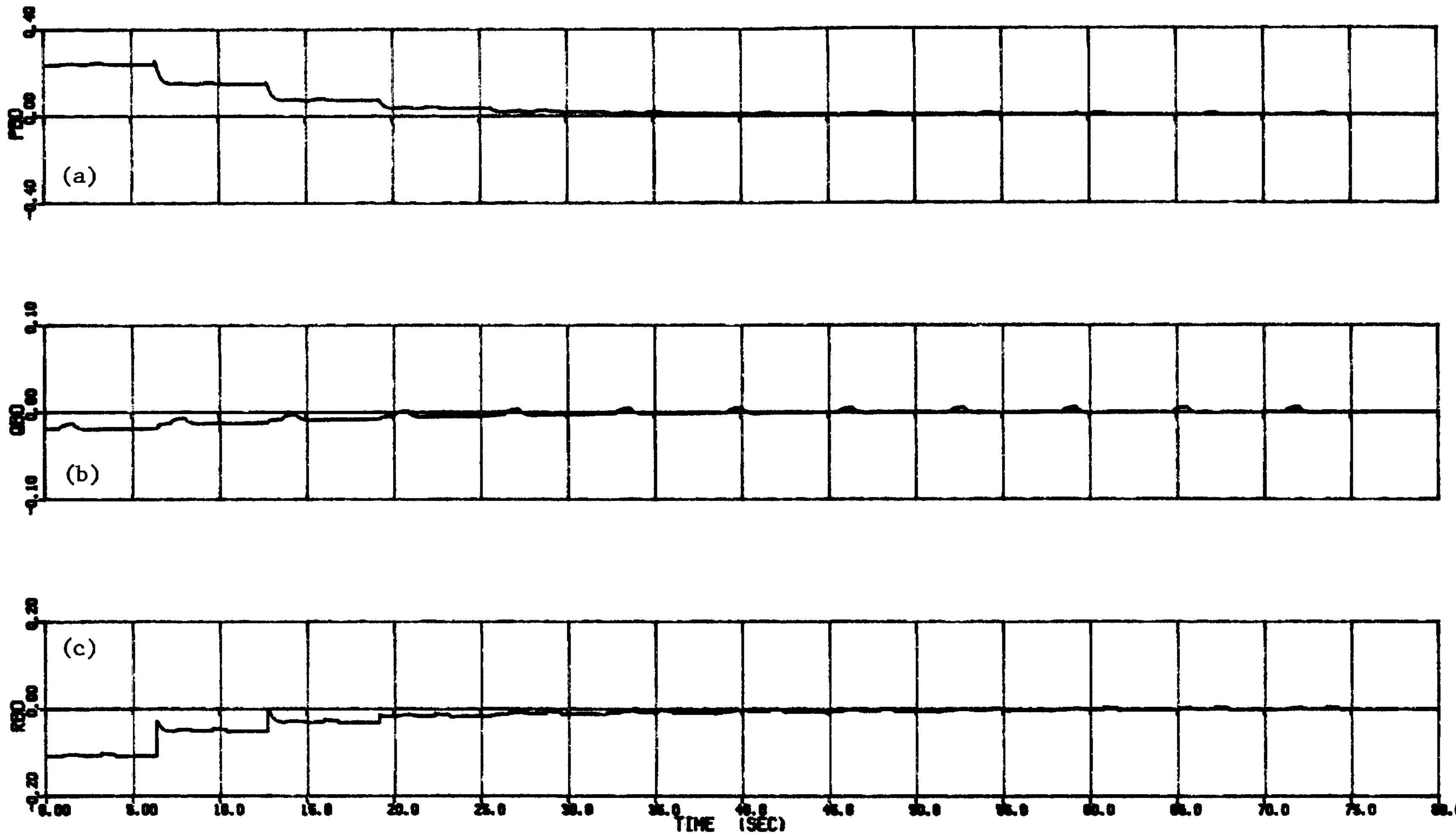

Figure 6.- Underdetermined states, complete trim. (a) Roll acceleration PBD. (b) Pitch acceleration QBD. (c) Yaw acceleration RBD.

