Structural Dynamic Analysis in Rocket Propulsion and Launch Vehicles

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ER41/Propulsion Structures &
Dynamic Analysis

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• Introduction to NASA’s new SLS and Artemis Program to the Moon!
• Description of Structural Dynamics, and how it applies to a rocket.
• Application of Structural Dynamics in all phases of the mission of a launch vehicle and its components:
  – Turbine Blades, Rocket Nozzle
  – Rocket Engine Loads
  – High Cycle Fatigue in Main Propulsion System
  – Launch Vehicle Structural Dynamic Characterization and Test Validation
  – Launch Vehicle Loads
• Will need to introduce various Structural Dynamics Analysis Methods throughout presentation.
• Questions
The Power of SLS and Orion

ORION
The only spacecraft capable of carrying and sustaining crew on missions to deep space, providing emergency abort capability, and safe re-entry from lunar return velocities.

SLS
The only rocket with the power and capability required to carry astronauts to deep space onboard the Orion spacecraft.

- SLS Block 1 is 321 feet high, 8.8 million lb thrust.
- 57,000 lbs to lunar orbit
- 100% more Payload. Volume, 50% more Mass than any other current launch vehicle (including SpaceX Falcon heavy).

5 main sections joined
Sep. 19, 2019
Artemis Phase 1: To The Lunar Surface by 2024

2021
Artemis I: First human spacecraft to the Moon in the 21st century

2023
Artemis II: First humans to orbit the Moon in the 21st century
Artemis Support Mission: First high-power Solar Electric Propulsion (SEP) system

2024
Artemis Support Mission: Human Landing System delivered to Gateway
Artemis III: Crewed mission to Gateway and lunar surface
Humans on the Moon - 21st Century
First crew leverages infrastructure left behind by previous missions

Early South Pole Mission(s)
- First robotic landing on eventual human lunar return and In-Situ Resource Utilization (ISRU) site
- First ground truth of polar crater volatiles

Commercial Lunar Payload Services
- CLPS-delivered science and technology payloads

Large-Scale Cargo Lander
- Increased capabilities for science and technology payloads

LUNAR SOUTH POLE TARGET SITE
What is Structural Dynamics and Why do we care?

• What is Structural Dynamics?
  – Quantify dynamic characteristics of structures
  – Enable prediction of response of structures to dynamic environment
  – Assess uncertainties in predictions.

• Why do we care?
  – Excessive vibration can cause excessive deformation.
    • Car noise, vibration, and harshness
    • Turbomachine rotordynamics (whirl)
    • Computer disk drives
    • Cutting machine chatter
    • Astronaut eyeballs and other sensitive parts due to thrust oscillation.
  – Excessive vibration can cause structural failure due to high dynamic stresses.
    • **Turbine blades**, other flow-path hardware
    • Buildings under earthquake load
  – Any structure or system undergoing dynamic loading responds differently than that system undergoing static loading only, which may be good or bad.
    • Space Vehicle loads
    • Airplane control surfaces
Free Vibration, Undamped Single Degree of Freedom System

\[ \sum F_x = m \ddot{u} \]
\[ m \ddot{u} + Ku = 0 \]

1) Steady State, simplest, worth remembering:
Assume solution \( u = u(t) \) is of form
\[ u(t) = A \cos(\omega t) \]
\[ \dot{u}(t) = -A\omega \sin(\omega t) \]
\[ \ddot{u}(t) = -A\omega^2 \cos(\omega t) \]

Now plug these equalities into eq of motion:
\[ m(-A\omega^2 \cos \omega t) + k(A \cos \omega t) = 0 \]
\[ A \cos \omega t(k - \omega^2 m) = 0 \]

For \( A \cos \omega t = 0 \), \( A \) has to = 0, i.e., no response (“trivial solution”)

Therefore, \( k - \omega^2 m = 0 \)

\[ \omega^2 = \frac{k}{m} \quad \Rightarrow \quad \omega = \sqrt{\frac{k}{m}} \text{ Rad/sec} \]

Define \( \lambda \equiv \text{Eigenvalue} = \omega^2 \equiv \text{Natural Frequency}^2 \)

So, solution for \( u = u(t) \) is
\[ u(t) = A \cos\left(\sqrt{\frac{k}{m}} t\right) \]

where \( A \) depends on the initial conditions
Add damping and forcing function

\[ m \ddot{u} + c \dot{u} + ku = F_o \cos(\Omega t) \]

