Modeling delamination migration: quasi-static and fatigue loading

N. V. De Carvalho  
nelson.carvalho@nasa.gov  
National Institute of Aerospace

B.Y. Chen  
Imperial College London  
National University of Singapore

J.G. Ratcliffe  
NASA Langley

S. T. Pinho, P. Baiz  
Imperial College London

T. E. Tay  
National University Singapore
Motivation

*Migration:* The process by which a propagating delamination relocates to a new ply interface via matrix cracking

**Impact**

![Image](image1.png)


**Skin-stringer pull off**

![Image](image2.png)

Contents

1 Experiments
2 Modeling approach
3 Validation
4 Summary
1 Experiment: delamination migration test

2 Modeling approach

3 Validation

4 Summary
Experiments: delamination migration

Test Setup - Premise

Delamination
(“positive” shear stress)

Migration
(“negative” shear stress)

*adapted from Greenhalgh, 2009

Experiments: delamination migration test

Test setup

- Cross-ply laminate
- “2D” migration process
- Pre-crack (Teflon insert) between 0° and 90° ply
- Variable load position (L)

Experiments: delamination migration test
Test setup - overview
Experiments: delamination migration test

Test setup - overview
Experiments: delamination migration test

Test setup – validation data

- Damage morphology
- Load - displacement
- Migration location

Damage morphology

Load - displacement

Migration location

- Force, N
- Displacement, mm
- \( D_m \) mm
- \( L/a_0 \)
Contents

1 Experiments: delamination migration test

2 Modeling approach: Floating Node Method (FNM) and Virtual Crack Closure Technique (VCCT)

3 Validation

4 Summary
Floating Node Method

Same implementation strategy suitable for standard finite element architecture

X-FEM
Phantom Node Method (PNM)
Floating Node Method (FNM)
Remeshing

Same solution
Same solution

Floating Node Method (FNM)

Real node
Floating node
Coordinates of crack positions

\[ K_q = Q \]
Floating Node Method (FNM)

Real node
Floating node
Coordinates of crack positions

\[ K_q q = Q \]
\[ K_A q_A = Q_A \]
\[ K_B q_B = Q_B \]
Floating Node Method (FNM)

Key Characteristics:

- Floating Nodes are topologically related to each element with no initial position assigned.
- The position of the floating nodes is assigned only after the crack path is determined.
- The floating nodes are used to form sub-elements within the original element and accommodate crack networks.
- Ideally suited to represent multiple cracks and their intersection.
- Can be coupled with Virtual Crack Closure Technique (VCCT) and cohesive zone crack formulations to model crack propagation.
Virtual Crack Closure Technique (VCCT):

\[
	ext{Mode I:} \quad G_I = \frac{1}{2\Delta a_1} F_n[q_n] \left( \frac{\Delta a_1}{\Delta a_2} \right)^{\frac{1}{2}}
\]

\[
	ext{Mode II:} \quad G_{II} = \frac{1}{2\Delta a_1} F_t[q_t] \left( \frac{\Delta a_1}{\Delta a_2} \right)^{\frac{1}{2}}
\]
FNM & VCCT applied to cross-ply laminates:

Laminate

\[[0^\circ/90^\circ_2/0^\circ]\]

\[\Omega\]

\[0^\circ\]

\[90^\circ\]

\[0^\circ\]

\[\Delta\]

\[\Delta\]

\[\Delta\]

\[\Delta\]

\[\Delta\]

\[\Delta\]

\[\Delta\]

Real node

Floating node (DoF)

Coordinates of crack positions

FNM & VCCT applied to cross-ply laminates:

Laminate

\[ [0^\circ/90^\circ_2/0^\circ] \]

1 FNM Element

\[ [0^\circ/90^\circ_2/0^\circ] \]

- Real node
- Floating node (DoF)
- Coordinates of crack positions
Mixed Mode exponential law:

\[
f (G_I, G_{II}) = \frac{G_T}{G_{c}^{Int}} - 1 = 0
\]

Fracture Criterion:

\[
G_{c}^{Int} = G_{Ic} + (G_{IIc} - G_{Ic}) \left( \frac{G_{II}}{G_T} \right)^{\eta}
\]

