Damage Instability and Transition from Quasi-Static to Dynamic Fracture

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Quasi-Static Loading and Rupture

Loading Phases:

- 0) to A) – Quasi-static (QS) loading
- A) to B) – Dynamic response

Snapback behavior:

- More strain energy available than necessary for fracture
Failure Criteria and Material Degradation

Progressive Failure Analysis

Benefits
- Simplicity (no programming needed)
- Convergence of equilibrium iterations

Drawbacks
- Mesh dependence
- Dependence on load increment
- Ad-hoc property degradation
- Large strains can cause reloading
- Errors due to improper load redistributions
Failure Criteria and Material Degradation

Progressive Failure Analysis

Elastic property

Failure criterion

Residual = E/100

Progressive Damage Analysis – Regularized Softening Laws

Elastic property

Increasing $l_{elem}$
Before damage

\[ F = A \sigma = E A \frac{\Delta}{L + \frac{E}{K}} \]

After damage

\[ F = A \sigma = E A \frac{\Delta - \frac{2G_c}{\sigma_c}}{L - \frac{2EG_c}{\sigma_c^2} + \frac{E}{K}} \]

For stable fracture under \( \Delta \) control:

\[ \frac{\partial F}{\partial \Delta} \leq 0 \rightarrow L \leq \frac{2EG_c}{\sigma_c^2} \]

For “long” beams, the response is unstable, dynamic, and independent of \( G_c \)
Fracture-Dominated Failure

\[ G = \frac{\pi \sigma^2 a}{E} \]

Crack propagates unstably once driving force \( G(\sigma, a_0) \) reaches \( G_{lc} \)
Fracture-Dominated Failure

Crack propagates **stably** when driving force \( G(\sigma, a_0) > G_{\text{Init}} \)

**Unstable** propagation initiates at \( G_{\text{Init}} < G \leq G_c \)
Mechanics of Crack Arrest

Crack arrest due to decreasing $G$

$$R = G_{ic}$$

$R$ represents the critical energy release rate, $G_{ic}$ is the fracture toughness, $\Delta a$ is the crack extension, and $\sigma_{max}$ is the maximum stress.
Large strain rates often result in lower fracture toughness and delayed arrest
Griffith growth criterion

\[
\frac{\partial \Pi_{\text{total}}}{\partial a_i} = \frac{\partial (\Pi_{\text{int}} + \Pi_{\text{ext}})}{\partial a_i} + G_{c,i} = \begin{cases} 
> 0 & \text{no growth} \\
0 & \text{equilibrium growth} \\
< 0 & \text{dynamic growth}
\end{cases}
\]

Stability of equilibrium propagation

\[
\frac{\partial^2 \Pi_{\text{total}}}{\partial a_i^2} = \begin{cases} 
> 0 & \text{stable} \\
< 0 & \text{unstable}
\end{cases}
\]

Wimmer & Pettermann
J of Comp. Mater, 2009
Stability of Propagation with Multiple Crack Tips

Curved laminate with through-the-width delamination

\[
\frac{\partial^2 \Pi_{\text{total}}}{\partial a_i^2} = \begin{cases} 
> 0 & \text{stable} \\
< 0 & \text{unstable}
\end{cases}
\]

Wimmer & Pettermann
J of Comp. Mater, 2009
Scaling: The Effect of Structure Size on Strength

Scaling from test coupon to structure

Scaling Laws
(Z. Bažant)

Structural size, in.

log $\sigma_n$

Yield or Strength Criteria

Linear Elastic Fracture Mechanics (LEFM)

Normal testing

Range of main practical interest for size effect

Extrapolation that must be anchored in a theory

86% of data

Number of Tests

0 10 20 30 40 50 100 200 250

log D
Cohesive Laws

Two material properties:
- $G_c$ Fracture toughness
- $\sigma_c$ Strength

Bilinear Traction-Displacement Law

$$\int_0^{\delta_c} \sigma(\delta) \, d\delta = G_c$$

Characteristic Length:

$$l_p = \gamma \frac{E G_c}{\sigma_c^2}$$
Crack Length and Process Zone

Brittle:  
\[ a_0 > 100 \ l_p \]

Quasi-brittle:  
\[ 100 \ l_p \geq a_0 \geq 5l_p \]

Ductile:  
\[ 5 \ l_p > a_0 \]
Brittle: 
\[ a_0 > 100 \, l_p \]

Quasi-brittle: 
\[ 100 \, l_p \geq a_0 \geq 5l_p \]

Ductile: 
\[ 5 \, l_p > a_0 \]
Strength and Process Zone

As the strength $\sigma_c$ decreases,

1. the length $l_p$ of the process zone increases
2. the error of the Linear LEFM solution increases