Define

\[ \zeta = \frac{c}{c_{critical}} \text{, if } <1, \text{ underdamped} \]

where

\[ c_{critical} = 2\sqrt{k \cdot m} = \text{no oscillation in response} \]

Dynamic response as function of \( \Omega/\omega \) is

\[ |U(\Omega)| = \frac{F_o}{k} \sqrt{\frac{1}{\left(1 - \left(\frac{\Omega}{\omega}\right)^2\right)^2 + \left(2\zeta \left(\frac{\Omega}{\omega}\right)\right)^2}} \]
Modal Analysis of Multiple DOF Systems

Solutions for Undamped, Free Vibration of MDOF Systems with N dof's.

\[
[M]\{\ddot{u}\} + [K]\{u\} = \{0\}
\]

Assume solution of form (1 spatial solutions = eigenvectors = modes)

\[
\{u\}_i = \{\phi\}_i e^{j(\omega t + \alpha_i)}
\]

Examples: [https://www.youtube.com/watch?v=kvG7OrjBirI](https://www.youtube.com/watch?v=kvG7OrjBirI)

Continuous MDOF

Discrete MDOF

\[ w(x) = U_i(x) \iff \{\phi\}_i \]

\[ \phi_{1i} \quad \phi_{2i} \quad \phi_{4i} \quad \phi_{6i} \]

\[ \bar{u}_{2i}(t) = \phi_{2i} e^{j(\omega t + \alpha_i)} \]

\[ u_{2i}, u_{4i}, u_{6i} \]
• Liquid Fuel (LH2, Kerosene) and Oxidizer (LO2) are stored in fuel tanks at a few atmospheres.
• Turbines, driven by hot gas created by mini-combustors, tied with shaft to pump, suck in propellants, increases their pressures to thousands of psi, producing substantial harmonic forces at specific frequencies.
• High pressure propellants sent to Combustion Chamber, which ignites mixture with injectors, produces large forces in a wide band of frequencies, most of which are random.
• Hot gas directed to converging/diverging nozzle to give flow very high velocity for thrust.
• Both random and harmonic loads propagate through every component on the engine and last throughout engine operation, requiring SD analysis to verify structural integrity.
Main Tool is Finite Element Analysis

- Discretize continuous structures into hundreds of thousands elements.
- Structural response of each element calculated by differential equation of motion.
- Can model very small turbine blades to complete launch vehicles.
• Structural Dynamics play a critical role in design of Turbomachinery, Nozzles, and System Hardware. (ignition 13 sec, vibration 44sec)
Structural Dynamics of Flowpath Components in Turbopumps

Inlet Guide Vanes

Turbine Blades particularly problematic since they have tremendous KE.

SSME/RS25 Powerhead

Turbine Bladed-Disk

Stators

Inlet Guide Vanes
Harmonic forcing function results from interaction of stationary and rotating components in flow-path. (wakes from upstream, potential field from downstream)

CFD needed to provide forcing functions

CFD mesh region of J2X fuel turbine
Modal Analysis is first step in Turbine Bladed-Disk Structural Dynamic Analysis

- Identify natural frequencies and mode shapes, compare with frequencies of forcing functions.
- Try to avoid resonant conditions (“triple crossover”) during design.
- If can’t avoid, frequently have to perform forced response analysis.
For Cyclically Symmetric Structures with Coupling, Identification of Nodal Diameters in Modes also Required

• Each blade mode on the previous chart exists within a family associated with blade-disk “Nodal Diameter” modes.

• The Tyler-Sofrin Blade-Vane Interaction Chart tells us which Nodal Diameter family of blade modes can be excited.

  • E.G., 2 x Nozzles excites the 5ND family.

<table>
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<th>2x37=</th>
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<th>148</th>
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<tr>
<td>Nozzle Multiples</td>
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<table>
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<th>Blade multiples</th>
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<th>32</th>
<th>-5</th>
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<td></td>
<td>138</td>
<td>N/A</td>
<td>27</td>
<td>-10</td>
<td></td>
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<tr>
<td></td>
<td>207</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Modal Testing verifies Numerical Predictions

- Use Instrumented Impact Hammer to impart a quick impact onto structure (force time history measured), which contains broadband frequency content (Fourier!).
- Response is measured using an accelerometer or laser vibrometer, which measures velocity.
- A frequency analyzer performs Fourier Transforms of the excitation and response to get frequency domain.
- Mode shapes obtained from

\[
\phi_{ijm} = \text{Im} \left( \overline{FRF}_{ij}(\Omega) \right) = \frac{u_i}{F_j} = \frac{\text{Response at dof } i}{\text{Harmonic excitation at dof } j}
\]