Quasi-static

Fatigue

\[
\frac{da}{dN} = A \left( G_{Tmax} \right)^{n} \\
n = n_I + (n_{II} - n_I) \left( \frac{G_{IImax}}{G_T} \right) \\
A = A_I + (A_{II} - A_I) \left( \frac{G_{IImax}}{G_T} \right)
\]
FNM & VCCT applied to cross-ply laminates: Migration onset

Quasi-static

\[ \frac{G_T}{G_c^i(F_t)} > \frac{G_T}{G_{\text{Inter}}} \geq 1 \]

\[ G_c^i = \begin{cases} G_c^A, & F_t < 0 \\ G_c^B, & F_t > 0 \end{cases} \]

Fatigue

\[ \left( \frac{da}{dN} (F_t) \right)_i > \left( \frac{da}{dN} \right)_{\text{Inter}} \]

\[ \left( \frac{da}{dN} \right)_i = \begin{cases} \left( \frac{da}{dN} \right)_A, & F_t < 0 \\ \left( \frac{da}{dN} \right)_B, & F_t > 0 \end{cases} \]

- Real node
- Floating node (DoF)
- Coordinates of crack positions
FNMT & VCCT applied to cross-ply laminates:
Migration onset – quasi-static

\[
\frac{G_T}{G_c^i(F_t)} > \frac{G_T}{G_{\text{Inter}}} \geq 1
\]

\[
G_c^i = \begin{cases} 
G_c^A, & F_t < 0 \\
G_c^B, & F_t > 0
\end{cases}
\]
FNM & VCCT applied to cross-ply laminates: Migration onset – quasi-static

\[
\frac{G_T^{i}}{G_{c}^{i}(F_t)} > \frac{G_T}{G_{\text{Inter}}} \geq 1
\]

\[
G_c^{i} = \begin{cases} G_c^A, & F_t < 0 \\ G_c^B, & F_t > 0 \end{cases}
\]

![Graph showing the relationship between migration onset and quasi-static conditions for different materials.](image-url)
FNM & VCCT - application to composites: Migration onset - fatigue

\[
\left( \frac{da}{dN} \right)_{i}^{(F_t)} > \left( \frac{da}{dN} \right)_{Inter}
\]

\[
\left( \frac{da}{dN} \right)_{i} = \begin{cases} 
\left( \frac{da}{dN} \right)_{A}, & F_t < 0 \\
\left( \frac{da}{dN} \right)_{B}, & F_t > 0 
\end{cases}
\]
FNM & VCCT applied to cross-ply laminates:

**Quasi-static**

\[ f(G_I, G_{II}) = \frac{G_T}{G_{Ic}} - 1 = 0 \]

**Fatigue**

\[ \frac{da}{dN} = A_I (G_{Tmax})^{n_I} \]

**Maximum tangential stress criterion:**

\[ \theta = 2 \tan^{-1} \left( \frac{1}{4} \left[ \left( \frac{G_I}{G_{II}} \right) \pm \sqrt{\left( \frac{G_I}{G_{II}} \right)^2 + 8} \right] \right) \]
• Topological criterion
  - local delamination is onset when matrix crack reaches interface
Fatigue algorithm

1. **Determine the growth rate for each crack**
   
   \[ G_{I_{\text{max}}}^i, G_{II_{\text{max}}}^i \]
   
   \[ \left( \frac{\text{da}}{\text{d}N} \right)^i \]

2. **Determine the number of cycles needed to propagate each crack by one element, and the crack which propagates in fewest cycles**
   
   \[ \delta N_{\text{inc}}^i = \frac{\delta a_{1\text{el}}}{\left( \frac{\text{da}}{\text{d}N} \right)^i} - \delta N_{\text{acc}}^i \]
   
   \[ \delta N_{\text{inc}}^n = \min \left\{ \delta N_{\text{inc}}^i \right\} \]

3. **Propagate the crack**
   
   \[ a^n = a^n + \delta a_{1\text{el}}^n \]

