ANALYSIS AND TESTING OF HIGH ENTRAINMENT SINGLE-NOZZLE JET PUMPS WITH VARIABLE-AREA MIXING TUBES

by Kenneth E. Hickman, Philip G. Hill, and Gerald B. Gilbert

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In this investigation, an analytical model was developed to predict the performance characteristics of axisymmetric single-nozzle jet pumps with variable area mixing tubes. The primary flow may be subsonic or supersonic. The computer program presented in this report uses integral techniques to calculate the velocity profiles and the wall static pressures that result from the mixing of the supersonic primary jet and the subsonic secondary flow.

An experimental program was conducted to measure mixing tube wall static pressure variations, velocity profiles, and temperature profiles in a variable area mixing tube with a supersonic (M = 2.72) primary jet. Static pressure variations were measured at four different secondary flow rates. These test results were used to evaluate the analytical model. The analytical results compared well to the experimental data. Therefore, the analysis is believed to be ready for use to relate jet pump performance characteristics to mixing tube design.
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ANALYSIS AND TESTING OF HIGH ENTRAINMENT
SINGLE-NOZZLE JET PUMPS WITH VARIABLE-AREA MIXING TUBES

By Kenneth E. Hickman, Philip G. Hill, and Gerald B. Gilbert

SUMMARY

The use of jet pumps is of increasing interest for boundary layer control or control force augmentation in V/STOL aircraft. In typical applications, a small mass flow of primary air at pressures up to 400 psia can be used to entrain a much larger mass flow of secondary air at ambient conditions. The primary nozzle flow is supersonic while the secondary flow is subsonic. The jet pump system design objectives may be maximum entrainment, maximum thrust augmentation, or some combination of the two. Little information is available in the literature to guide the designer of jet pumps for such applications.

In this investigation, an analytical model was developed to predict the performance characteristics of axisymmetric single-nozzle jet pumps with variable area mixing tubes. The primary flow may be subsonic or supersonic. In the region upstream of the section where the central jet reaches the wall, the analysis is based upon the hypothesis that the mixing phenomenon is fundamentally similar to the mixing of a free turbulent jet with the surrounding fluid. The eddy viscosity values used in the analysis are adjusted to allow for the effect of the duct walls on the mixing process. Integral techniques are employed in a computer program to solve the continuity, momentum, and energy equations to determine the variation of flow properties along the mixing tube. Wall boundary layer effects are included in the analysis.

Downstream of the section where the jet reaches the wall, the velocity profile is assumed to approach asymptotically the profile for fully developed turbulent flow in a pipe. Viscous forces are present throughout the flow so no distinct boundary layer analysis is employed. The eddy viscosity is assumed to approach the fully-developed flow value asymptotically. Wall friction forces are calculated from the fully-developed pipe flow friction coefficient. Integral techniques are employed as before to determine the variation of flow properties along the mixing tube.
An experimental program was conducted to measure mixing tube wall static pressure variations, velocity profiles, and temperature profiles in a variable area mixing tube with a supersonic ($M = 2.72$) primary jet. Static pressure variations were measured at four different secondary flow rates. These test results were used to evaluate the analytical model.

The analytical model yields good predictions of wall static pressure distributions, velocity profiles, and temperature profiles along the mixing tube. Therefore, the analysis is believed to be ready for use to relate jet pump performance characteristics to mixing tube design.
Section 1

INTRODUCTION

1.1 Background

A number of STOL aircraft boundary layer control systems now under consideration employ jet pumps to entrain large flows of secondary air and direct them over deflected flaps to achieve lift augmentation. Some proposed VTOL aircraft systems also employ jet pumps for direct lift or control force augmentation. The primary, high-pressure flow for the jet pumps can be provided either by bleed from main engine compressors or by an auxiliary power unit.

The use of jet pumps as primary components of V/STOL aircraft systems makes necessary the development of new design techniques for these devices. In aircraft applications, it is essential to be able to minimize the size of jet pumps for particular primary and secondary flow conditions. Jet pumps for boundary layer control systems generally must have high entrainment ratios—the secondary flow often must be over 10 times larger than the primary flow—but pressure rises of only a few psf are needed. The primary flow may be highly supersonic. Thus, design procedures which have been developed in the past for the more conventional low-entrainment, high-pressure rise industrial jet pumps are not suitable for V/STOL aircraft jet pump design.

1.2 Previous Work

In an earlier program, Dynatech R/D Company carried out an analytical and experimental investigation of high-entrainment ratio air-to-air jet pumps for the Ames Research Center of NASA (reference 1). This investigation was limited to jet pumps with constant-diameter mixing tubes. An analytical procedure and a computer program were developed to predict the performance of such a jet pump over a range of operating conditions. The accuracy of the analysis was confirmed by comparing predicted performance to test results for a number
of multiple-nozzle jet pump configurations at different primary flow pressure and
temperature levels. Procedures were demonstrated for matching a jet pump to
its duct system for maximum entrainment or thrust augmentation.

The selection of the constant-diameter mixing tube configuration allowed
considerable simplification of the analysis, design, and construction of the jet pump. However, it is unlikely that constant-area jet pumps give the best performance for all applications. Almost no information is available to indicate the extent of performance improvements which can be achieved with other mixing tube configurations.

A method for predicting the flow behavior in jet pumps with arbitrary
mixing tube shapes and incompressible flows was reported by P.G. Hill (reference
2). The method is based upon the hypothesis that the mixing phenomenon in a jet pump has a fundamental similarity to the mixing of a free turbulent jet with the surrounding fluid. Therefore, as in a free jet, the turbulent Reynolds number--

\[ \text{Re}_T = \frac{\text{jet velocity } \times \text{ duct radius}}{\text{eddy viscosity}} \]

--will remain constant with distance as mixing occurs. This is a rather gross simplifying assumption but the resulting flow predictions are good. Static pressure variations and velocity profiles computed on this basis agreed well with test data for Helmbold's converging-diverging mixing tube. Once the static pressure distribution is known, the jet pump performance can be predicted without further difficulty.

The analytical methods of reference 2 are limited in application to incompressible flow in axisymmetric jet pumps having a single primary jet. These analytical methods must be modified to include compressible flow effects if the methods are to be useful for the designer of V/STOL aircraft jet pump systems.
1.3 Objectives of This Investigation

The specific objectives of this investigation are as follows:

- to develop an analytical procedure for predicting the performance of high-entrainment-ratio compressible flow jet pumps with arbitrary mixing tube geometry.

- to obtain test results with jet pumps having variable-area mixing tubes so that the analytical methods can be checked.

The analytical procedure is formulated to allow prediction of the performance of a particular jet pump nozzle and mixing tube combination over a range of primary and secondary flow conditions. To select the best jet pump design for a particular application, the analysis can be used to predict the performance for a number of different mixing tube shapes. Comparison of the performance characteristics will show which geometry is best. The off-design performance of the jet pump can be determined by using the same analytical procedures.
Section 2

SYMBOLS

\[ A \quad \text{area, ft}^2 \]
\[ b \quad \text{diameter of jet at which } U = U_0 + \frac{U_j}{2}, \text{ ft} \]
\[ C_f \quad \text{wall friction coefficient} \]
\[ C_w \quad \text{nozzle flow coefficient} \]
\[ E \quad \text{dimensionless eddy viscosity } = \frac{\epsilon}{UR} \]
\[ f_0 \quad \text{free jet profile value } = f_0 (\eta); \text{ equation (1)} \]
\[ f_2 (\eta) \quad \text{velocity profile at the end of Part 1, equation (7)} \]
\[ g_0 \quad \text{dimensional constant } = 32.2, \text{ lbm-ft/lbf-sec}^2 \]
\[ g_2 (\eta) \quad \text{auxiliary velocity profile, equation (7)} \]
\[ g_0 (\eta) \quad \text{velocity profile for fully-developed flow in a pipe} \]
\[ H \quad \text{boundary layer shape factor} \]
\[ k \quad \text{specific heat ratio} \]
\[ K_L \quad \text{suction duct loss coefficient} \]
\[ m \quad \text{entrainment ratio } = W_o/W_1 \]
\[ n_\text{s} \quad \text{number of equal-radial-increment annuli used in integral analyses, equation (36)} \]
\[ p \quad \text{static pressure, lbf/ft}^2 \]
\[ P \quad \text{stagnation pressure, lbf/ft}^2 \]
\[ P_{o2} \quad \text{secondary flow stagnation pressure after correction for suction duct losses, lbf/ft}^2 \]
\[ R \quad \text{tube radius, ft} \]
\[ R_0 \quad \text{radius at nozzle exit section, ft} \]
\[ R_g \quad \text{gas constant } \times g_0, \text{ ft}^2/\text{sec}^2\cdot \text{R} \]
\( \text{Re}_m \) Reynolds number based on mean velocity; equation (50)

\( \text{Re}_T \) turbulent Reynolds number

\( R_\theta \) momentum thickness Reynolds number

\( S_{oo}, S_{20} \) parameters defined by equations (29) and (30)

\( T_o \) stagnation temperature at any radius in mixing zone, \( ^\circ \text{R} \)

\( T_{oj} \) relative stagnation temperature at centerline of jet, \( ^\circ \text{R} \)

\( T_{oo} \) stagnation temperature of flow adjacent to the duct, \( ^\circ \text{R} \)

\( \Delta T_o \) difference between stagnation temperature at any radius in the jet and the stagnation temperature of the surrounding flow, \( ^\circ \text{R} \)

\( \mathcal{T} \) temperature ratio = \( T_{oj}/T_{oo} \)

\( U \) velocity, ft/sec

\( U_c \) velocity at centerline of jet, ft/sec

\( U_j \) velocity at centerline of jet relative to \( U_0 \), ft/sec

\( U_{jo} \) relative velocity at centerline of jet at end of transition section, ft/sec

\( U_{joo} \) relative velocity at centerline of jet at beginning of transition section, ft/sec