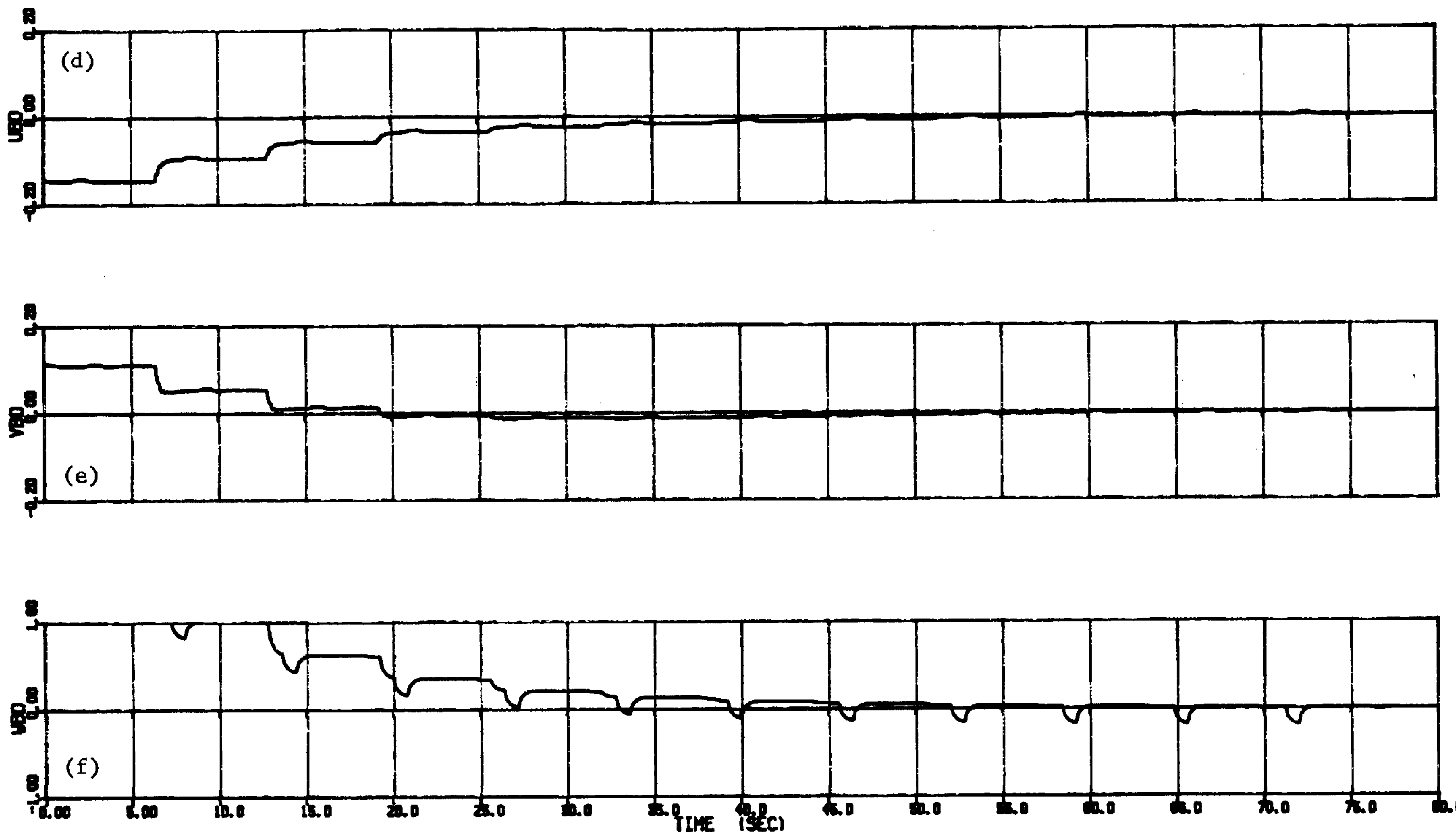

Figure 6.- Concluded. (d) Forward acceleration UBD. (e) Side acceleration VBD. (f) Downward acceleration WBD.

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15. Supplementary Notes

Point of Contact: Richard E. McFarland, MS 243-5, Ames Research Center, Moffett Field, CA 94035-5000, (415)694-6171 or FTS 464-6171

16. Abstract

Real-time piloted aircraft simulations with digital computers have been performed at Ames Research Center (ARC) for over two decades. For the simulation of conventional aircraft models, the establishment of initial vehicle and control orientations at various operational flight regimes has been adequately handled by either analog techniques or simple inversion processes. However, exotic helicopter configurations have been recently introduced that require more sophisticated techniques because of their expanded degrees of freedom and environmental vibration levels.

At ARC, these techniques are used for the backward solutions to real-time simulation models as required for the generation of trim points. These techniques are presented in this paper with examples from a blade-element helicopter simulation model.

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