Towards a Formal Semantics for Ada 9X

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Chapter 1

Introduction

The Ada 9X Language Precision Team (LPT) was formed in 1990 to study portions of the design of Ada 9X from a mathematical perspective. The first LPT project studied small parts of the language in isolation, formulating fairly simple models to explore the ramifications of the design. The idea was to avoid spending time studying the conventional parts of the language (where, we felt, little would be gained by the analysis) and to focus on the novel proposals such as the object-oriented features, overload resolution, and exception mechanisms. The results of this first project appear in two reports [9, 2].

The second LPT project had two separate goals. The first, similar in approach to the first project, was to study the rules of allowed optimizations. The second goal was rather different. Instead of defining many unrelated small models, and studying new features in depth, the plan was to formulate a broad model to cover a large part of the language. In this way, we hoped to find problems arising from the interactions between different features.

The level of effort of the project was clearly insufficient to define a complete model of the language, and our plan was not to make a complete model. Instead, we planned to define the framework for such a model, and to fill in the details of the framework only in certain areas. Furthermore, the framework was not intended to be complete; we did not expect to be able to describe concurrency or distributed programming. We did, however, hope for the framework to cover most of the sequential features.

Our expectation was not that the resulting incomplete model would necessarily be useful to anyone (although we hope that it could be extended to form the basis for analysis tools). Instead, we expected that the activity of making this model would allow us to influence the design of Ada 9X by identifying problems with specific language features or with the interactions between different features, or by suggesting improvements in the ways that parts of the language are described. As the design of Ada 9X was nearing completion when our project began, it was important to move as quickly as possible. So, the model is quite sketchy in areas. Moreover, the formal definition presented in this report omits several partial models that we built in the course of the project, as we did not have the time to integrate them with the overall framework. None the less, these models have played a useful role in helping us to understand and comment on various language features, and to influence (however slightly) the design of Ada 9X.

This report is organized as follows. The method used for defining the model is described in Chapter 2. A discussion of the model also appears there. The model itself appears in two parts. The “domains” (sets describing the values assumed by different entities in the language) are described in Chapter 3. The “judgements” describing possible executions of a program are described in Chapter 4. Our study of the optimization freedoms described in [11.6] does not fit into the main framework; it is described separately in Chapter 5. Conclusions are drawn in Chapter 6.

Two appendices give some additional information. Appendix A lists the official language com-
ments that were submitted by the LPT; most of these apply to Version 4.0 of the Draft Standard [3], and several have resulted in changes that appear in Version 5.0 [4]. Appendix B describes the intermediate syntax that results from overload resolution (and other static checks).

References to the Ada 83 Reference Manual [10] appear in the form [RM-83 c.s(p)], where c is the chapter number, s the section number, and p the paragraph number. References to version 5.0 of the Annotated Draft Standard for Ada 9X [4] appear in the form [c.s(p)]; this standard as a whole is often referred to as “the Reference Manual”.

2
Chapter 2

Method of Description

The "broad semantic framework" is defined using *Natural Semantics* [5]. The general idea of natural semantics is to define semantics as one or more relations between syntactic phrases and semantic values. These relations are defined using axioms and rules of inference. As a trivial example, a simple language of arithmetic expressions can be defined by the grammar

\[
\text{expression} ::= \text{numeral} \mid \langle\text{expression}\rangle \text{"plus"} \langle\text{expression}\rangle
\]

We can define the semantics of these expressions using a relation (between expressions and numbers) of "evaluates to", which we will write as expression \( \Rightarrow \) value, and a relation (between numerals and numbers) of "denotes", which we will write as numeral \( \rightarrow \) value.

Two rules suffice to define the semantics of expressions (although additional rules not shown here are needed to define the "denotes" relation). The first covers the base case of numerals:

\[
\frac{n \rightarrow v}{n \Rightarrow v} \quad [n \in \text{numeral}]
\]

The second gives the semantics of sums:

\[
\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2}{e_1 \text{ plus } e_2 \Rightarrow v_1 + v_2}
\]

The method is easily able to handle nondeterminism, where a phrase can have many possible results. If we extend the expression grammar above to include ranges, with the meaning that any number in the range is allowed, we have the grammar

```
expression ::= numeral
\mid \langle\text{expression}\rangle \text{"plus"} \langle\text{expression}\rangle
\mid \langle\text{expression}\rangle "." \langle\text{expression}\rangle
```

Only one additional semantic rule is needed:

\[
\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2}{e_1..e_2 \Rightarrow v} \quad [v_1 \leq v \leq v_2]
\]

Using these rules, we can deduce

\[(0..2) \text{ plus } (10..20) \Rightarrow x\]
for any $x$ between 10 and 22 inclusive.

In practical applications of Natural Semantics, the judgements used are often more complex than in this simple example. Usually, there is a certain amount of contextual information (such as the definitions of functions or procedures); this is represented by an environment. Often a store is used to record the values of variables. Furthermore, the evaluation of a phrase may have an effect on the environment or store. So the judgements often have many components, and there are a number of auxiliary domains of semantic values. We write most judgements in a standardized form

$$S, E \vdash_{pt} p \Rightarrow v, \ldots,$$

where $S$ is a store, $E$ is an environment, $p$ is a phrase, the subscript $pt$ gives the kind of phrase (e.g., whether $p$ is a declaration, statement, expression, or name), $v$ is a possible result of executing (or evaluating or elaborating) $p$, and the "..." are any other results that the execution may have. Usually, there is a final state that reflects any side-effects that the execution may have had.

We describe the domains of semantic values using the $Z$ notation [8], which furnishes a standard toolkit of notations for sets, functions, relations, and "freely constructed" sets.

The Natural Semantics definitions have been written in a machine-readable form in Prolog. Judgements are represented directly as Prolog predicates, and semantic rules as Prolog rules. There is a slight difficulty in transcribing uses of functions, but a simple translation to relations is possible. The Prolog representation has two main advantages. Firstly, we were able to apply a type-checking package developed by Reddy and Lakshman [7] to the definition; this found a number of simple errors in the rules. Secondly, we are able to run the Prolog and determine what outcomes are predicted by the semantic definition. This allows the semantics definition to be tested on small examples.

Several aspects of Ada 9X are tricky to define properly. In the remainder of this chapter, some of the awkward parts of the language and some of the more intricate aspects of the Natural Semantics definition will be explained. Not everything described below has been implemented in the Natural Semantics; some of the discussion describes our plan for dealing with a feature even though we did not have time to include that feature in the semantics definitions.

2.1 External Effects and Nonterminating Programs

Ultimately, the meaning of an Ada program is defined in terms of its sequence of "external effects", as described in [1.1.3(8)]. We can readily define several types of external effects, such as operations on files using the standard I/O packages, propagation of an exception, or return from the main program. Other effects are not covered.

An outside observer can see these external effects during the execution of a program, and does not need to wait until (and if) the program terminates. Therefore, we use a small trick to allow the semantics to assert that a certain sequence of external effects can be viewed whether or not the program terminates. We use a special $incomplete$ condition that is treated like an exception that cannot be handled. For any operation having an external effect, one possible result is to "raise" this condition. The semantic definition then propagates this condition out of the main call. Thus, we are able to infer judgements of the form

$Library \vdash program\_name \Rightarrow e,$

where $e$ is a sequence of external effects, for every sequence $e$ that might be observed during a run of the program.

2.2 Semantic Simplifications

Some language features appear to be very difficult to incorporate into this model. For these features, we have introduced a notion of an $unpredicted$ result. When our definition allows the deduction that
unpredicted is a possible result of an execution, it means that the particular program includes a language feature, or encounters a situation, that we decided not to account for in our model. This is similar to erroneous executions, where the language standard does not predict the results of a program.

For example, Section [11.6] of the Ada 9X Reference Manuals allows implementations to produce results at variance with the language rules described elsewhere in the manual (in situations where a language-defined check would fail if those rules were followed). The freedoms allowed by Section [11.6] appear to be very difficult to incorporate into the model defined here. Therefore, we have kept the model simple by treating the failure of a language-defined check as an unpredicted execution.

There are some rules new to Ada 9X that constrain the result of an execution that Ada 83 classes as erroneous. These bounded error situations can be difficult to model. For example, reading the value of an uninitialized scalar variable is erroneous in Ada 83. In Ada 9X, it is a bounded error, which can result in an exception or an implementation-defined result. Version 4.0 of the proposed Standard introduced the concept of invalid values to describe these results. Unfortunately, the introduction of invalid values complicates the semantics of the language considerably, as it is necessary to provide rules for computing with these values. The draft standard does not always provide the complete details of these rules. For example, what is the result of a comparison involving invalid values? Are the ordering operators transitive, even when applied to invalid values? We decided to keep our model simple, and to avoid these questions, by refusing to predict the outcome of a program that reads an uninitialized scalar variable. Version 5.0 of the Standard has changed the description of this situation, but once again the exact rules are vague. Therefore, we are continuing to use the simple model that refuses to predict the outcome of a program in these situations.

2.3 Static Checks and Overload Resolution

It is conventional to process Ada in two (or more) steps; the first step checking syntax, applying all of the “legality” checks, and resolving any overloading. We planned to define the semantic model in a similar way, with two distinct definitions. The first static semantics takes Ada source text, and produces a program in intermediate syntax. The intermediate syntax differs from Ada source text in several significant ways:

- Intermediate syntax is in the form of a tree, rather than a linear string of characters. Therefore intermediate syntax does not need to be parsed.

- Overloading has been resolved. Identifiers, characters, and operator names have been replaced by intermediate identifiers (in the set $Id$) in such a way that no two distinct declarations declare the same intermediate identifier. Any use of an identifier, character literal, or operator name has been replaced by a use of the appropriate element of $Id$. (Some names using selection, e.g., package components, are also replaced by intermediate identifiers.)

- Many of the notational conveniences of Ada have been eliminated. For example, infix operators are replaced by function calls.

- Generics are eliminated. Generic instantiations are expanded to a sequence of declarations.

- Some additional information is included. For example, a completing declaration is explicitly marked as such.

The intermediate syntax is described in Appendix B.

We have not formally defined the static analysis, although we believe that a Natural Semantics formulation of the static rules is possible.
2.4 Environments, Entities, and Stores

The Natural Semantics definition is faithful to the Reference Manual in its treatment of entities. We use several different sorts of "entity locations", which serve as unique names or references for entities. When a declaration is elaborated, new names are generated for any of the entities that need to be "created", and the environment is updated to reflect the association of the declared Id with a view of one of these entities. The store is a collection of mappings indexed by these different entity locations, which associates a value or meaning with each entity.

This indirect representation, using references to entity locations rather than the meanings of entities, makes it fairly easy to handle situations where an entity has a declaration that is separate from its definition. Between the declaration and definition, any references to the entity cannot make use of the definition (because it is not yet known). The location associated with the entity is known, and can be used.

2.5 Ordering

The Ada Standard gives implementations considerable freedom to select the order in which actions are performed. For example, in evaluating a sum, either the left or the right operand might be evaluated first. It is easy to write a program that gives different results depending on which order is chosen.

In order to write a concise description of the possible effects of the evaluation of constructs allowing a choice of orders, we define a set of actions, and several ways of combining actions. One combination, written by enclosing the actions in braces, allows the actions to be carried out in an arbitrary order; another, written by enclosing the actions in square brackets, requires the actions to be carried out in strict sequential order.

Actions are similar to judgements, except that the states do not appear explicitly. When actions are executed in some order, suitable states are added and the corresponding judgement is used. We write the actions in a notation that makes obvious the judgement for the corresponding execution. For example, corresponding to the judgement $S_1, E \vdash_{\text{stm}} \text{Stm} \Rightarrow S_2$ is an action written as $'E \vdash_{\text{stm}} \text{Stm} \Rightarrow V'$; corresponding to the judgement $S_1, E \vdash_{\text{exp}} \text{Exp} \Rightarrow V, S_2$ is an action written as $'E \vdash_{\text{exp}} \text{Exp} \Rightarrow V'$.

We also use states, which are combinations of stores and control flow information (for example, whether an exception has been raised, whether a return command has been executed, whether execution is normal.) The rules defining the execution of a combined action check the control flow information to skip some actions if that is appropriate. For example, in executing "a followed by b", if the execution of "a" propagates an exception, the action "b" is not executed.

One advantage to this approach is that the actions themselves look simpler than the corresponding judgements, because the flow of control through them is described by the way they are combined. For example, in defining the possible results of a sum using explicit ordering, we would need several judgements, including

\[
s, E \vdash_{\text{exp}} e_1 \Rightarrow v_1, s_1
\]
\[
s_1, E \vdash_{\text{exp}} e_2 \Rightarrow v_2, s_2
\]
\[
s, E \vdash_{\text{exp}} e_1 + e_2 \Rightarrow v_1 + v_2, s_2
\]

and

\[
s, E \vdash_{\text{exp}} e_2 \Rightarrow v_2, s_1
\]
\[
s_1, E \vdash_{\text{exp}} e_1 \Rightarrow v_1, s_2
\]
\[
s, E \vdash_{\text{exp}} e_1 + e_2 \Rightarrow v_1 + v_2, s_2
\]
and others to account for exceptions. Instead, using actions, we can write

$$S_1 \vdash \begin{cases} E \vdash_{\text{exp}} e_1 \Rightarrow v_1 \\ E \vdash_{\text{exp}} e_2 \Rightarrow v_2 \end{cases} S_2$$

$$S_1, E \vdash_{\text{exp}} e_1 + e_2 \Rightarrow v_1 + v_2, S_2$$

which accounts for the different possible orders of evaluation and for the propagation of an exception from one of those evaluations. (We still need something extra to account for an overflow in the addition.)

### 2.6 Types

It is awkward to define a domain of “type values” that describe types, because the exact characteristics of a type can change through its scope. For example, a type may be limited in some parts of its scope, and nonlimited in other parts (such as the body of a package defining the type of a component); a type can be private in some places and not in others. Furthermore, an incompletely defined type can be used in various ways (such as a record component or designated type of an access type); the characteristics of the using type can change after the incompletely defined type’s full definition.

In order to simplify the treatment of these situations, types are described by descriptors that refer to other types by their locations (see Section 2.4), rather than by their descriptors. This has the disadvantage that descriptors are not meaningful in isolation, but only with respect to the store that associates descriptors with type locations. However, it has several advantages:

- it gives a simple characterization of when types are the same; each type location represents a distinct type;
- when the characteristics of a type of a component change, that change can be reflected in just one descriptor; and
- circularities in type descriptors are easily handled (without needing any tricky domains allowing for infinite data structures).

An example illustrating these advantages is

```plaintext
type A;
type B is access A;
type A is access B;
```

### 2.7 Values

We expected it to be easy to describe the domain of values that objects might assume. It was surprising that this was not so. As mentioned above, the addition of invalid values to scalar types adds several complications, as the nature of such values is not completely specified. The latest version (5.0) of the proposed Standard no longer uses the term “invalid value”; instead, a variable may have an “invalid representation” [13.9.1].

We also argued whether “abnormal” values would be needed in order to model the concept of abnormal objects. We were able to avoid this, since the circumstances that can lead to abnormal objects are being treated as unpredictable executions.

There are a few situations where it is difficult to determine the set of values associated with a type. For example, given the declaration
subtype Void is Integer range 1 .. O; -- an empty range

type R is record
  v: Void
end R;

There are no values of subtype Void. However, there are values of type R. For example, a variable of type R can be declared, and it is not an error to "read" the value of such a variable, or to assign this value to a second variable of type R.

The problems with this type are related to those for uninitialized scalar variables, and we adopt a simple approach to solve them. We use a special indicator to denote an uninitialized scalar value. A scalar subcomponent of an object can have this value. If this "uninitialized" value is read, execution is unpredicted.

The set of values of an enumeration type is not obvious. Given the declarations

type E is (red, green);
for E use (red => 1; green => 100);

version 4.0 of the Draft Standard suggested that there were two "valid values" of type E, and (at least) 98 "invalid values" between them. The number of elements in an array indexed by E, then, is open to question. Are there elements corresponding to the invalid members of the base range?

Types declared with per-object constraints do not have obvious sets of values (since the constraint applied to a subcomponent might depend on the specific object of the type). Our model simply does not cover the kinds of per-object constraints that lead to this difficulty.

Our model for values uses integers to represent discrete values (even if the value is of an enumeration type). This means that the values of different types are not necessarily different. It would certainly be possible to mark values in such a way that no value belongs to more than one type, but there seems to be no benefit to doing so. An Ada program cannot directly compare values of different types, so there is no way for this detail to influence the outcome of a program.

2.8 Objects

It is normal in semantic definitions to use location semantics for variables, but different approaches can be used in accounting for structured (composite) variables and their components.

The approach that seems most convenient for us is to associate locations with entire variables (that is, variables that are not subcomponents of other variables). Every object is characterized by its location and a selector indicating which component of the entire variable it is.

2.8.1 Actual Subtypes

The actual subtype of an object is sometimes different from the nominal subtype in its declaration. This is an issue for assignments [5.2(11)] and formal parameters of mode in out or out that are passed by copy [6.4.1(17)]. So it is important only for variables.

The actual subtype of a variable differs from its nominal subtype in the following circumstances:

• the object is a declared object, and is constant, aliased, or has an indefinite nominal subtype [3.3.1(9)].

• the object is a formal parameter. [6.4.1(16)] states

A formal parameter of mode in out or out with discriminants is constrained if either its nominal subtype or the actual parameter is constrained.

[6.4.1(12-15)] gives additional rules for out parameters.
• the object is a generic formal object of mode in out. [12.4(8)] states that the nominal subtype is taken from the declaration of the formal, while the actual subtype is taken from the actual.

• the object is declared by an allocator, and the designated type of the result subtype of the allocator is indefinite or has discriminants (3.3(23) and 3.10(9)).

• the object is a view of another object, and the subtype of the view is indefinite [3.3(23)].

This leads to the question of how the actual subtype of an object is determined, and where, if needed, the actual subtype information is stored. There are two reasonable choices: the actual subtype might be associated with the object, or with each view of the object. However, two views of an object need not have the same type. Obviously, in such a case the actual subtypes must be different. Moreover, in a procedure call, the actual subtype of the view denoted by the formal can be different from the actual subtype of the actual parameter, even when the views have the same type. For example, the actual might be of an unconstrained discriminated type and the formal constrained. Therefore, we have decided to store the actual subtype as part of every view of an object.

2.8.2 Initialization

The calculation of the implicit initial value for an object is difficult to describe, as there is considerable freedom in the order of evaluation of default expressions used to initialize subcomponents. It is particularly awkward for subcomponents with discriminants; discriminants must be evaluated before any subcomponent that depends on them, but other subcomponents may have their initial values evaluated before then. So, given the declarations

```ada
type T(a: D := e0, b: D := e1) is record
  u: Integer := e2;
  v: U(a) := e3;
  w: U(b) := e4;
  x: S(a,b) := e5;
end record;
```

we must evaluate e0 before e3 and e1 before e4 and e5. The order of evaluation is otherwise unrestricted (unless references to a or b occur in e2, e3, or e4.)

In order to accommodate this flexibility, we use a variation on the “in some order” rules described in Section 2.5. We add an additional datum to the left and right of the turnstile; this datum records which discriminants have had their initializing expressions evaluated (and what the resulting value is). The individual actions record their prerequisites (that is, which discriminants must be evaluated before the action can be executed).

In the record of evaluated discriminants, we cannot simply use the name of the discriminant, as two subcomponents might have the same type, and thus have discriminants of the same name. Instead, for each discriminant subcomponent to be evaluated we generate a unique identifier. A “discriminant environment”, associating discriminant identifiers with discriminant names, is therefore also used in the judgements for initialization.

Another difficulty in describing the initialization of objects concerns per-object constraints. Ada 9X allows the name of a type to be used in its own definition, in which case it stands for the “current instance” of the type. Thus, a constraint on a component can refer to the containing object. Describing this formally can be difficult: the object might not exist until its subtype can be determined (which involves elaborating per-object constraints), yet the elaboration of a per-object constraint may refer to the object. We decided not to consider per-object constraints that refer to the “current instance” (but we allow them to refer to discriminants).
2.9 Aliasing

Some rules concerning aliasing look difficult to model. [6.2(12)] states

If one name denotes a part of a formal parameter, and a second name denotes a part of a distinct formal parameter or an object that is not a formal parameter, then the two names are considered distinct access paths. If an object is of a type for which the parameter passing mechanism is not specified, then it is a bounded error to assign to the object via one access path, and then read the value of the object via a distinct access path while the first access path still exists. The possible consequences are that Program_Error is raised, or the newly assigned value is read, or some old value of the object is read.

If we are to allow for accurate predictions of the effects of procedure calls (or to refuse to predict the outcome of calls that might involve aliasing), we need to be able to recognize, at a minimum, when the above rule might apply. It is not enough just to say that parameters of certain types may be passed by copy or by reference at the whim of an implementation, because the above paragraph allows for results that might not be produced under either of the two passing mechanisms. We might refuse to predict the result of any call with aliasing, but that can be hard to recognize if access values are used. Unfortunately, the notion of "access paths" is not well defined by the Reference Manual, and the precise meaning of the aliasing rule is unclear.

We have submitted several official comments on the aliasing rules, and had some discussions with the Mapping Team on possible interpretations of these rules. One model that may work can be sketched as follows: we would define a function access_path on names, which gives an element of optional Id. This “access path” gives the Id associated with the declaration of some variable denoting the object denoted by the name. This might be the declaration that created the object, or might be the name of a formal parameter. If the object was dynamically created, the access path is null.

In most cases, the definition of access_path is simple, e.g.,

\[
\begin{align*}
\text{access_path}(\text{Id}) &= \text{some}(\text{Id}) \text{ if Id is not declared by a renaming declaration} \\
\text{access_path}((\text{Nam}.\text{Id}) &= \text{access_path}(\text{Nam}) \\
\text{access_path}((\text{Nam}\text{.exp}, . . .) &= \text{access_path}(\text{Nam}) \\
\text{access_path}((\text{Nam}.\text{all} &= \text{none}
\end{align*}
\]

For Ids declared by renaming declarations, we would want to use the access path of the renamed object.

In order to state the rule of 6.2(12), we would associate a "last update path" with every object (including subobjects). Whenever an object is updated by an assignment, the access path of the name used in the assignment statement is recorded in the object (and every subobject and containing object). In addition, formal parameter objects are updated in a call with the formal parameter Id as the last update path, and after a call, any in out or out parameter objects are updated by the access paths of the corresponding actual parameter names. It would be a bounded error to evaluate a name denoting all or part of a formal parameter for which the parameter passing mode is unspecified, if the access path differs from the last update path of the object it denotes. Similarly, it would be a bounded error to evaluate a name if the last update path of the object it denotes is a formal parameter for which the parameter passing mode is not specified.

These rules account for most of the situations described by 6.2(12), but probably need refinement to deal properly with access values created by Access attributes.
Chapter 3

Semantic Domains

In this chapter, we define the domains of values used in the semantics. These sets describe the values assumed by the various entities of Ada 9X.

These definitions have been used as the basis for the Prolog representation of the Natural Semantics definitions presented in Chapter 4. However, some of the definitions defined here have not yet been incorporated into the Natural Semantics definition, and some small inconsistencies between the two definitions have not yet been eliminated.

3.1 Basic Notations

In this section, we define some basic notions that will be used in the model.

An association provides a finite function with an enumeration of its domain. It is convenient to represent such a function by enumerating (domain, range) pairs; the finite function is then the range of the sequence.

\[ X \rightarrow Y == \{ s : \text{seq} \times Y \mid \text{ran} s \in X \rightarrow Y \land \# \text{ran} s = \# s \} \]

Functions \textit{adom} and \textit{aran} give the domain and range of an association. Function \textit{\_ \_} is used to apply an association to an argument (as though it were a finite function).

\[
\begin{align*}
\text{adom}_{-} &: (X \rightarrow Y) \rightarrow P X \\
\text{aran}_{-} &: (X \rightarrow Y) \rightarrow P Y \\
\text{\_ \_} &: (X \rightarrow Y) \times X \rightarrow Y \\
\forall a : X \rightarrow Y \bullet \text{adom } a &= \text{dom}(\text{ran } a) \\
\forall a : X \rightarrow Y \bullet \text{aran } a &= \text{ran}(\text{ran } a) \\
(A, x) \in \text{dom}_{\_ \_} \Leftrightarrow x \in \text{adom } A \\
x \in \text{adom } A \Rightarrow A \cdot x &= (\text{ran } a) x
\end{align*}
\]

We sometimes use optional values:

\[
\text{optional } X ::= \text{none | some}(X)
\]

Function \textit{maximal} returns the set of maximal values of an arbitrary relation, where a maximal value of the relation \( R : X \leftrightarrow X \) is defined as an element \( z \) of \( X \) such that no \( y \neq z \) satisfies \( zRy \).
maximal : \( (X \rightarrow X) \rightarrow P X \)

\[
\text{maximal}(R) = X \setminus \text{dom}(\text{dom}(R) \setminus \text{id} X)
\]

Function restrict restricts both the domain and the range of a relation to some given set.

\[
\text{restrict} : (P X \times (X \rightarrow X)) \rightarrow (X \rightarrow X)
\]

\[
\text{restrict}(S, R) = (S \times S) \cap R
\]

Equivalently,

\[
\text{restrict}(S, R) = (S \triangleleft R) \triangleright S
\]

### 3.2 Entities and the Environment

After overload resolution, every occurrence of an identifier in an Ada program can be replaced by an \textit{Id}, so that each \textit{Id} has at most one declaration in the program.

The elaboration of a declaration creates an entity, and the \textit{Id} of the declaration then denotes a \textit{view} of this entity. A declaration might be elaborated many times (e.g., if it appears in a subprogram body), denoting a different entity each time.

\[
\text{Environment} == \text{Id} \rightarrow \text{View}
\]

A \textit{view} identifies an entity and provides some characteristics that affect the use of the entity. For example, there can be several views of the same subprogram, each having different parameter names and default expressions. Views refer to entities by using \textit{locations} of various types.

\[
\text{[Type\_location, Subtype\_location, Object\_location, Subprogram\_location]}
\]

There are no views associated with packages or generic declarations. Packages are significant in their provision of information hiding and modularization, but those aspects concern the static semantics, not the dynamic semantics. Generic declarations are expanded by the static semantics, so that only ordinary (non-generic) declarations appear in the intermediate syntax.

Exceptions are unusual entities. No matter how often an exception declaration is elaborated, the same exception is denoted. This exception is represented by an \textit{Exception\_Id} that is determined by the static semantics. (The \textit{Exception\_Id} could be chosen to be the \textit{Id} of the declaration, for example).

\[
\text{View ::= object\_view(}[\text{Object\_location} \times \text{Subtype})]
| \text{subtype\_view(}[\text{Subtype\_location})]
| \text{subprogram\_view(}[\text{Subprogram\_location} \times \text{Profile})]
| \text{exception(}[\text{Exception\_Id})]
| \text{constant(}[\text{Value})]
\]

Most kinds of entity are held in a \textit{store}. Assigning a \textit{location} to refer to the entity, and placing an entity at that location in the store, corresponds to what the Reference Manual calls "creating" the entity. This activity happens when a declaration is elaborated.

Several kinds of entities are used in the semantics definition:

- objects, which have values;
• subtypes, with their associated type, constraint, and attributes;
• types, with descriptors and optional parents;
• subprograms, with formals and bodies; and
• operations (representing the “predefined operations” of the Reference Manual).

| Store 
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>types : Type_location → Type</td>
</tr>
<tr>
<td>subtypes : Subtype_location → Subtype</td>
</tr>
<tr>
<td>objects : Object_location → Value</td>
</tr>
<tr>
<td>subprograms : Subprogram_location → SubprogramOrOp</td>
</tr>
</tbody>
</table>

SubprogramOrOp ::= subprogram((Environment x (seq Id) x Dcl x Stm))
| operation(...) |

The different sorts of entities are described in the following sections.
Evaluation is defined in terms of a state, which (usually) includes a store, as well as certain control information. States are defined below in Section 3.6. Function the_store, giving the store associated with a state, is used in some of the definitions below.

3.3 Thunks

In several situations it is necessary to record an expression together with the environment in which it appears, so that the expression can be evaluated in some other context. The environment is retained so that any Ids appearing in the expression have their correct denotation. We call this combination of an expression and an environment a thunk.

Thunk == Exp × Environment

Thunks appear in record type descriptors (where they describe the initializing values of any explicitly initialized components), and in parameter lists (where they describe any default values for parameters).

3.4 Values

There are several kinds of values of interest:
• discrete values (represented by integers)
• real values (represented by rationals)
• access values (represented by views of objects or of subprograms)
• record values (represented by partial functions)
• array values (represented by partial functions)

It is possible to use a model where the values of each type are distinct; however, the benefit of doing so is not completely clear. The rules of the language do not allow for comparisons of values of different types, so there is no way of telling whether these sets are disjoint.
3.4.1 Ranges

Ranges have two bounds, and determine a set of values of a scalar type.

\[ \text{Range} ::= \text{discrete_range}(\mathbb{Z} \times \mathbb{Z}) \mid \text{real_range}(\text{Rational} \times \text{Rational}) \]

\[ \text{Discrete_range} \equiv \text{ran discrete_range} \]

\[ \text{Real_range} \equiv \text{ran real_range} \]

(The definition of the bounds functions contains a forward reference to functions \text{discrete_value} and \text{real_value}, defined in Section 3.4.5.)

- \text{low_bound}, \text{high_bound} : \text{Range} \rightarrow \text{Value}
  - \text{low_bound(} \text{discrete_range}(l, h) \text{)} = \text{discrete_value}(l)
  - \text{high_bound(} \text{discrete_range}(l, h) \text{)} = \text{discrete_value}(h)
  - \text{low_bound(} \text{real_range}(l, h) \text{)} = \text{real_value}(l)
  - \text{high_bound(} \text{real_range}(l, h) \text{)} = \text{real_value}(h)

- \text{make_range} : \text{Value} \times \text{Value} \rightarrow \text{Range}
  - \text{make_range(} \text{discrete_value}(v), \text{discrete_value}(v') \text{)} = \text{discrete_range}(v, v')
  - \text{make_range(} \text{real_value}(v), \text{real_value}(v') \text{)} = \text{real_range}(v, v')

- \_ \text{belongs_to} \_ : \text{Value} \rightarrow \text{Range}
- \text{values_of_range} : \text{Range} \rightarrow \mathcal{P} \text{Value}
- \_ \text{is_included_in} \_ : \text{Range} \rightarrow \text{Range}
  - \_ \text{belongs_to} \_ R \Leftrightarrow v \in \text{values_of_range}(R)
  - \text{values_of_range}(\text{discrete_range}(l, h)) = \text{discrete_value}(l \ldots h)
  - \text{values_of_range}(\text{real_range}(l, h)) = \text{real_value}(l \ldots h)
  - R \text{ is_included_in } R' \Leftrightarrow \text{values_of_range}(R) \subseteq \text{values_of_range}(R')

3.4.2 Index Ranges

Arrays are indexed by sequences of discrete values. Index ranges are determined by a sequence of discrete ranges.

\[ \text{Array_bounds} ::= \text{seq}(\text{Discrete_range}) \]

Each sequence of bounds determines a set of indices:

\[ \text{indices} : \text{Array_bounds} \rightarrow \mathcal{P}(\text{seq_value}) \]

\[ \forall B : \text{Array_bounds} \Rightarrow \text{indices}(B) = \{ s \in \text{seq_value} \mid \#s = \#B \wedge \forall n \in \text{dom}s \cdot s(n) \text{ belongs_to } B(i) \} \]

3.4.3 Tags

We use a set of tags. The precise nature of this set is immaterial.

[Tag]
3.4.4 Bindings

Bindings are simply partial mappings from IDs to values. Most often these IDs are the names of record fields or discriminants.

\[ \text{Binding} = \text{Id} \rightarrow \text{Value} \]

3.4.5 Values

Although it seems redundant, we include the bounds as part of an array value. This is because two arrays with no components (thus, with the same mapping function) can have different bounds.

Record values are furnished with optional tags, discriminants, and other components. This allows the descriptions of the various language rules concerning tagged records, discriminated records, and normal records to be combined.

A special value, \text{uninitialized_value}, is used for uninitialized scalar objects. This is used to detect when such an object is read (in which case the result of the execution is \text{unpredicted}).

\[
\begin{align*}
\text{Value} &::= \text{uninitialized_value} \\
&\quad | \ \text{discrete_value}(\mathbb{Z}) \\
&\quad | \ \text{real_value}(\mathbb{R}) \\
&\quad | \ \text{access_value}(\text{optional View} \times \text{optional SubprogramLabel}) \\
&\quad | \ \text{record_value}(\text{optional Tag} \times \text{Binding} \times \text{Binding}) \\
&\quad | \ \text{array_value}(\{ B : \text{Array bounds}, v : (\text{seq Value}) \rightarrow \text{Value} \mid \text{dom } v = \text{indices}(B) \})
\end{align*}
\]

We can define various sets of values referred to in the language rules:

\[
\begin{align*}
\text{Discrete_value} &::= \text{ran } \text{discrete_value} \\
\text{Real_value} &::= \text{ran } \text{real_value} \\
\text{Access_value} &::= \text{ran } \text{access_value} \\
\text{Scalar_value} &::= \text{Discrete_values} \cup \text{Real_values} \\
\text{Elementary_value} &::= \text{Scalar_values} \cup \text{Access_values} \\
\text{Composite_value} &::= \text{ran record_value} \cup \text{ran array_value}
\end{align*}
\]

3.5 Types and Subtypes

Every type has an associated type descriptor giving the characteristics of the type (and possibly referring to other types via their type locations).

The descriptors are defined here, but described in the sections that follow.

\[
\begin{align*}
\text{Type Descriptor} &::= \text{enumeration_dsc}(\mathbb{N}_1) \\
&\quad | \ \text{signed_integer_dsc}(\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}) \\
&\quad | \ \text{modular_integer_dsc}(\mathbb{N}_1) \\
&\quad | \ \text{universal_integer_dsc}(\mathbb{Z} \times \mathbb{Z}) \\
&\quad | \ \text{float_dsc}(\mathbb{N}_1 \times \mathbb{R} \times \mathbb{R} \times \text{Float Implementation}) \\
&\quad | \ \ldots \text{something for fixed-point types} \\
&\quad | \ \text{array_dsc}(\text{seq, Subtype} \times \text{Subtype}) \\
&\quad | \ \text{record_dsc}(\text{optional Tag} \times \text{Discriminant descriptor} \times \text{Component list descriptor}) \\
&\quad | \ \text{class_dsc}(\text{Type location}) \\
&\quad | \ \text{access_dsc}(\text{Subtype location} \times \text{Access Mode}) \\
&\quad | \ \ldots
\end{align*}
\]
A type is then a combination of an optional parent (in case the type was derived) plus a descriptor:

\[
\text{Type ::= (optional Type\_location) x Type\_Descriptor}
\]

For each kind of type in Ada 9X, we define a type descriptor. Additionally, for each descriptor we define the set of values of the type:

\[
\text{descriptor\_values : Type\_Descriptor x State } \rightarrow \text{ Value}
\]

Function \(\text{type\_values}\) gives the set of values associated with a type location given a state:

\[
\forall l : \text{Type\_location}; S : \text{State} \Rightarrow \text{type\_values}(l, S) = \text{descriptor\_values}(\text{snd(\text{the\_store}(S).types}(u)), S)
\]

Note that these sets of values might change over time as the information about a type is updated.

### 3.5.1 Subtypes and Constraints

A subtype is a combination of a type, a constraint, and certain attributes [3.2(8)]. There are in fact two sorts of subtypes; we will call them “partial” subtypes and “true” subtypes. Partial subtypes can contain unevaluated per-object constraints, for example, references to discriminants. These partial subtypes appear as the subtypes of components of a record with discriminants. When an object of a subtype is created, some of these per-object constraints are elaborated and the true subtype of the object and its components is determined.

There are several cases where a partial subtype cannot be elaborated: in a variant record, per-object constraints in initially unused components are not elaborated; in an initialized object, none of the per-object constraints are elaborated. The Reference Manual is not completely clear on this point, and for now we will only consider per-object constraints that are references to discriminants.

Partial subtypes appear only as the subtypes of components of types with discriminants; named subtypes, and the subtypes of objects, will always be “true” subtypes.

The Reference Manual identifies three kinds of constraints: range constraints, index constraints, and discriminant constraints. In fact, the last two can also be applied indirectly, to an access type having a designated subtype to which the constraint would apply directly. (Only one level of indirection is allowed; given

\[
\text{type A1 is access String;}
\text{type A2 is access A1;}
\]

an index constraint can be applied to A1, but not to A2.) We may find it useful to distinguish these indirect constraints from their direct counterparts.

\[
\text{Constraint ::= no\_constraint}
\text{range\_constraint(}\!(\text{Range})\!\text{)}
\text{index\_constraint(}\!(\text{seq}\!\text{ Discrete}\!\text{ range})\!\text{)}
\text{discriminant\_constraint(}\!(\text{Binding})\!\text{)}
\text{indirect\_index\_constraint(}\!(\text{seq}\!\text{ Discrete}\!\text{ range})\!\text{)}
\text{indirect\_discriminant\_constraint(}\!(\text{Binding})\!\text{)}
\]

Some values satisfy a constraint:
- satisfies - : Value \rightarrow Constraint

\forall v : Value \bullet v \text{ satisfies no_constraint}
\forall v : Value \bullet v \text{ satisfies range_constraint}(R) \iff v \text{ belongs_to } R
array_value(B, a) \text{ satisfies index_constraint}(S) \iff B = S
record_value(t, d, r) \text{ satisfies discriminant_constraint}(d') \iff d = d'

Now we can define subtypes:

```
Subtype
  type : Type_location
  constraint : Constraint
  attributes : Attributes
```

For every subtype, there is an associated set of values, namely the values of the associated type that satisfy the constraint.

```
subtype_values : Subtype \rightarrow State \rightarrow P Value
\forall S : State, s : Subtype \mid s.type \in \text{dom the_store}(S).types \bullet
  subtype_values(s, S) = \{ v : Value \mid v \in \text{type_values}(s.type, S) \land v \text{ satisfies } s.constraint \}
```

We use function subtype to create subtype values:

```
subtype : Type_location \times Constraint \times Attributes \rightarrow Subtype
subtype(t, c, a).type = t
subtype(t, c, a).constraint = c
subtype(t, c, a).attributes = a
```

### 3.5.2 Partial Subtypes

Partial constraints are similar to constraints, except they may contain references to discriminants in place of values.

```
Partial_value ::= value((Value)) | discriminant_ref((Id))
Partial_discrete_range ::= Partial_value \times Partial_value
Partial_constraint ::= no_constraint
  | range_constraint((Partial_discrete_range))
  | index_constraint((seq1 Partial_discrete_range))
  | discriminant_constraint((Id \rightarrow Partial_value))
  | indirect_index_constraint((seq1 Partial_discrete_range))
  | indirect_discriminant_constraint((Id \rightarrow Partial_value))
```

A partial subtype combines a subtype and a partial constraint. (Using the subtype allows us to do a “dependent compatibility check” at the right time, and also gives us the needed attributes when we actualize.)

```
Partial_subtype ::= Subtype \times Partial_constraint
```

Given a mapping from discriminant names to values, a partial constraint can be turned into a true constraint.
3.5.3 Derived Types and Classes

A derived type is a new entity, but it generally uses a copy of the parent descriptor. The set of values of the derived type is then the same as the set of values of the parent type. However, when a derived type definition furnishes new discriminants or defines a type extension, a new descriptor is needed.

The store records some information about derivation, by recording the (optional) parent type [3.4(1)] of every known type.

\[
parent: State \rightarrow (Type_location \leftrightarrow Type_location)
\]

\[
parent(S) = \{ t, t' : Type_location | fst(the_store(S).types(t)) = some(t') \}
\]

Type extensions are considered in Section 3.5.8. Abstract types are considered in Section 3.5.8.1.

