Current Return: 
The Path of Least IMPEDANCE

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Kirchhoff’s Current Law

- All currents return to their sources following path of least **IMPEDANCE**
  - NOT always path of least resistance
- **Currents do NOT return to ground**
  - They may use ground as the return path to their source, if that is the path of least impedance

**BIG Rule of Thumb for EMC: “Follow the current”**
Which path will the return current take?

$I_C = center conductor current$

$I_S = shield return current$

$I_G = ground shunt wire return current$
$I_C = center\ conductor\ current$

$S = shield\ return\ current$

$G = ground\ wire\ return\ current$

$C = S + G$
$M = \text{mutual inductance between center conductor and shield}$

$M = L_C = L_S$
A Word About Mutual Inductance

Mutual inductance between 2 circuits means:
Current in circuit 1 induces emf (potential) in circuit 2
- Magnitude according to equation above
- Sign according to dot convention above

\[ V_2 = M \frac{dI_1}{dt} = j\omega MI_1 \]
\[ V_0 = I_C (R_C + R_L + j\omega L_C) - I_S (j\omega M) + I_S (R_S + j\omega L_S) - I_C (j\omega M) \]

\[ V_0 = I_C (R_C + R_L + j\omega L_C) - I_S (j\omega M) + I_G (R_G + j\omega L_G) \]

\[ 0 = I_S (R_S + j\omega L_S) - I_C (j\omega M) - I_G (R_G + j\omega L_G) \]

\[ I_S (R_S + j\omega L_S) = I_C (j\omega M) + I_G (R_G + j\omega L_G) \]

\[ I_C = I_S + I_G \quad M = L_S \]
Solving for $I_S/I_C$:

$$I_S(R_s + j\omega L_s) = I_C(j\omega L_s) + (I_C - I_S)(R_G + j\omega L_G)$$

$$I_S[(R_s + R_G) + j\omega(L_s + L_G)] = I_C[(R_G + j\omega(L_s + L_G))]$$

$$\frac{I_S}{I_C} = \frac{[R_G + j\omega(L_s + L_G)]}{[(R_s + R_G) + j\omega(L_s + L_G)]}$$

Solving for $I_G/I_C$:

$$(I_C - I_G)(R_s + j\omega L_s) = I_C(j\omega L_s) + I_G(R_G + j\omega L_G)$$

$$I_C R_s = I_G[(R_s + R_G) + j\omega(L_s + L_G)]$$

$$I_G = \frac{R_s}{I_C} = \frac{R_s}{[(R_s + R_G) + j\omega(L_s + L_G)]}$$
Shield return current:
\[
\frac{I_S}{I_C} = \frac{R_G}{(R_S + R_G)} + j \omega (L_S + L_G)
\]

At low frequencies:
\[
\frac{I_S}{I_C} \approx \frac{R_G}{R_S + R_G}
\]

At high frequencies:
\[
\frac{I_S}{I_C} \approx \frac{j \omega (L_S + L_G)}{j \omega (L_S + L_G)} \approx 1
\]

Ground wire return current:
\[
\frac{I_G}{I_C} = \frac{R_S}{(R_S + R_G)} + j \omega (L_S + L_G)
\]

At low frequencies:
\[
\frac{I_G}{I_C} \approx \frac{R_S}{R_S + R_G}
\]

At high frequencies:
\[
\frac{I_G}{I_C} \approx \frac{j \omega (L_S + L_G)}{j \omega (L_S + L_G)} \approx 0
\]

Resistive divider
Virtually all current returns on shield
Ratio of shield current to ground wire current:

\[
\frac{I_S}{I_G} = \frac{[R_G + j \omega (L_S + L_G)]}{R_S}
\]

\[I_S = I_G\] when:

\[
\sqrt{R_G^2 + [2\pi f_c (L_S + L_G)]^2} = R_S
\]

\[
f_c = \frac{\sqrt{R_S^2 - R_G^2}}{2\pi (L_S + L_G)}
\]
Test #1 Setup Parameters

\textbf{Shield return path:}

\( R_S \text{ (measured) } = 96 \text{ m}\Omega \)

\( L_S/l = 0.25 \mu H/m \) \text{(from RG-58 coaxial cable datasheet)}

\( l = 4.8 \text{ m} \)

\( L_S = 1.2 \mu H \)

\textbf{Ground wire return path:}

\( R_G \text{ (measured) } = 15 \text{ m}\Omega \)

\( L_G = 24.4 \mu H \) \text{(next slides)}
Inductance of Circular Loop

- From Missouri University of Science and Technology inductance calculator
  - http://emclab.mst.edu/inductance/

\[
L_{\text{loop}} \approx N^2 \mu_r \mu_0 R \left( \ln \frac{8R}{r_w} - 2 \right)
\]

- \(N = \sim 5.5\)
- \(\mu_r = 1\)
- \(\mu_0 = 4\pi \times 10^{-7} \text{ H/m}\)
- \(R = 6 \text{ inches} = 15 \text{ cm}\)
- \(r_w = 0.25 \text{ cm (shield radius)}\)

\[L_{\text{coil}} = \sim 23.8 \mu\text{H}\]
Coiled cable in series with loop completing connections to signal generator:

\[ L_{\text{loop}} \approx N^2 \mu_r \mu_0 R \left( \ln \frac{8R}{r_w} - 2 \right) \]

- \( N = 1 \)
- \( \mu_r = 1 \)
- \( \mu_0 = 4\pi \times 10^{-7} \, \text{H/m} \)
- \( R_{\text{eff}} = 12 \, \text{cm} \)
- \( r_w = 0.25 \, \text{cm} \) (shield radius)

\[ L_{\text{gen}} = \sim 0.6 \, \mu\text{H} \]

\[ L_G = L_{\text{coil}} + L_{\text{gen}} = 24.4 \, \mu\text{H} \]
Why Do We Use the Shield Radius?