- Match results with analysis, improve model (eg. SSME)
Frequency and Transient Response Analysis uses Concept of Modal Superposition using Generalized (or Principal Coordinates).

\[
[M] \{\ddot{u}\} + [C] \{\dot{u}\} + [K] \{u\} = \{P(t)\}
\]

- **Mode Superposition Method** – transforms to set of uncoupled, SDOF equations that we can solve using SDOF methods.
- First obtain \([\Phi]_{\text{mass}}\). Now, introduce coordinate transformation:

\[
\{u\} = N^{\text{T}} [\Phi] \{\eta\}^{\text{T}}
\]

\[
[M][\Phi] \{\ddot{\eta}\} + [C][\Phi] \{\dot{\eta}\} + [K][\Phi] \{\eta\} = \{P(t)\}
\]

\[
\]

Now, if resonance, forced response required, need to know about **Generalized Coordinates/Modal Superposition**.
Frequency Response of MDOF Systems can be Reduced to Solutions of multiple SDOF’s

\[ |\eta_m(t)| = \frac{\phi_m^T \{F\}}{\lambda_m} \left(\frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_m}\right)^2\right)^2 + \left(2\zeta_m \frac{\omega}{\omega_m}\right)^2}}\right) \]
For Example, Turbine Blade Forced Response Results

- Obtain alternating stresses during modal resonance.
- Combine with mean stress to give High Cycle Fatigue Safety Factor, $SF_{HCF}$
- Analysis for AMDE blades showed a blade damper needed.

Without Damper:
- Without external damper
- $\zeta = 0.077\%$
- $SF_{HCF} = 0.70$

With Damper:
- With damper
- $\zeta = 0.251\%$
- $SF_{HCF} = 2.27$
“Side Loads” in Rocket Nozzles is Major Fluid/Structural Dynamic Interaction Issue

- Start-up, shut-down, or sea-level testing of high-altitude engines, ambient pressure higher than internal nozzle wall pressures.
- During transient, pressure differential moves axially down nozzle.
- At critical $\frac{p_{\text{wall}}}{p_{\text{ambient}}}$, flow separates from wall - Free Shock Separation (FSS), induces “Side Load”.

Mach number simulation from CFD
In-rushing ambient pressure at uneven axial locations causes large transverse shock load.

Caused failures of both nozzle actuating systems (Japanese H4 engine) and sections of the nozzle itself (SSME).

Existing Side Load calculation method

- Assumes separation at two different axial stations, integrates the resultant $\Delta P \cdot dA$ loads.

Method calibrated to maximum and minimum possible separation locations to be intentionally conservative.
Primary Nozzle Failure Mode for most Rocket Engines is Buckling due to Side Loads during Start-Up and Shut-Down.

Vulcain engine test, DLR Germany
• FASTRAC engine designed to operate in overexpanded condition during ground test.
• Didn’t have funding to pay for vacuum clamshell.
• Test/analysis program initiated with goal of obtaining physics-based, predictable value.
• Strain-gauge measurements taken on nozzle during hot-fire test
• Flow separation clearly identified at Steady-State Operation.

- Video, Pressure and strain-gage data from thin-wall nozzle show self-excited vibration loop tying structural 2\textsuperscript{ND} mode and flow separation.
Engine System Structural Dynamic Loads

- During steady-state operation, two types of dynamic force environments: sinusoidal (resulting from turbomachinery) and random (from combustion), which typically dominate.
- Structural dynamic model of entire engine required to calculate response “loads”.
- With current level technology, impossible to quantify the forces with enough precision to conduct a true transient dynamic analysis.
- Methodology: measure dynamic environment (i.e., accelerations) at key locations in the engine. For a new engine, data from “similar” previous engine designs is scaled to define an engine vibration environment.

Key locations near the primary vibration sources.