\( U_0 \) velocity of outer stream, ft/sec

\( U_r \) velocity ratio for transition zone = \( U_{jo}/U_{joo} \)

\( V(J) \) terms in equation (37)

\( W_o \) mass flow rate, secondary flow, lbm/sec

\( W_1 \) mass flow rate, primary flow, lbm/sec

\( W(J,K) \) coefficient matrix; equation (37)

\( x \) axial position along mixing tube, ft

\( x_{core} \) length of the transition zone, ft
\( y \)  
radius, ft

\( Y(K) \)  
derivatives in equation (37)

\( \gamma \)  
velocity profile shape parameter

\( \delta \)  
width of shear layer, ft

\( \delta^* \)  
boundary layer displacement thickness, ft

\( \epsilon \)  
eddy kinematic viscosity, ft\(^2\)/sec

\( \eta \)  
dimensionless radius = \( y/\delta \) or \( y/R \)

\( \Theta \)  
boundary layer momentum thickness, ft

\( \lambda \)  
velocity ratio \( U_0/U_j \)

\( \rho \)  
density, lbm/ft\(^3\)

\( \tau \)  
shear stress, lbf/ft\(^2\)

\( \nu \)  
kinematic viscosity, ft\(^2\)/sec

**Subscripts**

\( oo \)  
value at top-hat section

\( 1 \)  
primary flow

\( \text{core} \)  
dimension at end of transition zone

\( \text{eff} \)  
effective radius or area of mixing tube

\( m \)  
value at mean area of transition zone

\( \text{noz} \)  
primary nozzle exit area

\( \text{SD} \)  
suction duct upstream of mixing tube
3.1 **Purpose**

The purpose of the analysis developed in this section is to predict the performance characteristics of compressible flow jet pumps with variable-area mixing tubes. The jet pumps may have supersonic or subsonic primary flow issuing from a single nozzle located along the axis of an axisymmetric cylindrical mixing tube. The secondary flow and the mixed flow downstream must remain subsonic. The primary and secondary flows are taken to be the same perfect gas.

A particular objective of the analysis is to predict the variation in static pressure along the length of the mixing tube. Knowledge of this pressure variation allows calculation of the thrust augmentation of the jet pump, an essential parameter for jet pump application studies.

3.2 **General Description of the Analytical Model**

The analysis is based upon the incompressible flow jet pump analytical model developed by Dr. P.G. Hill (reference 2). This analytical model, with its associated computer program, was modified in the present study to account for compressible flow effects. The formulation of the analytical model is described in this section. The computer program which is based upon the compressible flow model is described in Appendix B of this report.

The following initial assumptions are made for the analysis:

1. The primary and secondary flows are the same perfect gas.

2. No heat is transferred across the wall of the jet pump.
3. The jet pump consists of an axisymmetric, cylindrical, variable-area mixing tube with a single primary nozzle located along the axis.

4. The primary and secondary flow conditions and the nozzle geometry are assumed to be such that no normal shocks or moisture condensation shocks occur in the primary flow.

5. The secondary flow and the combined flows after mixing are assumed to remain subsonic throughout the mixing tube.

6. The velocity of the primary jet at the nozzle exit is greater than the velocity of the secondary flow.

7. The static pressure is constant across any section perpendicular to the axis of the jet pump.

Dr. Hill's analysis identifies three distinct flow regimes in a jet pump. These regimes are shown in figure 1; they may be described as follows:

**Part 1** - A region in which the jet is approximately self-preserving and is immersed in a potential outer stream which may be accelerating or decelerating, depending on the shape of the duct and the rate of entrainment of mass into the jet.

**Recirculation Zone** - A possible region in which recirculation occurs, following a deceleration of the outer stream. At the beginning of this zone the "edge" of the jet has not yet diffused to the wall and the secondary fluid recirculates through the jet. The pressure gradient is generally observed to be negligible in this zone.
Part 2 - The region downstream of the point (fairly distinct in many cases) at which the jet attaches to the wall. An adverse pressure gradient is generally established but the relatively high shearing forces near the wall tend to accelerate the fluid against the pressure gradient. If there is a zone of recirculation, it is terminated in a short axial distance by these high shearing forces.

In addition to these three regions, there is a relatively short transition zone between the nozzle exit and the section at which a subsequently self-preserving velocity profile is attained.

In Part (1) the jet velocity profile can be approximated well by

\[
\frac{U - U_o}{U_j} = f_0 \left( \frac{y}{\delta} \right) \text{ at any } x
\]

where

- \(U\) = velocity at radius \(y\)
- \(U_o\) = outer stream velocity
- \(U_j\) = jet relative velocity at centerline
- \(x\) = axial position along mixing tube
- \(y\) = radius
- \(\delta\) = width of shear layer (see sketch)

![Velocity Profile at Station x](image-url)
The functional relationship $f_0 (y/\delta)$ is determined quantitatively from velocity profiles measured in axisymmetric jets discharging into free space.

$$f_0 (\eta) = 1.0004 - 0.0175\eta - 8.3821\eta^2 + 16.5806\eta^3 - 12.7877\eta^4 + 3.608\eta^5$$

(2)

where $\eta = y/\delta$  (Part 1)

The same relationship holds in the recirculation zone but the axial pressure gradient in this region is assumed to be zero.

The relationship above is used to describe the velocity profile at a particular axial station in Part 1 of the mixing tube flow. The continuity, momentum, moment-of-momentum, energy, and boundary layer equations are used to determine the changes in $U_j$, $U_o$, $\delta$, temperature, and pressure which occur from station to station along the mixing tube. To solve these equations, the temperature profile must be known so that the density variations across the section can be determined. Following Abramovich (reference 3), the stagnation temperature profile is taken to be the square-root of the velocity profile.

$$\frac{T_o - T_{00}}{T_{0j}} = f_0^{1/2} \left( \frac{y}{\delta} \right)$$

(3)

where $T_o$ = stagnation temperature at any radius in the mixing zone

$T_{00}$ = stagnation temperature of surrounding secondary flow

$T_{0j}$ = relative stagnation temperature at center of jet
The solution of the moment-of-momentum equation requires shear stress values to be known as a function of radius. These values are obtained as follows:

\[ \tau = \epsilon \rho \frac{\partial U}{\partial y} \]  

where \( \tau = \) shear stress  
\( \epsilon = \) eddy kinematic viscosity,  
\( \rho = \) density

The value of the turbulent Reynolds number is assumed to remain constant across the flow at any axial station in Part 1. This allows calculation of the eddy viscosity from the following equation:

\[ \epsilon = \frac{U_j \delta}{Re_T} \]  

where \( Re_T = \) turbulent Reynolds number

At the beginning of Part 1, the jet mixing process is not significantly affected by the presence of the mixing tube walls. Therefore, the value \( Re_{TF} = 147 \), from incompressible free jet mixing tests, can be used. Further downstream in Part 1, as the jet approaches the walls, the mixing process is altered from a free jet to a free wake type of mixing. The change in the mixing process is accounted for by using the following equation to determine the eddy viscosity at any station in Part 1:

\[ \epsilon = \frac{U_j \delta}{Re_{TF}} \left[ 1 + \frac{3}{2} (1-e^{-1.1\lambda}) \right] \]  

where \( \lambda = \frac{U_o}{U_j} \)

Boundary layer growth must be taken into account in order to predict wall static pressure variations with accuracy. Boundary layer displacement thickness variations are obtained in the analysis by using the methods of Moses (reference 4). The equations used are described in Section 3.4 in this report.
In Part 2, the jet has reached the wall. The free jet mixing velocity profile is no longer appropriate. Instead, the velocity profile is assumed to follow the relationship:

\[
\frac{U}{U_c} = f_2(\eta) + \gamma g_2(\eta) \tag{7}
\]

where

- \(U_c\) = jet velocity at centerline
- \(f_2(\eta)\) = velocity profile at the end of Part 1
- \(\eta = \frac{y}{R}\)
- \(R\) = mixing tube radius at the axial position considered
- \(\gamma = \gamma(x)\) adjustable shape parameter
- \(g_2(\eta)\) = auxiliary velocity profile

At the beginning of Part 2, \(\gamma\) is set equal to zero and the velocity profile matches the velocity profile at the end of Part 1. The auxiliary profile \(g_2(\eta)\) is chosen so that, as \(\gamma\) approaches 1.0, the \(U/U_c\) velocity profile approaches the profile for fully-developed flow in a pipe.

\[
g_2(\eta) = gg(\eta) - f_2(\eta) \tag{8}
\]

where

- \(gg(\eta)\) = velocity profile for fully-developed flow in a pipe

No boundary layer calculations are made in Part 2. Viscous forces are present throughout the flow so no distinct boundary layer exists. Wall friction forces are calculated from turbulent pipe flow correlations.

The continuity, momentum, moment-of-momentum, and energy equations are used to determine the changes in \(U_c\), \(\gamma\), temperature, and pressure which occur with distance along the mixing tube in Part 2. The solution of the moment-of-momentum equation requires determination of the eddy viscosity as a function of radius and axial position. Because the flow in Part 2 becomes asymptotic to fully-developed pipe flow, the eddy viscosity must be asymptotic to the fully-developed flow value.
\[ \tau / \tau_{\text{wall}} = \gamma/R = \eta \text{ as } \gamma(x) \text{ approaches } 1.0 \]  \hspace{1cm} (9)

\[ E_{2f} = -\frac{1}{2} \frac{C_{fd} \eta}{\frac{\partial}{\partial \eta} \lg(\eta)} \]  \hspace{1cm} (10)

where \( E_{2f} = \epsilon_{2f}/U_c R = \) dimensionless eddy viscosity distribution

\( \epsilon_{2f} = \) eddy kinematic viscosity for fully-developed pipe flow

\( C_{fd} = \frac{\tau_{\text{wall}}}{\frac{1}{2} \rho U_c^2} = \) wall friction coefficient

An arbitrary function is used to make the eddy viscosity distribution in Part 2 continuous with that at the end of Part 1.

\[ E_2 = E_1 (1 - \gamma^2) + \gamma^2 E_{2f} \]  \hspace{1cm} (11)

where \( E_1 = \) dimensionless eddy viscosity at the end of Part 1
calculated from equation (6)

The paragraphs above have described the basic approaches used for the analysis of flow behavior in the variable-area compressible flow jet pump. The fundamental assumptions for the analysis have been identified. The sections which follow present the sets of equations which must be solved in each of the three regions of the flow; the transition zone, the region upstream of jet attachment to the wall (Part 1), and the region downstream of the point of attachment (Part 2).