$G_c = \text{constant}$

$F, \Delta$

Force, $F$

$\sigma_c$

Decreasing

$\Delta$

$\Delta$

$F = \gamma \frac{E G_c}{\sigma_c^2}$

$G_c$

$\Delta$

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Size Effect and Material Softening Laws

Damage Evolution Laws:
Each damage mode has its own softening response

Two material properties:
- $\sigma_c$ Strength
- $G_c$ Fracture toughness

Material length scale

$$l_c \approx \gamma \frac{EG_c}{\sigma_c^2}$$
Damage Modes:
- Tension $F^+$
- Compression $F^-$

Damage Evolution:
Thermodynamically-consistent material degradation takes into account energy release rate and element size for each mode.

LaRC04 Criteria
- In-situ matrix strength prediction
- Advanced fiber kinking criterion
- Prediction of angle of fracture (compression)
- Criteria used as activation functions within framework of continuum damage mechanics (CDM)

$$d_i = 1 - \frac{1}{f_i} \exp(A_i(1 - f_i))$$

$f_i$: LaRC04 failure criteria as activation functions

$$i = F^+; F^-; M^{y+}; M^{y-}; M^s$$

Bazant Crack Band Theory:
$$A_i = \frac{2l^*X_i^2}{2E_iG_i - l^*X_i^2}$$

Critical (maximum) finite element size:
$$l^* \leq \frac{2E_iG_i}{X_i^2}$$
Prediction of size effects in notched composites

- Stress-based criteria predict no size effect
- CDM damage model predicts scale effects w/out calibration
  
  (P. Camanho, 2007)
Scale effect is due to relative size of process zone

(P. Camanho, 2007)
Short Tensile Test
Lexan Polycarbonate

Cohesive elements

Length of the Process Zone (Elastic Bulk Material)

\[ l_c \approx 0.6 \frac{E G_c}{\sigma_c^2} = 3.4 \text{ mm} \]

Symmetry

A

B

C

D

E

F

Symmetry

Maximum Load

\[ \frac{h}{a_0} = 1 \]

\[ l_{pz} = 4.7 \text{ mm} \]
Cohesive Laws - Prediction of Scale Effects

- The use of cohesive laws to predict the fracture in complex stress fields is explored.
- The bulk material is modeled as either elastic or elastic-plastic.

Lexan Plexiglass tensile specimens (CT Sun)

Observations:
- LEFM overpredicts tests for h/a<1

\[ \text{h/a} = 0.25 \quad \text{(long process zone)} \]

\[ \text{l}_{cz} = 4.7 \text{ mm.} \]

\[ \text{h/a} = 1 \quad \text{(short process zone)} \]
Study of size effect: measuring the R-curve

Double-notched compression specimens

By FEM analysis

\[ G = \phi \left( \frac{a}{w} \right) \frac{\sigma_u^2 a}{E_{eff}} \]

From test

\[ G = \frac{\pi \sigma_u^2 a}{E} \]

(Similar to)

\( \sigma_u^{-2} \), MPa\(^{-2} \times 10^{-5} \)

Catalanotti, et al., *Comp A*, 2014

\[ \sigma_u^{-2} \]

\[ \Delta a \]

Increasing \( \frac{a}{w} \)

\( a_0 \)

\( \Delta a_{stable} \)
Characterization of Through-Crack Cohesive Law

Compact Tension (CT) Specimen

Characterization Procedure:

1. Measure R-curve from CT test

\[ G_R = \frac{P^2 \partial C}{2t \partial a} \]

2. Assuming a trilinear cohesive law, fit analytical R-curve to the measured R-curve

\[ \eta = \sum_{i}^{n_s} |J_{fit}^i - G_R^i| \]

3. Obtain the cohesive law by differentiating the analytical R-curve

\[ \sigma(\delta) = \frac{\partial J_{fit}}{\partial \delta} \]

Experimental setup

Bergan, 2014

Trilinear cohesive law
Displacements measured through digital image correlation (DIC)

Plotting the R-curve as a function of the notch displacement removes the size-dependency.