Acceleration data is enveloped to capture uncertainties thus defining a vibration environment.
## Typical MC-1 Engine Load Set

| Glue Bracket 3 | Shear 1 (lbs) | Shear 2 (lbs) | Axial (lbs) | Bending 1 (in-lbs) | Bending 2 (in-lbs) | Torque (in-lbs) |
|---------------|---------------|---------------|-------------|-------------------|-------------------|----------------
| GB-3          |               |               |             |                   |                   |                |
| Sine X        | 97            | 7             | 0           | 3                 | 78                | 72             |
| Sine Y        | 91            | 7             | 0           | 3                 | 98                | 70             |
| Sine Z        | 119           | 5             | 0           | 2                 | 78                | 52             |
| **Sine Peak (RSS)** | **178** | **11** | **0** | **5** | **148** | **113** |
| 3 sig Random X | 450           | 113           | 0           | 16                | 25                | 1475           |
| 3 sig Random Y | 781           | 66            | 0           | 9                 | 41                | 828            |
| 3 sig Random Z | 155           | 1             | 0           | 4                 | 1101              | 6              |
| **Random Peak (RSS)** | **915** | **130** | **0** | **19** | **1102** | **1692** |

<table>
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<th>Stringer Bracket 3 (Lower Support)</th>
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<td>8</td>
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<td><strong>Sine Peak (RSS)</strong></td>
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<td><strong>15</strong></td>
<td><strong>17</strong></td>
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<td><strong>16</strong></td>
<td><strong>187</strong></td>
<td><strong>1556</strong></td>
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<tr>
<td>Sine X</td>
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<td>7</td>
<td>21</td>
<td>81</td>
<td>9</td>
<td>21</td>
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<tr>
<td>Sine Y</td>
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<td>80</td>
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<tr>
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<td>4</td>
<td>16</td>
<td>59</td>
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<tr>
<td><strong>Sine Peak (RSS)</strong></td>
<td><strong>93</strong></td>
<td><strong>9</strong></td>
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<td><strong>515</strong></td>
<td><strong>1029</strong></td>
<td><strong>371</strong></td>
<td><strong>1795</strong></td>
<td><strong>79</strong></td>
</tr>
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</table>
Oct. 28, 2014, Orbital ATK (now Northrup Grumman) launched Antares-130 with ISS cargo from Virginia.

15 seconds into flight, Main Propulsion System (engine and propellant feedlines) exploded. Damage to launch pad, no injuries.

MSFC called to help determine cause of failure.

AJ26 Engine, actually an old Russian NK-33 engine purchased by Aerojet with little understanding of design.
• Data taken on fuel propellant line close to tank showed very high acceleration levels, but unclear if that meant turbopumps themselves had excessive vibrations.

• MSFC Structural Dynamics team created/adapted finite element models of engines, feedlines, and tank, applied loads at turbopumps and reproduced measured response at accelerometer locations.

• Conclusion: rotating turbopump components contacted housing, rubbed, caused ignition (easy in LOX).

• Possible root causes were manufacturing defect, foreign object debris, or inadequate turbine-end bearings.
2002 – Cracks found in Orbiter Main Propulsion System Feedline Flowliner
Dynamic analysis determined source of cracking was several modes excited by upstream inducer blade count and cavitation.

Tested flowliner dynamic response to validate models.

Performed fracture analysis and computed expected service life based upon observed crack sizes. Solution was improved and more frequent inspections.

Complex Mode Shapes 1000 to 4000 Hz
Launch Vehicle Loads Analysis (Coupled Loads Analysis)

- **Purpose** – calculate “gpa” (grid point accelerations) and resolved forces (shears and moments) at all points along vehicle structure during all phases of mission. These are generically called “loads”.

- **First must generate estimates of forces on vehicle**
  - Transportation forces – ground, shipping
  - Launch – vibroacoustics (acoustic waves from engines rebounding off of launch area back onto vehicle structure)
  - Ascent – wind and aerodynamic forces
  - Thrust from Engines
  - Stage Separation & Pyrotechnic Events

- **Then calculate Structural Dynamic Response**
  - “Coupled Loads Analysis” using “Component Mode Synthesis” primary technique

- **Outputs**
  - Random Vibration & Shock Criteria
  - Component Accerations
  - Design Limit Loads
  - Aeroelasticity Assessments
  - Propellant Slosh Dynamics
### SLS Europa Clipper Configuration – Y & Z Bending & Axial Modes Comparison - Liftoff

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Mode Number</th>
<th>Freq (Hz)</th>
<th>Mode Shape</th>
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<tr>
<td>DAC1 SM-1</td>
<td>31</td>
<td>1.48</td>
<td>1st Y-Bending</td>
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<tr>
<td>DAC1 SM-1</td>
<td>32</td>
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<td>DAC1 SM-1</td>
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<td>143</td>
<td>11.16</td>
<td>Axial 2 (MPCV+ICPS)</td>
</tr>
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</table>