3.3 Transition Zone Analysis

The transition zone begins at the primary nozzle exit plane and has a length of approximately 20 jet nozzle diameters. At the nozzle exit plane, the static pressure in the supersonic primary flow may be different from the static pressure in the surrounding secondary flow. We assume that before mixing of the two flows begins, the primary jet expands or contracts isentropically until its static pressure matches that of the secondary flow. At the station where this accommodation is complete, the velocity profile is assumed to resemble a "top-hat" as shown in figure 2. Then mixing of the primary and secondary flows begins.
The transition zone continues downstream to the section where the potential core in the jet ends. At this point, the \( f_0 (y/\delta) \) profile has been attained and the stagnation pressure at the center of the jet begins to fall because of mixing with the secondary flow.

The flow conditions at the end of the transition zone are determined by solving three simultaneous non-linear algebraic equations which are developed from the continuity and momentum equations written for the transition zone as a control volume, and from the condition that the stagnation pressure remains constant along the centerline of the primary jet.

The length of the transition zone is measured from the primary nozzle exit section to the point where the \( f_0 (y/\delta) \) profile is attained. This length is designated as \( x_{\text{core}} \) and must be specified as input data for the analysis. For incompressible flow, equation (12) may be used (reference 3).

\[
x_{\text{core}} = 4.08 \delta_o \left(1 + \frac{U_{oo}}{U_{joo}} \right)
\]

where

- \( \delta_o \) = radius of primary jet at top-hat section
- \( U_{oo} \) = secondary flow velocity at top-hat section
- \( U_{joo} \) = primary jet relative velocity

For compressible flow with a supersonic primary jet, the value of \( x_{\text{core}} \) will depend on whether the jet is under- or over-expanded as it leaves the nozzle. A suitable replacement for equation (12) is not known to be available, so \( x_{\text{core}} \) was arbitrarily chosen to be equal to the mixing tube inlet diameter. This length is equivalent to about 18 primary nozzle diameters.

The transition from the top-hat profile to the \( f_0 (y/\delta) \) profile is assumed to occur in a control volume of essentially constant area. The effective mixing tube radius at \( x_{\text{core}} \) is calculated by taking the boundary layer thickness into account.
Ref \( f \) = \( R_{core} - \Theta H_o \) \hspace{1cm} (13)

where \( \Theta = \Theta_o + 0.001 \times \theta_{core} \) \hspace{1cm} (14)

\( R_{eff} \) = effective radius of mixing tube at \( x_{core} \)

\( R_{core} \) = radius of mixing tube at \( x_{core} \)

\( \Theta \) = boundary layer momentum thickness at \( x_{core} \)

\( \Theta_o \) = inlet boundary layer momentum thickness

\( H_o \) = inlet boundary layer shape factor = 1.4 assumed

The flow area available for the secondary flow at the top-hat section is given by equation (15).

\[ A_{eff} = \pi R_{eff}^2 - A_{noz} \quad A_{noz} \approx A_{primary \ flow} \] \hspace{1cm} (15)

where \( A_{eff} \) = secondary flow area at top-hat section

\( A_{noz} \) = area of primary nozzle exit section

The velocity of the secondary flow at the top-hat section is calculated from equation (16).

\[ U_{oo} = \frac{W_o}{\rho_o A_{eff}} \] \hspace{1cm} (16)

where \( U_{oo} \) = secondary flow velocity at top-hat section

\( W_o \) = mass flow rate of secondary flow

\( \rho_o \) = density of secondary flow

The value of \( \rho_o \) in equation (16) is the density corresponding to the local static pressure and temperature. It is computed by an iterative process using the known values of inlet stagnation pressure and temperature and the appropriate perfect gas relationships. The same calculation yields the value of the local static pressure.
The primary flow conditions at the top-hat section are calculated as follows:

\[ T_1 = T_{o1} \left( \frac{p_1}{p_{o1}} \right)^{\frac{k}{k-1}} \]  \hfill (17)

where

- \( T_1 \) = static temperature in primary flow at top-hat section
- \( T_{o1} \) = specified primary flow stagnation temperature
- \( p_{o1} \) = specified primary flow stagnation pressure
- \( p_1 \) = static pressure from secondary flow calculations

\[ U_1 = \sqrt{2 \frac{k}{k-1} R_g(T_{o1} - T_1)} \]  \hfill (18)

where

- \( U_1 \) = primary flow velocity at top-hat section

\[ U_{joo} = U_1 - U_{oo} \]  \hfill (19)

where \( U_{joo} \) = primary jet relative velocity

The flow conditions at the end of the transition zone are computed by using the continuity and momentum relationships and the assumption that the stagnation pressure is unchanged at the center of the jet. The stagnation pressure of the secondary flow outside the mixing region is assumed to remain constant during transition. The stagnation temperature of the secondary flow outside the mixing region, and the stagnation temperature at the center of the primary jet, are assumed to remain constant.

The values of \( U_{oo}, U_{joo}, p_1, W_0, \) and \( W_1 \) are known to begin the analysis which determines the velocity profile at the end of the transition zone. The continuity, momentum, and constant centerline pressure equations at the end of the zone may be written as follows on the next page.
Continuity: \[ 2\pi \int_0^{R_{\text{eff}}} \rho U y \, dy = W_0 + W_1 \quad (20) \]

Momentum: \[ 2\pi \int_0^{R_{\text{eff}}} \rho U^2 y \, dy + (p - p_\infty) A_m = (p_1 - p_\infty) A_m + \frac{W_1 U_1}{g_o} + \frac{W_0 U_\infty}{g_o} \quad (21) \]

Constant Stagnation Pressure Along Centerline: \[ P_{o1} = \text{constant} \quad (22) \]

where \( A_m = \pi R_{\text{eff}}^2 \)

The velocity profile at the end of the transition zone is given by equations (1) and (2). The temperature profile is given by equation (3).

To permit equations (20), (21), and (22) to be solved simultaneously using standard computer subroutines, these equations were rewritten in terms of the following dependent parameters:

\[ U_r = \frac{U_{jo}}{U_{joo}} \quad (23) \]

\[ \lambda = \frac{U_o}{U_{jo}} \quad (24) \]

\[ \frac{\delta}{R_{\text{eff}}} = \frac{\delta}{\sqrt{\frac{A_m}{\pi}}} \quad (25) \]
The continuity, momentum, and constant centerline stagnation pressure equations in final form are as follows:

**Continuity:**

Let \[ \frac{W_0 + W_1}{\pi R^2 \rho_{\text{eff}} \rho_{\infty} U_{j\infty}} \]  

It can be shown that

\[ C_{\text{mass}} = \frac{p}{\rho_{\infty}} \left( 1 - \frac{k-1}{k} \right) \]  

where

\[ \frac{p}{\rho_{\infty}} = (1 - S_{\infty} U^2) \]  

\[ S_{\infty} = \frac{k-1}{2} \frac{U^2_{j\infty}}{kR g_{\infty} T_{\infty}} \]  

\[ S_{20} = \frac{\lambda}{1 - S_{\infty} U^2} \]  

\[ Z_1 = \int_{0}^{1} \frac{(\lambda + f_0) 2 \eta d\eta}{1 + \mathcal{U} f_0^{1/2} - S_{\infty} U^2 r (\lambda + f_0)^2} \]  

\[ \mathcal{U} = \frac{T_{oj}}{T_{\infty}} \]

**Momentum:**

Let \[ \frac{g_o (p - \rho_{\infty}) A_m + W_1 U_1 + W_0 U_0}{\pi R^2 \rho_{\text{eff}} \rho_{\infty} U_{j\infty}^2} \]  

It can be shown that

\[ C_{\text{mom}} = \frac{(\frac{p}{\rho_{\infty}} - 1) R g_{\infty} T_{\infty}}{U_{j\infty}^2} \]  

\[ + \left( \frac{p}{\rho_{\infty}} \right) \left[ U_r^2 (\frac{\delta}{R})^2 (Z_2 - \lambda S_{20}) + U_r^2 \lambda S_{20} \right] \]
where
\[
Z_2 = \int_0^1 \frac{(\lambda + f_0)^2 \eta^2}{1 + \frac{f_0}{f_0} 1/2 - S_{oo} U_r^2 (\lambda + f_0)^2} \, d\eta
\]  

(34)

Constant Stagnation Pressure:
\[
1 - S_{oo} U_r^2 \frac{\lambda^2}{1 - \frac{S_{oo} T_{oo}}{T_{oo}} U_r^2 (1 + \lambda)^2} = \left( \frac{P_{oo}}{P_{oo} - \Delta P_{SD}} \right)^{k-1} k
\]

(35)

where \(\Delta P_{SD} = \) suction duct losses (see section 4.2.3)

The Z integrals are evaluated by using the following summation:
\[
Z_k = \frac{1}{n_s} \sum_{i=1}^{n_s} \left( \frac{N_i}{D_i} \right)^{k-1} k 2\eta_i
\]

(36)

where \(n_s = \) number of summation strips, each of the same \(\Delta \eta\)

\(N_i, D_i\) are defined as follows:

<table>
<thead>
<tr>
<th>Equation</th>
<th>(K)</th>
<th>(N_i)</th>
<th>(D_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuity</td>
<td>1</td>
<td>(\lambda + f_{oi})</td>
<td>(1 + \frac{f_{oi}}{f_{oi}} 1/2 - S_{oo} U_r^2 (\lambda + f_{oi})^2)</td>
</tr>
<tr>
<td>Momentum</td>
<td>2</td>
<td>((\lambda + f_{oi})^2)</td>
<td>same</td>
</tr>
</tbody>
</table>

Equations (27), (33), and (35) are solved simultaneously to yield values of \(U_r, \lambda,\) and \(\delta/R\) at the end of the transition zone. The value of the static pressure at the end of the zone is then determined from equation (28).
3.4 Flow Analysis Upstream of Jet Attachment

In Part 1, the zone between the end of the transition zone and the section where the jet reaches the wall, seven variables are determined by integral techniques. These dependent variables are $U_j, \lambda = U_\infty / U_j, \delta$, the static pressure $p$, the relative stagnation temperature at the jet centerline $T_{oj}$, the boundary layer momentum thickness $\theta$, and the boundary layer shape factor $H$.