3.5.3.1 Derivation Classes

The descriptor for a class-wide type has the form class_dsc(t), where t is the Type_location of the root of the class. The values of this type are the values of all types derived (directly or indirectly) from t:

\[
descriptor_values(class_dsc(t), S) =
\cup \{ t' : Type_location | (u', u) \in parent(S)^* \circ descriptor_values(t', S) \}
\]

The notions of [3.4.1(10)] are easily defined:
\[
\text{descendant}, \text{ancestor} : \text{State} \rightarrow (\text{Type}\_\text{location} \leftrightarrow \text{Type}\_\text{location}) \\
\text{ultimate}\_\text{ancestor} : \text{State} \times \text{Type}\_\text{location} \rightarrow \text{Type}\_\text{location}
\]

\[
\begin{align*}
\text{ancestor}(S) &= \text{parent}(S) \\ 
\text{descendant}(S) &= \text{ancestor}(S)^{-1} \\
\end{align*}
\]

\[ t \in \text{dom the}\_\text{store}(S).\text{types} \Rightarrow \\
\text{ultimate}\_\text{ancestor}(S, t) = ((\text{ran parent}(S)) \triangleleft \text{ancestor}(S)) \triangledown t \\
\]

### 3.5.4 Scalar Types

Every scalar type records a base range, in addition to any other needed information.

\[
\text{base}\_\text{range} : \text{Type}\_\text{Descriptor} \rightarrow \text{Range}
\]

Scalar types are either discrete types or real types. A value of discrete type is simply an integer; a value of a real type is a rational number.

Each scalar subtype determines a range, as specified in [3.5(6)]:

\[
\begin{align*}
\text{range}\_\text{of}\_\text{subtype} : \text{Subtype} \times \text{Store} \rightarrow \text{Range} \\
\sigma.\text{constraint} = \text{range}\_\text{constraint}(R) \Rightarrow \text{range}\_\text{of}\_\text{subtype}(\sigma, S) = R \\
\sigma.\text{constraint} = \text{no}\_\text{constraint} \Rightarrow \\
\text{range}\_\text{of}\_\text{subtype}(\sigma, S) &= \text{base}\_\text{range}(\text{snd}(\text{the}\_\text{store}(S).\text{types}(\sigma.\text{type})))
\end{align*}
\]

### 3.5.4.1 Enumeration Types

The type descriptor for an enumeration type has the form \( \text{enumeration}\_\text{dsc}(n) \), where \( n : \mathbb{N} \) gives the number of enumerands.

The base range of the enumeration is the set of discrete values with position numbers between 0 and \( n - 1 \) (inclusive). The values of an enumeration type are the values in the base range.

\[
\begin{align*}
\text{base}\_\text{range}(\text{enumeration}\_\text{dsc}(n)) &= \text{discrete}\_\text{range}(0, n - 1) \\
\text{descriptor}\_\text{values}(\text{enumeration}\_\text{dsc}(n), S) &= \text{discrete}\_\text{values}(0 \ldots n - 1)
\end{align*}
\]

### 3.5.4.2 Character Types

Character types are simply enumeration types.

### 3.5.4.3 Boolean Types

Boolean types are simply enumeration types.

### 3.5.4.4 Integer Types

There are three descriptors for integer types:

- \( \text{signed}\_\text{integer}\_\text{dsc}(bf, bl, f, l) \), with \( bf, bl, f, l : \mathbb{Z} \) satisfying (according to [3.5.4(7)]),
  1. \( bf \leq 0 \leq bl \),
  2. \( bf + bl \in \{-1, 0\} \), and
  3. \( \{f, l\} \subseteq bf \ldots bl \).
The base range of this type is $bf .. bl$.

- $modular\_integer\_dsc(m)$, with $m : \mathbb{N}_1$. (The base range is $0 .. (m - 1)$ [3.5.4(7)].

- $universal\_integer\_dsc(bf, bl)$ for $bf, bl : \mathbb{Z}$ (although the base range seems to be irrelevant here).

$root\_integer$ is just a particular signed integer type.

In every case, the set of valid values of the type consists of all discrete values [3.5.4(6)].

$$
\begin{align*}
\text{descriptor\_values}(\text{signed\_integer\_dsc}(bf, bl, f, l), S) &= \text{discrete\_value}([\mathbb{Z}] \cup \text{Invalid\_value}) \\
\text{descriptor\_values}(\text{modular\_integer\_dsc}(m), S) &= \text{discrete\_value}([\mathbb{Z}] \cup \text{Invalid\_value}) \\
\text{descriptor\_values}(\text{universal\_integer\_dsc}(bf, bl), S) &= \text{discrete\_value}([\mathbb{Z}] \cup \text{Invalid\_value}) \\
\text{base\_range}(\text{signed\_integer\_dsc}(bf, bl, f, l)) &= \text{discrete\_range}(bf, bl) \\
\text{base\_range}(\text{modular\_integer\_dsc}(m)) &= \text{discrete\_range}(0, m - 1) \\
\text{base\_range}(\text{universal\_integer\_dsc}(bf, bl)) &= \text{discrete\_range}(bf, bl)
\end{align*}
$$

### 3.5.4.5 Floating Point Types

As acknowledged in the Ada 9X Rationale, the core language leaves the semantics of floating point operations largely unspecified. By contrast, the floating point annex (Annex G) is quite precise—though some flaws in the annex will be noted below. Therefore our model has two parts, one for the core and the other for Annex G.

The semantics of the Reference Manual refers to the underlying machine values and operations, and makes features of them visible, for example, in the values of attributes. We have attempted to model this semantics directly, so that it will be clear how to tell whether an actual implementation satisfies the semantic rules. This provides a model from the point of view of the implementor. It would have been easier (and from some points of view, perhaps, preferable) to make a model from the user's point of view: e.g., take the values of the attributes as given and simply state axiomatically, in terms of the attributes, the resulting constraints on the values returned by the predefined operations. The user’s model is, of course, a consequence of the implementor’s model.

The descriptor for a floating point type has four components:

$$\text{float\_dsc}(\langle \mathbb{N}_1 \times \text{Rational} \times \text{Rational} \times \text{Implementation} \rangle)$$

The first three components are provided directly by the type's definition: the requested precision and the bounds of its constraint. The fourth component characterizes the chosen implementation of the type. The descriptor for an integer type contains a component with analogous information, namely, the bounds of the underlying base type. We could represent the fourth component in a finitary way by listing the values of a large number of floating point attributes determined by the implementation. Instead, this component of the descriptor will consist of a model of the implementation itself, from which the attributes can be calculated.

$$
\begin{align*}
\text{descriptor\_values}(\text{float\_dsc}(n, v, v', imp), S) &= \text{real\_value}(\text{Float}) \\
\text{base\_range}(\text{float\_dsc}(n, v, v', imp)) &= \inf(\text{imp\_ma\_numbers}) .. \sup(\text{imp\_ma\_numbers})
\end{align*}
$$

### 3.5.4.6 The core model of floating point

The values of a floating point type are rationals, with the possible addition of some extra things like signed zeroes or NaN's. For now these extra possibilities are ignored.
Float == Rational ∪ ...

There is a problem in the Reference Manual: The possibility of signed zeroes or NaN's is incompatible with [RM-83 3.5.7(8)], which says that the set of values for a floating point type is the set of rational numbers.

3.5.4.6.1 Machine arithmetics Elaboration of a floating point type declaration includes the choice of an appropriate Implementation (from some predefined non-empty finite set of them) to model the type. One component of an implementation is a machine arithmetic, which consists of a radix, a set of machine numbers, and relations modeling the predefined binary and unary floating point operations. Floating point operations will be modeled not as functions but as relations, in order to model their potential non-determinism. Some of the predefined binary floating point operations return floats and some return booleans; all unary operations return floats. It is convenient to add a special “return value,” overflow, to represent the possibility of overflow:

FloatResult ::= overflow | result(curr(Float))

BinOpFloat == Float^2 ← FloatResult

BinOpBool == Float^2 ← Boolean

UnOp == Float ← FloatResult

It will also be handy to have an operation that extracts the (non-overflow) Float values from a set of FloatResults:

floats_of : FloatResult → Float

floats_of(X) = result~(X)

In schema MachineArithmetic:

- radix is the radix of the machine representation

- numbers is the set of machine numbers—that is, the set of storable values that will “fit” in any variable of the type.

- plus, equals, ..., are relations modeling the predefined floating point operations; convert represents type conversion of an arbitrary real value to a machine number of this arithmetic.

Notice that operations like plus are not restricted to returning machine numbers of the type as values. The machine numbers represent the storable values of the type, but operations may return, e.g., extra-precision values that are not immediately rounded.
The axioms involving float_outcomes are technical conditions guaranteeing that the (non-overflow) results of any operation can legitimately be passed as arguments to any of the others.

The concluding inequality is all we can represent formally of paragraph 3.5.7(8):

The base range (see 3.5) of a floating point type is symmetric around zero, except that it can include some extra negative values in some implementations.

Note: It would probably be reasonable to suppose that the machine numbers are (roughly) symmetric in a stronger sense: the set of machine numbers between −sup(numbers) and sup(numbers) is closed under additive inverse. The Reference Manual does not require this.

Note: This definition could be shortened if we simply assumed that the relations modeling all the predefined operations were total. One reason for not making that assumption is the desire that there be an obvious relation between this definition and actual floating point implementations. In representing an actual implementation as a machine arithmetic two principles apply: First, the plus relation modeling an actual implementation of + should contain ((x, y), z) if z is the actual result (presumably computed by the hardware in some register) of summing x and y; and should also include ((x, y), z') for every possible “perturbation” z' of z obtained by moving z to and from registers of other precisions or to and from storage. (Similar considerations apply to all other operations.) Second, it is sound to model an implementation by using relations that are supersets of this “minimal” model.

3.5.4.6.2 Parameters of floating point implementations We need two kinds of specifications for describing aspects of floating point implementations: A MachineParam is a specification requiring that certain floating point numbers actually be machine numbers of an implementation. It does not constrain the semantics of the floating point operations. The “representation-oriented attributes” of a type will be defined to return, essentially, the “strongest” MachineParam satisfied by the type’s implementation. (Strictly speaking, we will define what it means for a machine arithmetic
to satisfy a MachineParam, and a machine arithmetic is only one component of an implementation.)
An AccurParam does constrain the behavior of the floating point operations. Annex G will define
the “model-oriented attributes” of a type to return, essentially, the strongest AccurParam satisfied
by the type’s implementation. The core of the Reference Manual says very little about the relation
between the implementation and these model attributes.

3.5.4.6.3 Machine parameters A MachineParam is a triple whose elements are interpreted
as, respectively, a mantissa length, a minimum exponent, and a maximum exponent.

\[
\text{MachineParam} = \langle \text{mantissa}, \text{emin}, \text{emax} \rangle
\]

where

\[
\text{mantissa} : N_1
\]
\[
\text{emin} : \{ i : N \mid i < 0 \}
\]
\[
\text{emax} : N_1
\]

A machine arithmetic satisfies a MachineParam if all the canonical numbers defined in terms of
these parameters (and of the machine arithmetic’s radix) are machine numbers:

\[
\text{sat_float_param} : \text{MachineArithmetic} \leftrightarrow \text{MachineParam}
\]

\[
(ma, fp) \in \text{sat_float_param} \iff
\text{BoundedCanonical}(ma, \text{radix}, fp, \text{mantissa}, fp, \text{emin}, fp, \text{emax}) \subseteq ma, \text{numbers}
\]

The BoundedCanonical numbers are defined in Section 3.5.4.6.4. Note that sat_float_param does
not constrain the operations of the machine arithmetic in any way.

A first MachineParam is “improved by” a second if the second is at least as restrictive a specifica-
tion as the first.

\[
\text{improved_by} : \text{MachineParam} \leftrightarrow \text{MachineParam}
\]

\[
(p_1, \text{emin}_1, \text{emax}_1) \text{improved_by} (p_2, \text{emin}_2, \text{emax}_2) \iff
p_1 \leq p_2 \land \text{emin}_1 \geq \text{emin}_2 \land \text{emax}_1 \leq \text{emax}_2
\]

A MachineParam \(fp\) for \(ma\) is maximal if \(ma\) satisfies \(fp\) but satisfies no strict improvement of
\(fp\). The function max_mach_params(ma) returns the set of all maximal MachineParams satisfied
by \(ma\).

\[
\text{max_mach_params} : \text{MachineArithmetic} \rightarrow F \text{MachineParam}
\]

\[
\text{max_mach_params}(ma) =
\text{maximal}(\text{restrict}({ mattr \mid \text{sat_float_param}(ma, mattr) }, \text{improved_by}_-))
\]

The generic constant maximal returns the set of maximal values of a relation. The generic constant
restrict returns the result of restricting both the domain and range of a relation to the same set.
These constants are defined in Section 3.1.

There is a problem in the Reference Manual: [RM-83 A.5.3] defines the representation-oriented
attributes of a floating point type. They are intended, collectively, to denote a “best” MachineParam
satisfied by the machine arithmetic of the type, but the definitions given there are not quite right.
In particular, if \(ma\) is the machine arithmetic chosen to implement type \(T\), the rules of the Reference
Manual do not guarantee that

\[
ma \text{ sat_float_param } (T'\text{machine_mantissa}, T'\text{machine_emin}, T'\text{machine_emax})
\]

although this is surely one intended consequence of the rules.
3.5.4.6.4 Representations of floating point

The canonical representations of floating point numbers are defined in the core semantics, Appendix A.

Our definitions will represent fractions with radix \( r \) and mantissa length \( m \) by length-\( m \) sequences of the "digits" 0, \ldots, \( r - 1 \). A normal representation is a representation whose first element is non-zero, or which consists solely of zeroes.

\[
\text{reps} : N_1 \times N_1 \rightarrow \mathbb{P}_{\text{seq}}(N)
\]

\[
\text{normal_reps} : N_1 \times N_1 \rightarrow \mathbb{P}_{\text{seq}}(N)
\]

\[
\text{reps}(r, m) = 1 \ldots m \rightarrow 0 \ldots (r - 1)
\]

\[
s \in \text{normal_reps}(r, m) \iff s \in \text{reps}(r, m) \land
s(1) = 0 \Rightarrow \forall i : \text{dom}(s) \bullet s(i) = 0
\]

The operation \( \text{fraction_value} \) returns the fraction represented by the sequence \( s \) in radix \( r \)—that is, the "decimal" \(.s(1)s(2)\ldots s(m)\), understood as a literal in base \( r \).

\[
\text{fraction_value} : N_1 \times \text{seq}(N_1) \rightarrow \mathbb{R}_{\text{ational}}
\]

\[
\text{fractions} : N_1 \times N_1 \rightarrow \mathbb{P}_{\text{ rational}}
\]

\[
\text{normal_fractions} : N_1 \times N_1 \rightarrow \mathbb{P}_{\text{ rational}}
\]

\[
\text{fraction_value}(r, s) = \sum_{i=1}^{s} s(i) \ast r^{-i}
\]

\[
\text{fractions}(r, m) = \text{fraction_value}(\text{normal_reps}(r, m))
\]

\[
\text{normal_fractions}(r, m) = \text{fraction_value}(\text{normal_reps}(r, m))
\]

The model floating point numbers are those suitably definable in scientific notation, i.e., as fractions times powers of the radix.

\[
\text{make_floats} : \mathbb{P}_{\text{ rational}} \times \mathbb{Z} \times N_1 \rightarrow \mathbb{P}_{\text{ rational}}
\]

\[
\text{make_floats}(\text{fracs}, \text{exps}, \text{rad}) =
\{ f \in \text{fracs} \mid e \in \text{exps} \bullet f \ast \pm \text{rad}^e \}
\]

We are principally interested in two classes of “canonical” floating point numbers:

\[
\text{Canonical} : N_1 \times N_1 \times N_1 \rightarrow \mathbb{P}_{\text{ rational}}
\]

\[
\text{BoundedCanonical} : N_1 \times N_1 \times N_1 \times N_1 \rightarrow \mathbb{P}_{\text{ rational}}
\]

\[
\text{Canonical}(\text{rad}, \text{mant}, \text{emin}) =
\text{make_floats}(\text{normal_fractions}(\text{rad}, \text{mant}), \{ i : \mathbb{Z} | \text{emin} \leq i \}, \text{rad})
\]

\[
\text{BoundedCanonical}(\text{rad}, \text{mant}, \text{emin}, \text{emax}) =
\text{make_floats}(\text{normal_fractions}(\text{rad}, \text{mant}),
\{ i : \mathbb{Z} | \text{emin} \leq i \leq \text{emax} \}, \text{rad})
\]

3.5.4.6.5 Implementations

An Implementation consists of a machine arithmetic, a MachineParam modeling the representation-oriented attributes of the arithmetic, and a boolean indicating the response to numeric overflow. The properties of the machine arithmetic do not uniquely determine the appropriate MachineParam.

\[
\text{Implementation}
\]

\[
\text{ma} : \text{MachineArithmetic}
\]

\[
\text{machine_attr} : \text{MachineParam}
\]

\[
\text{overflows} : \text{Boolean}
\]

\[
\text{machine_attr} \in \text{max_mach_params}(\text{ma})
\]
An AccurParam is a 6-tuple whose elements are interpreted as a radix, a mantissa length, a minimum exponent, the bounds for a safe interval, and an indication of whether overflows are to be reported. (The constraints defined by the other parameters are interpreted more strictly if the "overflows" flag is true.)

<table>
<thead>
<tr>
<th>AccurParam</th>
</tr>
</thead>
<tbody>
<tr>
<td>radix, mantissa : N;</td>
</tr>
<tr>
<td>emin : { i : N</td>
</tr>
<tr>
<td>sfirst, slast : Float</td>
</tr>
<tr>
<td>overflows : Boolean</td>
</tr>
</tbody>
</table>

It is convenient to have an abbreviation for the set of safe numbers that an AccurParam defines.

safe : AccurParam → P Float

safe(ap) = ap.sfirst .. ap.slast

We will formalize an essential notion of Annex G with _has_accuracy_, which says what it means for an implementation to satisfy an AccurParam. From the core model, we can extract only some minimal properties of this relation, expressed in the weaker notion _has_weak_acc_:

_has_weak_acc_ : Implementation → AccurParam
_has_accuracy_ : Implementation → AccurParam

imp has_weak_acc ap ↔
imp.radix = imp.ma.radix ∧
imp.mantissa ≤ imp.machine_attr.mantissa ∧
imp.emin ≥ imp.machine_attr.emin ∧
{ ap.sfirst, ap.slast } ⊆ ma.numbers ∧
imp.overflows = ap.overflows
imp has_accuracy ap ⇒ imp has_weak_acc ap

For any implementation imp, in Annex defines model_attr(imp), a unique "best" AccurParam satisfied by an implementation. All we can say in the core semantics is that imp satisfies the weak accuracy requirements imposed by model_attr(imp).

model_attr : Implementation → AccurParam

imp has_weak_acc model_attr(imp)

More precisely, Annex G defines model_attr(imp) to be a particular maximal ap such that imp has_accuracy ap.

The maximum number of decimal digits of accuracy is uniquely determined by the model-oriented attributes of the implementation.

digits : Implementation → N,

digits(imp) = ceiling(
(model_attr(imp).mantissa * log(10)/log(imp.ma.radix)) + 1)

There is a problem in the Reference Manual: This definition of "digits" is not given anywhere in the Reference Manual. It is surely the intended one, but it does not seem to follow from anything in the Reference Manual.
There is a problem in the Reference Manual: Section A.5.3(67-68) defines the value of \( S'\text{Model}_\text{Mantissa} \) in incompatible ways, depending on whether or not the implementation "supports" Annex G. The same is true for \( S'\text{Model}_\text{Emin} \). In consequence, an implementation can be valid for Core+G but not valid for the Core alone. (By contrast, the core semantics makes the semantics of the model-oriented attributes 'Safe_First' and 'Safe_Last' upward compatible by leaving them implementation-defined.)

The definitions of \( S'\text{Model}_\text{Mantissa} \) and \( S'\text{Model}_\text{Emin} \) given here are weaker than those of the core semantics. We require only that

\[
S'\text{Model}_\text{Mantissa} \leq S'\text{Machine}_\text{Mantissa} \\
S'\text{Model}_\text{Emin} \geq S'\text{Machine}_\text{Emin}
\]

(See the definition of \( \_\text{has\_weak\_acc} \).)

These definitions make the core semantics compatible with Annex G. In addition, they capture the only information that the present version of the Reference Manual allows a user to rely on across all implementations.

### 3.5.4.6.6 Satisfaction of a type definition

A floating point declaration supplies a requested precision (a value of \( N \)) and, optionally, a constraint (two \( \text{Rationals} \)). The accuracy of the type's implementation must be at least as great as the requested precision and the safe range of the implementation must include the interval defined by the constraint. This requirement is captured by the definition of \( \text{sat\_float\_def} \).

The Reference Manual says that any such implementation may be chosen. We represent the particular strategy that the implementation uses for choosing the implementation (such as choosing the coarsest acceptable implementation type) by the relation \( \text{implements\_float\_def} \). The judgement defining elaboration of a type definition selects an implementation satisfying \( \text{implements\_float\_def} \). All the reference manual requires of this relation is that it be consistent with \( \text{sat\_float\_def} \):

\[
\text{sat\_float\_def}, \text{implements\_float\_def} \ :
\begin{align*}
\text{Implementation} &\leftrightarrow (N \times \text{Rational} \times \text{Rational}) \\
(imp, (n, L, R)) \in \text{sat\_float\_def} &\Rightarrow \\
L \ldots R \subseteq \text{model\_attr}(imp).sfirst \ldots \text{model\_attr}(imp).slast \wedge \\
m \leq \text{digits}(imp) \\
\text{implements\_float\_def} &\subseteq \text{sat\_float\_def}
\end{align*}
\]

### 3.5.4.6.7 Attributes

If \( imp \) is the implementation chosen for type \( T \), then the basic implementation-oriented attribute values of \( T \) are given as follows, where we let \( \text{imp\_machine\_attr} = (\text{mant}, \text{emin}, \text{emax}) \):

\[
\begin{align*}
T'\text{Machine\_Radix} &= \text{imp\_ma.radix} \\
T'\text{Machine\_Mantissa} &= \text{mant} \\
T'\text{Machine\_Emin} &= \text{emin} \\
T'\text{Machine\_Emax} &= \text{emax} \\
T'\text{Base\_First} &= \min(\text{imp\_ma.numbers}) \\
T'\text{Base\_Last} &= \max(\text{imp\_ma.numbers})
\end{align*}
\]

The limits of the base range are the least and greatest machine numbers. This follows from 3.5(6)

The base range of a scalar type is the range of finite values of the type that can be represented in every unconstrained object of the type

and from 3.5.7(8)

The machine numbers of a floating point type are the values of the type that can be represented exactly in every unconstrained variable of the type.

26
The basic model-oriented attribute values of an implementation \( \text{imp} \) are given as follows (where we let \( \text{model\_attr}(\text{imp}) = (\text{Mant}, \text{Emin}, \text{sfirst}, \text{slast}) \)):

\[
\begin{align*}
T'\text{Mode\_Mant} &= \text{Mant} \\
T'\text{Model\_Emin} &= \text{Emin} \\
T'\text{Safe\_First} &= \text{sfirst} \\
T'\text{Safe\_Last} &= \text{slast} \\
T'\text{Base'Digits} &= \text{digits(\text{imp})}
\end{align*}
\]

The definition of \( T'\text{Base'Digits} \) is something of a guess.

Note: The definitions of various attributes, such as \( S'\text{Model} \) (for floating point subtypes) and \( S'\text{Machine} \) (for fixed and floating point subtypes) say that the value returned is obtained "by rounding or truncating" the operand "to either one of the adjacent" model or machine numbers, as appropriate. It is not clear from this language whether these operations are non-deterministic.

### 3.5.4.7 Annex G

Annex G defines some more precise constraints on the floating point operations.

#### 3.5.4.7.1 Model numbers and accuracy

An accuracy parameter determines a set of model intervals (intervals bounded by the associated canonical real numbers) and associates a model interval with each bounded set (namely, the smallest model interval that contains it):

\[
\begin{align*}
\text{ModelIntervals} : \text{AccurParam} &\rightarrow \mathbb{P} \text{ Float} \\
\text{ModelIntOf} : \mathbb{P} \text{ Float} \times \text{AccurParam} &\rightarrow \mathbb{P} \text{ Float}
\end{align*}
\]

\[
\begin{align*}
u \in \text{ModelIntervals}(\text{ap}) &\iff \\
&\exists \lo, \hi : \text{Float} \cdot \\
&\ u = \lo \ldots \hi \\
&\ \{ \lo, \hi \} \subseteq \text{Canonical}(\text{ap}.\text{radix}, \text{ap}.\text{mantissa}, \text{ap}.\text{emin}) \\
\text{ModelIntOf}(X, \text{ap}) &= \bigcap \{ u \in \text{ModelIntervals}(\text{ap}) \mid X \subseteq u \}
\end{align*}
\]

Given a "paradigm" operation \( f \) and an accuracy specification \( \text{ap} \), we define for each \( x \) the set of results that approximate \( f(x) \) to within the demands of \( \text{ap} \)—namely, the model interval of the set that results from applying \( f \) to the model interval of \( x \). (The same applies, mutatis mutandis, to the binary operations.) All definitions follow the same pattern, but the type restrictions of \( \mathbb{Z} \) require us to provide separate definitions for unary operations, binary operations returning floats, and binary operations returning booleans.

\[
\begin{align*}
\text{ResultUnOp} : (\text{Float} \rightarrow \text{Float}) \times \text{AccurParam} &\rightarrow (\text{Float} \rightarrow \mathbb{P} \text{ Float}) \\
\text{ResultBinOpFloat} : (\text{Float}^2 \rightarrow \text{Float}) \times \text{AccurParam} &\rightarrow (\text{Float}^2 \rightarrow \mathbb{P} \text{ Float}) \\
\text{ResultBinOpBool} : (\text{Float}^2 \rightarrow \text{Bool}) \times \text{AccurParam} &\rightarrow (\text{Float}^2 \rightarrow \mathbb{P} \text{ Bool})
\end{align*}
\]

\[
\begin{align*}
\text{ResultUnOp}(f, \text{ap}) &= \\
&\lambda x : \text{Float} \cdot \\
&\text{ModelIntOf}(f[\text{ModelIntOf}\{\{x\}, \text{ap}\}], \text{ap}) \\
\text{ResultBinOpFloat}(f, \text{ap}) &= \\
&\lambda x, y : \text{Float} \cdot \\
&\text{ModelIntOf}(f[\text{ModelIntOf}\{\{x\}, \text{ap}\} \times \text{ModelIntOf}\{\{y\}, \text{ap}\}], \text{ap}) \\
\text{ResultBinOpBool}(f, \text{ap}) &= \\
&\lambda x, y : \text{Float} \cdot \\
&\text{ModelIntOf}(f[\text{ModelIntOf}\{\{x\}, \text{ap}\} \times \text{ModelIntOf}\{\{y\}, \text{ap}\}], \text{ap})
\end{align*}
\]

Operands are "safe for" a paradigm operation if they and all their approximate results are safe.
A \textit{UnOp} approximates a "paradigm" function to within some accuracy specification if it associates all operands with results that are acceptable approximations to that function. The same goes for \textit{BinOpFloats} and \textit{BinOpBools}. In particular, safe operands may not return an \textit{overflow}; and if the "overflows" flag is true, unsafe operands must return either an approximately correct result or the \textit{overflow} token.
imp has_accuracy ap \iff
imp has_weak_acc ap ∧
imp.ma.plus Approx BinOpFloat (+, ap) ∧

...  
imp.ma.equals Approx BinOpBool (=, ap) ∧

...  
imp.ma.convert Approx UnOp (id Float, ap)

3.5.4.7.2 Model-oriented attributes To define model_attr we choose a particular maximal AccurParam satisfied by an implementation. (Note: The definitions below do not follow the definition in version 5.0, which is incorrect, but Ken Dritz's subsequent reworking of version 5.0.)

\[
\begin{align*}
\text{best_mant} : & \ P \text{AccurParam} \rightarrow \ P \text{AccurParam} \\
\text{best_emin} : & \ P \text{AccurParam} \rightarrow \ P \text{AccurParam} \\
\text{best_first} : & \ P \text{AccurParam} \rightarrow \ P \text{AccurParam} \\
\text{best_last} : & \ P \text{AccurParam} \rightarrow \ P \text{AccurParam} \\
\text{best_ap} : & \ P \text{AccurParam} \rightarrow \ P \text{AccurParam}
\end{align*}
\]

\[
\text{best_ap}(X) = \text{best_last}(\text{best_first}(\text{best_emin}(\text{best_mant}(X))))
\]

\[
\begin{align*}
\text{model_attr}(\text{imp}) = \\
\mu\ ap : \text{AccurParam} \cdot \text{ap} & \in \text{best_ap}(\text{ap}') \cdot \text{AccurParam} \mid \text{imp has_accuracy ap'} \Rightarrow \\
\text{ap} & \in X \land \forall \text{ap'} : X \cdot \text{ap}.mantissa \leq \text{ap'}.mantissa \\
\text{ap} & \in \text{best_emin}(X) \\
\text{ap} & \in X \land \forall \text{ap'} : X \cdot \text{ap}.emin \leq \text{ap'}.emin \\
\text{ap} & \in \text{best_first}(X) \\
\text{ap} & \in X \land \forall \text{ap'} : X \cdot \text{ap}.sfirst \leq \text{ap'}.sfirst \\
\text{ap} & \in \text{best_last}(X) \\
\text{ap} & \in X \land \forall \text{ap'} : X \cdot \text{ap}.slast \geq \text{ap'}.slast
\end{align*}
\]

There is, of course, exactly one best_ap:

\[
\#\text{best_ap}(\{ \text{ap} : \text{AccurParam} \mid \text{imp has_accuracy ap} \}) = 1
\]

### 3.5.5 Array Types

The descriptor for an array type has the form array_dsc(i, c), where i : seq Subtype is a sequence of discrete subtypes (the index subtypes), and c : Subtype is the component subtype.

\[
\text{descriptor_values}(\text{array_dsc}(i, c), S) = \\
\{ \text{B} : \text{Index_bounds}, \text{v} : \text{(seq Value)} \leftrightarrow \text{subtype_values}(c, S) \\
| \text{Array}(\text{B}, \text{v}) \in \text{Value} \land B = \text{range_of_subtype} \circ i \\
| \text{Array}(\text{B}, \text{v}) \}
\]

### 3.5.5.1 String Types

String types are just particular array types.

### 3.5.6 Discriminants

Discriminants are specialized components of some composite types. We incorporate discriminants (which might be null) into the descriptor for every type that can have discriminants, in order to avoid a tedious duplication of definitions in similar cases.
This model does not account for access discriminants.

\[
\textbf{Discriminant} \_\textbf{descriptor} \equiv \text{Id} \leadsto \text{Subtype} \times \text{optional Thunk}
\]

\[
\text{discriminant-values} : \text{Discriminant} \_\textbf{descriptor} \times \text{State} \rightarrow \text{P Binding}
\]

\[
\text{discriminant-values}(d, S) = \\
\{ f : \text{dom } d \rightarrow \text{Values } | \forall n \in \text{dom } d \bullet f(n) \in \text{subtype-values} (\text{first}(d \cdot n), S) \}
\]

### 3.5.7 Record Types

It seems like a waste of effort to describe records with discriminants separately from records without discriminants, as there is a good deal of overlap in the two cases. Thus, we give every record type descriptor discriminants (which may be null).

Similarly, it seems like a big duplication of effort to describe tagged record types separately. Thus, we will give every record descriptor a tag (which may be null in the case of an untagged record).

\[
\text{Component-list} \_\textbf{descriptor} \equiv \\
(\text{Id} \leadsto \text{Partial-subtype} \times \text{optional Thunk}) \times \text{optional Variant} \_\textbf{descriptor}
\]

A record type descriptor consists of a description of the discriminants (if any), and a component list description. The values of such a descriptor are records with fields for the discriminants, and fields for the other components. The subtypes of these latter fields (and even the exact fields present) may depend on a value of a discriminant.

\[
\text{descriptor-values}(\text{record} \_\textbf{dsc}(t, DA, CL), S) = \\
\{ d, r : \text{Binding } | d \in \text{discriminant-values}(DA, \rho) \land r \in \text{CL-values}(CL, d, S) \\
\bullet \text{record}(t, d, r) \}
\]

A binding giving the values of the discriminants is given to function \(CL\_values\), so that the actual subtype of each component can be determined.

\[
\text{CL-values} : \text{Component-list} \_\textbf{descriptor} \times \text{Binding} \times \text{State} \rightarrow \text{P Binding}
\]

\[
\text{CL-values}((A, V), d, S) = \{ f, v : \text{Binding } | \text{dom } f = \text{dom } A \land \\
(\forall n \in \text{dom } f \bullet f(n) \in \text{Subtype-values} (\text{actualize-subtype} (\text{first}(A \cdot n), d), S)) \\
\bullet f \cup v \}
\]

### 3.5.7.1 Variant Parts and Discrete Choices

A variant descriptor has the Id of the discriminant of the variant part, and a mapping from the possible values of this discriminant to component list descriptors.

\[
\text{Variant} \_\textbf{descriptor} \equiv \text{Id} \times (\text{Discrete-value} \leftrightarrow \text{Component-list} \_\textbf{descriptor})
\]

\[
\text{Variant-values} : (\text{optional Variant} \_\textbf{descriptor}) \times \text{Binding} \times \text{State} \rightarrow \text{P Binding}
\]

\[
\text{Variant-values}(\text{none}, d, S) = \{\emptyset\}
\]

\[
d(n) \in \text{dom } f \Rightarrow \text{Variant-values}(\text{some}(n, f), d, S) = \text{CL-values}(f(d(n)), d, S)
\]

\[
d(n) \notin \text{dom } f \Rightarrow \text{Variant-values}(\text{some}(n, f), d, S) = \{\emptyset\}
\]

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3.5.8 Tagged Types and Type Extensions

The model for tagged types has not yet been developed.

3.5.8.1 Abstract Types and Subprograms

[AARM 3.9.3(8.a)] asserts that there are no values of an abstract type. But it is possible to have subprograms for such subtypes, and for non-abstract descendents to inherit them. If we want to say something about the meanings of such subprograms, we probably need to talk about the values of the parameters. (In some sense, these are parameters of type T’Class.)

On the whole, it probably seems easiest to use a model of values as though the type were not abstract; we can also treat abstract subprograms as though their bodies raised Program_Error.

3.5.9 Access Types

The descriptor for an access type has the form access_dsc(s, m), where s : Subtype_location describes the designated subtype of the access type, and m : Access_mode describes the access mode.

Access_Mode ::= constant_access | all_access | pool_access

A value of an access-to-object type is either a null value, or a view of an object of the designated subtype:

descriptor_values(access_dsc(s, m), S) =  
{ access_value(none) } U  
{ access_value(some(object_view(l, s')) |  
the_store(S).objects(l) E subtype_values(the_store(S).subtypes(s), S) }  

3.5.9.1 Incomplete Type Declarations

The descriptor incomplete describes an incomplete type. When the full type definition for the type is elaborated, the type environment is updated to reflect the appropriate descriptor for the type.

Note that there is an issue about the first subtype of an incomplete type; it is constrained if there is no discriminant part. However, the first subtype corresponding to the full definition may be unconstrained. (See comment 94-3901.c.)

There are no values of an incomplete type.

3.6 States

A state combines a store, an external state, and control flow information (in the usual cases), or is erroneous or unpredicted. We use the state unpredicted in those cases where our semantic definition, in the interests of simplicity, makes no prediction about the effect of a program (even though the Standard defines the effect, or calls it implementation-defined).

State ::= normal(Store)  
| exception(ExceptionId x Store)  
| exit(Loop_Id x Store)  
| proc_return(Store)  
| func_return(Value x Store)  
| intermediate(Store)  
| erroneous  
| unpredicted

Function the_store gives the store associated with a state.
Chapter 4

Judgements

This section summarizes the domains and judgements used in the definition of Ada 9X semantics. The details of the domain definitions are given in Chapter 3. The formal definitions of these judgements are given in later sections.

4.1 Domains

The definition is based on the concepts, domains and support functions, introduced in Chapter 3. Specifically, it uses the domains listed in Table 4.1. All of these domains are generated by term algebras (subject to the constraints defined in Chapter 3). Constructors for these domains are given below. In addition, the domains of component associations and environments are defined as follows:

\[
\text{CompAssoc} = \text{list}((\text{Id} \times (\text{Subtype} \times \text{optional(Thunk)))))
\]
\[
\text{Environment} = (\text{Id} \leftrightarrow \text{View})
\]

Also the signatures for all functions (other than constructors) and predicates are given. Note that these signatures are those used in the Prolog representation of judgements and may differ from those given in Chapter 3.

4.1.1 Types

The structure of a type is given by a type descriptor of the form:

\[
type \quad \text{Type}
\]
\[
\text{enum\_type} : \text{integer} \rightarrow \text{Type}
\]
\[
\text{modular\_type} : \text{integer} \rightarrow \text{Type}
\]
\[
\text{signed\_integer\_type} : \text{integer} \times \text{integer} \times \text{integer} \times \text{integer} \rightarrow \text{Type}
\]
\[
\text{universal\_integer\_type} : \text{integer} \times \text{integer} \rightarrow \text{Type}
\]
\[
\text{array\_type} : \text{list(Subtype)} \times \text{Subtype} \rightarrow \text{Type}
\]
\[
\text{class\_type} : \text{Type\_location} \rightarrow \text{Type}
\]
\[
\text{access\_type} : \text{Subtype\_location} \times \text{Access\_modifier} \rightarrow \text{Type}
\]
\[
\text{func\_profile} : (\text{Id} \rightarrow^\alpha \text{Parameter}) \times \text{Subtype} \rightarrow \text{Type}
\]
\[
\text{proc\_profile} : (\text{Id} \rightarrow^\alpha \text{Parameter}) \rightarrow \text{Type}
\]
\[
\text{incomplete\_type} : \text{Discriminant} \rightarrow \text{Type}
\]
\[
\text{record\_type} : \text{optional(Type\_location)} \times \text{Discriminant} \times \text{Record\_fields} \rightarrow \text{Type}
\]
<table>
<thead>
<tr>
<th>Access_modifier</th>
<th>access (type) modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>executable actions</td>
</tr>
<tr>
<td>Attributes</td>
<td>subtype attributes</td>
</tr>
<tr>
<td>Bool</td>
<td>truth values (non-Ada)</td>
</tr>
<tr>
<td>Choice</td>
<td>choices</td>
</tr>
<tr>
<td>Constraint</td>
<td>constraints</td>
</tr>
<tr>
<td>Discriminant</td>
<td>discriminants</td>
</tr>
<tr>
<td>ExId</td>
<td>internal names for exception identifiers</td>
</tr>
<tr>
<td>Record_fields</td>
<td>record fields including variants</td>
</tr>
<tr>
<td>LValue</td>
<td>L-values (addresses)</td>
</tr>
<tr>
<td>LoopId</td>
<td>internal names for loop identifiers</td>
</tr>
<tr>
<td>Mode</td>
<td>parameter modes</td>
</tr>
<tr>
<td>Object_location</td>
<td>addresses of top-level objects</td>
</tr>
<tr>
<td>Object_mode</td>
<td>indication of constancy and aliasing</td>
</tr>
<tr>
<td>Partial_constraint</td>
<td>partial constraints</td>
</tr>
<tr>
<td>Partial_subtype</td>
<td>partial subtypes</td>
</tr>
<tr>
<td>Partial_value</td>
<td>partial values</td>
</tr>
<tr>
<td>Parameter</td>
<td>formal parameters</td>
</tr>
<tr>
<td>Range</td>
<td>discrete and real ranges</td>
</tr>
<tr>
<td>State</td>
<td>state of a computation</td>
</tr>
<tr>
<td>Store</td>
<td>model of memory</td>
</tr>
<tr>
<td>Subprogram</td>
<td>subprogram values</td>
</tr>
<tr>
<td>Subprogram_label</td>
<td>unique tags for subprogram access values</td>
</tr>
<tr>
<td>Subprogram_location</td>
<td>addresses for subprogram values</td>
</tr>
<tr>
<td>Subtype</td>
<td>subtypes</td>
</tr>
<tr>
<td>Subtype_location</td>
<td>addresses for subtypes</td>
</tr>
<tr>
<td>Thunk</td>
<td>an expression with its declaration environment</td>
</tr>
<tr>
<td>Type</td>
<td>type descriptors</td>
</tr>
<tr>
<td>Type_location</td>
<td>address space for type information</td>
</tr>
<tr>
<td>Value</td>
<td>runtime values</td>
</tr>
<tr>
<td>Variant</td>
<td>variants</td>
</tr>
<tr>
<td>View</td>
<td>views</td>
</tr>
</tbody>
</table>

Table 4.1: Domains used in the judgements
Descriptors of this form are stored in the state and are accessed by unique addresses of sort \textit{Type\_location}. A derived type is represented by a new name for an existing type descriptor. The use of type names has the advantage that it is easy to deal with cyclic types and type completion. It has the disadvantage that types can be understood only in the context of a state.

