A Space-Time Discontinuous-Galerkin Approach for Separated Flows

Scott M. Murman, Laslo T. Diosady, Anirban Garai, Marco Ceze
NASA ARC
Background

• Developing a spectral-element DG capability for separated flows over the past few years
  • Led by investment from TTT/RCA
  • *Diosady & Murman AIAA 2013-2870, 2014-2784, 2015-0294*
• Effort has grown recently w/ collaboration from other projects
  • Desired synergy between R&D and engineering
• Opportunity to share broader vision of effort & fill in technical gaps for AIAA audience
Context

• No single optimal algorithm/method/solver
  • Pareto front of optimal choices

• Different groups prioritize differently

• Our priorities are derived from the needs of numerous projects within NASA - ARMD, HEOMD, STMD - and industrial partners

• Current NASA technology based primarily on RANS/DES
  • Works well for many engineering tasks

• Supplement existing capability w/ scale-resolving methods
Target Applications

- Complex geometry - unstructured mesh
- Complex physics - scale-resolving methods
- High-Re, combustion, chemistry - fully implicit methods
- Computational intensive - high-order, adaptive methods
- Multi-disciplinary, multi-physics - robust, extensible methods
Approach

- DG spectral-element formulation
  - Unstructured arbitrary order
  - Variational Multiscale Method (VMM) for scale-resolving
- Fully implicit space-time
  - Entropy-stable, consistent all-speed scheme
  - $h$-$p$ adaptation in space and time
- Galerkin formulation
  - Demonstrated success for relevant applications
Approach

• Three main thrusts
  • New algorithms and methods
  • Optimized for next-gen exascale hardware
  • Novel physical models
• Informally known as the eddy solver
• Currently lower TRL than production methods
Turbomachinery Benchmarks

- Developed PML approach for non-reflective BC
  - Garai et al. AIAA 2016-1338
- Developed physics-based approach for freestream turbulence
  - Garai et al., ASME GT2015-42773, GT2016-56700
What is high-order?

- We want to approach spectral limit in space and time
- Leads to efficiency gains and improved physical models
- Better match for current/future hardware
  - Less data movement, more flops for the same level of accuracy
Hardware-optimized Kernels

- Current algorithms achieve < 5% of machine peak
- Spectral elements a good match for current & future hardware
Hardware-optimized Kernels

- Tensor-product sum-factorization linear algebra kernels
- Benchmark represents ~20% of code

Sandybridge (2.6GHz) vs Haswell (2.5GHz)

Theoretical Peak

Gflops vs Order of Accuracy (N)
• Exploit multiple levels of parallelism
  • Parallel in space across nodes (MPI)
  • Parallel in time within node (OpenMP)
  • Parallel within loops on chip (SIMD vectorization)

• 500 Gflops per Haswell node for 8th-order
Scale-resolving Models

- Improved numerics changes how we \textit{do} CFD
  - Efficiency, automation, error estimates
- Consistent predictive models would change how we \textit{use} CFD
  - Certification through simulation
- Need to prioritize new modeling approaches
  - Tighter coupling of numerics and modeling
- Current work is not a DG solver development it is a framework for examining scale-resolving models and methods
• Explicit separation of scales (Hughes et al., 1998, Collis, 2001)
• Filtering is variational projection operator
• Assume unresolved scales only interact w/ finest resolved scales
• Extended VMM to dynamic procedure, varying coefficient in space & time
Dynamic Modeling

- Dynamic (parameter-free) models are a necessity for complex flows
  - Automatically adjust to physics, numerics
  - $C_S = 0.18$ for HIT, $C_S = 0.065$ for shear flow - 10x change in eddy viscosity
- Successful approaches have been built upon strong physical understanding
  - Scale similarity, homogeneity, local isotropy, near-wall asymptotics
- New approaches need to leverage these lessons learned
Dynamic VMM Model

\( \tau (u, \bar{w}) \simeq -2 \left( (C_1 \Delta)^2 \| \tilde{S}_{i,j} \| \tilde{S}_{i,j}, \bar{w}_{i,j} \right) \)

- Variational Leonard stresses
  - Requires high-order \((N \geq 4)\)

\[
\left( \bar{u}^h_i \bar{u}^h_j - \bar{u}^H_i \bar{u}^H_j, \bar{w}^H_i, \bar{w}^H_j \right) = -2 \left( (C_1 \Delta)^2 \| \tilde{S}^h_{i,j} \| \tilde{S}^h_{i,j}, \bar{w}^H_{i,j} \right) + 2 \left( (C_1 \Delta)^2 \| \tilde{S}^H_{i,j} \| \tilde{S}^H_{i,j}, \bar{w}^H_{i,j} \right)
\]

- Using state as test function gives variational analogue to Germano procedure
- Can also provide analogue to Lilly’s least-square
- Entropy-stable compressible formulation in full paper
• Previous work demonstrates dynamic model converges to DNS w/ sufficient resolution (consistency)

• Practical simulations never have sufficient resolution
• Examine behavior on realistic coarse mesh
  • Re_τ = 544
  • 4th-order in time, 8th-order in space
  • Δt^+ = 1, Δx^+ = 100, Δy^+ = 1, Δz^+ = 50
• VMM w/ fixed coefficient degrades performance

**Channel Flow**

**Mean Velocity**

- DNS
- ILES
- ILES+VMM ($C_1 = 0.1$)

**Reynolds Stress**

- DNS
- ILES
- ILES+VMM ($C_1 = 0.1$)
**Idealized Behavior**

- ILES always resolves lower Re
- Dynamic approach resolves inertial range uses model for dissipation scales
- Requires non-dissipative scheme (*e.g.* skew symmetry)
- Entropy-stable schemes inherently dissipative
- Completely remove numerical dissipation as first test
Channel Flow

- Dynamic procedure least sensitive to current mesh resolution
- Examine trends w/ changing resolution through higher Re

**Mean Velocity**

**Reynolds Stress**

![Graphs showing Mean Velocity and Reynolds Stress](image)
Dynamic VMM Model

- Expected value in log layer
- Approach zero towards wall
- Decays towards centerline

Subgrid-stress Coefficient
Summary

• Working prototype to experiment w/ scale-resolving methods for complex flows
• Existence proof that spectral-elements can take advantage of modern hardware
• Initial experiments w/ VMM encouraging
• Current work is extending to relevant flight geometry and conditions
  • Wall-modeled LES/VMM
  • Complex geometry (AIAA 2015-0294)
  • Relative motion/FSI capability
Backup
• Variable-order produces lower error at same cost
  • Ceze et al. AIAA 2016-0833
• Currently extending to space-time $h$-$p$ adaptation and error estimates
Current Status

- Laslo Diosady - moving body, shock capturing
- Anirban Garai - turbomachinery, LES - AIAA 2016-XXXX
- Marco Ceze - adjoint, mesh adaptation - AIAA 2016-XXXX
- Corentin Carton de Wiart - wall modeling, hybrid-RANS
Hardware-optimized Kernels

- Optimization counterbalances increase in cost for high order

AIAA 2013-2870
Residual

AIAA 2016-XXXX
Jacobian

Overflow