The values of these variables are obtained by solving seven simultaneous equations of the following general form:

$$\sum_{K=1}^{7} W(J,K) \times Y(K) = V(J)$$

(37)

where $W(J,K) =$ a coefficient matrix

$Y(K) =$ the derivatives of the dependent variables with respect to $x/R_o$

$V(J) =$ a set of terms not containing any of the dependent variables, evaluated at the $x/R_o$ station of interest
The $Y(K)$ values are listed below:

$$Y(1) = \frac{\partial \left( \frac{U_j}{U_j0} \right)}{\partial \left( \frac{x}{R_0} \right)}$$

$$Y(5) = \frac{\partial \left( \frac{T_{oj}}{T_{oo}} \right)}{\partial \left( \frac{x}{R_0} \right)}$$

$$Y(2) = \frac{\partial \lambda}{\partial \left( \frac{x}{R_0} \right)}$$

$$Y(6) = \frac{\partial \left( \frac{\theta}{R_0} \right)}{\partial \left( \frac{x}{R_0} \right)}$$

$$Y(3) = \frac{\partial \left( \frac{\delta}{R_0} \right)}{\partial \left( \frac{x}{R_0} \right)}$$

$$Y(7) = \frac{\partial H}{\partial \left( \frac{x}{R_0} \right)}$$

$$Y(4) = \frac{\partial \left( \frac{p}{P_{oo}} \right)}{\partial \left( \frac{x}{R_0} \right)}$$

The $J$ simultaneous equations used to evaluate these derivatives are as follows:

$$J = 1: \ p_{oo} = \text{stagnation pressure in the flow outside the jet} = \text{constant}$$

$$2: \ \text{momentum equation for the complete flow}$$

$$3: \ \text{continuity equation}$$

$$4: \ \text{energy equation}$$

$$5: \ \text{moment-of-momentum equation}$$

$$6: \ \text{boundary layer momentum equation}$$

$$7: \ \text{boundary layer moment-of-momentum equation}$$

These equations and the $W(J, K)$ coefficients are given in detail in Appendix A.
The velocity profile for the jet in Part 1 is given by equations (1) and (2), with the distribution function $f_0$ taken from free jet data (reference 5). The jet temperature profile is given by equation (3). In the jet ($0 \leq r \leq \delta$) the shear is obtained from equation (4) with the eddy viscosity given by equation (6).

Outside the jet ($\delta < y < R$), wall shear forces are assumed to be negligible in the momentum equation for the complete flow. The boundary layer momentum thickness $\Theta$ and the shape factor $H$ are calculated from the following equations:

$$\frac{d\Theta}{dx} + (2 + H) \frac{\Theta}{U_o} \frac{dU_o}{dx} = \frac{C_f}{2} \quad (40)$$

$$\frac{dH}{dx} = \frac{-H(H + 1)(H^2 - 1)}{2} \frac{1}{U_o} \frac{dU_o}{dx} + \frac{H^2 - 1}{\Theta} \left[ \frac{HC_f}{2} - \frac{0.06(H - 1)}{(H + 3)R_\Theta^{0.1}} \right] \quad (41)$$

The friction coefficient in these equations is taken from the Ludwieg-Tillman skin friction equation (reference 6):

$$C_f = 0.246 R_\Theta^{-0.268} 10^{-0.678H} \quad (42)$$

where $R_\Theta = $ Reynolds number based on momentum thickness

These equations are based on the assumption that the outer (potential) flow at velocity $U_o$ is incompressible, and use has been made of the relation between the boundary layer and jet mixing parameters given in equation (43):

$$\frac{1}{U_o} \frac{dU_o}{dx} = \frac{1}{\lambda} \frac{d\lambda}{dx} + \frac{1}{U_j} \frac{dU_j}{dx} \quad (43)$$
The equations above allow the boundary layer development to be calculated simultaneously with the main flow mixing. Thus, the boundary layer displacement thickness is taken into account when the momentum, continuity, and energy equations are integrated across the mixing tube cross section.

The seven equations (39) are solved simultaneously to yield the values of the derivatives (38). Then the derivatives are integrated using Runge-Kutta-Merson techniques. This integration yields the desired values of \( U_j, U_o, \delta, p, T_{oj}, \Theta, \) and \( H \) at selected values of \( x/R_o \) along the mixing tube.

If a region of recirculation is present, the value of \( U_o \) becomes negative. The development of an analysis for the flow behavior in a recirculation zone was not included in this investigation.

### 3.5 Flow Analysis Downstream of Jet Attachment (Part 2)

After the jet reaches the wall, the jet velocity profile is assumed to follow the relationship given in equation (7). At the beginning of Part 2, the value of the shape parameter \( \gamma(x) \) is set equal to zero and the velocity profile is given by \( f_2(\eta) \). The functional relationship \( f_2 \) is defined so as to be identical to the final velocity distribution in Part 1.

\[
f_2(\eta) = \frac{f_o(\eta) + \lambda f_{b\ell}(\eta)}{1 + \lambda}
\]  

(44)

In this equation, \( f_{b\ell}(\eta) \) is the boundary layer profile at the end of Part 1, estimated by using a power law for the boundary layer:

\[
u = U_o \left( \frac{y}{\delta_{b\ell}} \right)^n
\]  

(45)

The exponent \( n \) must satisfy the values of momentum and displacement thickness calculated for the boundary layer at the end of Part 1. The resulting equation for \( f_{b\ell} \) follows.
\[ f_{bl} = \frac{R (H_1 - 1)}{\Theta_1 H_1 (H_1 + 1)} \left[ 1 - \frac{H_1 - 1}{2} \right] \]  

where \( \Theta_1 = \) boundary layer momentum thickness at the end of Part 1  
\( H_1 = \) boundary layer shape factor at the end of Part 1

The velocity profile in Part 2 includes the auxiliary profile \( g_2 (\eta) \). This profile is defined by equation (8) so that, as \( \gamma \) approaches 1.0, the Part 2 velocity profile asymptotically approaches the profile for fully-developed turbulent flow in a pipe.

In Part 2, the values of five variables are determined by integral techniques. These dependent variables are the velocity profile values \( U_c \) and \( \gamma \), the static pressure \( p \), the temperature ratio \( T \), and the temperature of the flow at the outer radius of the mixing tube, \( T_{oo} \). The values of these variables are obtained by solving six simultaneous equations of the general form given by equation (37). The dependent \( Y(K) \) variables are listed below:

\[
\begin{align*}
Y(1) &= \frac{\partial (U_c)}{\partial (\frac{x}{R_o})} \\
Y(2) &= \frac{\partial (U_0)}{\partial (\frac{x}{R_o})} \text{ (not used)} \\
Y(3) &= \frac{\partial \gamma}{\partial (\frac{x}{R_o})} \\
Y(4) &= \frac{\partial (\frac{P}{P_{ooi}})}{\partial (\frac{x}{R_o})} \\
Y(5) &= \frac{\partial (\frac{T_0}{T_{ooi}})}{\partial (\frac{x}{R_o})} \\
Y(6) &= \frac{\partial (\frac{T_{oo}}{T_{ooi}})}{\partial (\frac{x}{R_o})}
\end{align*}
\]

where \( P_{ooi} \) and \( T_{ooi} \) are the constant values of \( P_{oo} \) and \( T_{oo} \) in Part 1. The variable \( Y(2) \) above remains zero throughout the Part 2 analysis; this variable is a redundant parameter which remains from an earlier version of the computer program.
The equations used to evaluate these derivatives are as follows:

\[ J = 1 \quad = \quad \text{continuity equation} \]
\[ J = 2 \quad = \quad \text{energy equation} \]
\[ J = 3 \quad = \quad \text{momentum equation for the complete flow} \]
\[ J = 4 \quad = \quad \text{moment-of-momentum integral equation} \]
\[ J = 5 \quad = \quad \text{centerline velocity-temperature relationship} \]
\[ J = 6 \quad = \quad \text{wall velocity} = 0 \]

These equations and the \( W(J, K) \) coefficients are given in detail in Appendix A.

The form of the stagnation temperature profile must be known in order to solve the first four equations (48). For simplicity, the temperature profile was assumed to be the same as in a free jet.

\[ \frac{T_0 - T_{oo}}{T_{oj}} = \sqrt{f_0(\eta)} \quad \text{in Part 2} \quad (49) \]

This approximation is justified by the test results in Section 4.3 of this report.