The domain \textit{Type\_location} contains constants that represent the predefined types of the language:

\begin{verbatim}

type Type\_location
  boolean\_tn : Type\_location
  character\_tn : Type\_location
  universal\_integer\_tn : Type\_location
  universal\_real\_tn : Type\_location
  root\_integer\_tn : Type\_location

In the case of tagged types, values of \textit{Type\_location} are used as unique tags. The optional \textit{Type\_location} component of a record type descriptor defines the tag of the parent type. Values of sort \textit{Type\_location} are related by the ancestor relation which represents both the derivation and class hierarchy:

\begin{verbatim}
  ancestor(State, Type\_location, Type\_location)
  descendant(State, Type\_location, Type\_location)
  ultimate\_ancestor(State, Type\_location, Type\_location)
\end{verbatim}

Access modifiers are used in the descriptors of access types with the obvious meaning.

\begin{verbatim}

type Access\_modifier
  constant\_access : Access\_modifier
  all\_access : Access\_modifier
  pool\_access : Access\_modifier
\end{verbatim}

Discriminants are represented as follows:

\begin{verbatim}

type Discriminant
  discr : (Id \rightarrow (Subtype \times \text{optional}(Thunk))) \rightarrow Discriminant
  () : Discriminant
\end{verbatim}

Two discriminants can be combined using:

\begin{verbatim}

discriminant\_union : Discriminant \times Discriminant \rightarrow Discriminant
\end{verbatim}

A thunk represents an expression together with its declaration environment.

\begin{verbatim}

type Thunk
  thunk : Environment \times Exp \rightarrow Thunk
\end{verbatim}

A component list represents actualized record fields, i.e., record fields that do not contain partial information that depends on discriminant values.

\begin{verbatim}

type CompAssoc = list((/(/x (Subtype \times \text{optional}(Thunk)))))
\end{verbatim}

The fields of a, possibly discriminated, record type are represented by the domain

\begin{verbatim}

type Record\_fields
  fields : (Id \rightarrow (Partial\textunderscore subtype \times \text{optional}(Thunk))) \times \text{optional}((Id \times Variant)) \rightarrow Record\_fields
\end{verbatim}

which includes the proper fields (maybe partial) as well as any variant.
Component associations can be constructed from discriminant values and partial component lists or variants:

\[
\text{actualized_complist} : (\text{Id} \rightarrow \text{Value}) \times \text{list}((\text{Id} \times (\text{Partial subtype} \times \text{optional(Thunk)}))) \rightarrow \text{CompAssoc}
\]

\[
\text{actualized_components} : (\text{Id} \rightarrow \text{Value}) \times \text{Record fields} \rightarrow \text{CompAssoc}
\]

\[
\text{actualized_variants} : (\text{Id} \rightarrow \text{Value}) \times (\text{Id} \times \text{Variant}) \rightarrow \text{CompAssoc}
\]

\[
\text{append_components} : \text{CompAssoc} \times \text{CompAssoc} \rightarrow \text{CompAssoc}
\]

A variant part is represented by a pair \((\text{Id} \times \text{Variant})\) where the identifier specifies the name of the discriminant and the second component represents the actual fields:

\[
\text{type Variant}
\]

\[
\text{variant} : \text{list}((\text{Choice} \times \text{Record fields})) \rightarrow \text{Variant}
\]

\[
\langle \rangle : \text{Variant}
\]

Given a discriminant value, the actual record fields of a variant are defined by the predicate

\[
\text{the_variant( Variant, Value, Record fields)}
\]

Variant parts are combined using function

\[
\text{variant_union} : \text{optional}(\text{Id} \times \text{Variant}) \times \text{optional}(\text{Id} \times \text{Variant}) \rightarrow \text{optional}(\text{Id} \times \text{Variant})
\]

The representation of subprogram access types uses parameter descriptors of the form:

\[
\text{type Parameter}
\]

\[
\text{formal} : \text{Mode} \times \text{Subtype} \times \text{optional(Thunk)} \rightarrow \text{Parameter}
\]

where modes are given by:

\[
\text{type Mode}
\]

\[
\text{in_mode} : \text{Mode}
\]

\[
\text{out_mode} : \text{Mode}
\]

\[
\text{in_out_mode} : \text{Mode}
\]

The following predicates define the set of values of a given type (descriptor):

\[
\text{cLvalue(State, Record fields, (Id \rightarrow Value), (Id \rightarrow Value))}
\]

\[
\text{discriminant_value(State, Discriminant, (Id \rightarrow Value))}
\]

\[
\text{variant_values(State, optional((Id \times Variant)), (Id \rightarrow Value), (Id \rightarrow Value))}
\]

\[
\text{descriptor_value(State, Type, Value)}
\]

### 4.1.2 Values

The representation of values is straightforward using the definition

\[
\text{type Value}
\]

\[
\text{invalid_val : Value}
\]

\[
\text{discrete_val : integer \rightarrow Value}
\]

\[
\text{real_val : real \rightarrow Value}
\]

\[
\text{access_val : View \times \text{optional(Subprogram label)} \rightarrow Value}
\]

\[
\text{null : Value}
\]

\[
\text{record_val : optional(Type location) \times (Id \rightarrow Value) \times (Id \rightarrow Value) \rightarrow Value}
\]

\[
\text{array_val : list(Range) \times (list(Value) \rightarrow Value) \rightarrow Value}
\]

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The term access_val(...) is used for access to object and access to subprogram values. In the latter case, each time a subprogram access is computed a new unique subprogram label

```
type Subprogram_label
```

is generated. This label is needed to properly model equality of access to subprogram values.

Note that access values are views. This representation is used to determine the actual subtype of an access value.

Ranges are represented as pairs of values in the obvious way.

```
type Range
discrete_rng : integer × integer → Range
real_rng : real × real → Range
```

The following functions are defined for ranges.

```
low_bound : Range → Value
high_bound : Range → Value
make_range : Value × Value → Range
base_range : State × Type → Range
indices : list(Range) → set(list(Value))
```

The latter function defines the set of all index vectors that fit a list of (index) ranges.

Sets of (scalar) values are represented using the domain

```
type Choice
choice_range : Range → Choice
choice_value : Value → Choice
choice_lst : list(Choice) → Choice
choice_default : Choice
```

The following are predicates on values

```
access_value(Value)
array_value(Value)
belongs_to(Value, Range)
composite_value(Value)
covers(Value, Choice, Bool)
discrete_value(Value)
elementary_value(Value)
real_value(Value)
satisfies(Value, Constraint)
scalar_value(Value)
```

### 4.1.3 Subtypes

The definition of subtypes follows the description provided by the Reference Manual:

```
type Subtype
subtype : Type_location × Constraint × Attributes → Subtype
```

Access to the underlying type is indirect through a type location. The present version of the semantics does not use any subtype attributes.

```
type Attributes
not_used : Attributes
```
Functions defined for subtypes are

\[
\begin{align*}
\text{range_of subtype} & : \text{State} \times \text{Subtype} \rightarrow \text{Range} \\
\text{ranges_of_subtypes} & : \text{State} \times \text{list(Subtype)} \rightarrow \text{list(Range)} \\
\text{type_struct} & : \text{State} \times \text{Subtype} \rightarrow \text{Type}
\end{align*}
\]

Constraints are

\[
\begin{align*}
\text{type} & \quad \text{Constraint} \\
\text{no_constraint} & : \text{Constraint} \\
\text{range_constraint} & : \text{Range} \rightarrow \text{Constraint} \\
\text{index_constraint} & : \text{list(Range)} \rightarrow \text{Constraint} \\
\text{discriminant_constraint} & : (\text{Id} \leftrightarrow \text{Value}) \rightarrow \text{Constraint} \\
\text{indirect_index_constraint} & : \text{list((Partial_value \times Partial_value))} \rightarrow \text{Constraint} \\
\text{indirect_discriminant_constraint} & : (\text{Id} \leftrightarrow \text{Value}) \rightarrow \text{Constraint}
\end{align*}
\]

In the case of discriminated record types, component subtypes can be partial if they depend on discriminant values. This leads to the following definitions of partial counterparts of values, constraints, and subtypes.

\[
\begin{align*}
\text{type} & \quad \text{Partial_value} \\
\text{p_value} & : \text{Value} \rightarrow \text{Partial_value} \\
\text{discriminant_ref} & : \text{Id} \rightarrow \text{Partial_value} \\
\text{type} & \quad \text{Partial_constraint} \\
\text{p_no_constraint} & : \text{Partial_constraint} \\
\text{p_range_constraint} & : (\text{Partial_value} \times \text{Partial_value}) \rightarrow \text{Partial_constraint} \\
\text{p_index_constraint} & : \text{list((Partial_value \times Partial_value))} \rightarrow \text{Partial_constraint} \\
\text{p_discriminant_constraint} & : (\text{Id} \leftrightarrow \text{Partial_value}) \rightarrow \text{Partial_constraint} \\
\text{p_indirect_index_constraint} & : \text{list((Partial_value \times Partial_value))} \rightarrow \text{Partial_constraint} \\
\text{p_indirect_discriminant_constraint} & : (\text{Id} \leftrightarrow \text{Partial_value}) \rightarrow \text{Partial_constraint} \\
\text{type} & \quad \text{Partial_subtype} \\
\text{p_subtype} & : \text{Subtype} \times \text{Partial_constraint} \rightarrow \text{Partial_subtype}
\end{align*}
\]

Given a discriminant constraint, partial entities can be actualized. This process is defined by the functions:

\[
\begin{align*}
\text{actualized_partial_range} & : (\text{Id} \leftrightarrow \text{Value}) \times (\text{Partial_value} \times \text{Partial_value}) \rightarrow \text{Range} \\
\text{actualized_range_list} & : (\text{Id} \leftrightarrow \text{Value}) \times \text{list((Partial_value \times Partial_value))} \rightarrow \text{list(Range)} \\
\text{actualized_value} & : (\text{Id} \leftrightarrow \text{Value}) \times \text{Partial_value} \rightarrow \text{Value} \\
\text{actualized_constraint} & : (\text{Id} \leftrightarrow \text{Value}) \times \text{Partial_constraint} \rightarrow \text{Constraint} \\
\text{actualized_binding_list} & : \text{binding( Id \leftrightarrow Value) \times list((Id \times Partial_value))} \rightarrow \text{list((Id \times Value))}
\end{align*}
\]
The following predicates on subtypes define the taxonomy of RM 3.2:

- `is_access_to_object_type(State, Subtype)`
- `is_access_to_subprogram_type(State, Subtype)`
- `is_access_type(State, Subtype)`
- `is_array_type(State, Subtype)`
- `is_boolean_type(State, Subtype)`
- `is_by_copy_type(State, Subtype)`
- `is_by_reference_type(State, Subtype)`
- `is_character_type(State, Subtype)`
- `is_composite_type(State, Subtype)`
- `is_discrete_type(State, Subtype)`
- `is_elementary_type(State, Subtype)`
- `is_enumeration_type(State, Subtype)`
- `is_integer_type(State, Subtype)`
- `is_modular_integer_type(State, Subtype)`
- `is_protected_type(State, Subtype)`
- `is_real_type(State, Subtype)`
- `is_record_type(State, Subtype)`
- `is_scalar_type(State, Subtype)`
- `is_signed_integer_type(State, Subtype)`
- `is_string_type(State, Subtype)`
- `is_tagged_type(State, Subtype)`
- `is_task_type(State, Subtype)`

Other predicates used in the semantics are:

- `component_type((Id → Value), Type, Id, Subtype)`
- `convert_return_value(State, View, Subtype, View)`
- `range_constraints(State, list(Subtype), list(Range))`
- `select_component_type((Id → Value), Record_fields, Id, Subtype)`
- `subtype_value(State, Subtype, Value)`
- `test_in(State, Value, Subtype)`
- `type_constraint(Environment, Subtype, Constraint, Type)`
- `null_range(Range)`
- `included_in(Range, Range)`
- `values_in_range(Range, set(Value))`
- `discrete_range(Range)`
- `real_range(Range)`

Finally, the following predicates describe state transformers related to various checks and conversions related to subtypes.

- `slice_check(State, Range, Range, State)`
- `view_convert(State, Environment, Subtype, View, View, State)`
- `subtype_convert(State, Environment, Subtype, Value, Value, State)`
- `index_list(State, list(Range), list(Value), list(Value), State)`
- `return_check(State, list(Range), list(Value), list(Value), State)`

### 4.1.4 Environments and Views

Environments map identifiers to views.

\[
\text{type } \text{Environment} = (\text{Id} \rightarrow \text{View})
\]
An environment is updated using the usual syntax for bindings.

\[ \_ \leftarrow \_ : \text{Environment} \times \text{Id} \times \text{View} \rightarrow \text{Environment} \]

Note that identifiers are assumed to be unique and that all overloading and qualified names have been resolved to unique identifiers.

The lookup of an identifier \( I \) in an environment \( E \) is written as

\[ E \leadsto \text{lookup} I \Rightarrow V \]

A view describes entities denoted by identifiers as follows:

**type** View

\- \text{object\_view} : \text{LValue} \times \text{Subtype} \times \text{Object\_mode} \rightarrow \text{View}
\- \text{loop\_view} : \text{Id} \rightarrow \text{View}
\- \text{constant\_view} : \text{Value} \rightarrow \text{View}
\- \text{subtype\_view} : \text{Subtype\_location} \rightarrow \text{View}
\- \text{subprogram\_view} : \text{Subprogram\_location} \times (\text{Id} \rightarrow \text{Parameter}) \times \text{optional(Subtype)} \rightarrow \text{View}
\- \text{undefined\_view} : \text{View}

Object views include a mode description of the form:

**type** Object\_mode

\- \text{variable\_object} : \text{Object\_mode}
\- \text{constant\_object} : \text{Object\_mode}
\- \text{aliased\_object} : \text{Object\_mode}

Whether or not an object may be assigned to depends on its mode:

\text{assignable(Object\_mode)}

Loop views are used in defining exit from named and unnamed loops.

**type** Loop\_Id

\- \text{unnamed} : \text{Loop\_Id}
\- \text{loop\_id} : \text{Id} \rightarrow \text{Loop\_Id}

The following predicate defines how the bindings of a parameter association are added to an environment creating the environment in which a subprogram body is executed. Note that the names used in the given parameter association may differ from the names of the formal parameters (due to renaming). The names of the formals are provided by a separate argument.

\text{bind\_actuals(Environment, (Id a View), list(Id), Environment)}

The following constants of sort \text{ExId} denote the language-defined exceptions (others may be added).

**type** ExId

\- \text{constraint\_error} : \text{ExId}
\- \text{program\_error} : \text{ExId}
\- \text{ex\_id} : \text{Id} \rightarrow \text{ExId}

New unique names for user-defined exceptions are introduced by static semantics.
4.1.5 Memory Model

Values of sort Store describe the current binding of addresses to Ada run-time values (of sort Value). In our model, there are four different types of addresses (locations). They include type location (see above) as well as locations for objects, subprograms and subtypes.

```haskell
  type Object_location
  loc : integer → Object_location

  type Subprogram_location

  type Subtype_location
```

Object locations are associated only with objects that are not components of other objects. Components are specified by the address of the containing object together with a selector sequence. An address together with a selector sequence is a L-value (sort LValue).

```haskell
  type LValue
  location : Object_location → LValue
  array_component : LValue × list(Value) → LValue
  array_slice : LValue × Range → LValue
  record_component : LValue × Id → LValue
```

Thus, the domain of stores is defined as a 4-tuple as follows:

```haskell
  type Store
  (,-,-,-,-) : (Object_location → Value) ×
              (Subprogram_location → Subprogram) ×
              (Type_location → (optional(Type_location) × Type)) ×
              (Subtype_location → Subtype) → Store
```

Individual components of a store can be updated using the following notation:

- `[→ 1] : Store × Object_location × Value → Store
- `[→ 2] : Store × Subprogram_location × Subprogram → Store
- `[→ 3] : Store × Type_location × (optional(Type_location) × Type) → Store
- `[→ 4] : Store × Subtype_location × Subtype → Store

Values of sort State describe a current store, together with the current status of program execution. A state may represent the propagation of an exception, exit from a subprogram, or exit from a loop.

```haskell
  type State
  exception : ExId × Store → State
  exit : LoopId × Store → State
  proc_return : Store → State
  func_return : View × Store → State
  normal : Store → State
```
The following functions are defined to access and manipulate the store embedded in a state.

\[\begin{align*}
&\text{v--*l} : \text{State} \times \text{Object}_\text{location} \times \text{Value} \rightarrow \text{State} \\
&\text{State} \times \text{Subprogram}_\text{location} \times \text{Subprogram} \rightarrow \text{State} \\
&\text{State} \times \text{Type}_\text{location} \times (\text{optional}(\text{Type}_\text{location}) \times \text{Type}) \rightarrow \text{State} \\
&\text{State} \times \text{Subtype}_\text{location} \times \text{Subtype} \rightarrow \text{State} \\
&\text{State} \times \text{Object}_\text{location} \rightarrow \text{Value} \\
&\text{State} \times \text{Subprogram}_\text{location} \rightarrow \text{Subprogram} \\
&\text{State} \times \text{Type}_\text{location} \rightarrow (\text{optional}(\text{Type}_\text{location}) \times \text{Type}) \\
&\text{State} \times \text{Subtype}_\text{location} \rightarrow \text{Subtype} \\
&\text{make\_state} : \text{State} \times \text{Store} \rightarrow \text{State} \\
&\text{the\_store} : \text{State} \rightarrow \text{Store}
\end{align*}\]

The allocation of new location of the four different kinds is defined by the predicates:

\[\begin{align*}
&\text{new\_type}(\text{State}, (\text{optional}(\text{Type}_\text{location}) \times \text{Type}), \text{Type}_\text{location}, \text{State}) \\
&\text{new\_subtype}(\text{State}, \text{Subtype}, \text{Subtype}_\text{location}, \text{State}) \\
&\text{new\_object}(\text{State}, \text{Value}, \text{Object}_\text{location}, \text{State}) \\
&\text{new\_subprogram}(\text{State}, \text{Subprogram}, \text{Subprogram}_\text{location}, \text{State})
\end{align*}\]

For a given state and L-value, the following predicate defines the current value. The definition of this predicate includes access of the appropriate subobjects denoted by an L-value.

\[\text{content}(\text{State}, \text{L Value}, \text{Value})\]

States are classified as normal or abnormal. Abnormal states will, in general, alter the control flow.

\[\begin{align*}
&\text{abnormal\_state}(\text{State}) \\
&\text{normal\_state}(\text{State})
\end{align*}\]

The return from a subprogram may be indicated by an abnormal state. The following predicates deal with the cases of procedure and function returns, respectively.

\[\begin{align*}
&\text{proc\_exit}(\text{State}, \text{State}) \\
&\text{return\_value}(\text{State}, \text{View}, \text{State})
\end{align*}\]

Values stored in the subprogram component of a store are of the form:

\[\begin{align*}
\text{type} & \text{ Subprogram} \\
&\text{subprogram} : \text{Environment} \times \text{list}(\text{Id}) \times \text{Dcl} \times \text{Stm} \rightarrow \text{Subprogram} \\
&\text{operator} : \text{Operator} \rightarrow \text{Subprogram} \\
&\text{unelaborated} : \text{Subprogram}
\end{align*}\]

A user-declared subprogram is represented by the declaration environment, the names of the formal parameters and the declarations and statements that comprise the body. Predefined operators of the language are enumerated by a domain \text{Operator}. The definition of the semantics of the operators is not covered. An attempt to execute a subprogram of the form \text{unelaborated} will raise program error.

4.1.6 Other Predicates

The following predicates deal with the selection of elements from parameter lists and record component associations.

\[\begin{align*}
&\text{the\_parameter}(\text{list}(\text{Pss}), \text{Id}, \text{Nam}) \\
&\text{given\_parameter}(\text{list}(\text{Pss}), \text{Id}, \text{Exp}) \\
&\text{find\_component}(\text{Id}, \text{list}(\text{Rca}), \text{Exp})
\end{align*}\]
4.2 Judgements

Judgements for various syntactic domains define the effect of the elaboration or execution of language phrases of this domain. In most cases, the effect of a phrase depends on the current state and the current environment. The effect typically consists of a change in state and the possible return of a result. Depending on the kind of phrase, the result may be a value, a type, a new environment and so on. The meaning of some phrases depends on additional context beyond the state and the environment. For instance, the meaning of a type definition depends on the discriminant (which is part of the enclosing type declaration).

The general form of a judgement is

\[ \text{state, environment, context} \vdash \text{language-phrase} \Rightarrow \text{result, new-state} \]

The following is a list of the judgements used in the definition. For each syntactic domain, the signature of the judgement is given together with an informal rationale.

If the evaluation of a construct raises an exception then the final state represents this information. The propagation of exceptions is described as part of the sequencing of actions described below.

The following conventions are used throughout the description of judgements:

- The initial state.
- The environment.
- The final state.

4.2.1 Declarations

The effect of elaborating a (sequence of) declaration(s) is to add bindings corresponding to the newly introduced identifiers to the environment. The elaboration of a declaration may also affect the state.

\[ S_1 \vdash_{\text{dcl}} Dc \Rightarrow E_2, S_2 \]

\[ S_1 : \text{State} \quad E_1 : \text{Environment} \quad Dc : \text{Dcl} \quad E_2 : \text{Environment} \]

The initial state

- The initial environment

A declaration.

A possibly modified environment.

4.2.2 Parameter lists

Elaboration of subprogram declarations is defined in terms of the judgements

\[ S_1, E_1 \vdash_{\text{pas}} Pas \Rightarrow A, S_2 \]

\[ Pas : \text{Pms}^* \quad A : \text{Id} \overset{\alpha}{\Rightarrow} \text{Parameter} \]

- Formal parameter list.
- Parameter signature.

and

\[ \vdash_{\text{mod}} Mod \Rightarrow M \]

\[ Mod : \text{Mde} \quad M : \text{Mode} \]

- Parameter mode.
- Mode representation.

The latter judgement uses neither states nor the environment.
4.2.3 Type Definitions

A type definition is elaborated in the context of a (possible) discriminant association. The result is the first-named subtype. Elaborating a type definition may affect the state and the environment because subexpressions may have side-effects and new internal type names may be bound in the environment.

\[ S_1, E_1, D \vdash \text{tdf} \Rightarrow T, E_2, S_2 \]

- **D**: Discriminant – The discriminant.
- **Tdf**: Tdf – A type definition.
- **T**: Subtype – The resulting subtype.
- **E_2**: Environment – The modified environment.

4.2.4 Variant Parts

\[ S_1, E, D \vdash \text{vern} \Rightarrow V, S_2 \]

- **D**: Discriminant – A discriminant.
- **Vrn**: Vnt* – A list of variant clauses.
- **V**: Variant – The resulting variant.

The judgement always uses a discriminant that may be empty.

4.2.5 Discriminant Parts

The evaluation of a discriminant part results in an association that maps discriminant identifiers into their subtype and optional initialization. As with record component lists, the initialization is represented by a thunk.

\[ S_1, E \vdash \text{dsc} \Rightarrow D, S_2 \]

- **Dcp**: Dcp – A discriminant part.
- **D**: Discriminant – The resulting association.

4.2.6 Component Lists

The result of evaluating a record component list is an association that maps each component identifier into a pair consisting of the component’s subtype and an optional initialization. The initialization is given by a *thunk* that represents the initialization expression, together with the environment in which this expression is to be evaluated.

\[ S_1, E \vdash \text{cmp} \Rightarrow C, S_2 \]

- **Cmp**: Cmp – A component list.
- **C**: CompAssoc – The result.

4.2.7 Subtype Indications

There are three flavors of judgements dealing with subtype indications. The normal case is covered by the declaration:

\[ S_1, E \vdash \text{sid} \Rightarrow T, S_2 \]
A special case is needed for subtype indications that appear inside discriminated records because the result will be a partial subtype.

\[ S_1, E_1, D \vdash_{ps} Sid \Rightarrow T, E_2, S_2 \]

\( D :\text{Discriminant} \quad -\quad \text{A discriminant.} \)

\( Sid : Sid \quad -\quad \text{A subtype indication.} \)

\( T : Partial\_subtype \quad -\quad \text{The denoted partial subtype.} \)

Finally, a third form of this judgement is needed to deal with access types to allow references to incomplete types. Instead of a subtype, this judgement returns a subtype location.

\[ S_1, E \vdash_{slf} Sid \Rightarrow L, S_2 \]

\( Sid : Sid \quad -\quad \text{A subtype indication.} \)

\( L : Subtype \quad -\quad \text{A subtype location.} \)

A completing type declaration will initialize the subtype location.

### 4.2.8 Statements

The execution of a statement only affects the state and has no result.

\[ S_1, E \vdash_{stm} Stm \Rightarrow S_2 \]

\( Stm : Stm \quad -\quad \text{A statement.} \)

### 4.2.9 Elsif Clauses

An elsif-clause consists of a condition and a sequence of statements. The judgement for this construct describes the evaluation of the condition followed by the conditional execution of the statements. There is a boolean result that indicates whether the condition was true. This result is used in the definition of cascaded elsif clauses.

\[ S_1, E \vdash_{elf} Eif \Rightarrow B, S_2 \]

\( Eif : Eif \quad -\quad \text{An elsif clause.} \)

\( B : Bool \quad -\quad \text{The boolean result.} \)

### 4.2.10 Case Alternatives

The evaluation of a list of case alternatives depends on the value of the case selector:

\[ S_1, E, V \vdash_{ch} Alt \Rightarrow B, S_2 \]

\( V : Value \quad -\quad \text{The value of the case selector.} \)

\( Alt : Alt \quad -\quad \text{A sequence of case alternatives.} \)

\( B : Bool \quad -\quad \text{True, if one of the alternatives has matched the case selector.} \)

The boolean result indicates whether one of the alternatives has been executed. The execution of the enclosing case statement will have to terminate in an exception if this result is false.
4.2.11 Discrete Choice Lists

Discrete choice lists are used in variant, array aggregates and case statements. The following judgement describes the evaluation of a list of choices. The result is a representation of the choices.

\[ S_1, E \vdash \text{Dch} \Rightarrow C, S_2 \]

\( \text{Dch} : \text{Dch}^* \) - A discrete choice list.
\( C : \text{Choice} \) - The representation of the choice list.

4.2.12 Expressions

The evaluation of an expression results in a value and possible side-effects. In certain cases the meaning of an expression depends on the expected type (e.g., the evaluation of aggregates and string literals). Rather than adding this type information to the judgement, the abstract syntax provides such information where necessary.

\[ S_1, E \vdash \text{Exp} \Rightarrow V, S_2 \]

\( \text{Exp} : \text{Exp} \) - An expression.
\( V : \text{Value} \) - The resulting value.

The judgement for conditions differs from that for expressions by returning a truth value.

\[ S_1, E \vdash \text{Cnd} \Rightarrow V, S_2 \]

\( \text{Cnd} : \text{Cnd} \) - A condition.
\( B : \text{Bool} \) - A truth value.

4.2.13 Names

The evaluation of a name results in a view of the named entity.

\[ S_1, E \vdash \text{Nam} \Rightarrow W, S_2 \]

\( \text{N} : \text{Nam} \) - A name.
\( W : \text{View} \) - The view denoted by the name.

The evaluation of certain kinds of names cannot have side-effects (e.g., subtype indications). Rather than defining a separate judgement for this case the definition will require that the initial and final states are identical in these cases.

4.2.14 Ranges

The following judgement defines the evaluation of ranges. In the abstract syntax ranges include discrete subtype definitions. The judgement also deals with the definition of the range attribute.

\[ S_1, E \vdash \text{Rng} \Rightarrow R, S_2 \]

\( \text{Rng} : \text{Rng} \) - A range or discrete subtype definition.
\( R : \text{Range} \) - The resulting range value.
4.2.15 Record Aggregates

The following judgement defines the value of a record or extension aggregate.

\[ S_1, E, C \vdash_{agg} \text{Agg} \Rightarrow V, S_2 \]

- \( C : \text{CompAssoc} \) - The expected components of the aggregate.
- \( \text{Agg} : \text{Agg} \) - The aggregate.
- \( V : \text{Id} \mapsto \text{Value} \) - The resulting binding.

The expected type for the aggregate is provided in the abstract syntax by allowing only qualified aggregates. Qualification is added by static analysis where necessary.

4.2.16 Array Aggregates

A separate judgement is used for array aggregates. It has an additional sequence of index ranges as context.

\[ S_1, E, R, T \vdash_{agg} \text{Agg} \Rightarrow V, S_2 \]

- \( R : \text{list}(\text{Range}) \) - The index ranges of the aggregate.
- \( T : \text{Subtype} \) - The type of the elements.
- \( \text{Agg} : \text{Agg} \) - An array aggregate.
- \( V : \text{Value} \) - The array value.

4.2.17 Attributes

The following judgement defines the values of (parameterless) attributes as defined in Appendix A of the Reference Manual.

\[ S_1, E, W_1 \vdash_{att} \text{Id} \Rightarrow W_2, S_2 \]

- \( W_1 : \text{View} \) - A view.
- \( \text{Id} : \text{Id} \) - The attribute name.
- \( W_2 : \text{View} \) - The view of the attribute.

All core language-defined attributes are free of side-effects. This means that the final state will always equal the initial state. Note that some attributes return a subprogram view which, when called, may have a side-effect or raise an exception. But the effect of such calls is not part of evaluating the attribute itself.

In the case of the range attribute, the signature differs as follows:

\[ S_1, E, W_1 \vdash_{att} \text{range} \Rightarrow R, S_2 \]

- \( W : \text{View} \) - A view.
- \( R : \text{Range} \) - A range.

4.3 Actions

A sequence of statements is executed by sequentially executing each statement in the sequence. Execution is abandoned if one of the statements raises an exception or causes some other change in the flow of control. In the case of expressions, the language specifies that, in certain cases, several expressions are evaluated in arbitrary order.
In order to systematically define different kinds of order of execution of program parts, the notion of an action is introduced. An action can be viewed as a representation of a judgement without the state. Consider, for instance, the judgement for statements:

\[ S_1, E \vdash_{stm} Stm \Rightarrow S_2 \]

The corresponding action is a term

\[ \text{statement}_{-\text{fn}}(E, Stm) \]

that represents the environment and the statement. For convenience, we shall write actions just like judgements with the initial and final state omitted. In this case, the action is written as

\[ E \vdash_{stm} Stm \Rightarrow \]

Given an action \( A \), it is meaningful to talk about the effect of executing the action in a given state \( S_0 \). This is expressed by the predicate \( \text{run} \):

\[ \text{run}(S_0, A, S_1) \]

holds if and only if the execution of action \( A \) in state \( S_0 \) results in state \( S_1 \).

Using \( \text{run} \) it is possible to define different orders of evaluation of sets and sequences of actions. The notation

\[ S_0 \left[ \begin{array}{c} A_1 \\ \ldots \\ A_n \end{array} \right] S_1 \]

means that the actions \( A_1 \) through \( A_n \) are to be executed sequentially starting in state \( S_0 \) and leading to \( S_1 \). The definition of this notation needs to consider the propagation of exceptions by any of the actions.

Similarly, the notation

\[ S_0 \left\{ \begin{array}{c} A_1 \\ \ldots \\ A_n \end{array} \right\} S_1 \]

means that the actions \( A_1 \) through \( A_n \) are to be executed in arbitrary order.

The set of actions (sort \( \text{Action} \)) is given by the following terms. There is one action constructor for each judgement.
type Action

then : list(Action) → Action

new_object_fn : Value × Object_location → Action

attribute_fn : Environment × View × Id × View → Action

param_attribute_fn : Environment × View × Id × Value × View → Action

case_alternative_fn : Environment × Value × Alt × Bool → Action

choices_fn : Environment × Dch_lst × Choice → Action

component_list_fn : Environment

Discriminantx

Cmp_lst×

(Id = (Partial_subtype × optional(Thunk))) → Action

constraint_fn : Environment × Subtype × Cns × Constraint → Action

compatible_fn : Environment × Subtype × Constraint → Action

declaration_fn : Environment × Dcl × Environment → Action

default_value_fn : Environment × Subtype × Value → Action

discr_assoc_fn : Environment × Subtype × Dca × Constraint → Action

discriminant_part_fn : Environment × Dcp × Discriminant → Action

eelsif_clause_fn : Environment × Eif × Bool → Action

expression_fn : Environment × Exp × Value → Action

ame_fn : Environment × Nam × View → Action

new_type_fn : (optional(Type_location) × Type) × Type_location → Action

range_fn : Environment × Rug × Range → Action

subtype_indication_fn : Environment × Sid × Subtype → Action

p_subtype_indication_fn : Environment × Discriminant × Sid × Partial_subtype → Action

type_definition_fn : Environment × Discriminant × Tdf × Subtype × Environment → Action

variant_list_fn : Environment × Discriminant × Vnt_lst × Variant → Action

variant_part_fn : Environment × Discriminant × Vrp × optional(Id × Variant) → Action

subtype_convert_fn : Environment × Subtype × Value × Value → Action

view_convert_fn : Environment × Subtype × View × View → Action

content_fn : LValue × Value → Action

assign_fn : Environment × Subtype × LValue × Value → Action

raw_assign_fn : LValue × Value → Action

finalize_fn : Environment × Subtype × Value → Action

value_split_fn : Environment × Subtype × Value → Action

discrete_type_fn : Environment × Subtype × Bool → Action

covers_fn : Value × Choice × Bool → Action

statement_fn : Environment × Stm → Action

arb_fn : list(Action) → Action

subprogram_body_fn : (Id = View) × View → Action

The following predicates define a sequence of actions for a variety of different syntactic constructs:

component_actions( Environment, CompAssoc, list(Rca), list((Id × Value)), list(Action))

expression_list( Environment, list(Exp), list(Value), list(Action))

index_actions( Environment, list(Sid), (Subtype), list(Action))

parameter_action( Environment, list(Pss), (Id × Parameter), (Id × View), Action)

parameter_list( Environment, list(Pss), (Id = Parameter), (Id = View), list(Action))

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4.4 State

Values of sort State describe a current store, together with the current status of program execution. A state may represent the propagation of an exception, exit from a subprogram, or exit from a loop.

4.4.1 Classification of States

States are classified as normal or abnormal. The following judgements define the classification.

\[
\begin{align*}
\text{abnormal\_state}(\text{exception}(I_d, N)) \\
\text{abnormal\_state}(\text{exit}(I_d, N)) \\
\text{abnormal\_state}(\text{proc\_return}(N)) \\
\text{abnormal\_state}(\text{func\_return}(W, N)) \\
\text{normal\_state}(\text{normal}(N))
\end{align*}
\]

4.4.2 Accessing the Store of a State

The following judgements describe the store associated with different states:

\[
\begin{align*}
\text{the\_store}(\text{exception}(I_d, N)) &= N \\
\text{the\_store}(\text{exit}(I_d, N)) &= N \\
\text{the\_store}(\text{proc\_return}(N)) &= N \\
\text{the\_store}(\text{func\_return}(W, N)) &= N \\
\text{the\_store}(\text{normal}(N)) &= N
\end{align*}
\]

The following judgements describe the construction of states:

\[
\begin{align*}
\text{make\_state}(\text{exception}(I_d, N), N_1) &= \text{exception}(I_d, N_1) \\
\text{make\_state}(\text{exit}(I_d, N), N_1) &= \text{exit}(I_d, N_1) \\
\text{make\_state}(\text{proc\_return}(N), N_1) &= \text{proc\_return}(N_1) \\
\text{make\_state}(\text{func\_return}(W, N), N_1) &= \text{func\_return}(W, N_1) \\
\text{make\_state}(\text{normal}(N), N_1) &= \text{normal}(N_1)
\end{align*}
\]
4.4.3 Reading and Writing the Store

\[
\begin{align*}
\text{the\_store}(S) &= \langle B, ?, ?, ? \rangle \\
S^{\text{obj}}[I] &= B[I] \\
\text{the\_store}(S) &= \langle ?, B, ?, ? \rangle \\
S^{\text{typ}}[I] &= B[I] \\
\text{the\_store}(S) &= \langle ?, ?, B, ? \rangle \\
S^{\text{typ}}[I] &= B[I] \\
\end{align*}
\]

\[
S_1[I \mapsto V] = \text{make\_state}(S_1, \text{the\_store}(S_1)[I \mapsto V])
\]

\[
S_1[I \mapsto V] = \text{make\_state}(S_1, \text{the\_store}(S_1)[I \mapsto V])
\]

\[
S_1[I \mapsto V] = \text{make\_state}(S_1, \text{the\_store}(S_1)[I \mapsto V])
\]

\[
S_1[I \mapsto V] = \text{make\_state}(S_1, \text{the\_store}(S_1)[I \mapsto V])
\]

\[
\begin{align*}
\langle B_0, B_p, B_t, B_s \rangle[I \mapsto V] &= \langle B_0[I \mapsto V], B_p, B_t, B_s \rangle \\
\langle B_0, B_0, B_t, B_s \rangle[I \mapsto V] &= \langle B_0, B_0[I \mapsto V], B_t, B_s \rangle \\
\langle B_0, B_p, B_0, B_s \rangle[I \mapsto V] &= \langle B_0, B_p[B_0[I \mapsto V]], B_s \rangle \\
\langle B_0, B_p, B_t, B_0 \rangle[I \mapsto V] &= \langle B_0, B_p[B_t, B_0[I \mapsto V]] \rangle
\end{align*}
\]

4.4.4 The Content of a Location

This function differs from stored\_value because it works on a state rather than a store and because it allows L-values rather than just locations. In the case of a structured L-value, the appropriate component of a compound object will be returned.

\[
S \vdash \text{content}(\text{location}(I)) \Rightarrow S^{\text{obj}}[I]
\]

\[
S \vdash \text{content}(L_v) \Rightarrow \text{array\_val}(R_v, B) \\
S \vdash \text{content}(\text{array\_component}(L_v, V_z)) \Rightarrow B[V_z]
\]

\[
S \vdash \text{content}(L_v) \Rightarrow \text{array\_val}(\{ R_v \}, B) \\
S \vdash \text{content}(\text{array\_slice}(L_v, R_v)) \Rightarrow \text{array\_val}(\{ R_v \}, B)
\]

\[
S \vdash \text{content}(L_v) \Rightarrow \text{record\_val}(T_v, D_v, C_i) \\
S \vdash \text{content}(\text{record\_component}(L_v, I_d)) \Rightarrow C_i[I_d]
\]

\[
\text{the\_store}(S_1) = \langle B, ?, ?, ? \rangle \\
\neg I \in \text{dom}(B) \\
\text{new\_object}(S_1, V, I, S_1[I \mapsto V])
\]

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\[
\text{the\_store}(S_1) = (?, B, ?, ?) \\
\text{new\_subprogram}(S_1, V, I, S_1[I \mapsto V])
\]

\[
\text{the\_store}(S_0[I \mapsto T_e]) = (?, ?, B, ?) \\
\text{new\_type}(S_0, T_e, U, S_0[I \mapsto T_e])
\]

\[
\text{the\_store}(S_0[I \mapsto T_e]) = (?, ?, ?, B) \\
\text{new\_subtype}(S_0, T_e, U, S_0[I \mapsto T_e])
\]

### 4.5 Order of Execution

#### 4.5.1 Sequential Execution

\[
S \{ S \}
\]

\[
\text{run}\left(\text{normal}(N), A_0, S_2\right) \\
S_2 \begin{bmatrix} A_1 & \ldots & A_n \end{bmatrix} S_3
\]

\[
\text{normal}(N) \begin{bmatrix} A_1 & \ldots & A_n \end{bmatrix} S_3
\]

\[
\text{abnormal\_state}(S) \\
S \begin{bmatrix} A_1 & \ldots & A_n \end{bmatrix} S
\]

#### 4.5.2 Arbitrary Order Execution

\[
S \{ S \}
\]

\[
\text{abnormal\_state}(S) \\
S \begin{bmatrix} A_1 \\ \ldots \\ A_n \end{bmatrix} S
\]

\[
\text{pick}(A, A_1, A_1) \\
\text{run}(\text{normal}(N), A_1, S_1) \\
S_1 \begin{bmatrix} A_{11} & \ldots & A_{1n} \end{bmatrix} S_2
\]

\[
\text{normal}(N) \begin{bmatrix} A_1 \\ \ldots \\ A_n \end{bmatrix} S_2
\]
pick(A \cdot A_s, A, A_s)

\[ \frac{pick(A_s, A_x, A_t)}{pick(A \cdot A_s, A_z, A \cdot A_t)} \]