4. **Accumulate the cycles**
   
   \[ N = N + \delta N_{\text{inc}}^n \]
   
   \[ \delta N_{\text{acc}}^i = \delta N_{\text{acc}}^i + \delta N_{\text{inc}}^n \]
   
   \[ \delta N_{\text{acc}}^n = 0 \]
Verification – Static: DCB

BENCHMARK

SIMULATION

Load, N

Displacement, mm

Verification – Fatigue: DCB benchmark

1. **Experiments**: delamination migration test

2. **Modeling approach**: Floating Node Method (FNM) and Virtual Crack Closure Technique (VCCT)

3. **Validation**: modeling delamination migration

4. **Summary**
Validation: Delamination migration test

Model details

- Contact modeled between specimen and clamps/baseplate
- Clamping force applied in a first static step
- Abaqus/Standard (Implicit) + UEL
- All material properties obtained using standard/recommended test methods

Dimensions (mm)

<table>
<thead>
<tr>
<th>$B^*$</th>
<th>$2h$</th>
<th>$C$</th>
<th>$S$</th>
<th>$a_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.7</td>
<td>5.25</td>
<td>12.7</td>
<td>115</td>
<td>49</td>
</tr>
</tbody>
</table>

*B is the width of the specimen (out-of-the page);
90° - specimen width direction; 0° - specimen span direction
Validation: delamination migration test

Results - migration process

Observations

• Correct sequence of events: delamination followed by migration
• Failure morphology well captured – including crack path through-thickness
Validation: delamination migration test
Results – load vs displacement

\[ L = 1.0a_0: \]

**Observations**

- Max load: good agreement
- Delamination: unstable growth followed by arrest and subsequent unstable and stable growth
- Migration: predicted before delamination arrest
Validation: delamination migration test

Results – load vs displacement

\[ L = 1.1 a_0: \]

Observations

- Max load: good agreement
- Delamination: small region of stable growth prior to main load drop
- Migration: predicted within the main load drop
Validation: delamination migration test

Results – load vs displacement

\[ L = 1.2a_0: \]

**Observations**
- Max load: good agreement
- Delamination: stable delamination growth prior to main load-drop
- Migration: predicted within the main load drop
Validation: delamination migration test

Observations

- Max load: good agreement
- Delamination: stable growth prior to main load-drop
- Migration: predicted within the main load drop
Validation: delamination migration test

Results – Migration location

- **Trend well captured**
- **Conservative predictions**

**Observations**

- Trend well captured
- Conservative predictions
Fatigue - Preliminary results
Delamination growth and cycles to migration

Constant amplitude, $R = 0.1$ and $f = 5$ Hz:

- **Observations**
  - Load-offset affects fatigue life
1. **Experiments**: delamination migration test

2. **Modeling approach**: Floating Node Method (FNM) and Virtual Crack Closure Technique (VCCT)

3. **Validation**: modeling delamination migration

4. **Summary**
• Developed a finite element model based on the Floating Node Method combined with the Virtual Crack Closure Technique to capture the interaction between delamination and matrix-cracking

• Identified and applied migration criteria for both quasi-static and fatigue loading

• Compared simulations and experiments.
  – Good agreement observed for load-displacement, migration location and path

• Validation of the fatigue simulations are in progress
Modeling delamination migration: quasi-static and fatigue loading

N. V. De Carvalho
nelson.carvalho@nasa.gov
National Institute of Aerospace

B.Y. Chen
Imperial College London
National University of Singapore

J.G. Ratcliffe
NASA Langley

S. T. Pinho, P. Baiz
Imperial College London

T. E. Tay
National University Singapore
Backup Slides: cohesive zone elements

FNM

“Non-matching mesh”
Backup Slides: element integration
Backup Slides: Topological migration criterion, experimental evidence
Backup Slides: FNM vs PNM, convergence: $K_I$

Phantom Node Method (PNM) (Abaqus)
Backup Slides: FNM vs PNM, accuracy: $K_I$, $K_{II}$

$K_I$, $K_{II}$ (MPa mm$^{1/2}$)

$\sigma$

$L$

$W$

$a$

$\theta$

$\theta$ (degrees)

<table>
<thead>
<tr>
<th></th>
<th>FNM</th>
<th>Int. 1</th>
<th>Int. 2</th>
<th>PNM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode I</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode II</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Backup slides: MMB benchmark