Wall shear stresses are included in the momentum equation for the complete flow. The wall friction coefficient used in the analysis is based upon pipe flow correlations which yield equation (50).

\[ C_{fd_f} = \frac{\tau_{wall}}{\frac{1}{2} \rho \overline{U_c}^2} = 0.048 \left( \frac{\overline{U}}{\overline{U_c}} \right)^2 \text{Re}_m^{-0.20} \quad (50) \]

where
\[ \text{Re}_m = \frac{\overline{U} D}{\nu} \quad \text{Reynold's number based on mean velocity} \]
\[ \overline{U} = \text{mass-average mean velocity} \]
\[ \overline{U_c} = \text{centerline velocity} \]
\[ \nu = \text{kinematic viscosity} \]
This wall friction coefficient is only an approximation to the actual value because the velocity profile near the wall in Part 2 of the jet pump mixing tube is generally not identical to the fully-developed pipe flow velocity profile. Comparison of analytical predictions to measured wall static pressure values indicated that equation (50) gave values of \( C_{fd} \) which were too high. Therefore, the analysis now employs an arbitrarily reduced friction factor.

\[
C_{fd} = \frac{1}{2} C_{f_d} \tag{51}
\]

The moment-of-momentum integral equation includes a term which represents axial shear forces between adjacent stream tubes. These shear forces are determined from the eddy viscosity relationship given in equation (11).

The fifth equation in the set, the centerline velocity-temperature relationship, is based upon the test results obtained during this investigation. As shown in Section 4.3 and Figure 16, the following equation may be used to supplement the energy equation in Part 2.

\[
\frac{l}{T_j} \frac{dT_j}{dx} = \frac{l}{U_c} \frac{dU_c}{dx} \tag{52}
\]

The sixth equation (48) sets \( U_0 \), the velocity of the flow along the mixing tube surface, equal to zero. This equation was added to eliminate \( Y(2) \), the redundant variable in equation (47), during the solution of the six simultaneous equations (48). The solution of these equations yields the values of the derivatives (47). The derivatives then are integrated using Runge-Kutta-Merson techniques. This integration yields the desired values of \( U_c, \gamma, \rho, T_{oj}, \) and \( T_{oo} \) at selected values of \( x/R_o \) along the mixing tube in the region after the jet reaches the wall.
The objective of the test program was to provide data which could be used to evaluate the analytical model. The test conditions are summarized below:

**Primary Flow**

- stagnation pressure: 348 psia
- stagnation temperature: 807°F
- nozzle throat area: $1.587 \times 10^{-4}$ ft$^2$
- nozzle geometry: see figure 3
- mass flow rate: 6.76 lbm/min

**Secondary Flow**

- inlet stagnation pressure: laboratory ambient (30.06” Hg)
- inlet stagnation temperature: laboratory ambient (92°F)
- mixing tube geometry: see figure 4
- pressure rise: regulated by discharge throttling device

This section of the report describes the jet pump test arrangement, instrumentation and data reduction procedures, and the results which were obtained.

### 4.1 Test Arrangement

The jet pump test arrangement is shown in figure 5. The primary flow was supplied by a 2-stage reciprocating compressor. Electrical heaters were used to increase the temperature of the flow up to about 800°F. The primary flow was delivered to a single nozzle directed along the axis of the mixing tube.

The momentum of the primary flow entrains a secondary air flow from the room into the bellmouth inlet and then into the mixing tube. Here, the
two streams mix together and the stagnation pressure of the secondary stream is increased. The flow from the mixing tube passes through a conical diffuser and exhausts to the atmosphere through an adjustable throttling cone.

The individual components of the experimental jet pump are described below:

1. Calibrated bellmouth inlet section

   This component consists of a wooden bellmouth, metal connecting tube, and fiberglass primary flow inlet section. The bellmouth differential pressure was calibrated in terms of flow rate by using an orifice and blower available in the laboratory. The calibrated bellmouth permitted direct measurement of secondary mass flow rate for all jet pump tests.

2. Mixing tube

   The mixing tube geometry was chosen rather arbitrarily before the computer program became available as a design guide. The basic Helmbold mixing tube geometry (reference 7) was selected because this geometry has been tested thoroughly in the incompressible flow regime. The incompressible results provide a guide to the flow behavior which may be expected in the compressible flow regime.

   The Helmbold mixing tube was scaled down so that all dimensions were 0.892 times their original values. This scale was selected so that the mixing tube would match an existing discharge diffuser and the mixing tube throat velocity would remain subsonic for all flow rates expected in the test program. The smooth curving
profile of the Helmbold tube was approximated with cones and cylinders as shown in figure 4 for ease of fabrication.

3. Discharge diffuser

A conical diffuser with an area ratio of about 2.8 and a total included angle of 7.1° was added at the end of the mixing tube to maximize static pressure recovery and allow high entrainment ratios to be achieved. Changes in the axial positioning of the throttle cone in the diffuser exit produced a variable system resistance so that the jet pump could be tested over a range of secondary flow rates.

4. Nozzle geometry

Figure 3 shows the geometry of the converging-diverging primary flow nozzle. The area ratio from throat to exit section is 3.24, the area ratio corresponding to one-dimensional isentropic expansion from 350 psia to 14.7 psia. When the jet pump was assembled, the exit plane of the nozzle was at \( x = 0 \) where \( x \) is defined on the mixing tube drawing, figure 4. The mixing tube diameter at the nozzle exit plane is 5.341 in. The nozzle flow coefficient, according to the definition below, was measured to be 0.929.

\[
C_w = \frac{W_1}{W_{1\text{ideal}}} 
\]

where \( W_1 \) = measured nozzle flow rate at design pressure and temperature

\( W_{1\text{ideal}} \) = isentropic flow rate through nozzle throat at design pressure and temperature; based upon one-dimensional flow assumption
4.2 Instrumentation and Data Reduction Procedures

4.2.1 Instrumentation

The instrumentation used to determine the performance of the experimental jet pump is shown on figure 6 and described in table 1.

The jet pump inlet bellmouth was calibrated for use as a flowmeter. The calibration was accomplished by connecting the bellmouth and the suction duct to the inlet of a blower by means of an orifice run and throttling arrangement. The bellmouth flow equation follows:

\[ W_0 = 229.5 \sqrt{\rho_b \Delta h_b} \]  \hspace{3cm} (54)

where \( \Delta h_b \) = \( p_b \) differential pressure, in. H\(_2\)O gage

\( \rho_b \) = inlet density, lbm/ft\(^3\)

The mixing tube was provided with 21 static pressure taps along its length. Four additional static pressure taps were located in the discharge diffuser. Provision was made for traverse probe measurements at five of the static pressure tap sections. The location of all of these taps is given in table 2. The exact dimensions were measured at several stations in the mixing tube after its construction; these dimensions are also given in table 2.

The Kiel-temperature probe which was traversed to measure the velocity and temperature profiles had a stem diameter of 1/8". The probe was small enough so that probe blockage effects were negligible during the traversing.

4.2.2 Data Reduction Procedures

The measured data were used to calculate the following jet pump parameters:
The stagnation pressure and temperature profiles were measured at all traverse locations in a plane perpendicular to the axis of the primary flow feed pipe (see figure 6). At the station in the mixing tube throat \((x/R_0 = 9.25)\), a traverse also was made in the plane of the feed pipe to confirm that the flow was axisymmetric as desired.

The wall static pressure and the traverse probe stagnation pressure and temperature measurements were used with the appropriate compressible flow equations to allow calculation of the velocity profiles at traverse stations 2 through 6. As a result of a thermocouple failure during the test runs, no temperature data were obtained at traverse station 1. Because the temperature profile is required in order to calculate the velocity profile, it was necessary to prepare an approximate temperature profile for this station. The procedure used is described under Test Results in Section 4.3 of this report.

### 4.2.3 Suction Duct Losses

Results from previous tests of the bellmouth and suction duct assembly (reference 1) and static pressure data from the present test program indicate that stagnation pressure losses in the suction duct upstream of the mixing tube are of the order of 2 in. \(H_2O\) for the tested secondary flow rates. These losses may be
accounted for in the jet pump analysis by using equation (55) to calculate the secondary flow stagnation pressure at the primary nozzle exit section in the mixing tube:

\[ P_{o2} = P_{oo} - K_L \rho_{oo} \frac{U_{SD}^2}{2g_o} \]

or

\[ P_{o2} = P_{oo} - K_L \left( \frac{W_0^2}{2g_o \rho_{oo} A_{SD}^2} \right) \]  \hspace{1cm} (55)

where

- \( P_{o2} \) = stagnation pressure at primary nozzle exit plane
- \( P_{oo} \) = stagnation pressure at suction duct inlet (laboratory ambient)
- \( K_L \) = suction duct loss coefficient
- \( \rho_{oo} \) = density corresponding to suction duct inlet stagnation state
- \( U_{SD} \) = suction duct velocity (assumed uniform)
- \( A_{SD} \) = suction duct cross-sectional area

For the suction duct in the experimental jet pump, the value of \( K_L \) is 0.33.

4.3 Test Results

The jet pump was tested at four values of entrainment ratio, 17.0, 19.4, 21.0, and 23.6. The corresponding values of primary and secondary mass flow rates are given in table 3. The inlet pressures and temperatures were constant throughout the test and were as listed at the beginning of this Section 4.
Wall static pressure values measured along the mixing tube are listed for all four entrainment ratios on table 3. These values are plotted in figure 7.

The operation of the jet pump was reasonably steady (i.e., wall pressure fluctuations were small) when the entrainment ratio was 21.0. Therefore, this condition was selected for the velocity traverse measurements which require long periods of steady operation. The velocity profiles for traverse stations 2 through 6 are shown in figure 8. The associated temperature profiles are shown in figure 9.

Traverses 4 and 5 were taken at the same station in the constant-area throat section of the mixing tube. The axes of the traverse were 90° apart so that any departures from axial symmetry in the flow could be detected. The slight departures which were observed are due to heating of the secondary flow as it passes over the primary nozzle feed pipe upstream of the mixing tube inlet. These departures have a negligible effect on jet pump performance and will not interfere with our comparison of measured and predicted flow behavior through the jet pump.