**4.5.3 Executing Individual Actions**

Note that there are predicates that do not involve state. They are included as a matter of convenience. It is possible to include the appropriate terms in a sequential or arbitrary order execution where this makes the definition more readable.

\[
S_1 \begin{bmatrix} A_1 \\ \vdots \\ A_n \end{bmatrix} S_2 \\
\text{run}(S_1, \text{then}(A), S_2)
\]

\[
new\_object(S_1, V, L, S_2) \\
\text{run}(S_1, new\_object\_fn(V, L), S_2)
\]

\[
S_1, E, V \vdash \text{clt} \text{ Clt} \Rightarrow B, S_2 \\
\text{run}(S_1, E, V \vdash \text{clt} \text{ Clt} \Rightarrow B, S_2)
\]

\[
S_1, E \vdash \text{chc} \text{ Chc} \Rightarrow C, S_2 \\
\text{run}(S_1, E \vdash \text{chc} \text{ Chc} \Rightarrow C, S_2)
\]

\[
S_1, E \vdash \text{cmp} \text{ D} \Rightarrow \text{Cmp}, A \\
\text{run}(S_1, E \vdash \text{cmp} \text{ D} \Rightarrow \text{Cmp}, S_2)
\]

\[
S_1, E_1 \vdash \text{del} \text{ Dcl} \Rightarrow E_2, S_2 \\
\text{run}(S_1, E_1 \vdash \text{del} \text{ Dcl} \Rightarrow E_2, S_2)
\]

\[
S_1, E \vdash \text{elf} \text{ Elf} \Rightarrow R, S_2 \\
\text{run}(S_1, E \vdash \text{elf} \text{ Elf} \Rightarrow R, S_2)
\]

\[
S_1, E \vdash \text{exp} \text{ Exp} \Rightarrow V, S_2 \\
\text{run}(S_1, E \vdash \text{exp} \text{ Exp} \Rightarrow V, S_2)
\]

\[
S_1, E \vdash \text{nam} \text{ Nam} \Rightarrow D, S_2 \\
\text{run}(S_1, E \vdash \text{nam} \text{ Nam} \Rightarrow D, S_2)
\]

\[
S_1, E \vdash \text{rng} \text{ Rng} \Rightarrow R, S_2 \\
\text{run}(S_1, E \vdash \text{rng} \text{ Rng} \Rightarrow R, S_2)
\]

\[
S_1, E \vdash \text{stm} \text{ Stm} \Rightarrow S_2 \\
\text{run}(S_1, E \vdash \text{stm} \text{ Stm} \Rightarrow S_2)
\]

\[
S_1, E_1, D \vdash \text{tdf} \text{ Tdf} \Rightarrow S_1, E_2, S_2 \\
\text{run}(S_1, E_1, D \vdash \text{tdf} \text{ Tdf} \Rightarrow S_1, E_2, S_2)
\]

\[
S_1, E, S_t \vdash \text{subtype}\_convert(V_1) \Rightarrow V_2, S_2 \\
\text{run}(S_1, E, S_t \vdash \text{subtype}\_convert(V_1) \Rightarrow V_2, S_2)
\]

\[
S_1, E, S_t \vdash \text{subtype}\_convert(W_1) \Rightarrow W_2, S_2 \\
\text{run}(S_1, E, S_t \vdash \text{subtype}\_convert(W_1) \Rightarrow W_2, S_2)
\]

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\[ S \vdash \text{content}(L_v) \Rightarrow V \]

\[
\text{run}(S, \vdash \text{content}(L_v) \Rightarrow V, S)
\]

\[
S_1 \left\{ \begin{array}{c}
A_1 \\
\ldots \\
A_n
\end{array} \right\} S_2
\]

\[
\text{run}(S_1, \left\{ \begin{array}{c}
A_1 \\
\ldots \\
A_n
\end{array} \right\}, S_2)
\]

\[
\text{subprogram\_body}(S_1, A, W, S_2)
\]

\[
\text{run}(S_1, \text{subprogram\_body\_fn}(A, W), S_2)
\]

### 4.6 Values

#### 4.6.1 Ranges

\[
\text{discrete\_range}(\text{discrete\_rng}(I_1, I_2))
\]

\[
\text{real\_range}(\text{real\_rng}(R_1, R_2))
\]

\[
\text{low\_bound}(\text{discrete\_rng}(I, ?)) = \text{discrete\_val}(I)
\]

\[
\text{low\_bound}(\text{real\_rng}(R, ?)) = \text{real\_val}(R)
\]

\[
\text{high\_bound}(\text{discrete\_rng}(?, I)) = \text{discrete\_val}(I)
\]

\[
\text{high\_bound}(\text{real\_rng}(?, R)) = \text{real\_val}(R)
\]

\[
\text{make\_range}(\text{discrete\_val}(I_1), \text{discrete\_val}(I_2)) = \text{discrete\_rng}(I_1, I_2)
\]

\[
\text{make\_range}(\text{real\_val}(R_1), \text{real\_val}(R_2)) = \text{real\_rng}(R_1, R_2)
\]

\[
\begin{align*}
R_1 & \leq R \\
R & \leq R_2
\end{align*}
\]

\[
\text{belongs\_to}(\text{real\_val}(R), \text{real\_rng}(R_1, R_2))
\]

\[
\begin{align*}
I_1 & \leq I \\
I & \leq I_2
\end{align*}
\]

\[
\text{belongs\_to}(\text{discrete\_val}(I), \text{discrete\_rng}(I_1, I_2))
\]

\[
\begin{align*}
I_1 & \geq I_3 \\
I_4 & \geq I_2
\end{align*}
\]

\[
\text{included\_in}(\text{discrete\_rng}(I_1, I_2), \text{discrete\_rng}(I_3, I_4))
\]

\[
\begin{align*}
I_1 & > I_2
\end{align*}
\]

\[
\text{null\_range}(\text{discrete\_rng}(I_1, I_2))
\]

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\[
\begin{align*}
R_1 > R_2 & \quad \frac{\text{null_range(real_rng(R_1, R_2))}}{} \\
I_1 > I_2 & \quad \frac{\text{values_in_range(discrete_rng(I_1, I), set_of([discrete_val(I)]))}}{} \end{align*}
\]

\[
\begin{align*}
I_1 > I_2 & \quad \frac{\text{values_in_range(discrete_rng(I_1, I), set_of([])})}{}}{} \\
I_1 > I_2 & \quad \frac{\text{values_in_range(discrete_rng(I_1 + 1, I), set_of(V_s))}}{} \\
I_1 > I_2 & \quad \frac{\text{values_in_range(discrete_rng(I_1, I), set_of(discrete_val(I) \cdot V_s))}}{} \\
\end{align*}
\]

4.6.2 Index Ranges

\[
\begin{align*}
\text{indices([])} & = \text{set_of([])} \\
\text{values_in_range}(R, V_r) & \quad \frac{\text{index_pairing}(V_r, \text{indices}(R), V)}{}}{} \\
\text{indices}(R \cdot R_s) & = V \\
\text{index_pairing}(\text{set_of([]), V_1}, \text{set_of([])}) & = \text{set_of([])} \\
\text{index_pairing}(\text{set_of(V_r), V_1, V_2}) & \quad \frac{\text{prefix_set_with_element}(V, V_1, V_s)}{}}{} \\
\text{prefix_set_with_element}(V, \text{set_of([]), set_of([])}) & = \text{set_of([])} \\
\end{align*}
\]

4.6.3 Predicates of Values

\[
\begin{align*}
\text{discrete_value}(\text{discrete_val}(X)) & \quad \frac{\text{real_value}(\text{real_val}(X))}{}}{} \\
\text{access_value}(\text{access_val}(X, ?)) & = \text{scalar_value}(V) \\
\end{align*}
\]

\[
\begin{align*}
\text{real_value}(V) & \quad \frac{\text{scalar_value}(V)}{}}{} \\
\text{scalar_value}(V) & \quad \frac{\text{elementary_value}(V)}{}}{} \\
\end{align*}
\]
access_value(V) 

\[
\begin{align*}
\text{elementary_value}(V) \\
\text{composite_value}(\text{record_val}(X, Y, Z)) \\
\text{composite_value}(\text{array_val}(X, Y)) \\
\text{dom}(V) = \text{indices}(B) \\
\text{array_value}(\text{array_val}(B, V))
\end{align*}
\]

4.7 Types

4.7.1 Type Descriptors

\[
S^{\text{typ}}[U] = (? \times T_y) \\
\text{type_struct}(S, \text{subtype}(U, ?, ?)) = T_y
\]

4.7.2 Ancestry Relation

\[
\text{ancestor}(S, U, U) \\
S^{\text{typ}}[U] = (\text{some}(U_2) \times ?) \\
\text{ancestor}(S, U_2, U_1) \\
\text{ancestor}(S, U, U_1) \\
\text{descendant}(S, U_1, U_2) \\
\text{ancestor}(S, U, U) \\
S^{\text{typ}}[U] = (\text{nonex}?) \\
\text{ultimate_ancestor}(S, U, U) \\
S^{\text{typ}}[U] = (\text{some}(U_2) \times ?) \\
\text{ultimate_ancestor}(S, U_1, U_2) \\
\text{ultimate_ancestor}(S, U, U_2)
\]

4.7.3 Ranges of Scalar Types

\[
\text{base_range}(S, \text{enum_type}(N)) = \text{discrete_rng}(0, N)
\]

\[
\text{base_range}(S, \text{modular_type}(N)) = \text{discrete_rng}(0, N, -, 1)
\]

\[
\text{base_range}(S, \text{signed_integer_type}(B_f, B_t, ?, ?)) = \text{discrete_rng}(B_f, B_t)
\]

\[
\text{base_range}(S, \text{universal_integer_type}(B_f, B_t)) = \text{discrete_rng}(B_f, B_t)
\]

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\[
S_t = \text{subtype}(U, \text{no\_constraint}, A)
\]

\[
\text{range\_of\_subtype}(S, S_t) = \text{base\_range}(S, \text{type\_struct}(S, S_t))
\]

\[
\text{range\_of\_subtype}(S, \text{subtype}(U, \text{range\_constraint}(R), A)) = R
\]

\[
\text{ranges\_of\_subtypes}(S, []) = []
\]

\[
\text{ranges\_of\_subtypes}(S, S_t, S_r) = \text{range\_of\_subtype}(S, S_t) \cdot \text{ranges\_of\_subtypes}(S, S_r)
\]

### 4.7.4 Values of a Type

\[
\text{ancestor}(S, U, U_p)
\]

\[
S^{\text{type}}[U_p] = (? \times T_v)
\]

\[
\text{descriptor\_value}(S, T_v, V)
\]

\[
\text{descriptor\_value}(S, \text{class\_type}(U), V)
\]

\[
\text{belong\_to}(V, \text{base\_range}(S, \text{enum\_type}(N)))
\]

\[
\text{descriptor\_value}(S, \text{enum\_type}(N), V)
\]

\[
\text{descriptor\_value}(S, \text{enum\_type}(N), \text{invalid\_val})
\]

\[
\text{belong\_to}(V, \text{base\_range}(S, \text{modular\_type}(N)))
\]

\[
\text{descriptor\_value}(S, \text{modular\_type}(N), V)
\]

\[
\text{descriptor\_value}(S, \text{modular\_type}(N), \text{invalid\_val})
\]

\[
\text{descriptor\_value}(S, \text{signed\_integer\_type}(B_f, B_l, F, L), \text{discrete\_val}(N))
\]

\[
\text{descriptor\_value}(S, \text{signed\_integer\_type}(B_f, B_l, F, L), \text{invalid\_val})
\]

\[
\text{descriptor\_value}(S, \text{universal\_integer\_type}(B_f, B_l), \text{discrete\_val}(N))
\]

\[
\text{descriptor\_value}(S, \text{universal\_integer\_type}(B_f, B_l), \text{invalid\_val})
\]

\[
\text{array\_value}(\text{array\_val}(\text{ranges\_of\_subtypes}(S, I), B))
\]

\[
\text{descriptor\_value}(S, \text{array\_type}(I, C), \text{array\_val}(\text{ranges\_of\_subtypes}(S, I), B))
\]

\[
N \in \text{set\_of}\text{\_adom}(D))
\]

\[
D[N] = (S_t \times T_h)
\]

\[
\text{subtype\_value}(S, S_t, B[N])
\]

\[
\text{discriminant\_value}(S, \text{discr}(D), B)
\]

\[
\text{discriminant\_value}(S, D_a, D_v)
\]

\[
\text{cl\_value}(S, C_i, D_v, C_v)
\]

\[
\text{descriptor\_value}(S, \text{record\_type}(T_g, D_a, C_i), \text{record\_val}(T_g, D_v, C_v))
\]

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\[ \text{dom}(F) = \text{set_of}(\text{dom}(A)) \]
\[ A[I_d] = (P_e \times T_t) \]
\[ \text{subtype_value}(S, \text{actualize}(D_v, P_e), F[I_d]) \]
\[ \text{variant_values}(S, V_r, D_v, F_v) \]
\[ \text{cl_value}(S, \text{fields}(A, V_r), D_v, F_v) \]
\[ \text{variant_values}(S, \text{none}, D_v, \{\}) \]
\[ \text{the_variant}(F, D_v[I_d], C_t) \]
\[ \text{cl_value}(S, C_t, D_v, F_v) \]
\[ \text{variant_values}(S, \text{some}((I_d \times F)), D_v, V_v) \]

### 4.7.5 Record Fields

The following definitions are useful to deal with the types of record fields. Binding \( B \) represented the values of the discriminants.

\[ D_{a}[I_d] = (S_l \times T_l) \]
\[ \text{component_type}(B, \text{record_type}(T_g, \text{discr}(D_a), C_t), I_d, S_l) \]
\[ \text{select_component_type}(B, C_t, I_d, S_l) \]
\[ C_a[I_d] = (P_e \times T_t) \]
\[ \text{select_component_type}(B, \text{fields}(C_a, ?), I_d, \text{actualize}(B, P_e)) \]

Since component names have to be unique, we can quantify over values \( V \) in the following rule.

\[ \text{the_variant}(V_b, V, C_t) \]
\[ \text{select_component_type}(B, C_t, I_d, S_l) \]
\[ \text{select_component_type}(B, \text{fields}(C_a, \text{some}((I_v \times V_b))), I_d, S_l) \]
\[ \text{covers}(X, C, \text{true}) \]
\[ \text{the_variant}(\text{variant}((C \times C_t) \cdot L), X, C_t) \]
\[ \text{covers}(X, C, \text{false}) \]
\[ \text{the_variant}(\text{variant}(L), X, C_t) \]

### 4.7.6 Classification of Types

\[ \text{is_scalar_type}(S, T_y) \]
\[ \text{is_elementary_type}(S, T_y) \]
\[ \text{is_access_type}(S, T_y) \]
\[ \text{is_elementary_type}(S, T_y) \]
\[ \text{is_array_type}(S, T_y) \]
\[ \text{is_composite_type}(S, T_y) \]
\[
\begin{align*}
\text{is_record_type}(S, T_v) & \quad \frac{\phantom{\text{is_composite_type}(S, T_y)}}{\text{is_composite_type}(S, T_y)} \\
\text{is_tagged_type}(S, T_y) & \quad \frac{\phantom{\text{is_composite_type}(S, T_y)}}{\text{is_composite_type}(S, T_y)} \\
\text{is_protected_type}(S, T_v) & \quad \frac{\phantom{\text{is_composite_type}(S, T_y)}}{\text{is_composite_type}(S, T_y)} \\
\text{is_discrete_type}(S, T_y) & \quad \frac{\phantom{\text{is_composite_type}(S, T_y)}}{\text{is_composite_type}(S, T_y)} \\
\text{is_real_type}(S, T_y) & \quad \frac{\phantom{\text{is_composite_type}(S, T_y)}}{\text{is_composite_type}(S, T_y)} \\
\text{is_scalar_type}(S, T_y) & \quad \frac{\phantom{\text{is_composite_type}(S, T_y)}}{\text{is_composite_type}(S, T_y)} \\
\text{is_access_to_object_type}(S, T_v) & \quad \frac{\phantom{\text{is_access_type}(S, T_y)}}{\text{is_access_type}(S, T_y)} \\
\text{is_access_to_subprogram_type}(S, T_y) & \quad \frac{\phantom{\text{is_access_type}(S, T_y)}}{\text{is_access_type}(S, T_y)} \\
\text{is_enumeration_type}(S, T_y) & \quad \frac{\phantom{\text{is_access_type}(S, T_y)}}{\text{is_discrete_type}(S, T_y)} \\
\text{is_integer_type}(S, T_y) & \quad \frac{\phantom{\text{is_access_type}(S, T_y)}}{\text{is_discrete_type}(S, T_y)} \\
\text{is_modular_integer_type}(S, T_y) & \quad \frac{\phantom{\text{is_access_type}(S, T_y)}}{\text{is_discrete_type}(S, T_y)} \\
\text{type_struct}(S, T_y) = \text{enum_type}(N) & \quad \frac{\phantom{\text{is_integ}_{\text{er_type}(S, T_y)}}}{\text{is_enumeration_type}(S, T_y)} \\
\text{is_boolean_type}(S, \text{subtype}(\text{boolean_tn}, ?, ?)) & \quad \frac{\phantom{\text{is_integ}_{\text{er_type}(S, T_y)}}}{\text{is_character_type}(S, \text{subtype}(\text{character_tn}, ?, ?))} \\
\text{is_character_type}(S, \text{subtype}(\text{integer_tn}, ?, ?)) & \quad \frac{\phantom{\text{is_integ}_{\text{er_type}(S, T_y)}}}{\text{is_signed_integer_type}(S, T_y)} \\
\text{is_integer_type}(S, T_y) & \quad \frac{\phantom{\text{is_integ}_{\text{er_type}(S, T_y)}}}{\text{is_modular_integer_type}(S, T_y)} \\
\text{is_integ}_{\text{er_type}(S, T_y)} & \quad \frac{\phantom{\text{is_integ}_{\text{er_type}(S, T_y)}}}{\text{is_signed_integer_type}(S, \text{subtype}(\text{root_integer_tn}, ?, ?))} \\
\text{type_struct}(S, T_y) = \text{modular_type}(?) & \quad \frac{\phantom{\text{is_integ}_{\text{er_type}(S, T_y)}}}{\text{is_modular_integer_type}(S, T_y)} \\
\text{is_real_type}(S, T_y) & \quad \frac{\phantom{\text{is_integ}_{\text{er_type}(S, T_y)}}}{\text{not yet defined}} \\
\text{type_struct}(S, T_y) = \text{access_type}(S_t, A_m) & \quad \frac{\phantom{\text{is_integ}_{\text{er_type}(S, T_y)}}}{\text{is_access_to_object_type}(S, T_y)} \\
\text{type_struct}(S, T_y) = \text{func_profile}(P_a, S_t) & \quad \frac{\phantom{\text{is_integ}_{\text{er_type}(S, T_y)}}}{\text{is_access_to_subprogram_type}(S, T_y)}
\end{align*}
\]
type\_struct(S, T_y) = proc\_profile(P_a)
is\_access\_to\_subprogram\_type(S, T_y)

type\_struct(S, T_y) = array\_type(C, B)
is\_array\_type(S, T_y)

not yet defined
is\_string\_type(S, T_y)

type\_struct(S, T_y) = record\_type(T_x, D, C)
is\_record\_type(S, T_y)

type\_struct(S, T_y) = record\_type(some(I_t), D, C)
is\_tagged\_type(S, T_y)

not yet defined
is\_task\_type(S, T_y)

not yet defined
is\_protected\_type(S, T_y)

is\_elementary\_type(S, T_y)
is\_by\_copy\_type(S, T_y)

is\_tagged\_type(S, T_y)
is\_by\_reference\_type(S, T_y)

is\_task\_type(S, T_y)
is\_by\_reference\_type(S, T_y)

is\_protected\_type(S, T_y)
is\_by\_reference\_type(S, T_y)

4.8 Subtypes

4.8.1 Constraint Satisfaction

satisfies(V, no\_constraint)

belongs\_to(V, R)
satisfies(V, range\_constraint(R))

satisfies(array\_val(B, A), index\_constraint(B))

satisfies(record\_val(T_x, D, R), discriminant\_constraint(D))
4.8.2 Values of a Subtype

\[ S'_{typ}[U] = (P \times T_y) \]
\[ \text{descriptor}_\text{value}(S, T_y, V) \]
\[ \text{satisfies}(V, C) \]
\[ \text{subtype}_\text{value}(S, \text{subtype}(U, C, A), V) \]

4.8.3 Actualization

4.8.3.1 Values

\[ \text{actualized}_\text{value}(B, p\text{-value}(V)) = V \]
\[ \text{actualized}_\text{value}(B, \text{discriminant}_\text{ref}(I)) = B[I] \]

4.8.3.2 Ranges

\[ \text{actualized}_\text{value}(B, L) = \text{discrete}_\text{val}(L_a) \]
\[ \text{actualized}_\text{value}(B, H) = \text{discrete}_\text{val}(H_a) \]
\[ \text{actualized}_\text{partial}_\text{range}(B, (L \times H)) = \text{discrete}_\text{rng}(L_a, H_a) \]
\[ \text{actualized}_\text{range}_\text{list}(B, [ ]) = [ ] \]
\[ \text{actualized}_\text{range}_\text{list}(B, R \cdot R_s) = \text{actualized}_\text{partial}_\text{range}(B, R) \cdot \text{actualized}_\text{range}_\text{list}(B, R_s) \]

4.8.3.3 Binding Lists

\[ \text{actualized}_\text{binding}_\text{list}(B, [ ]) = [ ] \]
\[ \text{actualized}_\text{binding}_\text{list}(B, (I \times V) \cdot R) = (I \times \text{actualized}_\text{value}(B, V)) \cdot \text{actualized}_\text{binding}_\text{list}(B, R) \]

4.8.3.4 Constraints

\[ \text{actualized}_\text{constraint}(B, p\text{-no\_constraint}) = \text{no\_constraint} \]
\[ \text{actualized}_\text{constraint}(B, p\text{-range\_constraint}(R)) = \text{range\_constraint}(\text{actualized}_\text{partial\_range}(B, R)) \]
\[ \text{actualized}_\text{constraint}(B, p\text{-index\_constraint}(S)) = \text{index\_constraint}(\text{actualized}_\text{range\_list}(B, S)) \]

\[ C = \text{discriminant\_constraint}(\text{actualized}_\text{binding\_list}(B, S)) \]
\[ \text{actualized}_\text{constraint}(B, p\text{-discriminant\_constraint}(S)) = C \]

\[ C = \text{indirect\_index\_constraint}(\text{actualized}_\text{range\_list}(B, S)) \]
\[ \text{actualized}_\text{constraint}(B, p\text{-indirect\_index\_constraint}(S)) = C \]

\[ C = \text{indirect\_discriminant\_constraint}(\text{actualized}_\text{binding\_list}(B, S)) \]
\[ \text{actualized}_\text{constraint}(B, p\text{-indirect\_discriminant\_constraint}(S)) = C \]
4.8.3.5 Subtypes

actualize\(B, p_{\text{subtype}}(\text{subtype}(U, C, A), P_c)) = \text{subtype}(U, \text{actualized\_constraint}(B, P_c), A)\)

4.8.3.6 Components

\[A_1 + A_2 = A_1 + A_2\]

\[\text{actualized\_components}(B, \text{fields}(C_a, \text{some}(V_p))) = \text{actualized\_complist}(B, C_a) + \text{actualized\_variants}(B, V_p)\]

\[\text{actualized\_components}(B, \text{fields}(C_a, \text{none})) = \text{actualized\_complist}(B, C_a)\]

4.8.3.7 Component Lists

\[\text{actualized\_complist}(B, []) = []\]

\[\text{actualized\_complist}(B, A_0) \rightarrow \text{actualized\_complist}(B, (I \times P_1) \cdot A) = (I \times \text{actualize}(B, P_1)) \cdot A_0\]

\[\text{the\_variant}(V_a, B[I], C_l) \rightarrow \text{actualized\_variants}(B, (I \times V_a)) = \text{actualized\_components}(B, C_l)\]

4.9 Declarations

4.9.1 Declarations

4.9.2 Types and Subtypes

4.9.2.1 Type Declarations

The semantics of a type definition are determined in the context of a discriminant association. For types without discriminant, this association is empty.

\[S_1, E_1, () \vdash_{\text{tdf}} \text{Tdf} \Rightarrow S_t, E_2, S_2\]

\[\text{new\_subtype}(S_2, S_t, L_s, S_3)\]

\[S_1, E_1 \vdash_{\text{dcl}} \text{type } l_d \text{ is Tdf}; \Rightarrow E_2[l_d \mapsto \text{subtype\_view}(L_s)], S_3\]

\[S_1 \left[\begin{array}{c}
E \vdash_{\text{dcl}} \text{Dcp} \Rightarrow D \\
E, D \vdash_{\text{tdf}} \text{Tdf} \Rightarrow S_t, E_2
\end{array}\right] S_2\]

\[\text{new\_subtype}(S_2, S_1, L_s, S_3)\]

\[S_1, E \vdash_{\text{dcl}} \text{type } l_d \text{ Dcp is Tdf}; \Rightarrow E_2[l_d \mapsto \text{subtype\_view}(L_s)], S_3\]

For a given type descriptor, the following rule creates a new unique type name and constructs a first subtype.
4.9.2.2 Subtype Declarations

A subtype indication may be a named subtype or a subtype with a constraint. In the former case, evaluation of the subtype indication cannot have side-effects.

\[
S_1, E \vdash _{\text{nam}} \text{Nam} \Rightarrow \text{subtype\_view}(L_4), S_1
\]

\[
S_1, E \vdash _{\text{sid}} \text{Nam} \Rightarrow S_1^{\text{stp}}[L_4], S_1
\]

\[
S_1, E \vdash _{\text{nam}} \text{Nam} \Rightarrow \text{subtype\_view}(L_4), S_1
\]

\[
S_1, E, S_1^{\text{stp}}[L_4] \vdash \text{cns}, \text{Cns} \Rightarrow C, S_2
\]

\[
\text{compatible}(E, S_1^{\text{stp}}[L_4], C)
\]

\[
S_1^{\text{stp}}[L_4] = \text{subtype}(U, C_1, A)
\]

\[
S_1, E \vdash _{\text{sid}} \text{Nam} \Rightarrow \text{subtype\_view}(U, C, A), S_2
\]

\[
S_1, E \vdash _{\text{nam}} \text{Nam} \Rightarrow \text{subtype\_view}(L_4), S_1
\]

\[
S_1, E, D \vdash _{\text{psn}} \text{Nam} \Rightarrow p\text{-subtype}(S_1^{\text{stp}}[L_4],\text{p\_no\_constraint}), S_1
\]

\[
S_1, E \vdash _{\text{nam}} \text{Nam} \Rightarrow \text{subtype\_view}(L_4), S_1
\]

\[
p\_constraint(S_1, E, D, S_1^{\text{stp}}[L_4], \text{Cns}, P, S_2)
\]

\[
\text{compatible}(E, S_1^{\text{stp}}[L_4], C)
\]

\[
S_1^{\text{stp}}[L_4] = \text{subtype}(U, C_1, A)
\]

\[
S_1, E \vdash _{\text{sid}} \text{Nam} \Rightarrow \text{subtype\_view}(U, C, A), S_2
\]

\[
S_1, E \vdash _{\text{nam}} \text{Nam} \Rightarrow \text{subtype\_view}(L_4), S_1
\]

\[
S_1, E_1 \vdash _{\text{sid}} \text{Sid} \Rightarrow S_1, S_2
\]

\[
\text{new\_subtype}(S_2, S_1, L_1, L_3)
\]

\[
S_1, E_1 \vdash _{\text{decl}} \text{subtype}_{\text{id}} \Rightarrow \text{Sid}_1 \Rightarrow E_1[I_d \mapsto \text{subtype\_view}(L_4), S_3]
\]

The following syntax represents ranges in a discrete subtype definition. It applies only to constrained array type definitions.

\[
\text{not yet defined}
\]

\[
S_1, E \vdash _{\text{sid}} \text{Rng} \Rightarrow \text{subtype}(U, C, A), S_2
\]

\[
S_1, E_1 \vdash _{\text{sid}} \text{Sid} \Rightarrow S_1, S_2
\]

\[
\text{new\_subtype}(S_2, S_1, L_1, S_3)
\]

\[
S_1, E_1 \vdash _{\text{decl}} \text{subtype}_{\text{id}} \Rightarrow \text{Sid}_1 \Rightarrow E_1[I_d \mapsto \text{subtype\_view}(L_4), S_3]
\]

4.9.3 Objects and Named Numbers

4.9.3.1 Object Declarations

\[
S_1 \begin{cases}
E_1 \vdash _{\text{sid}} \text{Sid} \Rightarrow S_1 \\
\text{new\_object\_fn}(\text{invalid\_val}, L) \\
\text{default\_value\_fn}(E_1, S_1, V) \\
\text{assign\_fn}(E_1, S_1, \text{location}(L), V)
\end{cases} \Rightarrow S_2
\]

\[
S_1, E_1 \vdash _{\text{decl}} \text{Id} : \text{Sid}; \Rightarrow E_1[I_d \mapsto \text{object\_view(location}(L), S_1, \text{variable})], S_2
\]

\[
S_1 \begin{cases}
E_1 \vdash _{\text{sid}} \text{Sid} \Rightarrow S_1 \\
\text{new\_object\_fn}(\text{invalid\_val}, L) \\
\text{default\_value\_fn}(E_1, S_1, V) \\
\text{assign\_fn}(E_1, S_1, \text{location}(L), V)
\end{cases} \Rightarrow S_2
\]

\[
S_1, E_1 \vdash _{\text{decl}} \text{Id} : \text{constant} \text{Sid}; \Rightarrow E_1[I_d \mapsto \text{object\_view(location}(L), S_1, \text{constant})], S_2
\]

\[
S_1 \begin{cases}
E_1 \vdash _{\text{sid}} \text{Sid} \Rightarrow S_1 \\
\text{new\_object\_fn}(\text{invalid\_val}, L) \\
\text{default\_value\_fn}(E_1, S_1, V) \\
\text{assign\_fn}(E_1, S_1, \text{location}(L), V)
\end{cases} \Rightarrow S_2
\]

\[
S_1, E_1 \vdash _{\text{decl}} \text{Id} := \text{Exp}; \Rightarrow E_1[I_d \mapsto \text{object\_view(location}(L), S_1, \text{variable})], S_2
\]
\begin{array}{ll}
S_1, E_1 \vdash \text{decl } \text{Id} : \text{constant } \text{Sid} := \text{Exp} \Rightarrow E_1[\text{Id} \mapsto \text{object}_\text{view}(\text{location}(L), S_t, \text{constant})], S_2 \\
S_1, E_1 \vdash \text{decl } \text{Id} : \text{aliased } \text{constant } \text{Sid} ; \Rightarrow E_1[\text{Id} \mapsto \text{object}_\text{view}(\text{location}(L), S_t, \text{aliased})], S_2 \\
S_1, E_1 \vdash \text{decl } \text{Id} : \text{aliased } \text{constant } \text{Sid} := \text{Exp} \Rightarrow E_1[\text{Id} \mapsto \text{object}_\text{view}(\text{location}(L), S_t, \text{aliased})], S_2 \\
E_3 = E_1[\text{Id} \mapsto \text{object}_\text{view}(\text{location}(L), \text{subtype}(\text{universal}_\text{real}_\text{tn}, \text{no}_{-}\text{constraint}, \text{not}_{-}\text{used}), \text{constant})] \\
S_1, E_1 \vdash \text{decl } \text{Id} : \text{constant} := \text{Exp} \Rightarrow E_3, S_2 \\
S_0, E_1, D_0 \vdash \text{new } \text{Sid} \Rightarrow \text{subtype}(U, C, A), S_2 \\
\end{array}

4.9.4 Derived Types and Classes

\begin{array}{ll}
S_0, E_1 \vdash \text{id } \text{Sid} \Rightarrow \text{subtype}(P, C, A), S_1 \\
S_1^{\text{typ}}[P] = (P_p \times T_y) \\
\text{new}_\text{type}(S_1, (\text{some}(P) \times T_y), U, S_2) \\
S_0, E_1, D_0 \vdash \text{new } \text{Sid} \Rightarrow \text{subtype}(U, C, A), E_2, S_2
\end{array}
4.9.5 Scalar Types

\[
\begin{align*}
S_1 \left\{ \begin{array}{l}
E \vdash_{exp} \text{Exp}_1 \Rightarrow V_1 \\
E \vdash_{exp} \text{Exp}_2 \Rightarrow V_2
\end{array} \right\} S_2 \\
S_1, E \vdash_{rng} \text{Exp}_1..\text{Exp}_2 \Rightarrow \text{make\_range}(V_1, V_2), S_2 \\
S_1, E \vdash_{nam} \text{Nam} \Rightarrow W, S_2 \\
\text{abnormal\_state}(S_2) \\
S_1, E \vdash_{rng} \text{Nam'}\text{range} \Rightarrow R, S_2 \\
S_1, E \vdash_{nam} \text{Nam} \Rightarrow W, S_2 \\
\text{normal\_state}(S_2) \\
S_2, E, W \vdash_{attr} \text{range} \Rightarrow R, S_3 \\
S_1, E \vdash_{rng} \text{Nam'}\text{range} \Rightarrow R, S_3 \\
\text{not yet defined}
\end{align*}
\]

\[
S_1, E \vdash_{rng} \text{Nam'}\text{range}(\text{Exp}) \Rightarrow R, S_2
\]

4.9.5.1 Enumeration Types

4.9.5.2 Character Types

4.9.5.3 Boolean Types

4.9.5.4 Integer Types

\[
\begin{align*}
S_1, E_1 \vdash_{exp} \text{Exp}_1 \Rightarrow \text{discrete\_val}(V_1), S_1 \\
S_1, E_1 \vdash_{exp} \text{Exp}_2 \Rightarrow \text{discrete\_val}(V_2), S_1 \\
\text{new\_type}(S_1, (\text{some(\text{root\_integer\_tn}) \times \text{signed\_integer\_type}(V_1, V_2, V_1, V_2))), U, S_2) \\
S_1, E_1, \emptyset \vdash_{tdf} \text{range} \text{Exp}_1..\text{Exp}_2 \Rightarrow \text{subtype}(U, \text{no\_constraint}, \text{not\_used}), E_1, S_2 \\
S_1, E_1 \vdash_{exp} \text{Exp} \Rightarrow \text{discrete\_val}(V), S_1 \\
\text{new\_type}(S_1, (\text{some(\text{root\_integer\_tn}) \times \text{modular\_type}(V))), U, S_2) \\
S_1, E_1, \emptyset \vdash_{tdf} \text{mod} \text{Exp} \Rightarrow \text{subtype}(U, \text{no\_constraint}, \text{not\_used}), E_1, S_2
\end{align*}
\]

4.9.5.5 Operations of Discrete Types

4.9.5.6 Real Types

4.9.5.7 Floating Point Types

\[
\begin{align*}
\text{not yet defined}
\end{align*}
\]

\[
S_1, E_1, D \vdash_{tdf} \text{digits} \text{Exp} \text{Cns} \Rightarrow S_1, E_2, S_2
\]

\[
\text{not yet defined}
\]

\[
S_1, E_1, D \vdash_{tdf} \text{digits} \text{Exp} \Rightarrow S_1, E_2, S_2
\]

4.9.5.8 Operations of Floating Point Types

4.9.5.9 Fixed Point Types

\[
\text{not yet defined}
\]

\[
S_1, E_1, D \vdash_{tdf} \text{delta} \text{Exp digits} \text{Exp} \text{Cns} \Rightarrow S_1, E_2, S_2
\]

\[
\text{not yet defined}
\]

\[
S_1, E_1, D \vdash_{tdf} \text{delta} \text{Exp digits} \text{Exp} \Rightarrow S_1, E_2, S_2
\]

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4.9.5.10 Operations of Fixed Point Types

4.9.6 Array Types

The evaluation of a subtype mark has no side-effect. Therefore, the evaluation of lists of subtype marks is defined sequentially.

\[
S, E \vdash_{Stl} () \Rightarrow []
\]

\[
S, E \vdash_{nam} \text{Nam0} \Rightarrow \text{subtype_view}(L_s), S
\]

\[
S, E \vdash_{Stl} (\text{Nam}, \ldots) \Rightarrow S_t
\]

\[
S, E \vdash_{Stl} (\text{Nam0}, \text{Nam}, \ldots) \Rightarrow S^{dfp}[L_s] \cdot S_t
\]

\[
\text{index_actions}(E, (\text{nam}, [\text{range}]), [\text{type}])
\]

\[
\text{index_actions}(E, (\text{sid}, \ldots), S_t, A_c)
\]

\[
\text{index_actions}(E, (\text{sid0}, \text{sid}, \ldots), S_t \cdot S_t, E \vdash_{sid} \text{sid0} \Rightarrow S_t \cdot A_c)
\]

\[
\text{range_constraints}(S, [], [])
\]

\[
\text{type_constraints}(S, S_t, R)
\]

\[
\text{range_constraints}(S, \text{subtype}(?, \text{range_constraint}(R_0, ?)) \cdot S_t, R_0 \cdot R)
\]

\[
\text{not yet defined}
\]

\[
S_1, E_1, D \vdash_{tdf} \text{array}(\text{Rng_lst}) \Rightarrow S_t, E_2, S_2
\]

\[
\text{not yet defined}
\]

\[
S_1, E_1, D \vdash_{tdf} \text{array}(\text{Nam0}, \text{Nam}, \ldots) \Rightarrow S_t, E_2, S_2
\]

\[
\text{index_actions}(E, \text{idx}, S_t, A_c)
\]

\[
\begin{align*}
E_1 \vdash_{sid} \text{sid} & \Rightarrow S_t \\
S_1 & \begin{cases}
A_{c1} \\
\ldots \\
A_{cn}
\end{cases} \Rightarrow S_2 \\
\text{range_constraints}(S_2, S_t, R)
\end{align*}
\]

\[
\text{new_type}(S_2, (\text{none} \times \text{array_type}(S_t, S_t)), U, S_3)
\]

\[
S_1, E_1, () \vdash_{tdf} \text{array}(\text{idx}) \Rightarrow \text{subtype}(U, \text{index_constraint}(R), \text{not_used}), E_2, S_3
\]

\[
S_1, E_1 \vdash_{Stl} \text{idx} \Rightarrow S_t \\
S_1, E_1 \vdash_{sid} \text{sid} \Rightarrow S_t, S_2
\]

\[
\text{new_type}(S_2, (\text{none} \times \text{array_type}(S_t, S_t)), U, S_3)
\]

\[
S_1, E_1, () \vdash_{tdf} \text{array}(\text{idx}) \Rightarrow \text{subtype}(U, \text{no_constraint}, \text{not_used}), E_1, S_3
\]
4.9.7 Discriminants

4.9.8 Record Types

\[
\begin{align*}
S_1 & \quad \text{new_type_fn}((\text{none} \times \text{record_type}(\text{none}, D, \text{fields}(C, V))), U) \\
S_1, E, D \vdash_{\text{idf}} \text{Cmp} \Rightarrow \text{subtype}(U, \text{no_constraint}, \text{not_used}), E, S_2
\end{align*}
\]

\[
\begin{align*}
S_1 & \quad \text{new_type_fn}((\text{none} \times \text{record_type}(\text{some}(U), D, \text{fields}(C, V))), U) \\
S_1, E, D \vdash_{\text{idf}} \text{tagged} \text{Cmp} \Rightarrow \text{subtype}(U, \text{no_constraint}, \text{not_used}), E, S_2
\end{align*}
\]

\[
\begin{align*}
S, E \vdash_{\text{cmp}} D & \Rightarrow \{.,[\]
\]

\[
\begin{align*}
S_1, E \vdash_{\text{cmp}} D & \Rightarrow \text{ldn} : \text{aliased Sid; Cmp}, . . . , A \\
S_1 & \quad \text{E, D} \vdash_{\text{ps1}} \text{Sid} \Rightarrow P_4 \\
S_1, E \vdash_{\text{cmp}} D & \Rightarrow \text{Cmp}, . . . \\
S_1, E, D \vdash_{\text{urn}} \text{ldn is Vnt end case; } \Rightarrow \text{some}(\text{ldn} \mapsto (P_4 \times \text{none}))
\end{align*}
\]

\[
\begin{align*}
S_1, E \vdash_{\text{cmp}} D & \Rightarrow \text{ldn} : \text{aliased Sid} := \text{Exp; Cmp}, . . . , A \\
S_1 & \quad \text{E, D} \vdash_{\text{ps1}} \text{Sid} \Rightarrow P_4 \\
S_1, E \vdash_{\text{cmp}} D & \Rightarrow \text{Cmp}, . . . \\
S_1, E, D \vdash_{\text{urn}} \text{ldn} : \text{Sid} := \text{Exp; Cmp}, . . . , A
\end{align*}
\]

