Modeling delamination migration: quasi-static and fatigue loading

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**Motivation**

**Migration:** The process by which a propagating delamination relocates to a new ply interface via matrix cracking

**Impact**


**Skin-stringer pull off**
Contents

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2. Modeling approach
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4. Summary
1. **Experiments:** delamination migration test

2. Modeling approach

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4. Summary
Experiments: delamination migration

Test Setup - Premise

Delamination ("positive" shear stress) +

Migration ("negative" shear stress) -

*adapted from Greenhalgh, 2009

Experiments: delamination migration test

Test setup

- Cross-ply laminate
- “2D” migration process
- Pre-crack (Teflon insert) between 0° and 90° ply
- Variable load position (L)

All units in mm

Ls = 115

Experiments: delamination migration test

Test setup - overview

Delamination
Experiments: delamination migration test

Test setup - overview
Experiments: delamination migration test

Test setup – validation data

Damage morphology

Load - displacement

Migration location

![Diagram showing delamination migration test setup and damage morphology.](image)
1. **Experiments**: delamination migration test

2. **Modeling approach**: Floating Node Method (FNM) and Virtual Crack Closure Technique (VCCT)

3. Validation

4. Summary
Floating Node Method (FNM)

Real node
Floating node
Coordinates of crack positions

\[ K_q = Q \]
Floating Node Method (FNM)

Real node
Floating node
Coordinates of crack positions

\[ K_q = Q \]
\[ K_A q_A = Q_A \]
\[ K_B q_B = Q_B \]
Floating Node Method (FNM)

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Key Characteristics:

- Floating Nodes are topologically related to each element with no initial position assigned.
- The position of the floating nodes is assigned only after the crack path is determined.
- The floating nodes are used to form sub-elements within the original element and accommodate crack networks.
- Ideally suited to represent multiple cracks and their intersection.
- Can be coupled with Virtual Crack Closure Technique (VCCT) and cohesive zone crack formulations to model crack propagation.
Virtual Crack Closure Technique (VCCT):

\[ G_I = \frac{1}{2\Delta a_1} F_n [q_n] \left( \frac{\Delta a_1}{\Delta a_2} \right)^{\frac{1}{2}} \]

\[ G_{II} = \frac{1}{2\Delta a_1} F_t [q_t] \left( \frac{\Delta a_1}{\Delta a_2} \right)^{\frac{1}{2}} \]
FNM & VCCT applied to cross-ply laminates:

Laminate $[0^\circ/90^\circ_2/0^\circ]$

1 FNM Element (multiple plies)

FNMS & VCCT applied to cross-ply laminates:

Laminate

$[0^\circ/90^\circ_2/0^\circ]$
FNM & VCCT applied to cross-ply laminates:

**Quasi-static**

- Fracture Criterion:
  \[ f(G_I, G_{II}) = \frac{G_T}{G_{II}^{Int}} - 1 = 0 \]

- Mixed Mode exponential law:
  \[ G_c^{Int} = G_{Ic} + (G_{IIc} - G_{Ic}) \left( \frac{G_{II}^{max}}{G_T} \right)^n \]

**Fatigue**

\[ \frac{da}{dN} = A \left( G_{T_{max}} \right)^n \]

\[ n = n_I + (n_{II} - n_I) \left( \frac{G_{II_{max}}}{G_T} \right) \]

\[ A = A_I + (A_{II} - A_I) \left( \frac{G_{II_{max}}}{G_T} \right) \]
FNM & VCCT applied to cross-ply laminates:

Migration onset

**Quasi-static**

\[
\frac{G_T}{G_c^i(F_t)} > \frac{G_T}{G_{c_{Inter}}} \geq 1
\]

\[
G_c^i = \begin{cases} 
G_c^A, & F_t < 0 \\
G_c^B, & F_t > 0 
\end{cases}
\]

**Fatigue**

\[
\left( \frac{da}{dN} (F_t) \right)_i > \left( \frac{da}{dN} \right)_{Inter}
\]

\[
\left( \frac{da}{dN} \right)_i = \begin{cases} 
\left( \frac{da}{dN} \right)_A, & F_t < 0 \\
\left( \frac{da}{dN} \right)_B, & F_t > 0 
\end{cases}
\]

- **Real node**
- **Floating node (DoF)**
- **Coordinates of crack positions**
FNM & VCCT applied to cross-ply laminates: Migration onset – quasi-static

\[
\frac{G_T}{G_c^i(F_t)} > \frac{G_T}{G_{\text{Inter}}} \geq 1
\]

\[
G_c^i = \begin{cases} 
G_A^C, & F_t < 0 \\
G_B^C, & F_t > 0 
\end{cases}
\]
FNM & VCCT applied to cross-ply laminates: Migration onset – quasi-static

\[
\frac{G_T}{G_c^i(F_t)} > \frac{G_T}{G_c^{Inter}} \geq 1
\]

\[
G_c^i = \begin{cases} 
G_c^A, & F_t < 0 \\
G_c^B, & F_t > 0 
\end{cases}
\]
FNM & VCCT - application to composites: 
Migration onset - fatigue

\[
\left( \frac{da}{dN} \right)_{i} > \left( \frac{da}{dN} \right)_{Inter}
\]

\[
\left( \frac{da}{dN} \right)_{i} = \begin{cases} 
\left( \frac{da}{dN} \right)_{A}, & F_t < 0 \\
\left( \frac{da}{dN} \right)_{B}, & F_t > 0 
\end{cases}
\]

Diagram showing material A and material B with a comparison of the crack growth rate in the interfacial region.
FNM & VCCT applied to cross-ply laminates:

**Quasi-static**

\[ f(G_I, G_{II}) = \frac{G_T}{G_{Ic}} - 1 = 0 \]

**Fatigue**

\[ \frac{da}{dN} = A_I (G_{Tmax})^{n_I} \]

**Maximum tangential stress criterion:**

\[ \theta = 2 \tan^{-1} \left( \frac{1}{4} \left[ \left( \frac{G_I}{G_{II}} \right) \pm \sqrt{\left( \frac{G_I}{G_{II}} \right)^2 + 8} \right] \right) \]
• Topological criterion
  - local delamination is onset when matrix crack reaches interface
Fatigue algorithm

1. **DETERMINE THE GROWTH RATE FOR EACH CRACK**

\[ G_{I_{\text{max}}} \left| ^i, \ G_{II_{\text{max}}} \left| ^i \right. \]

\[ \left( \frac{\text{da}}{\text{d}N} \right) \left| ^i \right. \]

2. **DETERMINE THE NUMBER OF CYCLES NEEDED TO PROPAGATE EACH CRACK BY ONE ELEMENT, AND THE CRACK WHICH PROPAGATES IN FEWEST CYCLES**

\[ \delta N_{inc}^i = \frac{\delta a^i_{1el}}{\left( \frac{\text{da}}{\text{d}N} \right) \left| ^i \right.} - \delta N_{acc}^i \]

\[ \delta N_{inc}^n = \min \left\{ \delta N_{inc}^i \right\} \]

3. **PROPAGATE THE CRACK**

\[ a^n = a^n + \delta a^n_{1el} \]

4. **ACCUMULATE THE CYCLES**

\[ N = N + \delta N_{inc}^n \]

\[ \delta N_{acc}^i = \delta N_{acc}^i + \delta N_{inc}^n \]

\[ \delta N_{acc}^n = 0 \]
Verification – Static: DCB

Verification – Fatigue: DCB benchmark

1. **Experiments**: delamination migration test

2. **Modeling approach**: Floating Node Method (FNM) and Virtual Crack Closure Technique (VCCT)

3. **Validation**: modeling delamination migration

4. Summary
Validation: Delamination migration test

Numerical model

Model details

- Contact modeled between specimen and clamps/baseplate
- Clamping force applied in a first static step
- Abaqus/Standard (Implicit) + UEL
- All material properties obtained using standard/recommended test methods

Dimensions (mm)

<table>
<thead>
<tr>
<th>B</th>
<th>2h</th>
<th>C</th>
<th>S</th>
<th>( a_0 )</th>
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<td>12.7</td>
<td>5.25</td>
<td>12.7</td>
<td>115</td>
<td>49</td>
</tr>
</tbody>
</table>

*B is the width of the specimen (out-of-the-page);
90° - specimen width direction; 0° - specimen span direction
Validation: delamination migration test

Results - migration process

Observations
- Correct sequence of events: delamination followed by migration
- Failure morphology well captured – including crack path through-thickness
Validation: delamination migration test

Results – load vs displacement

$L=1.0a_0$:

Observations
- Max load: good agreement
- Delamination: unstable growth followed by arrest and subsequent unstable and stable growth
- Migration: predicted before delamination arrest
Validation: delamination migration test

Results – load vs displacement

\[ L = 1.1a_0 : \]

**Observations**
- Max load: good agreement
- Delamination: small region of stable growth prior to main load drop
- Migration: predicted within the main load drop
Validation: delamination migration test

Results – load vs displacement

\[ L = 1.2a_0: \]

![Diagram of load vs displacement](image)

**Observations**
- Max load: good agreement
- Delamination: stable delamination growth prior to main load-drop
- Migration: predicted within the main load drop
Validation: delamination migration test

Results – load vs displacement

Observations

- Max load: good agreement
- Delamination: stable growth prior to main load-drop
- Migration: predicted within the main load drop
Validation: delamination migration test

Results – Migration location

- Trend well captured
- Conservative predictions

\[ \Delta M, \text{ mm} \]

\[ L = a_0 \]

\[ u_2 = V \]

\[ \Delta M \]

\[ a_0 \]

\[ \frac{L}{a_0} \]
Fatigue - Preliminary results
Delamination growth and cycles to migration

Constant amplitude, $R = 0.1$ and $f = 5$ Hz:

Observations:
- Load-offset affects fatigue life
1. **Experiments:** delamination migration test

2. **Modeling approach:** Floating Node Method (FNM) and Virtual Crack Closure Technique (VCCT)

3. **Validation:** modeling delamination migration

4. **Summary**
Summary

• Developed a finite element model based on the Floating Node Method combined with the Virtual Crack Closure Technique to capture the interaction between delamination and matrix-cracking

• Identified and applied migration criteria for both quasi-static and fatigue loading

• Compared simulations and experiments.  
  – Good agreement observed for load-displacement, migration location and path

• Validation of the fatigue simulations are in progress
Modeling delamination migration: quasi-static and fatigue loading

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Backup Slides: cohesive zone elements

FNM

“Non-matching mesh”
Backup Slides: element integration
Backup Slides: Topological migration criterion, experimental evidence
Backup Slides: FNM vs PNM, convergence: $K_I$

The graph compares the convergence of the Phantom Node Method (PNM) (Abaqus) and FNM. The x-axis represents the number of DoF, and the y-axis represents the error in $K_I$. The graph shows that as the number of DoF increases, the error decreases, indicating improved convergence. The FNM method demonstrates a more rapid decrease in error with increasing DoF compared to the PNM method.
Backup Slides: FNM vs PNM, accuracy: $K_\|$, $K_{\|}$
Backup slides: MMB benchmark