Because of a thermocouple failure during testing, no temperature data were obtained at traverse station 1. The stagnation pressure measurements at this section cannot be used to determine the velocity profile unless the temperature profile is available. An approximate velocity profile for traverse station 1 was developed by using the analytically-predicted temperature profile together with the measured stagnation pressure values. The resulting velocity profile is given at the end of the next section of this report.

The mass flow rate through the jet pump as determined by the calibrated inlet bellmouth was compared to the mass flow rate obtained by integration of the velocity profiles for stations 4 and 5. Agreement was within 1% (149.8 lbm/min. from integration vs. 148.8 lbm/min. from the bellmouth). The measured velocity profile at station 6 was used for a similar comparison. Integration of this profile gave a mass flow rate of 158.8 lbm/min., about 7% greater than the bellmouth measurement.
The values of \( \frac{U_j}{U_{jo}} \) and \( \frac{T_j}{T_{jo}} \) calculated from the measured velocity and temperature profiles are plotted in figure 10 to show how the centerline velocities and temperatures vary with distance along the mixing tube. The velocity and temperature ratios are nearly identical over most of the mixing tube length.
Section 5

COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

5.1 Mixing Tube Wall Static Pressure Variations

The mixing tube wall static pressure measurements are the most valuable results for evaluating the accuracy of the analytical model for use in jet pump design. The prediction of this pressure variation is the primary purpose of this investigation because knowledge of this variation permits calculation of the pressure force on the mixing tube wall. This force must be known in order to solve the momentum equation during jet pump system optimization studies.

The analytical predictions of mixing tube static pressure variations are compared to test results for four entrainment ratios in figures 11, 12, and 13. The analyses were carried out with two different values assumed for $x_{\text{core}}$, the length of the transition region at the primary nozzle exit, and for two values of the secondary flow rates for each test; the values determined from the test results using the bellmouth calibration equation (54), and values 2% lower. A key to the three figures follows:

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>$x_{\text{core}} / R_o$</th>
<th>Secondary Flow Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>2.5</td>
<td>from (54), reduced by 2%</td>
</tr>
<tr>
<td>12</td>
<td>2.5</td>
<td>from (54)</td>
</tr>
<tr>
<td>13</td>
<td>2.0</td>
<td>from (54), reduced by 2%</td>
</tr>
</tbody>
</table>

The mass flow rates given by equation (54) and used to prepare figure 12 cause the analytical predictions of static pressure to fall below the measured values in the throat section of the mixing tube ($\frac{x}{R_o}$ from 7.34 to 10.7). The assumption that the secondary flow rates are 2% lower yields better agreement as shown in figures 11 and 13. The choice of $x_{\text{core}}$ to be 2.5 $R_o$ rather than 2.0 $R_o$ causes only a small difference in the predicted static pressure levels. The differences are largest in the diffuser sections downstream of the mixing tube throat.
From these results, it was concluded that further comparisons of analytical and experimental results should be based on the assumption that the true secondary flow rates are 2% lower than the flow rates given by equation (54). An uncertainty of ± 2% in flow rate is not unreasonable for the bellmouth calibration. The 2% flow correction brings the analytical predictions very close to the experimental results except for the static pressures downstream of the mixing tube throat.

5.2 Velocity and Temperature Profiles

The variation of predicted centerline velocity with distance along the mixing tube is shown in figure 14 for three alternative values of $x_{\text{core}}/R_o$: 1.0, 2.0, and 2.5. The measured values of centerline velocity at traverse stations 2-6 are also plotted in the figure. A value of $x_{\text{core}}/R_o$ between 2.0 and 2.5 appears to make the analytical prediction fit the test data most accurately.

The variation of predicted centerline stagnation temperature with distance along the mixing tube also is shown in figure 14. A value of $x_{\text{core}}/R_o$ between 2.0 and 2.5 will make the temperature predictions fit the test data upstream of the throat section of the mixing tube. At traverse stations 4, 5, and 6, the measured temperature levels fall about 30°F below the predicted centerline stagnation temperatures.

The analytical results ($U_c$, $U_o$, $\delta$, $f_o$, $g_o$, and $v$), together with the known free jet profile $f_o(\eta)$, allow direct comparison of the velocity and temperature profiles predicted by the analysis to the velocity and temperature profiles measured during the test program. The velocity profiles are compared in figure 15, and the temperature profiles are compared in figure 16. The predicted velocity profiles agree reasonably well with the measured profiles. The measured and predicted temperature profiles agree well for traverse stations 2 and 3, but the predicted temperatures near the centerline for stations 4, 5, and 6 are somewhat higher than they should be.

Stagnation pressure measurements only were obtained at traverse station 1. These measurements, coupled with the analytical temperature profiles predicted for this station, can be used to develop an approximate velocity profile.
The procedures used were as follows:

1. The stagnation pressure data from the traverse probe, together with the local static pressure tap reading, were used to determine the Mach number and $T/T_o$ ratio at each $y/R$ position in the mixing tube cross-section. This data is given in table 4.

2. From the analytical solution for $x_{\text{core}} = 2.5 R_0$, the predicted value of $\frac{\delta}{R}$ at the traverse station ($\frac{x}{R} = 2.5$) was found to be 0.2118. The local value of $\frac{R}{R_0}$ is 0.889. These results allow the $y/R$ positions of the traverse probe near the duct centerline to be interpreted in terms of the $y/\delta$ values for the free jet velocity profile of equations (1) and (2). Using the analytically predicted values $U_c = 3019$ ft/sec, $U_o = 268$ ft/sec, $T_{\infty} = 1267^\circ R$, and $T_{oo} = 552^\circ R$, the free jet velocity and temperature profiles can be used, through equations (1), (2), and (3), to determine the predicted values of velocity and stagnation temperature for each $y/R$ position within the jet mixing region. The corresponding static temperatures can be determined from the $T/T_o$ ratios in table 4. The speed of sound is calculated from the static temperature. The predicted flow velocities and speed of sound values are used to calculate Mach numbers for each $y/R$ position within the jet mixing region. These predicted Mach numbers are compared to the "measured" Mach numbers in table 4. If the predicted and measured numbers agree, the associated velocity and temperature profiles afford a good approximation to the true profiles.

3. The same calculation procedure was followed using the analytical solution for $x_{\text{core}} = 2.0 R_0$. The predicted value of $\frac{\delta}{R}$ at the traverse station was 0.287. The other predicted values employed in the analysis were as follows:

\[
U_c = 2227 \text{ ft/sec.} \quad T_{oc} = 1041^\circ R
\]
\[
U_o = 261 \text{ ft/sec.} \quad T_{oo} = 552^\circ R
\]
The predicted and "measured" Mach number profiles for traverse station 1 are compared in figure 17. The predicted profile based upon the assumption that $x_{\text{core}} = 2.0 \, R_o$ is closer to the measured profile than the $x_{\text{core}} = 2.5 \, R_o$ profile although the predicted centerline velocity is too high. The predicted velocity and temperature profiles for both $x_{\text{core}}$ assumptions are given in table 4. The predicted values were obtained using a secondary flow rate which was 2% less than the value given by equation (54).
An analytical method has been developed to predict the performance characteristics of axisymmetric single-nozzle compressible flow jet pumps with variable area mixing tubes. The primary flow may be either subsonic or supersonic. The analysis is divided into two parts. In part 1, the region between the primary nozzle exit and the point where the jet reaches the wall, the analysis is based upon the hypothesis that the mixing phenomena in the jet pump is fundamentally similar to the mixing of a free turbulent jet with the surrounding fluid. The eddy viscosity is adjusted to account for the influence of the duct walls as the jet approaches the walls. In part 2, downstream of the point where the jet reaches the wall, the velocity profile is allowed to vary from the free jet profile at the end of part 1 to a profile which asymptotically approaches the fully-developed turbulent flow profile in a pipe. Integral techniques are employed in both part 1 and part 2 to solve the continuity, momentum, moment-of-momentum, and energy equations to determine the variations of flow properties along the mixing tube.

An experimental program was conducted to measure mixing tube wall static pressure variations, velocity profiles, and temperature profiles in a variable area mixing tube with a supersonic \((M = 2.72)\) primary jet. Static pressure variations were measured at four different secondary flow rates. These test results were used to evaluate the analytical model.

Analytical predictions of wall static pressure distributions along the mixing tube generally agreed well with the test results for all four entrainment ratios. The predicted wall static pressure values differed slightly from the measured pressures downstream of the constant-area throat section. The velocity profiles along the mixing tube were predicted accurately by the analysis. The analytical temperature profiles were not as accurate; the predicted centerline temperatures downstream of the throat were too high. These discrepancies are considered to be minor in view of the comparatively extreme mixing tube geometry used for the test case. Thus, the analysis is ready for use to calculate the pressure force on the wall of a variable area mixing tube. This permits the momentum equation to be solved accurately in jet pump-duct system optimization and design studies.
The analysis in part 2 of the jet pump makes the assumption that the temperature profiles are similar to free jet temperature profiles. A very simple and approximate form of the energy equation is employed. A more accurate energy equation, perhaps augmented by assumption of a different form for the temperature profile, might lead to greater accuracy in the prediction of wall static pressures and temperature profiles in this region.
APPENDIX A

Equations for the Flow

A1 - Part 1 - Upstream of Jet Attachment

The general form of the flow equations, as described in Section 3.4, is as follows:

$$\sum_{k=1}^{7} W(J, K) \times Y(K) = V(J)$$

The 7 variables are tabulated below, using the convention that the superscript ('') represents \( \frac{\partial}{\partial \left( \frac{\alpha}{R_o} \right)} \).

<table>
<thead>
<tr>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y(K) )</td>
<td>( \left( \frac{U_j}{U_{jo}} \right)' )</td>
<td>( \left( \frac{\delta}{R_o} \right)' )</td>
<td>( \left( \frac{p}{p_{oo}} \right)' )</td>
<td>( \left( \frac{T_{jo}}{T_{oo}} \right)' )</td>
<td>( \left( \frac{\Theta}{R_o} \right)' )</td>
<td>H'</td>
<td></td>
</tr>
</tbody>
</table>

The \( W(J, K) \) coefficients and \( V(J) \) terms are determined in this section.