4.9.8.1 Variant Parts and Discrete Choices

\[
S, E, D \vdash_{\text{urn}} \Rightarrow \text{none, S}
\]

\[
S_1, E, D \vdash_{\text{urn}} \text{ldn is Vnt end case; } \Rightarrow \text{some}(\text{ldn} \mapsto (P_4 \times B)), S_2
\]

\[
(\{\} = \text{variant}([\})
\]

\[
S, E, D \vdash_{\text{urn}} (\{) \Rightarrow (\}, S
\]

\[
S_1, E \vdash_{\text{chc}} \text{Dch} \Rightarrow C_v, S_1 \\
S_1 & \quad \text{E, D} \vdash_{\text{cmp}} D \Rightarrow \text{Cmp} \\
S_1, E, D \vdash_{\text{urn}} \text{Vrp} \Rightarrow V \\
S_1, E, D \vdash_{\text{urn}} \text{Vnt; . . . } \Rightarrow \text{variant}(B_1)
\]

\[
S_1, E, D \vdash_{\text{urn}} \text{when Dch => CmpVrp; Vnt; . . . } \Rightarrow \text{variant}(C_v \times \text{fields}(C, V)) \cdot B_2), S_2
\]

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4.9.9 Tagged Types and Type Extensions

4.9.9.1 Type Extensions

\[ S_0, E_1 \vdash \text{id} \text{ Sid} \Rightarrow \text{subtype}(P, C, A), S_1 \]
\[ S_1^{\text{typ}}[P] = (P_p \times \text{record_type}(\text{some}(U_p), D_1, \text{fields}(C_1, V_1))) \]
\[ E_1 \vdash \text{cmp} D_0 \Rightarrow \text{Cmp} \]
\[ E_1, D_0 \vdash \text{vpn} V_p \Rightarrow V_2 \]
\[ \text{new_type_fn}((\text{some}(U_p) \times \text{record_type}(\text{some}(U), D_0, D_1), \text{fields}(C_1 \oplus C_2, V_1 \oplus V_2)), U) \]
\[ S_0, E_1, D_0 \vdash \text{idf} \text{ new Sid with Cmp} \Rightarrow \text{subtype}(U, \text{no_constraint}, \text{not_used}), E_2, S_2 \]

\[ \text{none} \oplus V = V \]
\[ V \oplus \text{none} = V \]

\[ \text{discriminant_union}(\text{discr}(A_1), \text{discr}(A_2)) = \text{discr}(A_1 \oplus A_2) \]

4.9.10 Access Types

\[ S_1, E_1 \vdash \text{id} \text{ Sid} \Rightarrow L_2, S_2 \]
\[ \text{new_type}(S_2, (\text{none} \times \text{access_type}(L_1, \text{pool_access})), U, S_3) \]
\[ S_1, E_1, D \vdash \text{idf} \text{ access Sid} \Rightarrow \text{subtype}(U, \text{no_constraint}, \text{not_used}), E_1, S_3 \]

\[ S_1, E_1 \vdash \text{id} \text{ Sid} \Rightarrow L_2, S_2 \]
\[ \text{new_type}(S_2, (\text{none} \times \text{access_type}(L_1, \text{all_access})), U, S_3) \]
\[ S_1, E_1, D \vdash \text{idf} \text{ access all Sid} \Rightarrow \text{subtype}(U, \text{no_constraint}, \text{not_used}), E_1, S_3 \]

\[ S_1, E_1 \vdash \text{id} \text{ Sid} \Rightarrow L_2, S_2 \]
\[ \text{new_type}(S_2, (\text{none} \times \text{access_type}(L_1, \text{constant_access})), U, S_3) \]
\[ S_1, E_1, D \vdash \text{idf} \text{ access constant Sid} \Rightarrow \text{subtype}(U, \text{no_constraint}, \text{not_used}), E_1, S_3 \]

\[ S_1, E_1 \vdash \text{id} \text{ procedure Pms} \Rightarrow A, S_2 \]
\[ S_1, E_1 \vdash \text{idf} \text{ access procedure Pms} \Rightarrow \text{subtype}(U, \text{no_constraint}, \text{not_used}), E, S_2[\text{proc_profile}(A) \mapsto U] \]

\[ S_1, E_1 \vdash \text{id} \text{ function Pms} \Rightarrow A, S_2 \]
\[ S_2, E_1 \vdash \text{idf} \text{ access function Pms} \Rightarrow \text{subtype}(U, \text{no_constraint}, \text{not_used}), E_2, S_2[\text{func_profile}(A, S_1^\text{typ}[S_1]) \mapsto U] \]

4.9.10.1 Incomplete Type Declarations

In the case of access type definitions, a subtype indication may denote an incomplete type.

\[ S_1, E \vdash \text{nam} \text{ Nam} \Rightarrow \text{subtype_view}(L_1), S_1 \]
\[ S_1, E \vdash \text{Sid}, \text{ Nam} \Rightarrow L_2, S_1 \]

\[ S_1, E \vdash \text{nam} \text{ Nam} \Rightarrow \text{subtype_view}(L_1), S_1 \]
\[ S_1, E, S_1^{\text{typ}}[L_2] \vdash \text{cmp}, \text{ Csn} \Rightarrow C, S_2 \]
\[ \text{compatible}(E, S_1^{\text{typ}}[L_2], C) \]
\[ S_1^{\text{typ}}[L_1] = \text{subtype}(U, C_1, A) \]
\[ \text{new subtype}(S_3, \text{subtype}(U, C, A), L_1, S_3) \]
\[ S_1, E \vdash \text{Sid}, \text{ Nam Cns} \Rightarrow L_1, S_3 \]
() = discr([])

For incomplete type declarations, a new incomplete type descriptor carries the discriminant information.

\[
\begin{align*}
\text{new_type}(S_0, (\text{none} \times \text{incomplete_type}([])), U, S_1) \\
\text{new__subtype}(S_1, \text{subtype}(U, \text{no_constraint}, \text{not_used}), L_2, S_2) \\
S_0, E_1 \vdash \text{type } I_d \leftrightarrow E_1[I_d \mapsto \text{subtype]\_view}(L_3)], S_2 \\
S_1, E_1 \vdash \text{dc } Dcp \Rightarrow D, S_2 \\
\text{new__type}(S_2, (\text{none} \times \text{incomplete_type}(D)), U, S_3) \\
\text{new__subtype}(S_3, \text{subtype}(U, \text{no_constraint}, \text{not_used}), L_3, S_4) \\
S_1, E_1 \vdash \text{dc } I_d Dcp; \Rightarrow E_1[I_d \mapsto \text{subtype}\_\text{view}(L_4)], S_3
\end{align*}
\]

Completing type declarations have their own abstract syntax.

\[
\begin{align*}
E_1 \vdash \text{lookup } \text{Id} \Rightarrow \text{subtype}\_\text{view}(L_3) \\
S_1, E_1(), \vdash \text{def } \text{Tdf} \Rightarrow S_2, E_2, S_3 \\
S_1, E_1 \vdash \text{dc } \text{completetype } I_d \text{ is } \text{Tdf}; \Rightarrow E_2, S_2[L, \mapsto 4 S_1] \\
E_1 \vdash \text{lookup } \text{Id} \Rightarrow \text{subtype}\_\text{view}(L_3) \\
S_1 \\
E_1 \vdash \text{dc } Dcp \Rightarrow D \\
S_1 \\
E, D \vdash \text{def } \text{Tdf} \Rightarrow E_3, S_4 \\
S_1, E_1 \vdash \text{dc } \text{completetype } I_d Dcp \text{ is } \text{Tdf}; \Rightarrow E_2, S_2[L, \mapsto 4 S_1]
\end{align*}
\]

4.10 Expressions

4.10.1 Names

\[
\begin{align*}
E \vdash \text{lookup } \text{Idn} \Rightarrow W \\
S, E \vdash \text{nam } \text{Idn} \Rightarrow W, S \\
S_1 \ \ \ \ E \vdash \text{nam } \text{Nam} \Rightarrow \text{object}\_\text{view}(L_1, S_2,\?) \\
\vdash \text{content}(L_1) \Rightarrow \text{access}\_\text{val}(W, \?) \\
S_1, E \vdash \text{nam } \text{Nam.\_all} \Rightarrow W, S_2
\end{align*}
\]

4.10.1.1 Indexed Components

The function expression\_list computes a sequence of actions that corresponds to the evaluation of a sequence of expressions.

\[
\begin{align*}
\text{expression}\_\text{list}(E, [], [], []) \\
expression\_\text{list}(E, \text{Exs}, V, A) \\
A = E \vdash \text{exp } \text{Exp} \Rightarrow V \\
\text{expression}\_\text{list}(E, \text{Exp} \cdot \text{Exs}, V \cdot V, A \cdot A)
\end{align*}
\]

\[
\text{index}\_\text{list}(S, [], [], [], S)
\]

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belongs_to( V, R)

\[\text{index_list}(S_1, R, V, I, S_2)\]

\[\text{index_list}(S_1, R \cdot R, V \cdot V, I \cdot I, S_2)\]

\[\neg \text{belongs_to}( V, R)\]

\[\text{index_list}(\text{normal}(N), R \cdot R, V \cdot V, I \cdot I, \text{exception}(\text{constraint_error}, N))\]

\[\text{expression_list}(E, \text{Exs}, V, A)\]

\[S_1 \begin{cases} E \vdash_{\text{nam}} \text{Prefix} \Rightarrow \text{object_view}(L_1, S_1, ?) \\ A_{s1} \\ \ldots \\ A_{sn} \end{cases} \rightarrow S_2 \]

\[\text{abnormal_state}(S_2)\]

\[S_1, E \vdash_{\text{nam}} \text{Prefix(Exs, ...) \Rightarrow undefined_view, S_2}\]

\[\text{expression_list}(E, \text{Exs}, V, A)\]

\[S_1 \begin{cases} E \vdash_{\text{nam}} \text{Prefix} \Rightarrow \text{object_view}(L_1, S_1, C) \\ A_{s1} \\ \ldots \\ A_{sn} \end{cases} \rightarrow S_2 \]

\[\text{normal_state}(S_2)\]

\[\text{type_struct}(S_2, S_1) = \text{array_type}(?, B)\]

\[S_2 \vdash \text{content}(L_1) \Rightarrow \text{array_val}(R, A_v)\]

\[\text{index_list}(S_2, R, V, I, S_3)\]

\[S_1, E \vdash_{\text{nam}} \text{Prefix(Exs, ...) \Rightarrow object_view(array_component}(L_1, I_z), B, C), S_3\]

4.10.1.2 Slices

\[\text{null_range}(R_s)\]

\[\text{slice_check}(S, R_s, R_a, S)\]

\[\text{included_in}(R_s, R_a)\]

\[\text{slice_check}(S, R_s, R_a, S)\]

\[\neg \text{null_range}(R_s)\]

\[\neg \text{included_in}(R_s, R_a)\]

\[\text{slice_check}(\text{normal}(N), R_s, R_a, \text{exception}(\text{constraint_error}, N))\]

\[S_1 \begin{cases} E \vdash_{\text{nam}} \text{Prefix} \Rightarrow W \\ E \vdash_{\text{rng}} \text{Rng} \Rightarrow R_s \end{cases} \rightarrow S_2 \]

\[\text{abnormal_state}(S_2)\]

\[S_1, E \vdash_{\text{nam}} \text{Prefix(Rng) \Rightarrow undefined_view, S_2}\]

\[S_1 \begin{cases} E \vdash_{\text{nam}} \text{Prefix} \Rightarrow \text{object_view}(L_v, S_t, C) \\ E \vdash_{\text{rng}} \text{Rng} \Rightarrow R_s \end{cases} \rightarrow S_2 \]

\[\text{normal_state}(S_2)\]

\[S_2 \vdash \text{content}(L_v) \Rightarrow \text{array_val}([R_a], A_v)\]

\[\text{constrain}(S_1, \text{range_constraint}(R_s), S_q)\]

\[\text{slice_check}(S_2, R_s, R_a, S_3)\]

\[S_1, E \vdash_{\text{nam}} \text{Prefix(Rng) \Rightarrow object_view(array_slice}(L_v, R_s), S_q, C), S_3\]

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4.10.1.3 Selected Components

$$S_1, E \vdash_{\text{nam}} \text{Nam} \Rightarrow W, S_2$$

$$\text{abnormal_state}(S_2)$$

$$S_1, E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{undefined_view}, S_2$$

$$S_1, E \vdash_{\text{nam}} \text{Nam}.\text{Idn} \Rightarrow \text{object_view}(L_v, S_t, ?), S_2$$

$$\text{component_type}(D_v, \text{type_struct}(S_2, S_t), \text{Idn}, S_q)$$

$$S_2 \models \text{content}(L_v) \Rightarrow \text{record_val}(T_g, D_v, C_v)$$

$$\text{new_object}(S_2, D_v[\text{Idn}], L, S_3)$$

$$S_1, E \vdash_{\text{nam}} \text{Nam}.\text{Idn} \Rightarrow \text{object_view}(\text{location}(L), S_q, \text{constant}), S_3$$

$$S_1, E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{object_view}(L_v, S_t, C), S_2$$

$$S_2 \models \text{content}(L_v) \Rightarrow \text{record_val}(T_g, D_v, C_v)$$

$$\text{component_type}(D_v, \text{type_struct}(S_2, S_t), \text{Idn}, S_q)$$

$$S_1, E \vdash_{\text{nam}} \text{Nam}.\text{Idn} \Rightarrow \text{object_view}(\text{record_component}(L_v, \text{Idn}), S_q, C), S_2$$

$$S, E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{object_view}(L_v, S_t, ?), \text{normal}(N)$$

$$\neg \text{component_type}(B, \text{type_struct}(\text{normal}(N), S_1), \text{Idn}, S_q)$$

$$S, E \vdash_{\text{nam}} \text{Nam}.\text{Idn} \Rightarrow \text{undefined_view, exception(constraint_error, N)}$$

4.10.1.4 Expanded Names

4.10.1.5 Attributes

$$S_1 \left[ \begin{array}{l} E \vdash_{\text{nam}} \text{Nam} \Rightarrow W_1 \\ E, W_1 \vdash_{\text{att}} \text{Idn} \Rightarrow W \end{array} \right] S_2$$

$$S_1, E \vdash_{\text{nam}} \text{Nam}.\text{Idn} \Rightarrow W, S_2$$

$$S_1, E \models_{\text{exp}} \text{Exp} \Rightarrow V, S_1$$

$$S_1 \left[ \begin{array}{l} E \vdash_{\text{nam}} \text{Nam} \Rightarrow W_1 \\ E, W_1, V \vdash_{\text{att}} \text{Idn} \Rightarrow W \end{array} \right] S_2$$

$$S, E \vdash_{\text{nam}} \text{Nam}.\text{Idn}(\text{Exp}) \Rightarrow W, S$$

Note that the abstract syntax distinguishes N'I(E) for static expression E from N'I(E) where N'I is a function-valued attribute.

4.10.2 Literals

$$S, E \models_{\text{exp}} \text{null} \Rightarrow \text{null, S}$$

In the abstract syntax, the representation of character literals is given by the numeric value of the position of the character.

$$S, E \models_{\text{exp}} 'C' \Rightarrow \text{discrete_val}(C), S$$

$$S, E \models_{\text{exp}} R \Rightarrow \text{real_val}(R), S$$

$$S, E \models_{\text{exp}} N \Rightarrow \text{discrete_val}(N), S$$
4.10.3 Aggregates

Only explicitly qualified aggregates are defined. Static semantics adds qualification where needed.

We assume a normalized representation using named associations. This is possible because of [4.3.1(14)].

\[ S_1, E \vdash \text{nam} \text{Nam} \Rightarrow \text{subtype_view}(L_s), S_1 \]
\[ \text{type} \_\text{struct}(S_1, S_1^{\text{typ}}[L_s]) = \text{record} \_\text{type}(T_g, \text{discr}(L), C_i) \]
\[ S_1^{\text{typ}}[L_s] = \text{subtype}(?, \text{no}\_\text{constraint}, ?) \]
\[ S_1, E, L \vdash \text{agg} \text{Agg} \Rightarrow B_d, S_2 \]
\[ S_2, E, \text{actualized_components}(B_d, C_i) \vdash \text{agg} \text{Agg} \Rightarrow B_c, S_3 \]
\[ S_1, E \vdash \text{exp} \text{Nam'}\text{Agg} \Rightarrow \text{record} \_\text{val}(T_g, B_d, B_c), S_3 \]

\[ S_1, E \vdash \text{nam} \text{Nam} \Rightarrow \text{subtype_view}(L_s), S_1 \]
\[ \text{type} \_\text{struct}(S_1, S_1^{\text{typ}}[L_s]) = \text{record} \_\text{type}(T_g, D_a, C_i) \]
\[ S_1^{\text{typ}}[L_s] = \text{subtype}(?, \text{discriminant}\_\text{constraint}(B_d), ?) \]
\[ S_1, E, \text{actualized_components}(B_d, C_i) \vdash \text{agg} \text{Agg} \Rightarrow B_c, S_2 \]
\[ S_1, E \vdash \text{exp} \text{Nam'}\text{Agg} \Rightarrow \text{record} \_\text{val}(T_g, B_d, B_c), S_2 \]

\[ \text{find\_component}(I_d, \text{others} => \text{Exp} \cdot \text{Rca}, \text{Exp}) \]

\[ \text{member}(I_d, \text{Lst}) \]
\[ \text{find\_component}(I_d, \text{Lst}, \ldots => \text{Exp} \cdot \text{Rca}, \text{Exp}) \]

\[ \text{component\_actions}(E, [], \text{Rca}, [], []) \]

\[ \text{find\_component}(I_d, \text{Rca}, \text{Exp}) \]
\[ \text{component\_actions}(E, B, \text{Rca}, B_c, A_s) \]

\[ \text{find\_component}(I_d \times ?) \cdot B, \text{Rca}, (I_d \times V) \cdot B_c, E \vdash \text{exp} \text{Exp} \Rightarrow V \cdot A_s \]

\[ \text{component\_actions}(E, B, \text{Rca}, B_c, A_s) \]

\[ \text{find\_component}(I_d, \text{Rca}, \text{Exp}) \]
\[ \text{component\_actions}(E, B, \text{Rca}, B_c, A_s) \]

\[ \text{find\_component}(I_d \times ?) \cdot B, \text{Rca}, (I_d \times V) \cdot B_c, E \vdash \text{exp} \text{Exp} \Rightarrow V \cdot A_s \]

\[ \text{component\_actions}(E, A, \text{Rca}, B, A_s) \]

\[ S_1 = \{ \begin{array}{l} A_{s1} \ldots \ A_{sn} \end{array} \} S_2 \]

\[ S_1, E, A \vdash \text{agg} (\text{Rca}, \ldots) \Rightarrow B, S_2 \]

\[ S, E, [], \text{agg} (\text{null record}) \Rightarrow [], S \]

\[ S_1, E, R_s, C_s \vdash \text{agg} (\text{Exp}_1, \text{Exp}_2, \ldots) \Rightarrow V, S_2 \]

\[ S_1, E, R_s, C_s \vdash \text{agg} (\text{Exp}_1, \text{Exp}_2, \ldots, \text{others} => \text{Exp}) \Rightarrow V, S_2 \]

\[ S_1, E, R_s, C_s \vdash \text{agg} (\text{Aca}_1, \text{Aca}_2, \ldots) \Rightarrow V, S_2 \]

\[ S_1, E, R_s, C_s \vdash \text{agg} (\text{Exp}_1, \text{Exp}_2, \ldots, \text{others} => \text{Exp}) \Rightarrow V, S_2 \]
4.10.4 Operators and Expression Evaluation

It is assumed that static semantics has resolved all operators into function calls as defined in [4.5] with the following exceptions:

- short-circuit operators
- in and not in operators

4.10.4.1 Logical Operators and Short-circuit Control Forms

\[
\begin{align*}
S_1, E \vdash \text{Exp}_1 \rightarrow \text{discrete_val}(1) \\
&\quad S_2 \\
S_1, E \vdash \text{Exp}_2 \rightarrow V \\
\hline
S_1, E \vdash \text{Exp}_1 \text{ and then } \text{Exp}_2 \rightarrow V, S_2
\end{align*}
\]

\[
\begin{align*}
S_1, E \vdash \text{Exp}_1 \rightarrow \text{discrete_val}(0), S_2 \\
S_1, E \vdash \text{Exp}_1 \text{ and then } \text{Exp}_2 \rightarrow \text{discrete_val}(0), S_2
\end{align*}
\]

\[
\begin{align*}
S_1, E \vdash \text{Exp}_1 \rightarrow \text{discrete_val}(0) \\
&\quad S_2 \\
S_1, E \vdash \text{Exp}_2 \rightarrow V \\
\hline
S_1, E \vdash \text{Exp}_1 \text{ or else } \text{Exp}_2 \rightarrow V, S_2
\end{align*}
\]

\[
\begin{align*}
S_1, E \vdash \text{Exp}_1 \rightarrow \text{discrete_val}(1), S_2 \\
S_1, E \vdash \text{Exp}_1 \text{ or else } \text{Exp}_2 \rightarrow \text{discrete_val}(1), S_2
\end{align*}
\]

4.10.4.2 Relational Operators and Membership Tests

\[
\begin{align*}
S_1, E \vdash \text{nam \_ subtype\_view}(L_s), S_1 \\
S_1, E \vdash \text{Exp} \rightarrow V, S_2 \\
\hline
\text{belongs\_to}(V, \text{range\_of\_subtype}(S_1, S_{\text{top}}[L_s])) \\
S_1, E \vdash \text{Exp \ in \ Nam} \rightarrow \text{discrete\_val}(1), S_2
\end{align*}
\]

\[
\begin{align*}
S_1, E \vdash \text{nam \_ subtype\_view}(L_s), S_1 \\
S_1, E \vdash \text{Exp} \rightarrow V, S_2 \\
\hline
\neg \text{belongs\_to}(V, \text{range\_of\_subtype}(S_2, S_{\text{top}}[L_s])) \\
S_1, E \vdash \text{Exp \ in \ Nam} \rightarrow \text{discrete\_val}(0), S_2
\end{align*}
\]

\[
\begin{align*}
S_1 \{ \\
E \vdash \text{rng \_ Rng} \Rightarrow R \\
E \vdash \text{Exp} \Rightarrow V \\
\hline
\text{belongs\_to}(V, R)
\} \quad S_2
\end{align*}
\]

\[
\begin{align*}
S_1 \{ \\
E \vdash \text{rng \_ Rng} \Rightarrow R \\
E \vdash \text{Exp} \Rightarrow V \\
\neg \text{belongs\_to}(V, R) \\
\hline
S_1, E \vdash \text{Exp \ in \ Rng} \rightarrow \text{discrete\_val}(0), S_2
\}
\end{align*}
\]

\[
\begin{align*}
\neg \text{is\_tagged\_type}(S, S_t) \\
S_t = \text{subtype}(U, C, ?) \\
\hline
\text{satisfies}(V, C) \\
\text{test\_in}(S, V, S_t)
\end{align*}
\]

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\[ S_t = \text{subtype}(U,C,?) \]
\[ \text{satisfies}(V,C) \]
\[ V = \text{record_val}(\text{some}(T_g),?,?) \]
\[ \text{ancestor}(S,U,T_g) \]
\[ \text{test_in}(S,V,S_t) \]

\[ S_1, E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{subtype_view}(L_s), S_1 \]
\[ S_1, E \vdash_{\text{exp}} \text{Exp} \Rightarrow V, S_2 \]
\[ \text{test_in}(S_2, V, S_t^{\text{step}}[L_s]) \]
\[ S_1, E \vdash_{\text{exp}} \text{Exp in } \text{Nam} \Rightarrow \text{discrete_val}(1), S_2 \]

\[ S_1, E \vdash_{\text{exp}} \text{Exp in } \text{Nam} \Rightarrow \text{discrete_val}(0), S_2 \]

4.10.5 Type Conversions

\[ S_1, E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{subtype_view}(L_s), S_1 \]
\[ S_1, E \vdash_{\text{exp}} \text{Exp} \Rightarrow V_1 \]
\[ E, S_t^{\text{step}}[L_s] \vdash \text{convert}(V_1) \Rightarrow V_2 \]
\[ S_1, E \vdash_{\text{exp}} \text{Nam}(\text{Exp}) \Rightarrow V_2, S_2 \]

\[ S_1, E \vdash_{\text{nam}} \text{Nam}_1 \Rightarrow \text{subtype_view}(L_s), S_1 \]
\[ E \vdash_{\text{nam}} \text{Nam}_2 \Rightarrow W_1 \]
\[ E, S_t^{\text{step}}[L_s] \vdash \text{convert}(W_1) \Rightarrow W_2 \]
\[ S_1, E \vdash_{\text{nam}} \text{Nam}_1(\text{Nam}_2) \Rightarrow W_2, S_2 \]
4.10.6 Qualified Expressions

\[
S_1, E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{subtype}_{-}\text{view}(L_3), S_1 \\
S_1, E \vdash_{\text{exp}} \text{Exp} \Rightarrow V, S_2 \\
\text{abnormal}_{-}\text{state}(S_2) \\
\overline{S_1, E \vdash_{\text{exp}} \text{Nam}'(\text{Exp}) \Rightarrow V, S_2}
\]

\[
S_1, E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{subtype}_{-}\text{view}(L_3), S_1 \\
S_1^{\text{step}}[L_3] = \text{subtype}(?, C, ?) \\
S_1, E \vdash_{\text{exp}} \text{Exp} \Rightarrow V, S_2 \\
\text{normal}_{-}\text{state}(S_2) \\
\text{satisfies}(V, C) \\
\overline{S_1, E \vdash_{\text{exp}} \text{Nam}'(\text{Exp}) \Rightarrow V, S_2}
\]

\[
S_1, E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{subtype}_{-}\text{view}(L_3), S_1 \\
S_1^{\text{step}}[L_3] = \text{subtype}(?, C, ?) \\
S_1, E \vdash_{\text{exp}} \text{Exp} \Rightarrow V, \text{normal}(N) \\
\neg \text{satisfies}(V, C) \\
\overline{S_1, E \vdash_{\text{exp}} \text{Nam}'(\text{Exp}) \Rightarrow V, \text{exception}('\text{constraint}\_\text{error}, N)}
\]

4.10.7 Allocators

\[
S_1 \left[ E \vdash_{\text{exp}} \text{Exp} \Rightarrow V \right]_{\text{new}_{-}\text{object}_{-}\text{fn}(invalid\_val, L)} S_2 \\
\overline{S_1, E \vdash_{\text{exp}} \text{new}_{-}\text{Exp} \Rightarrow \text{access}_{-}\text{val}(\text{object}_{-}\text{view}(\text{location}(L), ?, aliased), \text{none}), S_2}
\]

not yet defined
\[
\overline{S_1, E \vdash_{\text{exp}} \text{new}_{-}\text{Sid} \Rightarrow V, S}
\]

4.11 Statements

4.11.1 Statement Sequences

\[
S_0 \left[ E \vdash_{\text{stm}} \text{Stm}_1 \Rightarrow E \vdash_{\text{stm}} \text{Stm}_2 \ldots \Rightarrow \right] S_1 \\
\overline{S_0, E \vdash_{\text{stm}} \text{Stm}_1 \text{Stm}_2 \ldots \Rightarrow S_1}
\]

\[
S, E \vdash_{\text{stm}} () \Rightarrow S
\]

\[
S, E \vdash_{\text{stm}} \text{Stm} \Rightarrow S \\
\overline{S, E \vdash_{\text{stm}} \text{<}< \text{Nam} >> \text{Stm} \Rightarrow S}
\]

\[
S, E \vdash_{\text{stm}} \text{null;} \Rightarrow S
\]

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4.11.2 Assignment Statements

assignable(variable)

assignable(aliased)

\[
\begin{align*}
S_0 \{ & \quad E \vdash_{\text{nom}} \text{Nam} \Rightarrow \text{object\_view}(L, S_1, M) \\
& \quad E \vdash_{\text{exp}} \text{Exp} \Rightarrow V \\
& \quad \text{assignable}(M) \\
S_1 & \quad [ \quad E, S_1 \vdash \text{subtype\_convert}(V) \Rightarrow V_1 \\
& \quad \vdash \text{content}(L) \Rightarrow V_0 \\
& \quad \text{finalize\_fn}(E, S_1, V_0) \\
& \quad \text{assign\_fn}(E, S_1, L, V_1) \\
& \quad \text{value\_split\_fn}(E, S_1, V_1) \quad S_2 \\
\] \\
\hline
S_0, E \vdash_{\text{stm}} \text{Nam} := \text{Exp} \Rightarrow S_2 \\
\hline
\end{align*}
\]

4.11.3 If Statements

\[
\begin{align*}
S_1, E \vdash_{\text{stm}} & \text{if } \text{Exp then Stm}_1 \text{Eif else Stm}_2 \text{end if}; \Rightarrow S_2 \\
S_1, E \vdash_{\text{stm}} & \text{if } \text{Exp then Stm}_1 \text{Eif end if}; \Rightarrow S_2 \\
S_1 & \quad [ \quad E \vdash_{\text{exp}} \text{Exp} \Rightarrow \text{discrete\_val}(0) \\
& \quad \vdash \text{elf Eif} \Rightarrow \text{true} \quad S_2 \\
S_1 & \quad [ \quad E \vdash_{\text{exp}} \text{Exp} \Rightarrow \text{discrete\_val}(0) \\
& \quad \vdash \text{elf Eif} \Rightarrow \text{false} \quad S_2 \\
S_1 & \quad [ \quad E \vdash_{\text{exp}} \text{Exp} \Rightarrow \text{discrete\_val}(0) \\
& \quad \vdash \text{elf Eif} \Rightarrow \text{false} \quad S_2 \\
S_1, E \vdash_{\text{stm}} & \text{if } \text{Exp then Stm}_1 \text{Eif end if}; \Rightarrow S_2 \\
S_1, E \vdash_{\text{stm}} & \text{if } \text{Exp then Stm}_1 \text{Eif end if}; \Rightarrow S_2 \\
S_1 & \quad [ \quad E \vdash_{\text{exp}} \text{Exp} \Rightarrow \text{discrete\_val}(0) \\
& \quad \vdash \text{elf Eif} \Rightarrow \text{false} \quad S_2 \\
S_1, E \vdash_{\text{stm}} & \text{if } \text{Exp then Stm}_1 \text{Eif end if}; \Rightarrow S_2 \\
S_1, E \vdash_{\text{elf}} & () \Rightarrow \text{false}, S \\
S_1, E \vdash_{\text{elf}} & \text{Eif}_1 \Rightarrow \text{true}, S_2 \\
S_1, E \vdash_{\text{elf}} & \text{Eif}_1 \text{Eif}_2 \ldots \Rightarrow \text{true}, S_2
\end{align*}
\]
4.11.4 Case Statements

\[ S_1 \begin{cases} E \vdash \text{if } E_1 \Rightarrow \text{false} \\ E \vdash \text{if } E_2 \ldots \Rightarrow R \end{cases} \Rightarrow S_2 \]

4.11.4.1 Choices

\[ S_1 \begin{cases} E \vdash \text{if } E_1 \Rightarrow \text{false} \end{cases} \Rightarrow S_2 \]

\[ S_1, E \vdash \text{elseif } \text{Exp} \text{ then } \text{Stm} \Rightarrow \text{true}, S_2 \]

\[ S_1 \begin{cases} E \vdash \text{elseif } \text{Exp} \Rightarrow \text{discrete_val}(1) \end{cases} \Rightarrow S_2 \]

\[ S_1, E \vdash \text{elsif } \text{Exp} \Rightarrow \text{discrete_val}(0), S_2 \]

\[ S_1 \begin{cases} E \vdash \text{elseif } \text{Exp} \Rightarrow \text{V} \end{cases} \Rightarrow S_2 \]

4.11.4 Case Statements

\[ normal(N), E, V \vdash \text{cit} () \Rightarrow R, \text{exception(\text{constraint}\_\text{error}}, N) \]

\[ S_1, E, V \vdash \text{cit Alt_1} \Rightarrow \text{true}, S_2 \]

\[ S_1, E, V \vdash \text{cit Alt_1 Alt_2} \ldots \Rightarrow \text{true}, S_2 \]

\[ S_1 \begin{cases} E, V \vdash \text{cit Alt_1 Alt_2} \ldots \Rightarrow \text{false} \end{cases} \Rightarrow R, S_2 \]

\[ S_1 \begin{cases} E \vdash \text{cit Dch} \Rightarrow C \\ \text{covers_fn}(V, C, \text{true}) \end{cases} \Rightarrow S_2 \]

\[ S_1, E, V \vdash \text{cit when} \text{Dch} \Rightarrow \text{Stm} \Rightarrow \text{true}, S_2 \]

\[ S_1 \begin{cases} E \vdash \text{cit Dch} \Rightarrow C \\ \text{covers_fn}(V, C, \text{false}) \end{cases} \Rightarrow S_2 \]

\[ S_1, E, V \vdash \text{cit when} \text{Dch} \Rightarrow \text{Stm} \Rightarrow \text{false}, S_2 \]

\[ S, E \vdash \text{cit Stm} \Rightarrow \text{exit(\text{unnamed}}, N) \]

\[ S, E \vdash \text{cit loop Stm} \text{ end loop}; \Rightarrow \text{normal(N)} \]

\[ S_1, E \vdash \text{cit} \Rightarrow R, S_2 \]

\[ S, E \vdash \text{cit} \Rightarrow \text{choice_range}(R), S_2 \]

\[ S_1, E \vdash \text{cit} \Rightarrow \text{choice_value}(V), S_2 \]

\[ S, E \vdash \text{cit} \Rightarrow \text{choice_default}, S \]

\[ S, E \vdash \text{cit} () \Rightarrow \text{choice_lst([-]), S} \]

\[ S_1, E \vdash \text{cit Dch_1} \Rightarrow C_1, S_2 \]

\[ S_2, E \vdash \text{cit Dch_2} \ldots \Rightarrow \text{choice_lst(C_2), S_3} \]

\[ S_1, E \vdash \text{cit Dch_1 | Dch_2 | \ldots \Rightarrow choice_lst(C_1 \cdot C_2), S_3} \]
4.11.5 Loop Statements

\[
\begin{align*}
S_1 & \begin{cases}
E \vdash_{\text{stm}} \text{Stm} \Rightarrow S_2
\end{cases} \\
S_1, E \vdash_{\text{stm}} \text{loop Stm end loop}; \Rightarrow S_2
\end{align*}
\]

\[
S_1, E_1[I_{\text{ln}} \mapsto \text{loop_view}(X)] \vdash_{\text{stm}} \text{loop Stm end loop}; \Rightarrow \text{normal}(N)
\]

\[
S_1, E_1[I_{\text{ln}} \mapsto \text{loop_view}(X)] \vdash_{\text{stm}} \text{loop Stm end loop}; \Rightarrow \text{exit}(\text{ln}_{\text{id}}(X), N)
\]

\[
S \begin{cases}
E \vdash_{\text{exp}} \text{Exp} \Rightarrow \text{discrete_val}(1) \\
E \vdash_{\text{stm}} \text{Stm} \Rightarrow \text{exit}(\text{unnamed}, N)
\end{cases}
\]

\[
S, E \vdash_{\text{stm}} \text{while Exp loop Stm end loop}; \Rightarrow \text{normal}(N)
\]

\[
S_1 \left[
\begin{array}{c}
E \vdash_{\text{exp}} \text{Exp} \Rightarrow \text{discrete_val}(1) \\
E \vdash_{\text{stm}} \text{Stm} \Rightarrow
\end{array}
\right] S_2
\]

\[
S_1, E \vdash_{\text{stm}} \text{while Exp loop Stm end loop}; \Rightarrow S_2
\]

\[
S_1, E \vdash_{\text{exp}} \text{Exp} \Rightarrow \text{discrete_val}(0) S_2
\]

\[
S_1, E \vdash_{\text{stm}} \text{while Exp loop Stm end loop}; \Rightarrow S_2
\]

\[
S_1, E_1[I_{\text{ln}} \mapsto \text{loop_view}(X)] \vdash_{\text{stm}} \text{while Exp loop Stm end loop}; \Rightarrow \text{normal}(N)
\]

\[
S_1, E_1[I_{\text{ln}} \mapsto \text{loop_view}(X)] \vdash_{\text{stm}} \text{while Exp loop Stm end loop}; \Rightarrow \text{normal}(N)
\]
4.11.6 Block Statements

\[ S_1, E \vdash_{\text{stm}} \text{Hsm} \Rightarrow S_2 \]

\[ S_1, E \vdash_{\text{stm}} \text{begin Hsm end}; \Rightarrow S_2 \]

\[ S_1, E \vdash_{\text{stm}} \text{Nam} : \text{begin Hsm end}; \Rightarrow S_2 \]

\[ S_1 \left[ \begin{array}{l}
E_1 \vdash_{\text{dcl}} \text{Dcl} \Rightarrow E_2 \\
E_2 \vdash_{\text{stm}} \text{Hsm} \\
S_2, E_2 \vdash_{\text{stm}} \text{finalize}(B) \Rightarrow S_3
\end{array} \right] \Rightarrow S_3 \]

\[ S_1, E_1 \vdash_{\text{stm}} \text{declare Dcl begin Hsm end}; \Rightarrow S_3 \]

4.11.7 Exit Statements

\[ \text{normal}(N), E \vdash_{\text{stm}} \text{exit}; \Rightarrow \text{exit}(\text{unnamed}, N) \]

\[ S, E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{loop_view}(X), \text{normal}(N) \]

\[ S, E \vdash_{\text{stm}} \text{exit Nam}; \Rightarrow \text{exit}(\text{loop_id}(X), N) \]

\[ S, E \vdash_{\text{exp}} \text{Exp} \Rightarrow \text{discrete_val}(1), \text{normal}(N) \]

\[ S, E \vdash_{\text{stm}} \text{exit when Exp}; \Rightarrow \text{exit}(\text{unnamed}, N) \]

\[ S_1, E \vdash_{\text{exp}} \text{Exp} \Rightarrow B, S_2 \]

\[ S_1, E \vdash_{\text{stm}} \text{exit when Exp}; \Rightarrow S_2 \]

\[ S \left[ \begin{array}{l}
E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{loop_view}(X) \\
E \vdash_{\text{exp}} \text{Exp} \Rightarrow \text{discrete_val}(1)
\end{array} \right] \Rightarrow \text{normal}(N) \]

\[ S_1, E \vdash_{\text{stm}} \text{exit Nam when Exp}; \Rightarrow \text{exit}(\text{loop_id}(X), N) \]

\[ S_1 \left[ \begin{array}{l}
E \vdash_{\text{nam}} \text{Nam} \Rightarrow \text{loop_view}(X) \\
E \vdash_{\text{exp}} \text{Exp} \Rightarrow \text{discrete_val}(0)
\end{array} \right] \Rightarrow S_2 \]
4.12 Subprograms

4.12.1 Subprogram Declarations

\[
S_1, E_1 \vdash \text{pms \Rightarrow A, S}_2 \\
\text{new_subprogram}(S_2, \text{unelaborated}, U, S_3)
\]

\[
S_1, E_1 \vdash \text{dcl procedure Idn Pms; } \Rightarrow E_1[\text{Idn \mapsto subprogram_view}(U, A, \text{none})], S_3
\]

\[
S_1, E_1 \vdash \text{pms \Rightarrow A, S}_2 \\
S_2, E_1 \vdash \text{nam Nam \Rightarrow subtype_view}(S_1), S_2 \\
\text{new_subprogram}(S_2, \text{unelaborated}, U, S_3)
\]

\[
S_1, E_1 \vdash \text{dcl function Idn Pms; } \Rightarrow E_1[\text{Idn \mapsto subprogram_view}(U, A, \text{some}(S^\text{tp}[S_1])), S_3
\]

\[
S_1, E_1 \vdash \text{pms \Rightarrow A, S}_2 \\
E \vdash \text{lookup Idn \Rightarrow subprogram_view}(U, A, \text{none})
\]

\[
S_1, E \vdash \text{dcl procedure Idn Pms is Dcl begin Stm end; } \Rightarrow E, S_2[U \mapsto \text{subprogram}(E, \text{adom}(A), \text{Dcl}, \text{Stm})]
\]

\[
S_1, E \vdash \text{pms \Rightarrow A, S}_2 \\
S_2, E \vdash \text{nam Nam \Rightarrow subtype_view}(S_1), S_2 \\
E \vdash \text{lookup Idn \Rightarrow subprogram_view}(U, A, \text{some}(S^\text{tp}[S_1]))
\]

\[
S_1, E \vdash \text{dcl function Idn Pms is Dcl begin Stm end; } \Rightarrow E, S_2
\]

\[
S, E \vdash \text{pms } \Rightarrow [], S
\]

not yet defined

\[
S, E \vdash \text{pms Idn : access Nam; Pms; } \ldots \Rightarrow A, S
\]

not yet defined

\[
S, E \vdash \text{pms Idn : access Nam := Exp; Pms; } \ldots \Rightarrow A, S
\]

\[
1 \vdash \text{mod Mde } \Rightarrow M
\]

\[
S_1, E \vdash \text{nam Nam \Rightarrow subtype_view}(S_1), S_1 \\
S_1, E \vdash \text{pms Pms; } \ldots \Rightarrow A_1, S_2
\]

\[
S_1, E \vdash \text{pms Idn : Mde Nam := Exp; Pms; } \ldots \Rightarrow A_1, S_2
\]

\[
1 \vdash \text{mod Mde } \Rightarrow M
\]

\[
S_1, E \vdash \text{nam Nam \Rightarrow subtype_view}(S_1), S_1 \\
S_1, E \vdash \text{pms Pms; } \ldots \Rightarrow A_1, S_2
\]

\[
S, E \vdash \text{pms Idn : Mde Nam; Pms; } \ldots \Rightarrow A, S
\]

4.12.2 Formal Parameter Modes

\[
1 \vdash \text{mod in } \Rightarrow \text{in_mode}
\]

\[
1 \vdash \text{mod in out } \Rightarrow \text{in_out_mode}
\]

\[
1 \vdash \text{mod } \Rightarrow \text{in_mode}
\]

\[
1 \vdash \text{mod out } \Rightarrow \text{out_mode}
\]
4.12.3 Subprogram Bodies

There are no semantics associated with subprogram bodies. The declarations and the statement part of a subprogram body, together with the declaration environment, are stored as a subprogram value. The rules given below define the effect of executing a subprogram value.