We use the shield radius because the current in the large loop is the net current from the coax, i.e. that gets past the shield.
Return Currents – Model for Test #1

Predicted $f_c = 590$ Hz

All currents normalized to $I_C$

$I_{S}/I_C$

$I_{G}/I_C$

Frequency (Hz)

Return Current (fraction of $I_C$)

Shield current (model)

Ground wire current (model)
Updated Equivalent Circuit

\[ V_0 \]

\[ I_C \quad R_C \quad 1.2 \mu H \]

\[ I_S \quad 96 \text{ m}\Omega \quad 1.2 \mu H \]

\[ I_G \quad 15 \text{ m}\Omega \quad 24.4 \mu H \]

\[ 50 \ \Omega \]
Test #1: Measured vs. Model

Return Current (fraction of $I_C$)

Predicted $f_c = 590 \text{ Hz}$

Measured $f_c = 700 \text{ Hz}$

$\frac{I_S}{I_C}$

$\frac{I_G}{I_C}$

All currents normalized to $I_C$

- Shield current (model)
- Ground wire current (model)
- Shield current (measured)
- Ground wire current (measured)

Frequency (Hz)
Test #2: 3.5-Turn Loop

\[ L_{\text{loop}} \approx N^2 \mu_r \mu_0 R \left( \ln \frac{8R}{r_w} - 2 \right) \]

- \( N = \sim 3.5 \)
- \( \mu_r = 1 \)
- \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \)
- \( R = 9 \text{ inches} = 23 \text{ cm} \)
- \( r_w = 0.25 \text{ cm} \) (shield radius)

\[ L_{\text{coil}} = \sim 16.3 \mu\text{H} \]
\[ L_{\text{gen}} = \sim 0.6 \mu\text{H} \text{ (same as before)} \]

\[ L_G = L_{\text{coil}} + L_{\text{gen}} = 16.9 \mu\text{H} \]
Test #2: Measured vs. Model

Predicted $f_c = 834$ Hz

Measured $f_c = 900$ Hz

All currents normalized to $I_C$

$IS/I_C$

$IG/I_C$
Test #3: 1-Turn Loop

\[
L_{\text{loop}} \approx N^2 \mu_r \mu_0 R \left( \ln \frac{8R}{r_w} - 2 \right)
\]

- \(N = 1\)
- \(\mu_r = 1\)
- \(\mu_0 = 4\pi \times 10^{-7} \text{ H/m}\)
- \(R_{\text{eff}} = \frac{5 \text{ m}}{2\pi} = \sim 80 \text{ cm}\)
- \(r_w = 0.25 \text{ cm} \text{ (shield radius)}\)

\[
L_G = 5.9 \mu\text{H}
\]
Test #3: Measured vs. Model

Return Current (fraction of $I_C$)

- Predicted $f_c = 2.1$ kHz
- Measured $f_c = 2.1$ kHz

All currents normalized to $I_C$

- $I_{S}/I_C$
- $I_{G}/I_C$

Frequency (Hz)
Why Doesn’t the Ground Wire Current Go To Zero?

\[ L_{\text{loop}} \approx N^2 \mu_r \mu_0 R \left( \ln \frac{8R}{r_w} - 2 \right) \]

- \( N = 1 \)
- \( \mu_r = 1 \)
- \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \)
- \( R = \sim 5 \text{ cm} \)
- \( r_w = 0.05 \text{ cm} \)

\( L_G = \sim 300 \text{ nH} \)

THE INDUCTANCE OF THIS LOOP LIMITS THE SHIELD CURRENT AT HIGHER FREQUENCIES
Observations/Summary

- **Measured data shows good agreement with model**
  - At low frequencies, current forms resistive divider with all available paths
  - At high frequencies, impedance is dominated by inductance (loop area)

- **High inductance (large loop areas) increases likelihood of ground bounce and magnetic coupling (crosstalk) between circuits**

- **Practical implications for circuit and system design:**
  - Provide deliberate low inductance (small loop area) return paths for high frequency currents
  - Cabling: twisted pairs, coax
  - PC board: return (-) trace immediately adjacent to send (+) trace
  - Do NOT route traces over splits in ground plane
  - Video type signals
    - Video signals tend to be “bursty”; mix of high and low frequency content
    - High frequency content will return on the shield; low frequency content will not
    - Shield isn’t a shield at low frequencies, and thus the low frequency video signal mixes in with ground noise which pollutes the video signal
Questions/Comments

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