A1-1 Equation for \( J = 1 \); Constant stagnation pressure in the flow outside the jet.

$$dp = -\rho \frac{UdU}{g_o}$$

$$Rg_T \frac{dp}{p} + \lambda U_j d(\lambda U_j) = 0$$

Normalizing:

$$\frac{Rg_T}{U_{jo}} \frac{dp}{p} + \lambda \frac{U_j}{U_{jo}} \left( \frac{dU_j}{U_{jo}} + \frac{U_j}{U_{jo}} d\lambda \right) = 0$$
Let \[ BP = \frac{R_g T_{\infty}}{U_{jo}} = \frac{k-1}{2kS_o} \]

\[ S_o = \frac{U_{jo}^2}{2\frac{k}{k-1} R_g T_{\infty}} \]

Then \[ T_o = T_{\infty} \left[ 1 - S_o \frac{U_i^2}{U_{jo}^2} \lambda^2 \right] = \text{Static temperature in the flow outside the jet} \]

The final values follow:

\[ W(1, 1) = \lambda^2 \frac{U_i}{U_{jo}} = W(1, 5) = 0 \]

\[ W(1, 2) = \lambda \left( \frac{U_i}{U_{jo}} \right)^2 = W(1, 6) = 0 \]

\[ W(1, 3) = 0 = W(1, 7) = 0 \]

\[ W(1, 4) = \left( 1 - S_o \frac{U_i^2}{U_{jo}^2} \lambda^2 \right) \frac{BP}{p} = V(1) = 0 \]

A1-2 Equation for \( J = 2 \); Momentum equation for the flow

\[ -\pi R^2 \frac{dp}{dx} = \frac{d}{dx} \int_{-R}^{R} \rho U^2 2\pi y dy \]

\[ -R^2 \frac{dp}{dx} = \frac{d}{dx} \left\{ \frac{p}{R_g T_{\infty}} U_i^2 \left[ \delta^2 \int_{0}^{1} \frac{(\lambda + f_o)^2 2\eta d\eta}{1 + \frac{1}{2} - S_o \frac{U_i^2}{U_{jo}^2} (\lambda + f_o)^2} + \frac{(R^2 - \delta^2) \lambda^2}{1 - S_o \frac{U_i^2}{U_{jo}^2} \lambda^2} \right] \right\} \]
where \( \lambda + f_o = \frac{U}{U_j} \)

\[
1 + \left[ f_o \right]_{1/2} - S_0 \left( \frac{1}{2} \right) \left( \lambda + f_o \right)^2 = \frac{T_0}{T_\infty} = \text{Static Temperature} \quad \text{Stagnation Temperature} \quad \frac{T_0}{T_\infty} = 1.0
\]

\[
1 - S_o \left( \frac{1}{2} \right) \lambda^2 = \frac{T_0}{T_\infty} = \text{Static Temperature} \quad \text{Stagnation Temperature} \quad \text{at} \ \eta = 1.0
\]

Let

\[
Z_{12} = \int_0^1 \frac{(\lambda + f_o)^2}{1 + \left[ f_o \right]_{1/2} - S_0 \left( \frac{1}{2} \right) \left( \lambda + f_o \right)^2} \ d\eta
\]

In the computer analysis, this integration is approximated by a summation across the jet:

Let

\[
Z_{1J} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{N_{ij}}{D_{ij}} \eta_i
\]

In this equation \( N_i \) and \( D_i \) are average values of the numerator and denominator across the \( i \)th equal-radius annular segment of the jet. The following additional definitions will be used:

\[
Z_{2J} = \frac{2}{n_s} \sum_{i=1}^{n_s} \left( \frac{\alpha N_{ij}}{D_{ij}} \right)^2 \frac{3 D_{ij}}{\beta \left( \frac{U_i}{U_j} \right)} \eta_i
\]
\[
Z_{3J} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{1}{D_{ij}} \frac{\partial N_{ij}}{\partial \lambda} \eta_i
\]

\[
Z_{4J} = \frac{2}{n_s} \sum \left( -\frac{N_{ij}}{D_{ij}^2} \right) \frac{\partial D_{ij}}{\partial \lambda} \eta_i
\]

\[
Z_{5J} = \frac{2}{n_s} \sum \frac{1}{D_{ij}} \frac{\partial N_{ij}}{\partial U} \eta_i
\]

\[
Z_{6J} = \frac{2}{n_s} \sum \frac{-N_{ij}}{2} \frac{\partial D_{ij}}{\partial U} \eta_i
\]

Then the relations below may be used:

\[
\frac{\partial Z_{1J}}{\partial \left( \frac{U_j}{U_{j0}} \right)} = Z_{2J}
\]

\[
\frac{\partial Z_{1J}}{\partial \lambda} = Z_{3J} + Z_{4J}
\]

\[
\frac{\partial Z_{1J}}{\partial U} = Z_{5J} + Z_{6J}
\]

Additional parameters which simplify the equations are defined as follows:

\[
S_2 = \frac{\lambda}{1 - S_0 \frac{U_j}{U_{j0}} \lambda^2}
\]

\[
\frac{R}{R_0} = \frac{R_{\text{tube}}}{R_0} - \frac{\Theta}{R_0} H
\]

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Employing these definitions in the momentum equation, the following expression is obtained after reorganizing, normalizing, and differentiating:

\[- \frac{dp}{dx} \frac{R g T}{p U_j^2} = \left[ \frac{p'}{p} + 2 \frac{U_j'}{U_j} \right] \left[ \frac{\delta^2}{R^2} Z_{12} + \left(1 - \frac{\delta^2}{R^2}\right) \lambda \frac{S_2}{R} \right] + 2 \frac{\delta}{R} \frac{\delta'}{R} Z_{12} \]

\[+ \frac{\delta^2}{R^2} \left[ \frac{Z_{22}}{U_j} + (Z_{32} + Z_{42}) \lambda' + (Z_{52} + Z_{62}) \lambda'' \right] \]

\[+ \left[ \frac{2}{R} \left( R'_{\text{tube}} - \theta' \frac{H-H}{\Theta} \right) - 2 \frac{\delta}{R} \frac{\delta'}{R} \right] \lambda S_2 \]

\[+ \left(1 - \frac{\delta^2}{R^2}\right) \left( S_2 \lambda' + \lambda \frac{\partial S_2}{\partial \lambda} + \lambda \frac{\partial S_2}{\partial U_j} \right) \]

The final values follow:

\[W(2, 1) = \left( \frac{\delta}{R} \right)^2 \left[ \frac{2Z_{12}}{U_j - U_j} \right] + \frac{1 - \delta^2}{R^2} \left[ \frac{2S_2}{U_j} + \lambda \frac{\partial S_2}{\partial \lambda} \right] \]

\[W(2, 2) = \left( \frac{\delta}{R} \right)^2 \left( Z_{32} + Z_{42} \right) + \left(1 - \frac{\delta^2}{R^2}\right) \left( S_2 + \lambda \frac{\partial S_2}{\partial \lambda} \right) \]

\[W(2, 3) = 2 \frac{\delta}{R} \left( Z_{12} - \lambda S_2 \right) \]
\[ W(2,4) = \frac{P_\infty}{p} \left[ \frac{\delta^2}{R^2} Z_{12} + \left(1 - \frac{\delta^2}{R^2}\right) \lambda S_2 \right] + \frac{BP * P_\infty}{p \left( \frac{U_j}{U_{j0}} \right)} \]

\[ W(2,5) = \frac{\delta^2}{R^2} (Z_{52} + Z_{62}) \]

\[ W(2,6) = -2 \frac{H}{R} S_2 \lambda \]

\[ W(2,7) = -2 \frac{\Theta}{R} S_2 \lambda \]

\[ V(2) = -2 \frac{R'_\text{tube}}{R} S_2 \lambda \]

Table A1 lists the values of \( N_1, D_1 \), and their derivatives which are required to evaluate the \( Z \) parameters in the previous equations.

A1.3 Equation for \( J = 3 \); Continuity equation

\[ W_0 + W_1 = 2\pi \int_0^R \rho U y dy \quad \text{where} \quad R = (\text{Local Duct Radius} - \Theta H) \]

\[ W_0 + W_1 = \frac{P \pi}{R T \infty} U_j \left[ \frac{1}{\eta} \int_0^1 (\lambda + f_0) d\eta \right] ^2 \frac{1}{1 + \Theta_0^{1/2} \left( \frac{U_j}{U_{j0}} \right) (\lambda + f_0)^2} + \frac{R^2 - \delta^2}{\lambda} \]

or

\[ W_0 + W_1 = \frac{P \pi}{R T \infty} U_j \left[ \delta^2 Z_{13} + (R^2 - \delta^2) S_2 \right] \]

where

\[ Z_{13} = \int_0^1 \frac{(\lambda + f_0) 2\eta d\eta}{1 + \Theta_0^{1/2} \left( \frac{U_j}{U_{j0}} \right) (\lambda + f_0)^2} \]

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The continuity equation is normalized as follows:

\[
\frac{W_0 + W_1}{\pi R_0^2 U_{jo}} \frac{R T_{oo}}{P_{oo}} = \frac{U_1}{U_{jo}} \left[ \left( \frac{\delta}{R_0} \right)^2 Z_{13} + \left( \frac{R_0^2 - \delta^2}{R_0^2} \right) S_2 \right] \frac{p}{P_{oo}}
\]