The following definition creates an environment for the execution of a procedure body by binding the formal parameter names to the views of the actual parameters. The actual parameters are given as an association but, due to renaming, the names in the association may differ from those of the formal parameters.

\[
bind\_actuals(E, [], [], E)
\]

\[
bind\_actuals(E_{1}[I_{1} \leftarrow W_{1}], P, \text{lds}, E_{2})
\]

\[
bind\_actuals(E_{1}, (P_{1} \times W_{1}) \cdot P, I_{1} \cdot \text{lds}, E_{3})
\]

\[
\text{proc\_exit}(\text{exception}(X, N), \text{exception}(X, N))
\]

\[
\text{not yet defined}
\]

\[
\text{proc\_exit}(\text{exit}(?, N), ?)
\]

\[
\text{not yet defined}
\]

\[
\text{proc\_exit}(\text{func\_return}(?, N), ?)
\]

\[
\text{proc\_exit}(\text{proc\_return}(N), \text{normal}(N))
\]

\[
\text{proc\_exit}(\text{normal}(N), \text{normal}(N))
\]

\[
S_{1}^{PP}[L] = \text{subprogram}(E_{1}, \text{lds}, \text{dcl}, \text{stm})
\]

\[
bind\_actuals(E_{1}, A, \text{lds}, E_{2})
\]

\[
S_{1} [E_{2} \vdash_{\text{dcl}} \text{dcl} \Rightarrow E_{3}]
\]

\[
S_{1} [E_{3} \vdash_{\text{stm}} \text{stm} \Rightarrow S_{2}]
\]

\[
\text{proc\_exit}(S_{2}, S_{3})
\]

\[
\text{subprogram\_body}(S_{1}, A, \text{subprogram\_view}(L, A_{f}, \text{none}), S_{3})
\]

\[
\text{return\_check}(\text{exception}(X, N), S_{1}, \text{exception}(X, N))
\]

\[
\text{not yet defined}
\]

\[
\text{return\_check}(\text{exit}(?, N), S_{1}, S)
\]

\[
\text{not yet defined}
\]

\[
\text{return\_check}(\text{proc\_return}(N), S_{1}, S)
\]

\[
\text{convert\_return\_value}(\text{normal}(N), W_{1}, S_{1}, W_{2})
\]

\[
\text{return\_check}(\text{func\_return}(W_{1}, N), S_{1}, \text{func\_return}(W_{2}, N))
\]

\[
\text{return\_check}(\text{normal}(N), S_{1}, \text{exception}(\text{program\_error}, N))
\]
$S_1^{ppg}[L] = \text{subprogram}(E_1, \text{lds}, \text{Dcl}, \text{Stm})$

\[\begin{array}{c}
\text{bind_actuals}(E_1, A, \text{lds}, E_2) \\
S_1 \left[ \begin{array}{c}
E_2 \vdash \text{dcl Dcl} \\
E_3 \vdash \text{stm Stm} \\
\end{array} \right] S_2 \\
\text{return\_check}(S_2, S_t, S_3) \\
\text{subprogram\_body}(S_1, A, \text{subprogram\_view}(L, A_f, \text{some}(S_1)), S_3)
\end{array}\]

Appropriate rules need to be defined for all predefined operators.

$S_1^{ppg}[L] = \text{operator}(\text{Opn})$

\[\begin{array}{c}
\text{not\ yet\ defined} \\
\text{subprogram\_body}(S_1, A, \text{subprogram\_view}(L, A_f, ?), S_3)
\end{array}\]

A call to an unelaborated subprogram raises program error.

\[\begin{array}{c}
\text{normal}(N)^{ppg}[L] = \text{unelaborated} \\
\text{subprogram\_body}(\text{normal}(N), A, \text{subprogram\_view}(L, A_f, ?), \text{exception}(\text{program\_error}, N))
\end{array}\]

### 4.12.4 Subprogram Calls

The rules given here are incomplete and do not describe subtype and view conversions that are part of a call.

\[\text{return\_value}(\text{func\_return}(W, N), W, \text{normal}(N))\]

\[\text{return\_value}(\text{exception}(X, N), ?, \text{exception}(X, N))\]

\[\begin{array}{c}
\text{parameter\_list}(E, \text{Pss}, A_f, A_a, P) \\
W = \text{subprogram\_view}(?, A_f, ?) \\
\left[ \begin{array}{c}
E \vdash \text{nam Nam} \Rightarrow W \\
P_1 \\
\ldots \\
P_n \\
\end{array} \right] S_2 \\
\text{return\_value}(S_2, W_r, S_3) \\
S_1, E \vdash \text{nam Nam}(\text{Pss}, \ldots) \Rightarrow W_r, S_3
\end{array}\]

\[\begin{array}{c}
\text{parameter\_list}(E, \text{Pss}, A_f, A_a, P) \\
W = \text{subprogram\_view}(?, A_f, ?) \\
\left[ \begin{array}{c}
E \vdash \text{nam Nam} \Rightarrow W \\
P_1 \\
\ldots \\
P_n \\
\end{array} \right] S_2 \\
\text{return\_value}(S_2, A_a, W) \\
S_1, E \vdash \text{stm Nam}(\text{Pss}, \ldots), \Rightarrow S_2
\end{array}\]

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4.12.4.1 Parameter Associations

\[ \text{parameter_list}(E, \text{Pss}, [], [], []) \]

\[ \text{parameter_list}(E, \text{Pss}, F, R, A) \]

\[ \text{parameter_action}(E, \text{Pss}, F_1, R_1, A_1) \]

\[ \text{parameter_list}(E, \text{Pss}, F_1 \cdot F, R_1 \cdot R, A_1 \cdot A) \]

\[ \neg \text{given}_\text{parameter}(\text{Pss}, \text{ldn}, ?) \]

\[ \text{parameter_action}(E, \text{Pss}, (\text{ldn} \times \text{formal(in_mode, St, some(thunk(E_1, Exp))])), (\text{ldn} \times \text{constant_view}(V)), E \vdash_{exp} \text{Exp} \]

\[ \text{given}_\text{parameter}(\text{Pss}, \text{ldn}, \text{Exp}) \]

\[ \text{parameter_action}(E, \text{Pss}, (\text{ldn} \times \text{formal(in_mode, St, ?))), (\text{ldn} \times \text{constant_view}(V)), E \vdash_{exp} \text{Exp} \Rightarrow V) \]

\[ \text{the}_\text{parameter}(\text{Pss}, \text{ldn}, \text{Nam}) \]

\[ \text{parameter_action}(E, \text{Pss}, (\text{ldn} \times \text{formal(out_mode, St, ?))), (\text{ldn} \times W), E \vdash_{nam} \text{Nam} \Rightarrow W) \]

\[ \text{the}_\text{parameter}(\text{Pss}, \text{ldn}, \text{Nam}) \]

\[ \text{parameter_action}(E, \text{Pss}, (\text{ldn} \times \text{formal(out_mode, St, ?))), (\text{ldn} \times W), E \vdash_{nam} \text{Nam} \Rightarrow W) \]

\[ \text{given}_\text{parameter}(\text{ldn} \Rightarrow \text{Exp} \cdot \text{Pss}, \text{ldn}, \text{Exp}) \]

\[ \text{ldn}_1 \neq \text{ldn}_2 \]

\[ \text{given}_\text{parameter}(\text{Pss}, \text{ldn}_2, \text{Exp}) \]

\[ \text{given}_\text{parameter}(\text{ldn}_1 \Rightarrow \text{ldn}_2, \text{Pss}, \text{ldn}_2, \text{Exp}) \]

\[ \text{the}_\text{parameter}(\text{ldn} \Rightarrow \text{Nam} \cdot \text{Pss}, \text{ldn}, \text{Nam}) \]

\[ \text{ldn}_1 \neq \text{ldn}_2 \]

\[ \text{the}_\text{parameter}(\text{Pss}, \text{ldn}_2, \text{Nam}) \]

\[ \text{the}_\text{parameter}(\text{ldn}_1 \Rightarrow \text{ldn}_2, \text{Nam}) \]

4.12.5 Return Statements

\[ \text{normal}(N), E \vdash_{stm} \text{return}; \Rightarrow \text{proc}_\text{return}(N) \]

\[ S_1, E \vdash_{exp} \text{Exp} \Rightarrow V, \text{normal}(N) \]

\[ S_1, E \vdash_{stm} \text{return} \text{Exp}; \Rightarrow \text{func}_\text{return}(\text{constant_view}(V), N) \]

The following needs to be defined to describe the rules of [6.5(6)] through [6.5(21)].

\[ \text{not yet defined} \]

\[ \text{convert}_\text{return_value}(S, W_1, S_t, W_2) \]
4.13 Attributes

\[
is\text{\_scalar\_type}(S_1, S_\text{stp}[L_1])
\]
\[
\text{low\_bound}(\text{range\_of\_subtype}(S_1, S_\text{stp}[L_1])) = V
\]
new\_object($S_1, V, L, S_2$

$S_1, E, \text{subtype\_view}(L_1) \vdash_{\text{att}} \text{first} \Rightarrow \text{object\_view}(L, S_\text{stp}[L_1], \text{constant}), S_2$

\[
is\text{\_scalar\_type}(S_1, S_\text{stp}[L_1])
\]
\[
\text{high\_bound}(\text{range\_of\_subtype}(S_1, S_\text{stp}[L_1])) = V
\]
new\_object($S_1, V, L, S_2$

$S_1, E, \text{subtype\_view}(L_1) \vdash_{\text{att}} \text{last} \Rightarrow \text{object\_view}(L, S_\text{stp}[L_1], \text{constant}), S_2$

\[
S_\text{stp}[L_1] = \text{subtype}(U, C, A)
\]
new\_object($S_1, \text{low\_bound}(R), L, S_2$

$S_1, E, \text{subtype\_view}(L_1) \vdash_{\text{att}} \text{base} \Rightarrow \text{subtype\_view}(L_1), S_2$

\[
S_\text{stp}[L_1] = \text{subtype}($?, \text{index\_constraint}(R \cdot R_s, ?)
\]
new\_object($S_1, \text{low\_bound}(R), L, S_2$

$S_1, E, \text{subtype\_view}(L_1) \vdash_{\text{att}} \text{first} \Rightarrow \text{object\_view}(L, S_1, \text{constant}), S_2$

\[
S_\text{stp}[L_1] = \text{subtype}($?, \text{index\_constraint}(R \cdot R_s, ?)
\]
new\_object($S_1, \text{high\_bound}(R), L, S_2$

$S_1, E, \text{subtype\_view}(L_1) \vdash_{\text{att}} \text{last} \Rightarrow \text{object\_view}(L, S_1, \text{constant}), S_2$

\[
S_\text{stp}[L_1] = \text{subtype}($?, \text{index\_constraint}(R \cdot R_s, ?)
\]
low\_bound($R) = \text{discrete\_val}(N_i)$
\[
S_i = \text{subtype}(\text{universal\_integer\_tn}, \text{no\_constraint}, ?)
\]
high\_bound($R) = \text{discrete\_val}(N_h)$
new\_object($S_1, \text{discrete\_val}(N_h - N_l + 1), L, S_2$

$S_1, E, \text{subtype\_view}(L_1) \vdash_{\text{att}} \text{length} \Rightarrow \text{object\_view}(L, S_1, \text{constant}), S_2$

\[
S_1, E, W \vdash_{\text{att}} \text{first} \Rightarrow \text{object\_view}(L_1, ?, ?), S_2
\]
$S_2, E, W \vdash_{\text{att}} \text{last} \Rightarrow \text{object\_view}(L_2, ?, ?), S_3$
\[
S_3 \vdash \text{content}(L_1) \Rightarrow V_1
\]
$S_3 \vdash \text{content}(L_2) \Rightarrow V_2$

$S_1, E, W \vdash_{\text{att}} \text{range} \Rightarrow \text{make\_range}(V_1, V_2), S_3$
Chapter 5

Exceptions and Optimization

5.1 Introduction

Version 5.0 of the Annotated Draft Ada 9X reference manual [4] contains language that obviates many of the problems associated with section 11.6 of the Ada 83 reference manual [10]. The purpose of this chapter is twofold. The first is to examine the Ada revision as represented by Version 5.0 in light of the earlier Language Precision Team work in this area as published in the LPT Task 1 report [9]. The second is to discuss the consequences of the remaining problems that the semantics of Ada 9X present in the areas of predictability and to offer suggestions for accommodating them in practice. The report concludes with a brief commentary on the Annotated Draft used to support this research.

5.2 The Ada 9X revision of 11.6

Section 11.6 of the Ada 83 reference manual contained explicit permissions to reorder operations or to omit some checks that might propagate predefined exceptions. In Ada 83 the notion of the "effect" of a program or of an operation was not as clearly defined as it is in Ada 9X and the language of the section gave rise to endless discussions such as those captured in AI-315.

As revised, [11.6] contains two substantive paragraphs, (5) and (7). The first gives permission to avoid raising exceptions under some circumstances. The second permits more extensive reordering of operations than was generally considered permissible in Ada 83 by relaxing the requirements for state predictability when an exception handler is entered.

5.2.1 [11.6(5)]

This paragraph allows the implementation to avoid raising exceptions in the face of failures of predefined language exceptions under some circumstances. In the context of a clearer notion of "effect," it is somewhat of an improvement over the language of Ada 83. Even so, the language used in [11.6] is less clear than it might be. Consider the language of [RM-83 11.6(7)]:

A predefined operation need not be invoked at all, if its only possible effect is to propagate a predefined exception. Similarly, a predefined operation need not be invoked if the removal of subsequent operations by the above rule renders this invocation ineffective.

In Ada 83 the term effect is not defined\(^1\) and the meaning of the term is the subject of considerable discussion in AI-315 and elsewhere. The gist of many of the discussions concerns the case in which the

\(^{1}\text{The index entry for "effect" in [RM-83 Appendix I] is "[see: elaboration has no other effect]."}\)
A programmer has apparently written an operation that is sure to raise an exception as a "shorthand" for a raise statement. While this should be considered to be poor programming style, suppressing the operation leads to surprising effects.

We note that Chapter 14 of the Ada 83 rationale which deals with exceptions does not discuss the material contained in [RM-83 11.6] and a search through the text of the rationale for the root string "optimiz" does not provide any appropriate insight.

From the discussions contained in AI-315 it appears that the primary need that the language of [RM-83 11.6(7)] is attempting to capture is the desire to remove code that is dead along its normal execution path even if executing it may (or is certain to) raise an exception due to the failure of a language-defined check. According to [11.6.7.f], the language of [RM-83 11.6(7)] is now reflected in paragraph [11.6(5)] which reads:

An implementation need not always raise an exception when a language-defined check fails. Instead, the operation that failed the check can simply yield an undefined result. The exception need be raised by the implementation only if, in the absence of raising it, the value of this undefined result would have some effect on the external interactions of the program. In determining this, the implementation shall not presume that an undefined result has a value that belongs to its subtype, nor even to the base range of its type, if scalar. [Having removed the raise of the exception, the canonical semantics will in general allow the implementation to omit the code for the check, and some or all of the operation itself.]

5.2.1.1 Defining undefined

Unfortunately, the index for Version 5.0 contains exactly one entry for undefined result, i.e., [11.6(5)]. Although this reference purports to define undefined result, we are given no useful semantics to associate with the term. Thus we are left to attempt to define exactly what is meant by the phrase through other means. A search of the source text for Version 5.0 yields several additional uses of the word undefined. The ones that appear to be related to its use in [11.6(5)] are:

13.9.1 NOTES

19 Objects can become abnormal due to other kinds of actions that directly update the object's representation; such actions are generally considered directly erroneous, however.

In order to reduce the amount of erroneousness, we separate the concept of an undefined value into objects with invalid representation (scalars only) and abnormal objects.

Reading an object with an invalid representation is a bounded error rather than erroneous; reading an abnormal object is still erroneous. In fact, the only safe thing to do to an abnormal object is to assign to the object as a whole.

3.8.1 The discrete_choice others covers all values of its expected type that are not covered by previous discrete_choice_lists of the same construct.

Ramification: For case_statements, this includes values outside the range of the static subtype (if any) to be covered by the choices. It even includes values outside the base range of the case expression's type, since values of numeric types (and undefined values of any scalar type?) can be outside their base range.

7.4 Extensions to Ada 83

2 This document may be obtained by anonymous ftp from ajpo.sri.cmu.edu in the directory public/rationales.
The rules for too-early uses of deferred constants are modified in Ada 9X to allow more cases, and catch all errors at compile time. This change is necessary in order to allow deferred constants of a tagged type without violating the principle that for a dispatching call, there is always an implementation to dispatch to. It has the beneficial side-effect of catching some Ada-83-erroneous programs at compile time. The new rule fits in well with the new freezing-point rules. Furthermore, we are trying to convert undefined-value problems into bounded errors, and we were having trouble for the case of deferred constants. Furthermore, uninitialized deferred constants cause trouble for the shared variable/tasking rules, since they are really variable, even though they purport to be constant. In Ada 9X, they cannot be touched until they become constant.

The first item seems to be the key. The remaining two items use the word undefined in ways that seem to confirm the impressions given by [13.9.1] as a whole. Thus, we see that undefined either applies to a scalar object with an invalid representation or to an abnormal object. Abnormal objects can either be produced by disrupted assignments (with a reference from [13.9.1(5)] back to [11.6], presumably to [11.6(6)]) or (for non-scalars) by a return from a call to either a language defined input procedure or to an imported procedure. It is, perhaps, stretching things to call the latter an operation in the sense of the discussion of [3.2].

Discussion: An operation is a program entity that operates on zero or more operands to produce an effect, or yield a result, or both.

It seems more likely that the operations referred to are akin to the primitive operations partially defined in 3.2.

A type is characterized by a set of values, and a set of primitive operations which implement the fundamental aspects of its semantics.

This leads us to consider the invalid values that can be associated with scalar objects and the predefined operations on scalar types. These are discussed in general in [4.5] where the relevant language appears in [4.5(9)-4.5(12)].

For each form of type definition, certain of the above operators are predefined; that is, they are implicitly declared immediately after the type definition. For each such implicit operator declaration, the parameters are called Left and Right for binary operators; the single parameter is called Right for unary operators. [ An expression of the form X op Y, where op is a binary operator, is equivalent to a function_call of the form “op”(X, Y). An expression of the form op Y, where op is a unary operator, is equivalent to a function_call of the form “op”(Y). The predefined operators and their effects are described in subclauses 4.5.1 through 4.5.6. ]

Dynamic Semantics

[ The predefined operations on integer types either yield the mathematically correct result or raise the exception Constraint_Error. The predefined operations on real types yield results whose accuracy is defined in Annex G, or raise the exception Constraint_Error. ]

To be honest: Predefined operations on real types can “silently” give wrong results when the Machine_Overflows attribute is false, and the computation overflows.

Implementation Requirements
The implementation of a predefined operator that delivers a result of an integer or
fixed point type may raise Constraint_Error only if the result is outside the base range
of the result type.

The implementation of a predefined operator that delivers a result of a floating point
type may raise Constraint_Error only if the result is outside the safe range of the result
type.

Unfortunately, there is a reading of this language that would make it impossible for a predefined
operation on integer types to produce an invalid value. Paragraph (10) requires the operation to
either yield the mathematically correct result or to raise Constraint_error. Paragraph (11) states
that the implementation of the predefined operation may raise Constraint_error only if the result
is outside the base range of the result type. Now, if we assume that “result” in paragraph (11) is
the value produced by the implementation, it is almost certainly the case that this result will be
within the range of the base type; it just will not be mathematically correct. It is likely that what
is intended is

The implementation of a predefined operator that delivers a result of an integer or
fixed point type may raise Constraint_Error only if the [mathematically correct] result
[of the operation] outside the base range of the result type.

As the language is currently defined, there is a direct contradiction between the language of
[4.5(10-11)] and that of [11.6(5)]

If we assume the revised interpretation, then we have a class of operations that can produce
results that are not mathematically correct though they will typically be precisely defined by the
implementation. If 11.6(5) is to have any reasonable meaning, it must be the case that results of this
kind are the undefined results referred to. If this is the case, we have extended the notion of invalid
to include representations of scalar objects that do represent values of the object’s subtype but are
not the mathematically correct values that would be produced without the violated constraint.3
This is a fairly serious extension and deserves more consideration. We will return to this
shortly.

5.2.1.2 Use of undefined results

The implementation note associated with [11.6(5)] seems to raise two distinct points. One, allowing
the removal of dead code, is fairly obvious and seems to be the only clear-cut case. The other
discusses implementation assumptions and seems to involve the extension noted above.

Implementation Note: This permission is intended to allow normal “dead code re-
moval” optimizations, even if some of the removed code might have failed some language-
declared check. However, one may not eliminate the raise of an exception if subsequent
code presumes in some way that the check succeeded. For example:

if X * Y > Integer'Last then
    Put_Line("X * Y overflowed");
end if;

3Addition in a 2's complement n bit machine produces a result that is either a mathematically correct integer
result or the mathematically correct integer result minus 2^n. Consider a 2 bit, 2's complement machine. Its value set
is given as

<table>
<thead>
<tr>
<th></th>
<th>00 (0)</th>
<th>01 (1)</th>
<th>10 (-2)</th>
<th>11 (-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>00 (0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01</td>
<td>01 (1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10 (-2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>11 (-1)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where the values in italics are not

mathematically correct integer results but are both well-defined and have a valid integer representation.
exception
  when others =>
    Put_Line("X * Y overflowed");

If X*Y does overflow, you may not remove the raise of the exception if the code that does
the comparison against Integer'Last presumes that it is comparing it with an in-range
Integer value, and hence always yields False.

As another example where a raise may not be eliminated:

subtype Str10 is String(1..10);
type P10 is access Str10;
X : P10 := null;
begin
  if X.all'Last = 10 then
    Put_Line("Oops");
  end if;

In the above code, it would be wrong to eliminate the raise of Constraint_Error on the
"X.all" (since X is null), if the code to evaluate 'Last always yields 10 by presuming that
X.all belongs to the subtype Str10, without even "looking."

The first point is that if the result of an operation is not subsequently used, then we can ignore the
possibility that execution of an operation might have raised an exception. Examples that illustrate
this situation are somewhat contrived since programmers
generally do not try to write code that is
not useful. For example, we might illustrate the permission by writing something like:

subtype Str10 is String(1..10);
X : Str10 := " ";
begin
  X := "2 01";
  Put_line(X);
  X := "8 01234567";
  Put_line(X);
  X := "10 0123456789";
exception
  when others =>
    Put_Line("OOPS");
end;

Since the scope of X does not extend beyond the end of the block, the value produced by the last
assignment has no effect along the normal path of execution. [11.6(5)] gives permission to ignore the
possibility (in this case, a certainty) that the assignment will raise Constraint_Error. This, in turn,
allows the elision of the entire assignment statement using conventional, “dead code” elimination
techniques.

Note that without the extra permission of [11.6(5)], the code for the last assignment is not dead
since there is a well-defined “effect” along the exceptional path. The extra permission allows
the implementation to restrict its analysis to the normal path. This is important since exception
handlers are dynamically bound and an analysis that shows that an operation is dead along its
exceptional path is generally intractable, while one that shows that the operation is dead along its
normal path may require only local analysis.

While the example is contrived, the situation that it presents appears fairly frequently as the
result of other transformations during code generation and optimization. For example, unrolling

---

4 See the definitions of exceptional and normal paths below.
a loop may well leave dead code in the final iteration, e.g., the code intended to initialize the
next iteration. Value propagation and common subexpression elimination also serve to create dead
variables and dead code to manipulate them.

The language of [11.6(5)] does not ensure that the obviously intended meanings of some realistic
examples will be preserved. For example, in AI-315, Robert Dewar presents the following example:

```
function Add_overflows(A, B: Integer) return Boolean is
  T: Integer
begin
  T := A + B;
  return False;
exception
  when Constraint_Error => return True;
end Add_Overflows;
```

The writer of code like this might hope to detect a potential overflow situation and, perhaps,
use the knowledge to invoke an alternate more robust computation, however, it appears that the
permissions of [11.5] would allow the constraint_error to be ignored, rendering the assignment dead
and allowing its elimination permitting the function to always return False. This could cascade,
if for example, the function were to be inlined, eliminating the code for the alternate computation
which would also appear dead.

Note that both Dewar's example and the example of [11.6(5.e)] have the same intent, the detection
of overflow. They differ in minor details with respect to the way the overflow is detected. It is
probably unreasonable to expect a casual (or even experienced) user of the language to detect the
subtleties. Indeed, the casual observer ought to come to the conclusion that the example of [11.6(5.e)]
cannot work because the implementation result (as opposed to the mathematically correct result) of
X * Y cannot possibly be larger than Integer'Last so that the first Put_Line cannot appear. This
would lead the user towards an example similar to Dewar's which apparently will not work. It is
not clear that there is any easy fix. The approach offered with respect to code motion in [11.6(6)], a
compromise that allows local code motion with local analysis, but does not insist on global analysis
to ensure that the permitted code motion does not disrupt the canonical semantics might also
apply here. This would require that analysis proceed along both the normal and exceptional paths
following from an operation if there were an exception handler for the exception potentially raised
by the operation associated with the innermost sequence of statements containing the operation.

The examples given in (5.e) and (5.g) raise more subtle points. In the absence of [11.6(5)],
Ada's exception model is similar to that of Gypsy. If we use a Gypsy-like model to specify the
Ada operations, we get a possibility of two execution paths from each operation [6]. We will call
these paths the normal and exceptional paths. If none of the language-defined checks fail during
the performance of the operation, execution proceeds along the normal path. If performance of
the operation causes a language-defined check to fail, execution proceeds along the exceptional path.
Associated with each operation is an entry specification which is assumed to be true when the
operation is invoked. Associated with each exit path is an exit specification which is guaranteed to
hold if the path is followed.

For example, the implementation of the integer multiplication operation on a given machine
might be specified as follows:

```
function Machine_Mul(X, Y : Machine_Integer) return Machine_Integer
  entry
  return Machine_Integer
```

“Assumed” is with respect to the operator definition. The implementation is required to “prove” that the
assumption holds every time the operation is invoked. In the case of operations such as Machine_Mul and Machine_CMP
all possible bit patterns represent valid values of Machine_Integer and the entry condition is trivially satisfied.
X, Y in (Machine_Integer’First .. Machine_Integer’Last);

normal exit
  Machine_Mul(X, Y) = Integer_Mul(X, Y) and
  Machine_Mul(X, Y) in (Machine_Integer’First .. Machine_Integer’Last);

exceptional exit
  Machine_Mul(X, Y) /= Integer_Mul(X, Y) and
  Machine_Mul(X, Y) in (Machine_Integer’First .. Machine_Integer’Last) and
  Integer_Mul(X, Y) not in (Machine_Integer’First .. Machine_Integer’Last);

This says that for some (possibly empty) set of input values, machine multiplication is equivalent
to abstract integer multiplication and that execution will proceed along the normal path when this
is the case. When machine and integer multiplication do not produce the same result, we are told
that the true result is not representable as a Machine_Integer but that some result\(^6\) representable
as a machine integer is produced.\(^7\) Now, if we look at a possible specification for the comparison
operator, >, we see a potential problem with the language of the note.

```plaintext
function Machine_CMP(X, Y : Machine_Integer)
    return Machine_CC
entry
  X, Y in (Machine_Integer’First .. Machine_Integer’Last);
normal exit
  Machine_CMP(X, Y) = GT implies Integer_GT(X, Y) and
  ... -- Specifications for other return values
exceptional exit
  false;
```

In this case, we assume a comparison instruction at the machine level that sets some condition
codes to indicate the results of the comparison. GT is a condition code value that indicates the first
operand, interpreted as an abstract integer, was greater than the second operand, also interpreted
as an abstract integer. Note that the only entry condition assumes that the inputs are machine
integers. This condition is satisfied by the exit condition of the multiply operation under either its
normal or exceptional execution. Note also that this operation is defined to always exit normally.

We note that, in program verification, an operational semantics that allows exceptions to be
raised when a language-defined check fails is, in a sense a dual of an operational semantics that
produces an undefined result under the same circumstances. In the absence of a way to effect a
meaningful recovery from failed checks,\(^8\) we must show that the exceptional path is not taken. The
proofs involved are exactly those that are required to show that operations do not produce undefined
results. Languages such as Euclid and Verdi (and C for that matter) use an undefined semantics
while Ada (in the absence of [11.6]) and Gypsy use an exception-based semantics.

For formal reasoning, the differences are largely matters of style. From an implementation
standpoint, unless it can be shown that a given program will not have effects based on undefined
results, the choice is between being able to detect a departure from normal execution and not.
[11.6(5)] requires that exceptions not be suppressed if suppressing them would lead to a visible

\(^6\)For most machines, we could specify exactly what this result is, i.e., how to compute it as a function of X and Y.
It is not undefined in the sense that we know nothing about it.

\(^7\)The first conjunct is redundant since it could be deduced from the other two.

\(^8\)By a meaningful recovery, we mean undoing or overcoming the failed operation in such a way that computation
can resume execution along the normal path, satisfying all the implicit and explicit assumptions of that path. In
practice, this is extremely difficult unless the specification of the normal computation is extremely weak.
effect due to the subsequent use of an *undefined* result. We will examine the process of substituting
operator definitions that produce “undefined” results for those that raise exceptions.

The stated assumptions associated with the example of [11.6(5.e)] are not sufficiently strong. In
most machines, the result of an integer operation that fails an Ada implementation-defined check
will be a valid value of the base type of the operation’s result and, in many cases, it will be a valid
value of the appropriate subtype as well. Thus, the values supplied to the comparison operation
will always be “in-range integer value”s. The *value* is not the issue. If we allow the operation to
omit its exception check, we must consider the result to be more than a value for the purposes of
analysis. In the abstract, the result of an operation that yields an undefined result must be seen as
as a object having two attributes, ‘*value* and ‘*defined*. ‘*value* is of the base type of the result of the
operation while ‘*defined* is boolean.

Under this view, multiply and compare might be defined as follows:

```pascal
function Machine_Mul(X, Y : Machine_Integer)
return Machine_Integer
entry
X'value, Y'value
in (Machine_Integer'First .. Machine_Integer'Last) and
X'defined and Y'defined;
exit
if Machine_MUL'defined then
    Machine_Mul'value(X, Y) = Integer_Mul(X, Y) and
    Machine_Mul'value(X, Y) in
    (Machine_Integer'First .. Machine_Integer'Last) and
else
    Machine_Mul'value(X, Y) /= Integer_Mul(X, Y) and
    Machine_Mul'value(X, Y) in
    (Machine_Integer'First .. Machine_Integer'Last) and
    Integer_Mul(X, Y) not in
    (Machine_Integer'First .. Machine_Integer'Last);
end if
```

```pascal
function Machine_CMP(X, Y : Machine_Integer)
return Machine_CC
entry
X, Y in (Machine_Integer'First .. Machine_Integer'Last) and
X'defined and Y'defined;
exit
Machine_CMP'defined(X, Y) and
(Machine_CMP'value(X, Y) = GT implies Integer_GT(X, Y) and
... -- Specifications for other return values)
```

Under this view of operational semantics, the obligation to take appropriate action in the case
of exceptional operations has shifted from the operation making the check to the operation using
the result. The substitution of “undefined” semantics for “exception” semantics might be done as
follows:

1. Tentatively replace an operation using “exception” semantics with the equivalent operation
using “undefined” semantics. Note that this substitution is dependent on being able to prove
the stronger *entry* specification of the latter.\(^9\)

\(^9\)In the implementation of a language using the “undefined” semantics, we note that there is, in general, no way
to determine by inspection that a given bit string represents an “undefined” value. A two's complement machine
2. If it is possible to prove that the 'defined attribute of the operation's result is always true then the substitution is permitted (see the ramification [11.6(5.a)]) and no further analysis is required.

3. Locate all uses of the result of the replaced operation. If there are none, the substitution is permitted. If there are any, substitute the corresponding "undefined" semantics operation, if necessary, and check the entry specification for references to the 'defined attribute of the result in question. If any using operation assumes that the 'defined attribute of the result is true, the substitution cannot be made.

If the substitution can be made, the net effect is to remove from further consideration the execution path arising from the exception exit of the replaced operator definition. This, in turn, should enable additional program transformations, including removal of the replaced operation since it is known to be without an externally visible effect. The removal of the exception path may permit additional removals since dependencies along the path no longer require consideration. Removal of the operation may permit additional operations to be removed since its inputs are now referenced at fewer places.

Note that this is an analytical approach, not an implementation. Typically, there is no practical way to tag values with an indication that they represent an "undefined" result. When this is the case there is no way for subsequent operations in an implementation to check explicitly for the undefined property. In addition, the amount of analysis required to detect all uses of a result may require extensive reasoning about the values of index expressions, etc., when the values are components of arrays or other composite structures.

Implicit in the assumption of the last paragraph of [11.6(5.e)] appears to be an additional assumption that the values being compared are also the mathematically correct results of the operations that produced them, i.e., that they are not undefined.

It is probably the case that the only permission actually granted by [11.6(5)] is the removal of code that is "dead" along its normal exit path regardless of any effects along its exceptional exit path.

5.2.1.3 Bounded errors and erroneous executions

The Ada 9X revision has made a serious attempt to reduce the number and types of circumstances under which a program's execution can become erroneous. Since an erroneous execution can exhibit arbitrary behavior, this change is highly desirable. Recognizing that most implementations do reasonable things in the face of program errors that violate language semantics, the notion of a bounded error has been introduced. The bounded errors associated with invalid representations are discussed in [13.9.1]

**Bounded (Run-Time) Errors**

If the representation of a scalar object does not represent a value of the object's subtype (perhaps because the object was not initialized), the object is said to have an invalid representation. It is a bounded error to read the value of such an object. If the error is detected, either Constraint_Error or Program_Error is raised. Otherwise, execution continues using the invalid representation. The rules of the language outside this subclause assume that all objects have valid representations. The semantics of operations on invalid representations are as follows:

- If the representation of the object represents a value of the object's type, the value of the type is used.

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• If the representation of the object does not represent a value of the object's type, the semantics of operations on such representations is implementation-defined, but does not by itself lead to erroneous or unpredictable execution, or to other objects becoming abnormal.

Erroneous Execution

A call to an imported function or an instance of Unchecked_Conversion is erroneous if the result is scalar, and the result object has an invalid representation.

Ramification: In a typical implementation, every bit pattern that fits in an object of an integer subtype will represent a value of the type, if not of the subtype. However, for an enumeration or floating point type, there are typically bit patterns that do not represent any value of the type. In such cases, the implementation ought to define the semantics of operations on the invalid representations in the obvious manner (assuming the bounded error is not detected): a given representation should be equal to itself, a representation that is in between the internal codes of two enumeration literals should behave accordingly when passed to comparison operators and membership tests, etc. We considered requiring such sensible behavior, but it resulted in too much arcane verbiage, and since implementations have little incentive to behave irrationally, such verbiage is not important to have.

If a stand-alone scalar object is initialized to a an [sic] in-range value, then the implementation can take advantage of the fact that any out-of-range value has to be abnormal. Such an out-of-range value can be produced only by things like unchecked conversion, input, and disruption of an assignment due to abort or to failure of a language-defined check.

This depends on out-of-range values being checked before assignment (that is, checks are not optimized away unless they are proven redundant).

The language of the Ramification sounds reasonable, but it flies in the face of the conventions used in many of the logics used to reason about program behavior. Typically, undefined is a loaded term in these logics. Undefined is used to represent a distinguished value about which nothing can be proven. Thus if undefined = undefined. This is too strong for implementation semantics in most cases. In any implementation in which evaluating $x$ is free of side effects that could change its value, $x = x$ is true even if $x$ has an invalid representation or is undefined so long as the implementation of = simply involves comparing bit patterns. In the absence of a requirement to actually evaluate $x$, it should be unconditionally ok to substitute true for the equality.

On the other hand, this language seems to have the potential for conflicts with semantics of "undefined" results discussed above in connection with [11.6(5)]. The relationship between undefined as used in [13.9.1] and [11.6(5)] should be further clarified.

5.2.2 [11.6(7)]

This paragraph allows fairly arbitrary reordering of actions within the scope of an exception handler by reducing the expectations that the programmer may have concerning the state of the computation at the time that the handler is entered. This is essentially the "Undefined" execution order of [9, section 2.6.5]. We note that the only effective actions that a programmer can take when an exception handler is entered in the face of this kind of reordering is to assign normal values to all variables that might have become abnormal due to operations disrupted by the exception.

The first sentence of this paragraph is complex and convoluted and calls out to be simplified or clarified. The following discussion may aid in finding more suitable language. An exception_handler is optionally associated with a handled_sequence_of_statements which contains a sequence_of_statements
and is, among other things, the operational portion of a task_body. When an exception is raised, it will either be handled or cause the containing task_body to terminate. In either case, all that the user can expect to know is that the exception was raised somewhere in the code of the sequence_of_statements component of the handled_sequence_of_statements that contains the exception_handler just entered or that constitutes the operational part of the task being terminated. The reordering that can be done is limited in two respects.

1. The operation that raises the exception due to a failed language-defined check cannot have been moved into the code of an independent subprogram, and

2. The operation that raises the exception due to a failed language-defined check cannot have been moved into the code of some abort-deferred operation.

Just breaking up the sentence may help. Instead of

If an exception is raised due to the failure of a language-defined check, then upon reaching the corresponding exception_handler (or the termination of the task, if none), the external interactions that have occurred need reflect only that the exception was raised somewhere within the execution of the sequence_of_statements with the handler (or the task_body), possibly earlier (or later if the interactions are independent of the result of the checked operation) than that defined by the canonical semantics, but not within the execution of some abort-deferred operation or independent subprogram that does not dynamically enclose the execution of the construct whose check failed.

perhaps language similar to the following would be more understandable.