*Additional verification of this mode performed (see backup)*

---

**Liftoff**

- **DAC1 SM-1**
  - Mode 31: 1.48 Hz (1st Y-Bending)
  - Mode 32: 1.49 Hz (1st Z-Bending)
  - Mode 35: 2.09 Hz (2nd Y-Bending)
  - Mode 34: 2.01 Hz (2nd Z-Bending)
  - Mode 114: 9.95 Hz (Axial (ICPS+CS))

- **VAC1R EM-1**
  - Mode 30: 1.36 Hz (1st Y-Bending)
  - Mode 32: 1.4 Hz (1st Z-Bending)
  - Mode 39: 2.4 Hz (2nd Y-Bending)
  - Mode 37: 2.25 Hz (2nd Z-Bending)
  - Mode 126: 10.42 Hz (Axial 1 (CS))
  - Mode 143: 11.16 Hz (Axial 2 (MPCV+ICPS))

*Additional verification of this mode performed (see backup)*

---

**Images**

- **Rigid wall representation of centerline shape**
- **Modal analysis results**
- **Comparison of modes**
- **Visualization of modes**
Coupled Loads Cycles

- When Aerodynamic, Inertial, and other external forces applied to Structural Dynamic model and “ Loads” (Shear X, Y, Z, Bending Moment X, Y, Z) obtained, used for design updates.
First 3 SLS Vehicle fundamental modes drive primary structure loads

- SLS Primary Structure loads are dominated by the first 3 bending mode pairs.
- The first 3 bending mode pairs also represent the most important modes for GNC stability.
- The beam-like response of SLS at low frequencies increases the likelihood of accurately capturing these modes during the Integrated Modal Test.
Component Mode Synthesis is Theory behind Coupled Loads

- Used to dynamically couple together “substructures” built by different organizations.
- Partition displacement vectors into internal and boundary DOF’s.
Solution of CMS using Craig-Bampton Transformation

Partition M and K matrices of each substructure in same way:

\[
\begin{align*}
[M]_\text{ET} &= \begin{bmatrix} M_{ii} & M_{ib} \\ M_{bi} & M_{bb} \end{bmatrix}^{ET} \\
[K]_\text{ET} &= \begin{bmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{bmatrix}^{ET}
\end{align*}
\]

Craig-Bampton Transformation Matrix \([CB]^{ET}\)

\[
\begin{align*}
\begin{bmatrix} u_i \\ u_b \end{bmatrix}^{ET} &= \begin{bmatrix} \Phi_{\text{cantilevered}} & -K_{ii}^{ET}^{-1}K_{ib}^{ET} \\ 0 & I \end{bmatrix} \begin{bmatrix} \eta \\ u_b \end{bmatrix}^{ET}
\end{align*}
\]

\[
\begin{align*}
\{F\}_{\text{system}} &= [CB]^{ETT} \{F(t)\}^{ET} + \ldots + [CB]^{ETT} \{F(t)\}^{\text{orbiter}}
\end{align*}
\]

now solve

\[
[M]_\text{sys} \{\ddot{\eta}\}_\text{sys} + [B]_\text{sys} \{\dot{\eta}\}_\text{sys} + [K]_\text{sys} \{\eta\}_\text{sys} = \{F\}_\text{sys}
\]

as you would any other MDOF system.

i.e., transform to generalized coordinates using new system matrices to obtain uncoupled equations of motion in \{\eta\}, solve time response numerically, back transform to \{\eta\}_\text{sys}, back transform again to get \{u\}. 

\[
\begin{bmatrix} \eta^{\text{orbiter}} \\ \eta^{ET} \\ \vdots \\ \eta^{\text{satellite}} \\ u_b^{\text{orb-ET}} \\ \vdots \end{bmatrix}
\]
Each test builds upon the previous test results
What is the probability that the flight model is “good”, given that verification is only done on the ground?
• Structural Dynamics is one of the Critical Disciplines for the successful Design, Development, & Testing of Space Launch Vehicles.

• It is applied from the smallest component (turbine blades), all the way to the entire vehicle, and has to be calculated for every phase of a mission, from ascent and orbit to landing.

• Successful application of Structural Dynamics requires extensive knowledge of Fourier Techniques, Linear Algebra, Random Variables, Finite Element Modeling, and essentials of SDOF and MDOF vibration theory.

• Working knowledge of Fluid Dynamics, Statistics, and Data Analysis also extremely useful.

• This is fun!