Bergan, 2014
R-Curve Effect in Fiber Fracture

\[ J_R = \int_0^{\delta_c} \sigma(\delta) d\delta \]

Curve fit assuming bilinear \( \sigma(\delta) \)

Cohesive elements w/ characterized cohesive law

Cohesive response for fiber failure

Bergan, 2014
Mode II-Dominated Adhesive Fracture

Adhesive thickness: 0.13 mm

Tip of adhesive

Teflon
ENF J-Integral from DIC

Mode II J-integral vs. Displacement Jump

\[ J_{\text{ENF}} \approx \frac{9}{16} \frac{F^2 a_0^2}{E b^2 t^3} + \frac{3 F \delta_t}{8 b t} \]

Mode II cohesive law

\[ \frac{dJ}{d\delta_t} = \tau(\delta_t) \]
Nominal identical bonded MMB specimens sometimes fail in quasi-static mode and others dynamically. Why?
Double Delamination in MMB Tests

- Unexpected failure mechanism
- Two delamination fronts run in parallel: one in the adhesive, the other in the composite
- When the fiber bridge breaks, the crack grows unstably in the composite causing the drop in the load-displacement curve
Modeling the Double Delamination

- A model was developed to evaluate the observed double delamination phenomenon
- The model contains two additional cohesive layers within the composite arms

- This failure mechanism is often observed in bonded joints
Why Micromechanics?

Assumption:

“Micromechanics has more built-in physics because it is closer to the scale at which fracture occurs”

Why NOT Micromechanics? (Representative Volume Element [RVE])

- Problem of localization
- Randomness of unit cell configurations
- Lengthscales missing
- Characterization of material properties, especially the interface
- Computational expense
RVE: 1) Problem of Localization

- Linear
- Hardening
- Softening

Scale of RVE cannot be eliminated

RVE, Schapery Theory, homogenization
Localization; regularized CDM, nonlocal methods

Hardening
Softening
Linear

σ
ε

Smeared
Localized

Scale of RVE cannot be eliminated
RVE: 2) Randomness of Unit Cell Configurations


Fracture is a combination of interacting discrete and diffuse damage mechanisms

*Bloodworth, V., PhD Dissertation, Imperial College, UK, 2008.*
RVE: 3) Issue of Length Scales

RVE may not account for:
- Ply thickness
- Longitudinal crack length
- Crack spacing

Crack spacing = RVE

![Diagram showing RVE and crack spacing](image-url)
Matrix Cracking – In Situ Effect

Transverse Strength of 90° Ply, GPa

- T300/944
- [±25/90ₙ]ₙ
- [25₂/-25₂/90₂]ₙ
- [90ₙ]ₙ
- Onset of delamination
- [0/90ₙ/0]

Thin ply model

Potential crack plane, with crack nucleus

Thick ply model

Unidirectional


Inner 90° Ply Thickness 2a, mm

Thickness propagation

Longitudinal propagation
Transverse Matrix Cracks w/ One Element Per Ply

Multi-element model: correct crack evolution

Conventional single-element: no opening w/out delam.

Modified single-element: correct Energy Release Rate

\[ K \approx \frac{4E_2}{\pi^2 t} \]

Van der Meer, F.P. & Dávila, C., JCM, 2013
Crack Initiation, Densification, and Saturation

\[ \sigma = 182 \text{ MPa} \]

\[ \sigma = 273 \text{ MPa} \]

\[ \sigma = 372 \text{ MPa} \]

\[ \sigma = 679 \text{ MPa} \]

Van der Meer, F.P. & Dávila, C., JCM, 2013
Initial crack density in a uniformly stressed laminate is strictly a function of material inhomogeneity.

- Strength scaled by $f$, Fracture toughness scaled by $f^2$
- Constant $f$ along each crack path

Inhomogeneity applied to 3 levels of mesh refinement:

- 10 elts.
- 2 elts.
- 1 elt.

F Leone, 2015
Effect of Transverse Mesh Density on Crack Spacing

F Leone, 2015
Commercial finite element vendors and developers are providing more and more tools for progressive damage analysis.

But, if the load incrementation procedures do not converge…

… more analysis tools
= more rope!
Techniques for Achieving Solution Convergence

- Viscoelastic Stabilization
  - Delayed damage evolution
- Implicit dynamics or Explicit solution
- Arc-length techniques
  - Dissipation-based arc-length

\[ f \]
\[ u \]

Constant energy dissipation in each load increment

QS Solution of Unstable OHT Fracture

Van der Meer, Eng Fract Mech, 2010
Open Questions

- Is the QS solution physical?
- Are the dynamic effects necessary?
- Which solution provides more insight into failure modes?

![Diagram: Implicit vs Explicit](image)
Concluding Remarks

- A typical structural tests usually consist of three stages:
  1. QS elastic response without damage
  2. QS response with damage accumulation
  3. Dynamic collapse/rupture

- Most structural failures exhibit size effects that depend on load redistribution that occurs during the QS phases
  - Correct softening laws based on strength and toughness considerations are required

- Dynamic collapse/rupture is a result of the interaction between damage propagation and structural response
  - A stable equilibrium state often does not exist after failure under either load or displacement control
  - Onset of instability (failure) occurs when more elastic strain energy can be released by the structure than is necessary for damage propagation
  - Simulation of unstable rupture is often needed to ascertain mode of failure and to compare to test results