• If an exception is raised due to the failure of a language-defined check, then upon reaching the corresponding exception_handler (or the termination of the task, if none), the external interactions that have occurred need reflect only that the exception was raised somewhere within the execution of the sequence_of_statements with the handler (or the task_body). It may appear by the canonical semantics (or later if the checked operation). It may not appear as if the exception were raised within the execution of some abort-deferred operation or within the execution of an independent subprogram that does not dynamically enclose the execution of the construct whose check failed.

5.3 Living with the “Canonical Semantics”

The canonical semantics define a potentially very large family of valid executions. This is due to the numerous places in which the language definition allows operations to be performed in an arbitrary order. An implementation is free to select any order under these circumstances. In the absence of order dependencies and tasking considerations, all canonical executions should produce the same externally visible effect. Order-dependent side effects, including exceptions raised due to the failure of language-defined checks, can affect the effect of the program. The problem is twofold:

1. Reducing the potential effect space of the program, and

2. Determining which execution the implementation has selected.

First of all, it is worth noting that this kind of problem is not unique to Ada (both Ada 83 and Ada 9X). Most programming languages, including C and C++, admit similar behaviors, either implicitly or explicitly. Ada is more explicit about them. In general, the failure of languages to define or enforce restrictive canonical executions is attributed to a need for flexibility in order to achieve run-time efficiency. There is tension between this need and the requirements for predictable
program behavior, which are imposed by small segments of the user community, typically those users associated with safety and security critical applications. Predictable behavior is usually defined as having a rigorous semantic definition that allows the formal verification of programs written in the language, preferably using mechanical aids.

In theory, one could reason about Ada programs by enumerating the set of possible executions and reasoning individually about each one. A program could then be said to exhibit a given property if each of its possible executions could be shown to exhibit the property. In practice, the combinatorics of potential execution choices are likely to render this approach infeasible for any non-trivial program. If we assume that a compiler conforming to the language standard produces code that follows one of the set of canonical executions of a given program, it seems a waste of time to prove properties of the set as a whole unless there is a need to guarantee the behavior of the program under all possible conforming implementations. This is seldom the case. Further complications arise when it is possible to show that some, but not all, members of the canonical execution set exhibit the desired property. In this case, it is essential to determine whether the implementation being considered exhibits the property.

There are several possibilities. The first is to attempt to reduce the size of the set of canonical executions to a tractable size and possibly to a single member. The second is to discover the member of the canonical execution set that has been chosen by a particular implementation and to reason about that execution alone.

5.3.1 Restricting the execution set size

The size of the canonical execution set about which one must reason can be reduced by one of two methods; reducing the choices available to the implementation or finding equivalence classes within the set, or by some combination of the two. Ada 9X provides some means for imposing order. For example, the order of the association of operands with a sequence of operators of the same precedence can be controlled by the explicit use of parentheses. The introduction of explicit intermediate variables and assignments should have a similar effect. For example, suppose that side effects exist such that the value resulting from the evaluation of \(<exp1>\) depends on whether it is evaluated before or after \(<exp2>\), but that there are no other order dependencies between the expressions. Further assume that the evaluations produce results of some integer subtype.

\[
\begin{align*}
A &:= <exp1> + <exp2>; \\
A1 &:= <exp1>; \\
A2 &:= <exp2>; \\
A &:= A1 + A2;
\end{align*}
\]

Either of the possible results is a member of the canonical execution set for this fragment. If we want to ensure that \(<exp1>\) is evaluated first, we might write:

\[
\begin{align*}
A1 &:= <exp1>; \\
A2 &:= <exp2>; \\
A &:= A1 + A2;
\end{align*}
\]

It is not clear that the additional freedoms to reorder operations granted by [11.6(6)] allow an implementation to ignore structuring of this kind, but aggressive optimizations in compilers for other languages are known to do so in some cases. Presumably, the dependency between the two expressions either becomes explicit or the implementation will be forced to recognize that it cannot assume independence because the expressions invoke separately compiled routines and it will be forced to produce the intended result.\(^{10}\)

Note that the explicitly ordered code may still exhibit a family of canonical executions. In the expression \(A1 + A2\), the language allows \(A1\) or \(A2\) to be "evaluated" first. We claim that given

\(^{10}\)If the expressions are sufficiently complex and the dependencies between them limited, it may be possible to interleave their evaluations. This would be permitted under the general freedoms noted in [11.6(3)]
appropriate type declarations (and barring some pathological implementation of +) that both orders will be equivalent and trivial.

By a combination of these two techniques, forcing orders where order makes a difference and creating situations where it is easy to show that, at least locally, all canonical executions are equivalent, it should be possible to reduce the number of canonical executions associated with a program to a tractable number. In most cases, the analysis of the remaining canonical executions should show that language-defined checks will not fail, rendering moot the freedoms of [(11.6)]. The utility of this approach depends on the implementation or implementations of interest ensuring that the canonical semantics are honored.

If the notion of a subset of Ada 9X for High Integrity systems, as recently proposed by Brian Wichmann, is accepted, the subset definition could restrict the ordering freedoms permitted by the primary language definition. This approach would necessitate subset compilers to enforce the restrictions, but would offer a higher degree of assurance than the use of general purpose compilers. If a subset is adopted with the notion of supporting mechanical verification, it is not unreasonable to expect that integrated environments will be developed in which both the verification and implementation tools are based on the same semantic assumptions.

5.3.2 Discovering the execution

Another approach to the problem of a canonical execution set is the determination of the actual execution produced for a given program by a given implementation. This requires that the compiler output its object code in a form that allows the user to determine the actual execution that will occur when the program is executed. Implementations conforming to the Safety and Security Annex, in particular to section [(H.3)], will provide this kind of information. With an appropriate transformation of the object code back into an appropriate Ada or Ada-like source form, it should be possible to perform source level analysis or verification on the program while maintaining confidence that the results are, in fact, applicable to the compiled program.

It is clear that this approach requires facilities that are not present in many, if not all, existing compilers, but the Annex should encourage development of this facility.

5.4 Observations on the Reference Manual

In the course of using the Reference Manual in the preparation of this chapter, a number of general shortcomings have been observed. These have more to do with presentation than with substance and can be fixed prior to the release of the final document.

First of all, we wish to compliment the Mapping/Revision Team on the content and style of the manual. Not only is the wording a substantial improvement over the Ada 83 Reference Manual, but the inclusions of the annotations provide useful and substantive insight into the workings of the language. It is to be hoped that the annotated version will be maintained along with its "official" subcomponents and that it will see widespread use by serious students of Ada 9X.

This said, there are ways in which the Reference Manual could be further improved.

1. The index is not sufficiently comprehensive. On a number of occasions, an attempt to trace the consequences of a definition found that the defining occurrence was the only reference in the index. Fortunately, the source files are available and can be searched as necessary; however, any term important enough to be marked as a definition is important enough to have the consequences of that definition tracked. A presentation similar to that used in the index for syntactic constructs should be adapted for defined terms, i.e., a defining reference followed by using references.

2. The index does not appear to cover the annotations. Extending it to this level would greatly aid in the use of the annotated manual.
3. The Syntax Cross-Reference would be much more useful if references for the defining occurrence as well as the using occurrences were given. For example, we find from the cross-reference that a task_body is used in the definition of a proper_body in [3.11], but we must go to the main index to discover that a task_body is defined in [9.1(6)]. Extending the indexing to the numbered paragraph level as is done in the index would also be useful.

4. The marginal paragraph numbering is incomplete and inconsistent. For example, the paragraphs following the example codes of [11.6(5.e)] and [11.6(5.g)] are not numbered while similar paragraphs elsewhere, e.g., [8.3(29.o)] are. There is a similar problem in [13.9.1(12.b)] as well. This is probably the result of the mechanical approach taken to inserting the annotating scribe commands. In preparing the \texttt{\LaTeX} source for the Annotated version of the Ada 83 reference manual, I found it necessary to insert this material manually.

5. In some cases, precision seems to have been sacrificed for readability. This occurs when it is difficult to determine the antecedents for pronouns or where the same noun appears in an ambiguous context. An example is [4.5(11)] discussed in Section 5.2.1.1 on page 87 above. More liberal use of the @Redundant (or a similar) construct might alleviate this problem in the annotated version.
Chapter 6

Conclusions

We did not expect to formulate a complete semantic definition (even for the sequential part of Ada) in this project; there were simply not enough resources to do so. What we did expect was to gain some insights into the structure of the language, and to identify some problems either with the description in the Reference Manual or in the design of the language itself, and to contribute to the development of Ada 9X by suggesting improvements to the description or design. Those expectations were met to some degree; for example, we identified some flaws (that have now been fixed) in the design in the area of per-object constraints; we identified some conceptual and some wording problems in the area of floating point and developed a model that was used in the development of new wording for the Reference Manual; and we identified some incompleteness in the description of actual subtypes.

However, we did not make as much progress in the natural semantics definition as we had originally hoped. It was more difficult to understand the supposedly trivial parts of the language than we had imagined. Large-scale languages like Ada do not have neat, independent parts; rather, each feature is affected in some measure by the others. For example, the type system is affected by the concurrency mechanism (e.g., task types), by the packaging mechanism (e.g., private types), and in several ways by the object-oriented features (e.g., access discriminants, per-object expressions, class-wide types). So, indeed, there are no really trivial aspects of the language. In the original LPT project, we had felt that it would be a waste of effort to develop a formal model for things that "everyone understands". Our recent efforts, however, have shown us that there are interesting problems lurking at the fringes of even these areas.

Even though there are serious gaps in the definition, a considerable amount of groundwork has been done. We have identified most of the basic semantic domains that must be used in a full definition, we developed structuring mechanisms for the definition that allow us to describe many of the implementation freedoms, and we have several tools (such as the type checker for the Prolog representation of the definition, and the tool that derives \LaTeX source from the Prolog representation) that help in the production and documentation of the definition. So, we feel that we have made a good start in the direction of a complete description of the sequential part of the language.

We are not sure how easily our framework could be adapted to deal with concurrency. The influence of tasking in DDC's formal definition of Ada 83 [1] is pervasive, and we suspect that incorporating concurrency into our definition would similarly affect every part of the model.

6.1 Implementation Freedoms

One impediment to writing a formal definition like ours is the high degree of underspecification in the Reference Manual. This allows implementations considerable freedom to choose orders of actions, accuracy of results, base ranges of types, and so on. These freedoms can be difficult to
model; where an implementation need only produce one acceptable result, our semantics tries to
describe all acceptable results.

Modeling these freedoms sometimes forces our formal model to differ in significant ways from an
implementation (and to use representations that no implementation is likely to use). For a simple
example, consider the following rule about access-to-subprogram values:

Two access-to-subprogram values are equal if they are the result of the same evalu-
ation of an Access attribute_reference, or if both are equal to the null value of the
access type. Two access-to-subprogram values are unequal if they designate different
subprograms. It is unspecified whether two access values that designate the same sub-
program but are the result of distinct evaluations of Access attribute_references are
equal or unequal.

In order to model this, we are forced to use a representation of access-to-subprogram values that
consists of both a reference to the designated subprogram and an “instance” value that tells which
evaluation of an Access attribute gave the access value. Each evaluation of an Access attribute
increments this instance value, so that we can determine whether two access values derive from the
same evaluation or not. Our definition of the equality function checks both the subprogram reference
and the instance number, and can give a result of equal, unequal, or unknown. Such a representation
is unlikely to be used in any implementation of Ada 9X.

An implementor of the language need not be concerned with all these freedoms; just one particular
implementation choice needs to be made and the existence of other possible choices is irrelevant.
A programmer does not necessarily need to be concerned about all the alternative orders; it is
usually possible to write programs in such a way that the specific choice made by an implementa-
tion does not matter (for example, by avoiding side effects in functions, restricting the statements in a
package_body to affect only variables local to the package, and so on). On the other hand, anyone
trying to read an Ada program may indeed be concerned about the different possible outcomes of
an execution (especially if the writer has not been careful to avoid situations where the different
orders matter). So, our model, while unnatural if compared to an implementation, is quite natural
as a description of the complexities that careful readers must deal with.

6.2 Notation and Tools

The natural semantics framework seems to have worked fairly well, although there were a few
awkward aspects to our formalization of the language semantics. In particular, our need to introduce
explicit sequencing and arbitrary-order combinations of actions (in two slightly different variations)
seems somewhat artificial. However, this mechanism of actions allows us to present reasonably
concise descriptions of many language features.

The use of Prolog to make the definition executable (and type-checkable) was a great help. We
have been able to execute parts of the definition to confirm that it expresses what we intended. The
type checker was able to find a number of trivial errors in our semantics. There is some price to
be paid, however; it is sometimes inconvenient to express a rule in a manner acceptable to Prolog.
This is particularly evident in the descriptions of the various semantic domains and the primitive
functions acting over those domains. Prolog does not support defined functions (instead, relations
must be used). We used a program to convert the Prolog code into the \LaTeX{} source used for this
report. This program is able to introduce functional notation in places where we have instructed
it to, so at least our published form of the rules can use a more expressive notation than Prolog.
But this is still rather unsatisfactory. It is possible that other tools might be able to provide better
mechanical support.
6.3 Bounded Errors

A number of rules and concepts were added to Ada 83 in order to make programs more predictable. For example, many situations leading to erroneous executions in Ada 83 have been made into bounded errors. For these errors, a range of possible outcomes is described. This seems like a beneficial change. However, there is a price to be paid for this benefit: the model for a feature using bounded errors can be substantially more complex than a model using erroneous executions. For example, in order to change the evaluation of an uninitialized scalar variable from erroneous to a bounded error, it was necessary to introduce the notion of “invalid representations” of scalar objects. The addition of this notion has an influence on a number of other areas of the language (e.g., relational operators, membership tests, and type conversions). So, the formal model is more complex, which means that formal predictions about programs are harder to derive. On the other hand, the execution of programs is more predictable in the sense that these executions are more constrained (the old rules allowed any behavior, whereas the new rules are more specific).

6.4 Structure of Models

It does not seem possible, using our methods, to formulate a model for Ada 9X that is simultaneously concise, comprehensible, broad, and accurate. Accounting for all the special cases of features adds so much detail to the model that it becomes unusable.

Textbook writers face a similar dilemma; if too much detail is presented, readers will find the text impenetrable. Therefore, authors present simplified descriptions of parts of the language. These simplified descriptions, even when they lead the reader to draw incorrect conclusions about the behavior of some programs, are nevertheless useful to readers who are first learning the language. In a later part of a book, an author may elaborate on some of these missing details, and may need to contradict some of his earlier oversimplified assertions.

We do not know exactly how to make layered formal models using a similar structure. In most formal notations, it is not possible to override an earlier assertion with a more detailed assertion. Even if this were allowed, it is unclear how a user of such a layered formal model would know when the simpler part of model was applicable.

In the model developed in this report, we have tried to approach this ideal of structured models in a very modest way through our use of the “unpredicted” outcomes to simplify the formal model; we can certainly imagine a more complex version of this model that would, in fact, make predictions where this simpler model refuses to. However, we have not had the resources to develop the more complex model.
Bibliography


Appendix A
Official comments submitted

This appendix lists the official comments submitted by the LPT. For each comment, we give its official "key" number, its title, and a short description of the comment or its effect on the Standard.

Comment 93-3209.a: Too much extra permission to remove checks. The conceptual framework of section 11.6 has been changed.

Comment 93-3398.a: per-object constraints and the "current instance". A new rule [3.8(13)] has been added to avoid the problem described.

Comment 93-3511.a: What is the base range of an enumeration type?

Comment 93-3547.a: constants that aren't. The problem described has been identified as an erroneous execution in [13.9.1(13)]

Comment 93-3547.b: subcomponents that are constrained by their initial value. A new legality rule [3.6(11)] was added.

Comment 93-3547.c: conversion to a type with aliased components. A new legality rule [3.6(11)] was added.

Comment 93-3574.a: interleaving evaluation and conversion. Several paragraphs have been modified to clarify the rules.

Comment 93-3574.b: reassociation of sequences of predefined operators. A note [4.5(13.b)] has been added to the Annotated Reference Manual. It is unclear, however, that the note clarifies the issue raised in this comment.

Comment 93-3575.a: aliased subcomponents with per-object constraints. A new legality rule [3.6(11)] was added.

Comment 94-3621.a: Initialize a discriminant before any subcomponents that depend on it. This clause has been added to the rules of [3.3.1(20)].

Comment 94-3760.a: Phraseology. An inaccurate statement has been reworded in [M(1)].

Comment 94-3761.a: Phraseology. Some wording in [3.3] has been improved.

Comment 94-3762.a: Phraseology.

Comment 94-3763.a: Phraseology. The wording of [3.2.3] has been clarified.

Comment 94-3764.a: Phraseology. A clarifying cross-reference was added.

Comment 94-3765.a: Phraseology.

Comment 94-3901.a: Are first subtypes of enumeration types constrained? A clause has been added to [3.5.1(10)] to answer this question.

Comment 94-3901.b: First subtypes of discriminated types are unconstrained. This clause has been added to [3.7(26)].


Comment 94.3901.d: Predefined operators and invalid scalar components. The entire discussion of "invalid" scalars has been modified in version 5.0 of the Reference Manual. This comment and
the following two are cited in the changes.

Comment 94-4045.a: Model for valid and invalid values. See above.
Comment 94-4054.a: Model for valid and invalid values. See above.
Comment 94-4064.a: What is the actual subtype of a formal object? The question has been answered in [6.4.1(15)].
Comment 94-4065.a: Normalizing composite objects. An explicit statement about restoring objects to normal state appears in [13.9.1(7)].
Comment 94-4149.a: Incomplete definition of 'expected profile' and 'corresponding parameter'. Some rules have been clarified.
Comment 94-4171.a: Actual subtypes and aliased views. The rules in [3.10(9)] have been reworded.

The remaining comments were sent too late to affect version 5.0 of the Reference Manual. Some of them are addressed in the electronically-distributed version 5.3, as noted below. Furthermore, the floating-point annex is under revision to address some of the comments on the floating-point model.

Comment 94-4448.a: Signed zeroes not permitted as floating point values.
Comment 94-4454.a: Model-oriented floating point attributes.
Comment 94-4455.a: Relation between requested precision and model numbers.
Comment 94-4481.a: Inappropriate references to Annex G.
Comment 94-4482.a: Are S'Model, S'Machine deterministic?
Comment 94-4486.a: Symmetry of floating point types.
Comment 94-4489.a: Derivation from a floating point type.
Comment 94-4535.a: Derived types with new discriminants are extensions. A sentence has been added.
Comment 94-4535.b: Discriminants used in constraints in derived type definitions.
Comment 94-4535.c: Incorrect rules for uses of new discriminants in constraint on parent. A new rule has been added.
Comment 94-4535.d: Current instance of a derived type. A note has been added in a “to be honest” section of the AARM.
Comment 94-4572.a: An object that is not {a part of} a formal parameter.
Comment 94-4572.b: For aliasing, the type of the formal, not the part, matters.
Comment 94-4572.c: When does an access path exist?
Comment 94-4587.a: Incompatibility between semantics of the core and annex G?
Comment 94-4796.a: reading a composite with an uninitialized scalar component. The wording of this rule has been changed.
Appendix B

Intermediate Syntax

This appendix describes the decorated abstract syntax representation of Ada 9X programs. This representation includes all static semantic information needed for defining the dynamic semantics.

One approach, not taken here, is to define a representation of semantic information such as types, subtypes, and overload information and defining necessary tree attributes. Instead of using semantic attributes we chose to give a purely syntactic representation of necessary information by introducing new kinds of tree nodes and new synthetic names.

For instance, the results of overload resolution are captured by the introduction of new unique names and suitable renaming of overloaded entities and their use. Crucial type information is represented using existing syntax for qualified expressions. Thus the result of static semantic analysis is a normalized abstract syntax tree. The details of this normalization are described below.

Declarations that introduce multiple names are replaced by static analysis with multiple declarations introducing single names where this is legal.

Constructs for which no abstract syntax is provided are either not treated in this definition (e.g., tasking) or have not significance for the dynamic semantics (e.g., generics).

B.1 Syntactic Domains

The following is a complete listing of the term algebra used to represent abstract syntax trees. The constructors are grouped by sorts and are arranged alphabetically.

B.1.1 Component Associations (Aca)

array_compassoc : Aca* \to \text{Exp} \to \text{Aca}

B.1.2 Aggregates (Agg)

extagg : Exp, Rca* \to \text{Agg}
named_arrayagg : Aca* \to \text{Agg}
null_extagg : Exp \to \text{Agg}
nul_recordagg : \to \text{Agg}
other_arrayagg : Exp*, Exp \to \text{Agg}
pos_arrayagg : Exp* \to \text{Agg}
recordagg : Rca* \to \text{Agg}
B.1.3 Case Alternatives (Alt)

\[ \text{Alt\$list} : \text{list(Alt)} \rightarrow \text{Alt} \]
\[ \text{case\$Alt} : \text{Dch}, \text{Stm} \rightarrow \text{Alt} \]

B.1.4 Choice Lists (Ccl)

\[ \text{list\$choice} : \text{id}^* \rightarrow \text{Ccl} \]
\[ \text{others\$choice} : \rightarrow \text{Ccl} \]

B.1.5 Context Items (Cit)

\[ \text{with\$context} : \text{Nam}^* \rightarrow \text{Cit} \]

B.1.6 Component Declarations (Cmp)

\[ \text{aliased\_comp\$decl} : \text{Id}, \text{Sid} \rightarrow \text{Cmp} \]
\[ \text{comp\$decl} : \text{Id}, \text{Sid} \rightarrow \text{Cmp} \]
\[ \text{init\_aliased\_comp\$decl} : \text{Id}, \text{Sid}, \text{Exp} \rightarrow \text{Cmp} \]
\[ \text{init\_comp\$decl} : \text{Id}, \text{Sid}, \text{Exp} \rightarrow \text{Cmp} \]

B.1.7 Compilation Units (Cmp)

\[ \text{lib\$unit} : \text{Cit}^*, \text{Dcl} \rightarrow \text{cmu} \]
\[ \text{private\$unit} : \text{Cit}^*, \text{Dcl} \rightarrow \text{cmu} \]
\[ \text{sub\$unit} : \text{Cit}^*, \text{Nam}, \text{Dcl} \rightarrow \text{cmu} \]

B.1.8 Conditions (Cnd)

\[ \text{Exp\$condition} : \text{Exp} \rightarrow \text{Cnd} \]

B.1.9 Constraints (Cns)

\[ \text{constr\_delta\$constr} : \text{Exp}, \text{Cns} \rightarrow \text{Cns} \]
\[ \text{constr\_digits\$constr} : \text{Exp}, \text{Cns} \rightarrow \text{Cns} \]
\[ \text{delta\$constr} : \text{Exp} \rightarrow \text{Cns} \]
\[ \text{digits\$constr} : \text{Exp} \rightarrow \text{Cns} \]
\[ \text{discr\$constr} : \text{Dca}^* \rightarrow \text{Cns} \]
\[ \text{index\$constr} : \text{Rng}^* \rightarrow \text{Cns} \]
\[ \text{range\$constr} : \text{Rng} \rightarrow \text{Cns} \]

B.1.10 Discriminant Associations (Dca)

\[ \text{named\$assoc} : \text{Id}, \text{Exp} \rightarrow \text{Dca} \]

B.1.11 Discrete Choices (Dch)

\[ \text{discr\_other\$choice} : \rightarrow \text{Dch} \]
\[ \text{exp\$choice} : \text{Exp} \rightarrow \text{Dch} \]
\[ \text{range\$choice} : \text{Rng} \rightarrow \text{Dch} \]

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B.1.12 Declarations (Dcl)

\[
dcl\text{\$}list : \quad \text{list}(\text{Dcl}) \to \text{Dcl}
\]
\[
a\_c\_i\_obj\$decl : \quad \text{Id, Sid, Exp} \to \text{Dcl}
\]
\[
a\_c\_obj\$decl : \quad \text{Id, Sid} \to \text{Dcl}
\]
\[
a\_i\_obj\$decl : \quad \text{Id, Sid, Exp} \to \text{Dcl}
\]
\[
a\_obj\$decl : \quad \text{Id, Sid} \to \text{Dcl}
\]
\[
c\_i\_obj\$decl : \quad \text{Id, Sid, Exp} \to \text{Dcl}
\]
\[
c\_obj\$decl : \quad \text{Id, Sid} \to \text{Dcl}
\]
\[
\text{real}\_\text{const}\$decl : \quad \text{Id, Exp} \to \text{Dcl}
\]
\[
\text{int}\_\text{const}\$decl : \quad \text{Id, Exp} \to \text{Dcl}
\]
\[
d\_\text{ext}\$decl : \quad \text{Id, Dcp, Sid} \to \text{Dcl}
\]
\[
d\_\text{i}\_\text{type}\$decl : \quad \text{Id, Dcp} \to \text{Dcl}
\]
\[
\text{d}\_\text{type}\$decl : \quad \text{Id, Dcp, Tdf} \to \text{Dcl}
\]
\[
\text{exception}\$\text{renaming} : \quad \text{Id, Nam} \to \text{Dcl}
\]
\[
\text{except}\$\text{decl} : \quad \text{Id} \to \text{Dcl}
\]
\[
\text{ext}\$\text{decl} : \quad \text{Id, Sid} \to \text{Dcl}
\]
\[
i\_\text{obj}\$decl : \quad \text{Id, Sid, Exp} \to \text{Dcl}
\]
\[
i\_\text{i}\_\text{type}\$decl : \quad \text{Id} \to \text{Dcl}
\]
\[
o\_\text{obj}\$decl : \quad \text{Id, Sid} \to \text{Dcl}
\]
\[
o\_\text{i}\_\text{type}\$\text{renaming} : \quad \text{Id, Nam, Nam} \to \text{Dcl}
\]
\[
\text{s}\_\text{subp}\$\text{spec} : \quad \text{Sps} \to \text{Dcl}
\]
\[
\text{subp}\$\text{body} : \quad \text{Sps, Dcl, Stm} \to \text{Dcl}
\]
\[
\text{subp}\$\text{renaming} : \quad \text{Sps, Nam} \to \text{Dcl}
\]
\[
\text{subp}\$\text{spec} : \quad \text{Sps} \to \text{Dcl}
\]
\[
\text{subtype}\$\text{decl} : \quad \text{Id, Sid} \to \text{Dcl}
\]
\[
\text{type}\$\text{decl} : \quad \text{Id, Tdf} \to \text{Dcl}
\]
\[
c\_\text{type}\$\text{decl} : \quad \text{Id, Tdf} \to \text{Dcl}
\]
\[
c\_d\_\text{type}\$\text{decl} : \quad \text{Id, Tdf, Dcp} \to \text{Dcl}
\]

B.1.13 Discriminant Parts (Dcp)

\[
\text{box}\$\text{discr} : \quad \to \text{Dcp}
\]
\[
\text{list}\$\text{discr} : \quad \text{Dcs} \to \text{Dcp}
\]

B.1.14 Discriminant Specifications (Dcs)

\[
\text{acc}\$\text{discr} : \quad \text{Id, Nam} \to \text{Dcs}
\]
\[
\text{acc}\_\text{init}\$\text{discr} : \quad \text{Id, Nam, Exp} \to \text{Dcs}
\]
\[
\text{init}\$\text{discr} : \quad \text{Id, Nam, Exp} \to \text{Dcs}
\]
\[
\text{simple}\$\text{discr} : \quad \text{Id, Nam} \to \text{Dcs}
\]

B.1.15 Exception Choices (Ech)

\[
\text{named}\$\text{except} : \quad \text{Nam} \to \text{Ech}
\]
\[
\text{others}\$\text{except} : \quad \to \text{Ech}
\]
B.1.16 Else-If Clauses (\textit{Eif})

\texttt{\textbf{elif}list : Eif \rightarrow Eif}
\texttt{\textbf{elsif}clause : Cnd, Stm \rightarrow Eif}

B.1.17 Expressions (\textit{Exp})

\texttt{\textbf{and\_then}\$Exp : Exp, Exp \rightarrow Exp}
\texttt{\textbf{exp}\$alloc : Exp \rightarrow Exp}
\texttt{\textbf{in\_name}\$exp : Exp, Nam \rightarrow Exp}
\texttt{\textbf{in\_range}\$exp : Exp, Rng \rightarrow Exp}
\texttt{\textbf{in\_type}\$exp : Exp, Nam \rightarrow Exp}
\texttt{\textbf{name}\$exp : Nam \rightarrow Exp}
\texttt{\textbf{not\_in\_name}\$exp : Exp, Nam \rightarrow Exp}
\texttt{\textbf{not\_in\_range}\$exp : Exp, Rng \rightarrow Exp}
\texttt{\textbf{not\_in\_type}\$exp : Exp, Nam \rightarrow Exp}

\texttt{\textbf{null}\$exp : \rightarrow Exp}
\texttt{\textbf{integer}\$exp : integer \rightarrow Exp}
\texttt{\textbf{real}\$exp : real \rightarrow Exp}
\texttt{\textbf{char}\$exp : integer \rightarrow Exp}
\texttt{\textbf{or\_else}\$exp : Exp, Exp \rightarrow Exp}
\texttt{\textbf{paren}\$exp : Exp \rightarrow Exp}
\texttt{\textbf{qual}\$aggregate : Nam, Agg \rightarrow Exp}
\texttt{\textbf{qual}\$exp : Nam, Exp \rightarrow Exp}
\texttt{\textbf{type}\$alloc : Sid \rightarrow Exp}
\texttt{\textbf{type}\$conversion : Nam, Exp \rightarrow Exp}

B.1.18 Modes (\textit{Mde})

\texttt{\textbf{in}\$mode : \rightarrow Mde}
\texttt{\textbf{in\_out}\$mode : \rightarrow Mde}
\texttt{\textbf{no}\$mode : \rightarrow Mde}
\texttt{\textbf{out}\$mode : \rightarrow Mde}

B.1.19 Names (\textit{Nam})

\texttt{\textbf{access}\$attr : Nam \rightarrow Nam}
\texttt{\textbf{delta}\$attr : Nam \rightarrow Nam}
\texttt{\textbf{digits}\$attr : Nam \rightarrow Nam}
\texttt{\textbf{deref}\$name : Nam \rightarrow Nam}
\texttt{\textbf{direct}\$name : Id \rightarrow Nam}
\texttt{\textbf{func}\$call : Nam, Pss* \rightarrow Nam}
\texttt{\textbf{Id}\$attr : Nam, Id \rightarrow Nam}
\texttt{\textbf{indexed}\$comp : Nam, Exp* \rightarrow Nam}

\texttt{\textbf{name}\_\textbf{type}\$conversion : Nam, Nam \rightarrow Nam}
\texttt{\textbf{param}\$attr : Nam, Id, Exp \rightarrow Nam}
\texttt{\textbf{selected}\$comp : Nam, Id \rightarrow Nam}
\texttt{\textbf{slice}\$op : Nam, Rng \rightarrow Nam}
B.1.20 Parameter Specifications (Pms)

access$param : Id, Nam → Pms
access_default$param : Id, Nam, Exp → Pms
default$param : Id, Mde, Nam, Exp → Pms
normal$param : Id, Mde, Nam → Pms

B.1.21 Pragmas (Prg)

param$pragma : Id, Pss* → Prg
simple$pragma : Id → Prg

B.1.22 Parameter Associations (Pss)

named_exp$arg : Id, Exp → Pss
named_name$arg : Id, Nam → Pss

B.1.23 Record Component Associations (Rca)

choice$assoc : Ccl, Exp → Rca

B.1.24 Ranges (Rng)

attr$range : Nam → Rng
explicit$range : Exp, Exp → Rng
parm_attr$range : Nam, Exp → Rng

B.1.25 Subtype Indications (Sid)

constrained$subtype : Nam, Cns → Sid
named$subtype : Nam → Sid
subtype$range : Rng → Sid

B.1.26 Subprogram Specifications (Sps)

function$spec : Id, Pms*, Nam → Sps
procedure$spec : Id, Pms* → Sps

B.1.27 Statements (Stm)

stm$1st : list(Stm) → Stm
agg_code$stm : Nam, Agg → Stm
assign$stm : Nam, Exp → Stm
call$stm : Nam, Pss* → Stm
case$stm : Exp, Alt → Stm
cond$exit : Cnd → Stm
declare$block : Dcl, Stm → Stm
exp_code$stm : Nam, Exp → Stm
for$loop : Id, Rng, Stm → Stm
func_return$stm : Exp → Stm
B.1.28 Type Definitions (Tdf)

<table>
<thead>
<tr>
<th>Type Definition</th>
<th>Signature</th>
</tr>
</thead>
<tbody>
<tr>
<td>access$ertype</td>
<td>Sid → Tdf</td>
</tr>
<tr>
<td>aliased_array$type</td>
<td>Rng*, Sid → Tdf</td>
</tr>
<tr>
<td>aliased_uc_array$type</td>
<td>Nam*, Sid → Tdf</td>
</tr>
<tr>
<td>all_access$type</td>
<td>Sid → Tdf</td>
</tr>
<tr>
<td>array$type</td>
<td>Rng*, Sid → Tdf</td>
</tr>
<tr>
<td>const_access$type</td>
<td>Sid → Tdf</td>
</tr>
<tr>
<td>const_dec_fixed$type</td>
<td>Exp, Exp, Cns → Tdf</td>
</tr>
<tr>
<td>const_float$type</td>
<td>Exp, Cns → Tdf</td>
</tr>
<tr>
<td>dec_fixed$type</td>
<td>Exp, Exp → Tdf</td>
</tr>
<tr>
<td>der$type</td>
<td>Sid → Tdf</td>
</tr>
<tr>
<td>enum$type</td>
<td>Id* → Tdf</td>
</tr>
<tr>
<td>ext$type</td>
<td>Sid, rcd → Tdf</td>
</tr>
<tr>
<td>float$type</td>
<td>Exp → Tdf</td>
</tr>
<tr>
<td>func$type</td>
<td>Pms*, Nam→ Tdf</td>
</tr>
<tr>
<td>int$type</td>
<td>Exp, Exp → Tdf</td>
</tr>
<tr>
<td>mod$type</td>
<td>Exp → Tdf</td>
</tr>
<tr>
<td>named$type</td>
<td>Sid → Tdf</td>
</tr>
<tr>
<td>ord_fixed$type</td>
<td>Exp, Rng → Tdf</td>
</tr>
<tr>
<td>proc$type</td>
<td>Pms* → Tdf</td>
</tr>
<tr>
<td>record$type</td>
<td>Cmp*, Vrp→ Tdf</td>
</tr>
<tr>
<td>t_record$type</td>
<td>Cmp*, Vrp→ Tdf</td>
</tr>
<tr>
<td>uc_array$type</td>
<td>Nam*, Sid → Tdf</td>
</tr>
</tbody>
</table>
B.1.29 Variants (Vnt)

\[
\text{variant$\text{clause}} : \quad Dch^*, Cmp^*, Vrp \rightarrow Vnt
\]

B.1.30 Variant Parts (Vrp)

\[
\text{no$\text{variant}} : \quad Vrp
\]
\[
\text{variant$\text{part}} : \quad \text{Nam}, Vnt^* \rightarrow Vrp
\]

B.1.31 Exception Choices (Xhd)

\[
\text{choice$\text{handler}} : \quad Id, Ech^*, Stm \rightarrow Xhd
\]
\[
\text{expt$\text{handler}} : \quad Ech^*, Stm \rightarrow Xhd
\]

B.2 Lexical Elements

\[
\text{pragma} ::= \\
\quad \text{pragma identifier [ ( pragma_argument_association \{ , pragma_argument_association \} ) ] ;}
\]
\[
\begin{array}{|c|}
\hline
\text{param$\text{pragma}} : \quad Id, Pss^* \rightarrow Prg \\
\text{simple$\text{pragma}} : \quad \text{pragma}, Id \rightarrow Prg \\
\hline
\end{array}
\]

\[
\text{pragma_argument_association} ::= \\
\quad [ \text{identifier} => \ ] \text{name} \\
\quad | [ \text{identifier} => \ ] \text{expression}
\]
\[
\begin{array}{|c|}
\hline
\text{named_exp$\text{arg}} : \quad Id, \text{Exp} \rightarrow \text{Pss} \\
\text{named_name$\text{arg}} : \quad Id, \text{Nam} \rightarrow \text{Pss} \\
\hline
\end{array}
\]

B.3 Declarations and Types

B.3.1 Declarations

\[
\text{basic$\text{declaration}} ::= \\
\quad \text{type$\text{declaration} \\
\quad | \text{subtype$\text{declaration} \\
\quad | \text{object$\text{declaration} \\
\quad | \text{number$\text{declaration} \\
\quad | \text{subprogram$\text{declaration} \\
\quad | \text{abstract$\text{subprogram$\text{declaration} \\
\quad | \text{package$\text{declaration} \\
\quad | \text{renaming$\text{declaration} \\
\quad | \text{exception$\text{declaration} \\
\quad | \text{generic$\text{declaration} \\
\quad | \text{generic$\text{instantiation}
\]

It is convenient to treat sequences of declarations as a single declaration.
defining_identifier ::= identifier

B.3.2 Types and Subtypes

B.3.2.1 Type Declarations

type_declaration ::=  
  full_type_declaration | incomplete_type_declaration | private_type_declaration | private_extension_declaration

full_type_declaration ::=  
  type defining_identifier [ known_discriminant_part ] is type_definition ;  
  | task_type_declaration  
  | protected_type_declaration

d_type$decl : Id, Dcp, Tdf → Dcl

type$decl : Id, Tdf → Dcl

type_definition ::=  
  enumeration_type_definition | integer_type_definition | real_type_definition | array_type_definition | record_type_definition | access_type_definition | derived_type_definition

B.3.2.2 Subtype Declarations

subtype_declaration ::=  
  subtype defining_identifier is subtype_indication ;

subtype$decl : Id, Sid → Dcl

subtype_indication ::=  
  subtype_mark [ constraint ]

subtype_mark ::=  
  name
The form `subtype$range` applies only to discrete subtype definitions.

```
constraint ::= 
    scalar_constraint 
    | composite_constraint

scalar_constraint ::= 
    range_constraint 
    | digits_constraint 
    | delta_constraint

composite_constraint ::= 
    index_constraint 
    | discriminant_constraint
```

B.3.2.3 Classification of Operations

B.3.3 Objects and Named Numbers

B.3.3.1 Object Declarations

```
object_declaration ::= 
    defining_identifier_list : [ aliased ] [ constant ] subtype_indication [ := expression ] ; 
    | defining_identifier_list : [ aliased ] [ constant ] array_type_definition [ := expression ] ; 
    | single_task_declaration 
    | single_protected_declaration
```

All forms of object declarations are normalized such that each declaration defines exactly one name. This is always possible by [3.3.1].

```
a.c.i.obj$decl :  Id, Sid, Exp → Dcl
a.c.obj$decl :  Id, Sid → Dcl
a.i.obj$decl :  Id, Sid, Exp → Dcl
a.obj$decl :  Id, Sid → Dcl
c.i.obj$decl :  Id, Sid, Exp → Dcl
c.obj$decl :  Id, Sid → Dcl
i.obj$decl :  Id, Sid, Exp → Dcl
obj$decl :  Id, Sid → Dcl
```
B.3.3.2 Number Declarations

number_declaration ::= 
defining_identifier_list : constant := expression

| real_const$decl : Id, Exp → Dcl |
| int_const$decl : Id, Exp → Dcl |

Number declarations are disambiguated by static analysis into real and integer number declarations.

B.3.4 Derived Types and Classes

derived_type_definition ::= 
[ abstract ] new subtype_indication [ record_extension_part ]

| der$type : Sid → Tdf |
| ext$type : Sid, rcd → Tdf |

B.3.4.1 Derivation Classes

B.3.5 Scalar Types

range_constraint ::= 
range range

| range$constr : Rng → Cns |

range ::= 
range_attribute_reference
| simple_expression .. simple_expression

| attr$range : Nam → Rng |
| explicit$range : Exp, Exp → Rng |
| parm_attr$range : Nam, Exp → Rng |

B.3.5.1 Enumeration Types

enumeration_type_definition ::= 
( enumeration_literal_specification { , enumeration_literal_specification } )

| enum$type : Id* → Tdf |

enumeration_literal_specification ::= 
defining_identifier
| defining_character_literal
defining_character_literal ::= 
character_literal

| char$enum : char → Id |
| id$enum : Id → Id |

B.3.5.2 Character Types
B.3.5.3 Boolean Types
B.3.5.4 Integer Types

integer_type_definition ::= 
signed_integer_type_definition |
modular_type_definition

signed_integer_type_definition ::= 
range simple_expression .. simple_expression

| int$type : Exp, Exp → Tdf |

modular_type_definition ::= 
mod expression

| mod$type : Exp → Tdf |

B.3.5.5 Operations of Discrete Types
B.3.5.6 Real Types

real_type_definition ::= 
floating_point_definition |
fixed_point_definition

B.3.5.7 Floating Point Types

floating_point_definition ::= 
digits expression [ real_range_specification ]

| float$type : Exp → Tdf |
| const_float$type : Exp, Cns → Tdf |

real_range_specification ::= 
range simple_expression .. simple_expression

See scalar types (3.5).
B.3.5.8 Operations of Floating Point Types

B.3.5.9 Fixed Point Types

fixed_point_definition ::= 
    ordinary_fixed_point_definition 
    | decimal_fixed_pointq_definition

ordinary_fixed_point_definition ::= 
    delta expression real_range_specification

    ord_fixed$type : Exp, Rng -> Tdf

decimal_fixed_point_definition ::= 
    delta expression digits expression [ real_range_specification ]

    const_dec_fixed$type : Exp, Exp, Cns -> Tdf
    dec_fixed$type : Exp, Exp -> Tdf

decimal_digits_constraint ::= 
    digits expression [ range_constraint ]

    constr_digits$constr : Exp, Cns -> Cns
    digits$constr : Exp -> Cns

B.3.5.10 Operations of Fixed Point Types

B.3.6 Array Types

array_type_definition ::= 
    unconstrained_array_definition 
    | constrained_array_definition

unconstrained_array_definition ::= 
    array ( index_subtype_definition { , index_subtype_definition } ) of component_definition

    aliased_uc_array$type : Nam*, Sid -> Tdf
    uc_array$type : Nam*, Sid -> Tdf

index_subtype_definition ::= 
    subtype_mark range <>

constrained_array_definition ::= 
    array ( discrete_subtype_definition { , discrete_subtype_definition } ) of component_definition
discretesubtype_definition ::=  
   subtype_indication  
   | range

Discrete subtype definitions are subsumed under subtype indications (Sid).

component_definition ::=  
   [ aliased ] subtype_indication

B.3.6.1 Index Constraints and Discrete Ranges

index_constraint ::=  
   ( discrete_range { , discrete_range } )

index$constr : Rng* → Cns

discrete_range ::=  
   subtype_indication  
   | range

B.3.6.2 Operations of Array Types

B.3.6.3 String Types

B.3.7 Discriminants

discriminant_part ::=  
   unknown_discriminant_part  
   | known_discriminant_part

unknown_discriminant_part ::=  
   ( <> )

known_discriminant_part ::=  
   ( discriminant_specification { ; discriminant_specification } )

box$discr : → Dcp
list$discr : Des* → Dcp

discriminant_specification ::=  
   defining_identifier_list : subtype_mark [ := default_expression ]  
   | defining_identifier_list : access_definition [ := default_expression ]
default_expression ::= expression

| acc$discr  |  Id, Nam → Dcs |
| acc_init$discr | Id, Nam, Exp → Dcs |
| init$discr   |  Id, Nam, Exp → Dcs |
| simple$discr |  Id, Nam → Dcs |

B.3.7.1 Discriminant Constraints
discriminant_constraint ::= ( discriminant_association { , discriminant_association } )
discriminant_association ::= [ selector_name { | selector_name } => ] expression

B.3.7.2 Operations of Discriminated Types
B.3.8 Record Types
record_type_definition ::=
[[ abstract ] tagged ] [ limited ] record_definition

| record$type | Cmp*, Vrp→ Tdf |
| t_record$type | Cmp*, Vrp→ Tdf |

record_definition ::= record component_list end record | null record

component_list ::= component_declaration { component_declaration }
| { component_declaration } variant_part
| null.

component_declaration ::= defining_identifier_list : component_definition [ := default_expression ]
Component declarations with multiple identifiers are replaced by multiple component declarations.

B.3.8.1 Variant Parts and Discrete Choices

```plaintext
variant_part ::= 
    case direct_name is 
    variant 
    { variant } 
    end case ;

no$variant: Vrp 
variant$part: Nam, Vnt* -> Vrp 

variant ::= 
    when discrete_choice_list => 
    component_list

variant$clause: Dch*, Cmp*, Vrp -> Vnt 

discrete_choice_list ::= 
    discrete_choice { 
    | discrete_choice }

discrete_choice ::= 
    expression 
    | discrete_range 
    | others

discr_other$choice : ~ Dch 
Exp$choice : Exp -> Dch 
range$choice : Rng -> Dch
```

B.3.9 Tagged Types and Type Extensions

B.3.9.1 Type Extensions

```plaintext
record_extension_part ::= 
    with record_definition
```
B.3.9.2 Dispatching Operations of Tagged Types
B.3.9.3 Abstract Types and Subprograms
B.3.10 Access Types

\[
\text{access\_type\_definition} ::= \\
\quad \text{access\_to\_object\_definition} \\
\quad \text{access\_to\_subprogram\_definition}
\]

\[
\text{access\_to\_object\_definition} ::= \\
\quad \text{access} \ [\ \text{general\_access\_modifier} \ ] \ \text{subtype\_indication}
\]

\[
\text{general\_access\_modifier} ::= \\
\quad \text{all} \\
\quad \text{constant}
\]

\[
\begin{align*}
\text{access}\$\text{type} & : \ Sid \rightarrow Tdf \\
\text{all\_access}\$\text{type} & : \ Sid \rightarrow Tdf \\
\text{const\_access}\$\text{type} & : \ Sid \rightarrow Tdf
\end{align*}
\]

\[
\text{access\_to\_subprogram\_definition} ::= \\
\quad \text{access} \ [\ \text{protected} \ ] \ \text{procedure} \ \text{parameter\_profile} \\
\quad \text{access} \ [\ \text{protected} \ ] \ \text{function} \ \text{parameter\_and\_result\_profile}
\]

\[
\begin{align*}
\text{func}\$\text{type} & : \ Pms^*, Nam \rightarrow Tdf \\
\text{proc}\$\text{type} & : \ Pms^* \rightarrow Tdf
\end{align*}
\]

\[
\text{access}\_\text{definition} ::= \\
\quad \text{access} \ \text{subtype\_mark}
\]

B.3.10.1 Incomplete Type Declarations

\[
\text{incomplete\_type\_declaration} ::= \\
\quad \text{type} \ \text{defining\_identifier} \ [\ \text{discriminant\_part} \ ]:
\]

\[
\begin{align*}
\text{d\_i\_type}\$\text{decl} & : \ Id, Dcp \rightarrow Dcl \\
\text{i\_type}\$\text{decl} & : \ Id \rightarrow Dcl
\end{align*}
\]

B.3.10.2 Operations of Access Types

B.3.11 Declarative Parts

\[
\text{declarative\_part} ::= \\
\quad \{ \ \text{declarative\_item} \} 
\]
declarative_item ::= 
    basic_declarative_item | body

basic_declarative_item ::= 
    basic_declaration | representation_clause | use_clause

body ::= 
    proper_body | body_stub

proper_body ::= 
    subprogram_body | package_body | task_body | protected_body

B.3.11.1 Completions of Declarations

B.4 Names and Expressions

B.4.1 Names

name ::= 
    direct_name | explicit_dereference | indexed_component | slice | selected_component | attribute_reference | type_conversion | function_call | character_literal

String and character literals that denote operators are included as direct names. String literals that denote string values are represented as aggregates.

```
char$Exp : integer -> Exp
```

direct_name ::= 
    identifier | operator_symbol

```
direct$name : Id -> Nam
```

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Names that are represented as strings, character literals, and identifiers are all treated uniformly as elements of type \( Id \)

\[
\text{prefix ::= name | implicit_dereference}
\]

\[
\text{explicit_dereference ::= name . all}
\]

\[
\text{deref\$name : Nam \rightarrow Nam}
\]

The abstract syntax for dereferencing includes explicit as well as implicit dereferencing.

\[
\text{implicit_dereference ::= name}
\]

**B.4.1.1 Indexed Components**

\[
\text{indexed\_component ::= prefix ( expression \{ , expression \} )}
\]

\[
\text{indexed\$comp : Nam, Exp^* \rightarrow Nam}
\]

**B.4.1.2 Slices**

\[
\text{slice ::= prefix ( discrete\_range )}
\]

\[
\text{slice\$op : Nam, Rng \rightarrow Nam}
\]

**B.4.1.3 Selected Components**

\[
\text{selected\_component ::= prefix . selector\_name}
\]

\[
\text{selected\$comp : Nam, Id \rightarrow Nam}
\]

Static semantics separates expanded names from selected components.

\[
\text{selector\_name ::= identifier | character\_literal | operator\_symbol}
\]
B.4.1.4 Attributes

attribute_reference ::= prefix ` attribute_designator

attribute_designator ::= identifier [ ( expression ) ]
| access
| delta
| digits

range_attribute_reference ::= prefix ` range_attribute_designator

range_attribute_designator ::= range [ ( expression ) ]

Id$attr : Nam, Id → Nam
param$attr : Nam, Id, Exp → Nam

There is special abstract syntax needed for attributes that are reserved words.

B.4.2 Literals

B.4.3 Aggregates

aggregate ::= record_aggregate
| extension_aggregate
| array_aggregate

B.4.3.1 Record Aggregates

record_aggregate ::= ( record_component_association_list )

record_component_association_list ::= record_component_association { , record_component_association }
| null record

null_record$Agg : → Agg
record$Agg : Rca* → Agg

record_component_association ::= [ component_choice_list => ] expression
Positional parameter associations have been eliminated by static analysis and are represented with an explicit choice list. This normalization is possible since, by [4.3.1], discriminant values that determine variants are required to be static.

\[
\text{choiceassoc} : \text{Ccl}, \text{Exp} \rightarrow \text{Rca}
\]

B.4.3.2 Extension Aggregates

\[
\text{extension aggregate} ::= \\
(\text{expression with record component association list})
\]

\[
\begin{align*}
\text{extAgg} : & \quad \text{Exp, Rca}^* \rightarrow \text{Agg} \\
\text{null_extAgg} : & \quad \text{Exp} \rightarrow \text{Agg}
\end{align*}
\]

B.4.3.3 Array Aggregates

\[
\text{array aggregate} ::= \\
\text{positional array aggregate} \\
| \quad \text{named array aggregate}
\]

\[
\begin{align*}
\text{positional array aggregate} ::= \\
(\text{expression, expression }\{}\text{, expression}\}) \\
| \quad (\text{expression }\{}\text{, expression}\} , \text{others }\Rightarrow \text{expression})
\end{align*}
\]

\[
\begin{align*}
\text{named array aggregate} ::= \\
(\text{array component association }\{}\text{, array component association}\})
\end{align*}
\]

\[
\begin{align*}
\text{named array Agg} : & \quad \text{Aca}^* \rightarrow \text{Agg} \\
\text{other array Agg} : & \quad \text{Exp}^*, \text{Exp} \rightarrow \text{Agg} \\
\text{pos array Agg} : & \quad \text{Exp}^* \rightarrow \text{Agg}
\end{align*}
\]

\[
\begin{align*}
\text{array component association} ::= \\
\text{discrete choice list }\Rightarrow \text{expression}
\end{align*}
\]

\[
\text{array compassoc} : \quad \text{Dch}^*, \text{Exp} \rightarrow \text{Aca}
\]
B.4.4 Expressions

expression ::=  
  relation { and relation }  
  | relation { and then relation }  
  | relation { or relation }  
  | relation { or else relation }  
  | relation { xor relation }

\[
\begin{align*}
\text{and\_then$Exp} & : \ Exp, \ Exp \rightarrow Exp \\
\text{or\_else$Exp} & : \ Exp, \ Exp \rightarrow Exp
\end{align*}
\]

Short-circuit operators are non-strict and require explicit representation.

relation ::=  
  simple_expression [ relational_operator simple_expression ]  
  | simple_expression [ not ] in range  
  | simple_expression [ not ] in subtype_mark  

\[
\begin{align*}
\text{in\_name$Exp} & : \ Exp, \ Nam \rightarrow Exp \\
\text{in\_range$Exp} & : \ Exp, \ Rng \rightarrow Exp \\
\text{in\_type$Exp} & : \ Exp, \ Nam \rightarrow Exp \\
\text{not\_in\_name$Exp} & : \ Exp, \ Nam \rightarrow Exp \\
\text{not\_in\_range$Exp} & : \ Exp, \ Rng \rightarrow Exp \\
\text{not\_in\_type$Exp} & : \ Exp, \ Nam \rightarrow Exp
\end{align*}
\]

Static semantics distinguishes between membership tests where the name denotes an object and those where the name denotes a subtype.

simple_expression ::=  
  [ unary_adding_operator ] term { binary_adding_operator term }  

term ::=  
  factor { multiplying_operator factor }

factor ::=  
  primary [ ** primary ]  
  | abs primary  
  | not primary

primary ::=  
  numeric_literal  
  | null  
  | string_literal  
  | aggregate  
  | name  
  | qualified_expression  
  | allocator  
  | ( expression )
paren$Exp : Exp → Exp
qual$aggregate : Nam, Agg → Exp
name$Exp : Nam → Exp
null$Exp : → Exp
integer$Exp : integer → Exp
real$Exp : real → Exp

All aggregates are assumed to be qualified by static analysis. String literals are represented as qualified aggregates. It may be necessary for static analysis to introduce new type names for aggregates of anonymous array types.

Numeric literals are separated into integer and real literals.

### B.4.5 Operators and Expression Evaluation

All strict operators on ordinary values are represented in the abstract syntax as function calls. As with other function calls, the function designators specify the unique overload that applies.

Non-strict operators or operators that take subtypes as arguments (e.g., in) have an explicit representation given below.

#### logical_operator ::=  
| and  
| or  
| xor

#### relational_operator ::=  
| =  
| /=  
| <  
| <=  
| >  
| >=

#### binary_adding_operator ::=  
| +  
| -  
| &

#### unary_adding_operator ::=  
| -  
| +

#### multiplying_operator ::=  
| *  
| /  
| mod  
| rem
highest_precedence_operator ::= **
    | abs
    | not

B.4.5.1 Logical Operations and Short-Circuit Control Forms
B.4.5.2 Relational Operators and Membership Tests
B.4.5.3 Binary Adding Operators
B.4.5.4 Multiplying Operators
B.4.5.5 Highest Precedence Operators
B.4.6 Type Conversions

type_conversion ::= subtype_mark ( expression )
    | subtype_mark ( name )

| type$conversion : Nam, Exp → Exp
| name_type$conversion : Nam, Nam → Nam

B.4.7 Qualified Expressions

qualified_expression ::= subtype_mark' ( expression )
    | subtype_mark' aggregate

| qual$Exp : Nam, Exp → Exp

The abstract syntax for qualified aggregates is covered under aggregates.

B.4.8 Allocators

allocator ::= new subtype_indication
    | new qualified_expression

| Exp$alloc : Exp → Exp
| type$alloc : Sid → Exp

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B.4.9 Static Expressions and Static Subtypes
B.4.9.1 Statically Matching Constraints and Subtypes

B.5 Statements
B.5.1 Simple and Compound Statements – Sequences of Statements

sequence_of_statements ::= statement { statement }

Sequences of statements can be treated as single statements.

\[
Stm^\dagger Stm^* \rightarrow Stm
\]

statement ::= { label } simple_statement |
{ label } compound_statement

\[
\text{labeled } Stm : Id, Stm \rightarrow Stm
\]

simple_statement ::= null_statement |
assignment_statement |
exit_statement |
goto_statement |
procedure_call_statement |
return_statement |
entry_call_statement |
requeue_statement |
delay_statement |
abort_statement |
raise_statement |
code_statement

compound_statement ::= if_statement |
case_statement |
loop_statement |
block_statement |
accept_statement |
select_statement

null_statement ::= null ;

\[
\text{null } Stm : \rightarrow Stm
\]
label ::=  
    \textless \textless \text{statement_identifier} \textgreater \textgreater

\text{statement_identifier} ::=  
    \text{direct_name}

B.5.2 Assignment Statement

\text{assignment_statement} ::=  
    \text{name} := \text{expression}

\text{assign$stm} : N\text{am, Exp} \rightarrow \text{Stm}

B.5.3 If Statements

\text{if_statement} ::=  
    \text{if} \text{ condition then}  
        \text{sequence_of_statements}
    \{ \text{elsif condition then}  
        \text{sequence_of_statements}\}
    \text{else}  
        \text{sequence_of_statements} \}
    \text{end if ;}

\text{if$stm} : \text{Cnd, Stm, Eif} \rightarrow \text{Stm}
\text{if\_else$stm} : \text{Cnd, Stm, Eif, Stm} \rightarrow \text{Stm}
\text{Eif$lst} : \text{Eif} \rightarrow \text{Eif}
\text{elsif$clause} : \text{Cnd, Stm} \rightarrow \text{Eif}

\text{condition} ::=  
    \text{expression}

\text{Exp$condition} : \text{Exp} \rightarrow \text{Cnd}

B.5.4 Case Statements

\text{case_statement} ::=  
    \text{case} \text{ expression is}  
        \text{case_statement\_alternative}
    \{ \text{case_statement\_alternative} \}
    \text{end case ;}

\text{case$stm} : \text{Exp, Alt} \rightarrow \text{Stm}
case_statement_alternative ::=  
  when discrete_choice_list ->  
  sequence_of_statements

\[
\begin{array}{ll}
\text{Alt} & \text{Alt}^* \rightarrow \text{Alt} \\
\text{caseAlt} & \text{Dch}^*, \text{Stm} \rightarrow \text{Alt}
\end{array}
\]

### B.5.5 Loop Statements

loop_statement ::=  
  [ statement_identifier : ]  
  [ iteration_scheme ] loop  
  sequence_of_statements  
  end loop [ identifier ] ;

iteration_scheme ::=  
  while condition  
  | for loop_parameter_specification

loop_parameter_specification ::=  
  defining_identifier in [ reverse ] discrete_subtype_definition

| named_for$loop | Id, Id, Rng, Stm \rightarrow Stm |
| named_reverse$loop | Id, Id, Rng, Stm \rightarrow Stm |
| named_while$loop | Id, Cond, Stm \rightarrow Stm |
| named$loop | Id, Stm \rightarrow Stm |
| for$loop | Id, Rng, Stm \rightarrow Stm |
| reverse$loop | Id, Rng, Stm \rightarrow Stm |
| while$loop | Cond, Stm \rightarrow Stm |
| plain$loop | Stm \rightarrow Stm |

### B.5.6 Block Statements

block_statement ::=  
  [ statement_identifier : ]  
  [ declare  
    declarative_part ]  
  begin  
  handled_sequence_of_statements  
  end [ identifier ] ;

| simple$block | Stm \rightarrow Stm |
| declare$block | Decl, Stm \rightarrow Stm |
| named$block | Id, Stm \rightarrow Stm |
| named_declare$block | Id, Decl, Stm \rightarrow Stm |
B.5.7 Exit Statements

\[
\text{exit\_statement ::= exit [ name ] [ when condition ];}
\]

| exit$exit : \quad \text{Nam} \rightarrow \text{Stm} |
| name$exit : \quad \text{Nam} \rightarrow \text{Stm} |
| name\_cond$exit : \quad \text{Nam, Cnd} \rightarrow \text{Stm} |
| plain$exit : \quad \rightarrow \text{Stm} |
| cond$exit : \quad \text{Cnd} \rightarrow \text{Stm} |

B.5.8 Goto Statements

\[
goto\_statement ::= goto name ;
\]

| goto$stm : \quad \text{Nam} \rightarrow \text{Stm} |

B.6 Subprograms

B.6.1 Subprogram Declarations

\[
\text{subprogram\_declaration ::= subprogram\_specification ;}
\]

| subp$spec : \quad \text{Sps} \rightarrow \text{Dcl} |

| abstract\_subprogram\_declaration ::= subprogram\_specification is abstract ; |

| subp$spec : \quad \text{Sps} \rightarrow \text{Dcl} |

\[
\text{subprogram\_specification ::= procedure defining\_program\_unit\_name parameter\_profile}
\quad | \quad \text{function defining\_designator parameter\_and\_result\_profile}
\]

| function$spec : \quad \text{Id, Pms}^*, \text{Nam} \rightarrow \text{Sps} |
| procedure$spec : \quad \text{Id, Pms}^* \rightarrow \text{Sps} |

| designator ::= [ parent\_unit\_name . ] \text{identifier}
| \quad | \text{operator\_symbol} |

| defining\_designator ::= defining\_program\_unit\_name
| \quad | \text{defining\_operator\_symbol} |
defining_program_unit_name ::= 
   [ parent_unit_name . ] defining_identifier

operator_symbol ::= 
   string_literal

defining_operator_symbol ::= 
   operator_symbol

parameter_profile ::= 
   [ formal_part ]

\begin{align*}
\text{no$\text{params} :} & \rightarrow Psig \\
\text{param$\text{list} :} & Pms \rightarrow Psig
\end{align*}

parameter_and_result_profile ::= 
   [ formal_part ] return subtype_mark

formal_part ::= 
   ( parameter_specification \{ ; parameter_specification \} )

parameter_specification ::= 
   defining_identifier_list : mode subtype_mark [ := default_expression ]
   \| defining_identifier_list : access_definition [ := default_expression ]

\begin{align*}
\text{access$\text{param} :} & Id, \text{Nam} \rightarrow Pms \\
\text{access_default$\text{param} :} & Id, \text{Nam}, \text{Exp} \rightarrow Pms \\
\text{default$\text{param} :} & Id, \text{Mde}, \text{Nam}, \text{Exp} \rightarrow Pms \\
\text{normal$\text{param} :} & Id, \text{Mde}, \text{Nam} \rightarrow Pms
\end{align*}

mode ::= 
   [ in ]
   \| in_out
   \| out

\begin{align*}
\text{in$\text{mode} :} & \rightarrow Mde \\
\text{in_out$\text{mode} :} & \rightarrow Mde \\
\text{no$\text{mode} :} & \rightarrow Mde \\
\text{out$\text{mode} :} & \rightarrow Mde
\end{align*}
B.6.2 Formal Parameter Modes

B.6.3 Subprogram Bodies

\[ \text{subprogram\_body ::= subprogram\_specification is declarative\_part begin handled\_sequence\_of\_statements end [ designator ] ;} \]

\[
\text{subp$\text{body} : \ Sps, Dcl, Stm \rightarrow Dcl}
\]

B.6.3.1 Conformance Rules

B.6.3.2 Inline Expansion of Subprograms

B.6.4 Subprogram Calls

\[
\text{procedure\_call\_statement ::= name | prefix actual\_parameter\_part ;}
\]

\[
\text{call$\text{stm} : \ Nam, Pss^* \rightarrow Stm}
\]

\[
\text{function\_call ::= name | prefix actual\_parameter\_part}
\]

\[
\text{func$\text{call} : \ Nam, Pss^* \rightarrow Nam}
\]

\[
\text{actual\_parameter\_part ::= ( parameter\_association \{ , parameter\_association \})}
\]

\[
\text{parameter\_association ::= [ selector\_name => ] explicit\_actual\_parameter}
\]

\[
\text{explicit\_actual\_parameter ::= expression | name}
\]

\[
\text{named\_exp$\text{arg} : \ Id, Exp \rightarrow Pss}
\]

\[
\text{named\_name$\text{arg} : \ Id, Nam \rightarrow Pss}
\]

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B.6.4.1 Parameter Associations

B.6.5 Return Statements

\[
\text{return\_statement} ::= \\
\text{return}\ [\text{expression}] ;
\]

| func\_return\$stm : \( Exp \rightarrow Stm \) |
| return\$stm : \( \rightarrow Stm \) |

B.6.6 Overloading of Operators

B.7 Packages

B.7.1 Package Specifications and Declarations

\[
\text{package\_declaration} ::= \\
\text{package\_specification} ;
\]

\[
\text{package\_specification} ::= \\
\text{package defining\_program\_unit\_name is} \\
\{\text{basic\_declarative\_item}\} \\
[\text{private} \\
\{\text{basic\_declarative\_item}\} ] \\
\text{end}\ [\text{parent\_unit\_name} . \text{identifier}] 
\]

B.7.2 Package Bodies

\[
\text{package\_body} ::= \\
\text{package\ body defining\_program\_unit\_name is} \\
\text{declarative\_part} \\
[\text{begin} \\
\text{handled\_sequence\_of\_statements} \\
\text{end}\ [\text{parent\_unit\_name} . \text{identifier}] ;
\]

B.7.3 Private Type and Private Extensions

\[
\text{private\_type\_declaration} ::= \\
\text{type defining\_identifier [discriminant\_part] is [abstract] tagged [limited] private ;}
\]

\[
\text{private\_extension\_declaration} ::= \\
\text{type defining\_identifier [discriminant\_part] is} \\
[\text{abstract}] \text{new\ subtype\_indication with private ;}
\]

| d\_ext$decl : \( Id, Dep, Sid \rightarrow Dcl \) |
| ext$decl : \( Id, Sid \rightarrow Dcl \) |
B.7.3.1 Operations of Private Types and Private Extensions
B.7.4 Deferred Constants
B.7.5 Limited Types
B.7.6 User-Defined Assignment and Finalization
B.7.6.1 Completion and Finalization

B.8 Visibility Rules

B.8.1 Declarative Region
B.8.2 Scope of Declarations
B.8.3 Visibility
B.8.4 Use Clauses

use_clause ::= 
  use_package_clause 
  | use_type_clause

use_package_clause ::= 
  use name { , name } :

  use$clause : Nam* → Dcl

use_type_clause ::= 
  use type subtype_mark { , subtype_mark } :

  use_type$clause : Nam* → Dcl

B.8.5 Renaming Declarations

renaming_declaration ::= 
  object_renaming_declaration 
  | exception_renaming_declaration 
  | package_renaming_declaration 
  | subprogram_renaming_declaration 
  | generic_renaming_declaration

B.8.5.1 Object Renaming Declarations

object_renaming_declaration ::= 
  defining_identifier : subtype_mark renames name :

  object$renaming : Id, Nam, Nam → Dcl
B.8.5.2 Exception Renaming Declarations

exception_renaming_declaration ::=  
defining_identifier : exception renames name ;

```
exception$renaming : Id, Nam -> Dcl
```

B.8.5.3 Package Renaming Declarations

package_renaming_declaration ::=  
package defining_program_unit_name renames name ;

B.8.5.4 Subprogram Renaming Declarations

subprogram_renaming_declaration ::=  
subprogram_specification renames name ;

```
subp$renaming : Sps, Nam -> Dcl
```

B.8.5.5 Generic Renaming Declarations

generic_renaming_declaration ::=  
generic package defining_program_unit_name renames name ;  
generic procedure defining_program_unit_name renames name ;  
generic function defining_program_unit_name renames name ;

B.8.6 The Context of Overload Resolution

B.9 Tasks and Synchronization

B.9.1 Task Units and Task Objects

task_type_declaration ::=  
task type defining_identifier [ known_discriminant_part ] [ is task definition ] ;

task_definition ::=  
{ task_item }  
[ private  
{ task_item } ]  
end [ identifier ]

task_item ::=  
    entry_declaration  
    | representation_clause

task_body ::=  
    task body defining_identifier is  
    declarative_part  
    begin  
    handled_sequence_of_statements  
    end [ identifier ] :

B.9.2 Task Execution – Task Activation
B.9.3 Task Dependence – Termination of Tasks
B.9.4 Protected Units and Protected Objects

protected_type_declaration ::=  
    protected type defining_identifier [ known_discriminant_part ] is protected_definition ;

single_protected_declaration ::=  
    protected defining_identifier is protected_definition ;

protected_definition ::=  
    { protected_operation_declaration }  
    [ private  
    { protected_element_declaration } ]  
    end [ identifier ] ;

protected_operation_declaration ::=  
    subprogram_declaration  
    | entry_declaration

protected_element_declaration ::=  
    protected_operation_declaration  
    | component_declaration

protected_body ::=  
    protected body defining_identifier is  
    { protected_operation_item }  
    end [ identifier ] ;

protected_operation_item ::=  
    subprogram_declaration  
    | subprogram_body  
    | entry_body
B.9.5 Intertask Communication

entry_declaration ::= 
  entry defining_identifier [ ( discrete_subtype_definition ) ] parameter_profile ;

accept_statement ::= 
  accept direct_name [ ( entry_index ) ] parameter_profile [ do 
    handled_sequence_of_statements 
  end [ identifier ] ] ;

tentry_index ::= 
  expression
	entry_body ::= 
  entry defining_identifier entry_body_formal_part entry_barrier is 
  declarative_part 
  begin 
  handled_sequence_of_statements 
  end [ identifier ] ;
	entry_body_formal_part ::= 
  [ ( entry_index_specification ) ]parameter_profile
	entry_barrier ::= 
  when condition

tentry_index_specification ::= 
  for defining_identifier in discrete_subtype_definition

tentry_call_statement ::= 
  name [ actual_parameter_part ] ;

requeue_statement ::= 
  requeue name [ with abort ] ;

B.9.6 Delay Statements, Duration, and Time

delay_statement ::= 
  delay_until_statement 
  | delay_relative_statement

delay_until_statement ::= 
  delay until expression ;
delay_relative_statement ::= 
   delay expression ;

B.9.7 Select Statements
select_statement ::=
   selective_accept
   | timed_entry_call
   | conditional_entry_call
   | asynchronous_select

B.9.8 Selective Accept
selective_accept ::= 
   select 
       [ guard ] select_alternative 
       | [ or ] 
           [ guard ] select_alternative 
       [ else 
           sequence_of_statements ] 
   end select ;

guard ::= 
   when condition =>

select_alternative ::= 
   accept_alternative 
   | delay_alternative 
   | terminate_alternative

accept_alternative ::= 
   accept_statement [ sequence_of_statements ]

delay_alternative ::= 
   delay_statement [ sequence_of_statements ]

terminate_alternative ::= 
   terminate ;

B.9.9 Timed Entry Calls
timed_entry_call ::= 
   select 
       entry_call_alternative

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or
    delay_alternative
end select；

entry_call_alternative ::= entry_call_statement [ sequence_of_statements ]

B.9.10  Conditional Entry Calls

conditional_entry_call ::= select
    entry_call_alternative
else
    sequence_of_statements
end select；

B.9.11  Asynchronous Transfer of Control

asynchronous_select ::= select
    triggering_alternative
then abort
    abortable_part
end select；

triggering_alternative ::= triggering_statement [ sequence_of_statements ]

triggering_statement ::= entry_call_statement | delay_statement

abortable_part ::= sequence_of_statements

B.9.12  Abort of a Task – Abort of a Sequence of Statements

abort_statement ::= abort name { , name}；
B.9.13 Task and Entry Attributes
B.9.14 Shared Variables
B.9.15 Example of Tasking and Synchronization

B.10 Program Structure and Compilation Issues

B.10.1 Separate Compilation

B.10.1.1 Compilation Units – Library Units

\[
\text{compilation ::=}
\quad \{ \text{compilation\_unit} \}
\]

\[
\text{compilation\_unit ::=}
\quad \text{context\_clause library\_item}
\quad | \quad \text{context\_clause subunit}
\]

\[
\text{library\_item ::=}
\quad \text{[ private ] library\_unit\_declaration}
\quad | \quad \text{library\_unit\_body}
\]

\[
\text{library\_unit\_declaration ::=}
\quad \text{subprogram\_declaration}
\quad | \quad \text{package\_declaration}
\quad | \quad \text{generic\_declaration}
\quad | \quad \text{generic\_instantiation}
\quad | \quad \text{library\_unit\_renaming\_declaration}
\]

\[
\text{library\_unit\_renaming\_declaration ::=}
\quad \text{package\_renaming\_declaration}
\quad | \quad \text{generic\_renaming\_declaration}
\quad | \quad \text{subprogram\_renaming\_declaration}
\]

\[
\text{library\_unit\_body ::=}
\quad \text{subprogram\_body}
\quad | \quad \text{package\_body}
\]

\[
\text{parent\_unit\_name ::=}
\quad \text{name}
\]

\[
\begin{array}{|l|l|l|}
\hline
\text{name} & \text{unit} & \hline
\text{lib$\text{unit}$} & \text{CIt$^*$, Dcl \to cmu} & \hline
\text{private$\text{unit}$} & \text{CIt$^*$, Dcl \to cmu} & \hline
\text{sub$\text{unit}$} & \text{CIt$^*$, Nam, Dcl \to cmu} & \hline
\end{array}
\]

Renaming of library units is dealt with in static semantics.
B.10.1.2 Context Clauses – With Clauses

context_clause ::= 
{ context_item }

context_item ::= 
with_clause
| use_clause

with_clause ::= 
with name { , name } :

with$context : Nam* → Cit

B.10.1.3 Subunits of Compilation Units

body_stub ::= 
subprogram_body_stub
| package_body_stub
| task_body_stub
| protected_body_stub

subprogram_body_stub ::= 
subprogram_specification is separate :

s_subp$spec : Sps → Del

package_body_stub ::= 
package body defining_identifier is separate :

task_body_stub ::= 

task body defining_identifier is separate :

protected_body_stub ::= 
protected body defining_identifier is separate :

subunit ::= 
separate ( parent_unit_name ) proper_body
B.10.1.4 The Compilation Process
B.10.1.5 Pragmas and Compilations
B.10.1.6 Environment-Level Visibility Rules
B.10.2 Program Execution
B.10.2.1 Elaboration Control

B.11 Exceptions

B.11.1 Exception Declarations

exception_declaration ::= defining_identifier_list : exception ;

As with object declarations, only a single name is defined by each exception declaration.

B.11.2 Exception Handlers

handled_sequence_of_statements ::= sequence_of_statements
[ exception
exception_handler
{ exception_handler }
]

handled_statement : Stm, Xhd* → Stm
unhandled_statement : Stm → Stm

exception_handler ::= when [ choice_parameter_specification : ] exception_choice { | exception_choice } => sequence_of_statements

choice_handler : Id, Ech*, Stm → Xhd
except_handler : Ech*, Stm → Xhd

choice_parameter_specification ::= defining_identifier

exception_choice ::= name
| others

named_except : Nam → Ech
others_except : → Ech
B.11.3 Raise Statements

\[
\text{raise\_statement ::= raise [ name ];}
\]

\[
\begin{array}{ll}
\text{raisestm : Nam} & \rightarrow \text{Stm} \\
\text{reraisestm : } & \rightarrow \text{Stm}
\end{array}
\]

B.11.4 Exception Handling

B.11.5 Suppressing Checks

B.11.6 Exceptions and Optimization

B.12 Generic Units

B.12.1 Generic Declarations

\[
\text{generic\_declaration ::=}
\]

\[
\begin{array}{ll}
\text{generic\_subprogram\_declaration} & \\
\text{generic\_package\_declaration}
\end{array}
\]

\[
\text{generic\_subprogram\_declaration ::=}
\]

\[
\begin{array}{ll}
\text{generic\_formal\_part subprogram\_specification ;}
\end{array}
\]

\[
\text{generic\_package\_declaration ::=}
\]

\[
\begin{array}{ll}
\text{generic\_formal\_part package\_specification ;}
\end{array}
\]

\[
\text{generic\_formal\_part ::=}
\]

\[
\begin{array}{ll}
\text{generic \{} generic\_formal\_parameter\_declaration}
\end{array}
\]

\[
\begin{array}{ll}
\text{use\_clause \}}
\end{array}
\]

\[
\text{generic\_formal\_parameter\_declaration ::=}
\]

\[
\begin{array}{ll}
\text{formal\_object\_declaration}
\end{array}
\]

\[
\begin{array}{ll}
\text{formal\_type\_declaration}
\end{array}
\]

\[
\begin{array}{ll}
\text{formal\_subprogram\_declaration}
\end{array}
\]

\[
\begin{array}{ll}
\text{formal\_package\_declaration}
\end{array}
\]

B.12.2 Generic Bodies

B.12.3 Generic Instantiation

\[
\text{generic\_instantiation ::=}
\]

\[
\begin{array}{ll}
\text{package defining\_program\_unit\_name is}
\end{array}
\]

\[
\begin{array}{ll}
\text{new name [ generic\_actual\_part ];}
\end{array}
\]

\[
\begin{array}{ll}
\text{procedure defining\_program\_unit\_name is}
\end{array}
\]

\[
\begin{array}{ll}
\text{new name [ generic\_actual\_part ];}
\end{array}
\]

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function defining_designator is
  new name [ generic_actual_part ] :

generic_actual_part ::= ( generic_association { , generic_association } )

generic_association ::= [ selector_name => ] explicit_generic_actual_parameter

explicit_generic_actual_parameter ::= expression | name | subtype_mark

B.12.4 Formal Objects
formal_object_declaration ::= defining_identifier_list : mode subtype_mark [ := default_expression ] :

<table>
<thead>
<tr>
<th>init_formal$obj</th>
<th>Id*, Mde, Nam, Exp → Gpd</th>
</tr>
</thead>
<tbody>
<tr>
<td>formal$obj</td>
<td>Id*, Mde, Nam → Gpd</td>
</tr>
</tbody>
</table>

B.12.5 Formal Types
formal_type_declaration ::= type defining_identifier [ discriminant_part ] is formal_type_definition :

formal_type_definition ::= formal_private_type_definition
| formal_derived_type_definition
| formal_discrete_type_definition
| formal_signed_integer_type_definition
| formal_modular_type_definition
| formal_floating_point_definition
| formal_ordinary_fixed_point_definition
| formal_decimal_fixed_point_definition
| formal_array_type_definition
| formal_access_type_definition

B.12.5.1 Formal Private and Derived Types
formal_private_type_definition ::= [[ [ abstract ] tagged ] [ limited ] private
formal_derived_type_definition ::= 
   [ abstract ] new subtype_mark [ with private ]

B.12.5.2 Formal Scalar Types
formal_discrete_type_definition ::= 
   ( <> )

formal_signed_integer_type_definition ::= 
   range <>

formal_modular_type_definition ::= 
   mod <>

formal_floating_point_definition ::= 
   digits <>

formal_ordinary_fixed_point_definition ::= 
   delta <>

formal_decimal_fixed_point_definition ::= 
   digits <> delta <>

B.12.5.3 Formal Array Types
formal_array_type_definition ::= 
   array_type_definition

B.12.5.4 Formal Access Types
formal_access_type_definition ::= 
   access_type_definition

B.12.6 Formal Subprograms
formal_subprogram_declaration ::= 
   with subprogram_specification [ is subprogram_default ] ;

subprogram_default ::= 
   default_name | <>

default_name ::= 
   name
B.12.7 Formal Packages

formal_package_declaration ::= 
    with package defining_identifier is new name formal_package_actual_part ;

formal_package_actual_part ::= 
    ( <> ) 
    | [ generic_actual_part ]

B.12.8 Example of a Generic Package

B.13 Representation Clauses and Implementation-Dependent Features

representation_clause ::= 
    attribute_definition_clause 
    | enumeration_representation_clause 
    | record_representation_clause 
    | at_clause

attribute_definition_clause ::= 
    for direct_name ' attribute_designator use expression ; 
    | for direct_name ' attribute_designator use name ;

enumeration_representation_clause ::= 
    for direct_name use enumeration_aggregate ;

tenumeration_aggregate ::= 
    array_aggregate

record_representation_clause ::= 
    for direct_name use record [ mod_clause ] 
    { component_clause } 
    end record ;

component_clause ::= 
    component_clause_component_name at position range first_bit .. last_bit ;

component_clause_component_name ::= 
    direct_name 
    | direct_name ' attribute_designator
position ::= expression

first_bit ::= simple_expression

last_bit ::= simple_expression

code_statement ::= qualified_expression ;

restriction ::= identifier | identifier => expression

delta_constraint ::= delta expression [ range_constraint ]

at_clause ::= for directed_name use at expression ;

mod_clause ::= at mod expression ;

B.14 Ada 9X Input–Output
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Towards a Formal Semantics for Ada 9X

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The Ada 9X Language Precision Team was formed during the revisions of Ada 83, with the goal of analyzing the proposed design, identifying problems, and suggesting improvements, through the use of mathematical models.

This report defines a framework for formally describing Ada 9X, based on Kahn's "Natural Semantics", and applies the framework to portions of the language. The proposals for exceptions and optimization freedoms are also analyzed, using a different technique.