Taking the derivative with respect to \( \frac{x}{R_0} \):

\[
o = \left( \frac{U_1}{U_{jo}} \right) \frac{p}{P_{oo}} \left[ \left( \frac{\delta}{R_0} \right)^2 Z_{13} + \left( \frac{R_0^2 - \delta^2}{R_0^2} \right) S_2 \right] + \left( \frac{p}{P_{oo}} \right) \frac{U_1}{U_{jo}} \left[ \left( \frac{\delta}{R_0} \right)^2 Z_{13} + \left( \frac{R_0^2 - \delta^2}{R_0^2} \right) S_2 \right]
\]

\[
+ \left( \frac{\delta}{R_0} \right) \left[ 2 \frac{\delta}{R_0} \left( Z_{13} - S_2 \right) \right] \frac{U_1}{U_{jo}} \frac{p}{P_{oo}}
\]

\[
+ \frac{U_1}{U_{jo}} \frac{p}{P_{oo}} \frac{\delta^2}{R_0^2} \left[ Z_{23} \left( \frac{U_1}{U_{jo}} \right) ' + (Z_{33} + Z_{43}) \lambda' + (Z_{53} + Z_{63}) \mu' \right]
\]

\[
+ \frac{U_1}{U_{jo}} \frac{p}{P_{oo}} \left( \frac{R_0^2 - \delta^2}{R_0^2} \right) \lambda' \left( 1 - S_o \left( U_1^2 / U_{jo}^2 \right) \right)^2 + \lambda S_o \left[ 2 \lambda \lambda' \left( U_1^2 / U_{jo}^2 \right)' + 2 \lambda^2 \left( U_1 / U_{jo} \right) \left( U_1 / U_{jo} \right)' \right]
\]

\[
+ \frac{2 R}{R_0} \frac{U_1}{U_{jo}} \frac{p}{P_{oo}} S_2 \left( \frac{R_{tube}}{R_0} - \frac{\Theta}{R_0} \right) H' - \frac{H}{R_0} \Theta
\]
Collecting terms and dividing each by \( \left( \frac{R}{R_o} \right)^2 \frac{U_j}{U_{jo}} \frac{p}{p_{oo}} \):

\[
W(3, 1) = \frac{U_{jo}}{U_j} \frac{R_o}{R} \frac{R^2}{R_o^2} \left( \frac{\delta}{R_o} \right)^2 Z_{13} + \left( 1 - \frac{R_o}{R} \frac{\delta^2}{R_o^2} \right) S_2 \frac{U_{jo}}{U_j} + Z_{23} \frac{R_o}{R} \frac{\delta^2}{R_o^2} + \left( \frac{U_j}{U_{jo}} \right) 2 \lambda S_2^2 S_o \left( 1 - \frac{R_o}{R} \frac{\delta^2}{R_o^2} \right)
\]

\[
W(3, 2) = \frac{R_o}{R} \frac{\delta^2}{R_o^2} (Z_{33} + Z_{43}) + \left( 1 - \frac{R_o}{R} \frac{\delta^2}{R_o^2} \right) \frac{1 + S_o}{1 - S_o} \frac{U_j^2}{U_{jo}^2} \lambda^2
\]

\[
W(3, 3) = 2 \frac{\delta}{R_o} (Z_{13} - S_2) \frac{R_o^2}{R^2}
\]

\[
W(3, 4) = \frac{p_{oo}}{p} \left[ \frac{\delta^2}{R_o^2} \frac{R_o}{R} \frac{R^2}{R_o^2} Z_{13} + \left( 1 - \frac{R_o}{R} \frac{\delta^2}{R_o^2} \right) S_2 \right]
\]

\[
W(3, 5) = \frac{R_o}{R} \frac{\delta^2}{R_o} \frac{R^2}{R_o} (Z_{53} + Z_{63})
\]

\[
W(3, 6) = -2S_2 \frac{R_o}{R} H
\]
\[ W(3, 7) = -2 S_2 \frac{R_o}{R} \frac{\Theta}{R_o} \]

\[ V(3) = -2 \frac{R_o}{R} \left( \frac{R}{R_o} \right) ' S_2 \]

Table A1 lists the values of \( N_i \), \( D_i \), and their derivatives which are required to evaluate the \( Z \) parameters in the equations above.

A1-4 Equation for \( J = 4 \); Energy Equation

\[ W_o C_p T_\infty + W_1 C_p T_oj = 2\pi \int_0^R \rho U C_p T_o y dy \text{ where } R = \text{(local duct radius - } \Theta H) \]

or

\[ W_o T_\infty + W_1 T_oj = 2\pi \int_0^R \rho U T_o y dy \]

\[ W_o T_\infty + W_1 T_oj = \pi \frac{P_0}{R_g} U_j \delta^2 \int_0^1 \frac{1}{1 + \sqrt{f_o/2}} \frac{2 \eta d\eta}{1 + \sqrt{f_o/2} - S_o \left( \frac{U_j}{U_{oj}} \right)^2 (\lambda + f_o)^2} + (R^2 - \delta^2) S_2 \]

Let

\[ Z_{14} = \int_0^1 \frac{(\lambda + f_o) (1 + \sqrt{f_o/2}) 2 \eta d\eta}{1 + \sqrt{f_o/2} - S_o \left( \frac{U_j}{U_{oj}} \right)^2 (\lambda + f_o)^2} \]

Then, the normalized energy equation may be written as follows:
If this equation is compared to the normalized continuity equation in Section A1-3, it is seen that the right-hand sides are identical except for the substitution of $Z_{14}$ for $Z_{13}$. This means that all of the $W(4, K)$ coefficients are identical to the $W(3, K)$ coefficients except for the substitution of $Z_{14}$ for $Z_{13}$ in all expressions. Table A1 lists the values of $N_i$, $D_i$, and their derivatives which are required to evaluate the $Z_{14}$ parameters.

A1-5 Equation for $J = 5$; Moment-of-Momentum Integral Equation

The momentum equation for an annular section of the jet can be derived as follows.

\[ \tau 2\pi \, dydx - \frac{dp}{dx} \, dx \, (2\pi ydy) + 2\pi y \frac{\partial \tau}{\partial y} \, dydx = \rho \frac{\partial u}{\partial x} \, dx \cdot 2\pi ydy + \rho v \, 2\pi ydx \frac{\partial u}{\partial y} \, dy \]

or

\[ \tau - \frac{dp}{dx} \cdot y + y \frac{\partial \tau}{\partial y} = \rho \frac{\partial u}{\partial x} \cdot uy + \rho vy \frac{\partial u}{\partial y} \]
To derive the moment-of-momentum integral equation, this momentum equation is multiplied by $y\,dy$ and integrated across the jet:

$$
\int_0^\delta \rho u y \frac{\partial u}{\partial x} \, y\,dy + \int_0^\delta \rho v y \frac{\partial u}{\partial y} \, y\,dy = \int_0^\delta \frac{\partial (\tau y)}{\partial y} \, y\,dy - \int_0^\delta \frac{dp}{dx} \, y^2\,dy
$$

Noting that $u = U_j(\lambda + f_o)$:

$$
\frac{\partial u}{\partial x} = \frac{1}{R_0} U_j' (\lambda + f_o) + \frac{U_j}{R_0} \left[ \lambda' + \frac{\partial f_o}{\partial \eta} \left( -\frac{\eta \delta'}{\delta} \right) \right]
$$

$$
\frac{\partial u}{\partial y} = \frac{U_j}{\delta} \frac{\partial f_o}{\partial \eta}
$$

then

$$
\int_0^\delta \rho u y \frac{\partial u}{\partial x} \, y\,dy = \int_0^1 \frac{p}{R T} \frac{1}{1 + f_o \sqrt{f_o} - S_0 \frac{U_j^2}{U_{jo}^2} (\lambda + f_o)^2} \, 3 \eta^2 d\eta
$$

$$
= \frac{p U_j^2 \delta^2}{R T} \int_0^1 \frac{1}{U_j} \frac{(\lambda + f_o) \frac{\partial u}{\partial x} \eta^2}{1 + f_o \sqrt{f_o} - S_0 \frac{U_j^2}{U_{jo}^2} (\lambda + f_o)^2} \, d\eta
$$

$$
= \frac{p U_j^2 \delta^2}{R T} \int_0^1 \frac{1}{U_j} \left[ \frac{U_j}{U_j} Q_1 + \lambda' Q_2 + \frac{\delta'}{\delta} Q_3 \right]
$$

in which

$$
Q_1 = \int_0^1 \frac{(\lambda + f_o)^2 \eta^2}{D} \, d\eta
$$

$$
Q_2 = \int_0^1 \frac{(\lambda + f_o) \eta^2}{D} \, d\eta
$$

$$
Q_3 = -\int_0^1 \frac{\delta f_o}{\partial \eta} (\lambda + f_o) \eta^3 \, d\eta
$$

$$
D = 1 + f_o \sqrt{f_o} - S_0 \frac{U_j^2}{U_{jo}^2} (\lambda + f_o)^2
$$

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In order to evaluate the radial velocity, \(v\), it is necessary to use the continuity relation. Employing a control volume of radius \(y\) and length \(dx\), the continuity equation may be written as follows:

\[
\rho \nu 2 \pi y dx = - \int_0^y \frac{\partial}{\partial x} (\rho u 2 \pi y dy) dx
\]

so

\[
\rho v y = - \int_0^y \frac{\partial}{\partial x} (\rho u y) dy
\]

\[
= - \int_0^y \frac{\partial}{\partial x} \left[ \frac{p}{R^2} T \frac{U_j (\lambda + f_\sigma)}{D} \right] ydy
\]

\[
\rho v y = - \int_0^\frac{\delta^2}{R_0 R^2 T \infty} \left\{ \left[ \frac{p U_j (\lambda + f_\sigma)}{D} + \frac{p U_j (\lambda + f_\sigma)}{D} + \frac{p U_j (\lambda + f_\sigma)}{D} \right] \left[ \frac{\partial D}{\partial \eta} + \frac{\lambda' \partial D}{\partial \lambda} + \frac{\partial D}{\partial \lambda} - \eta \frac{\partial^2 D}{\partial \eta^2} \right] \right\} \eta d \eta
\]

\[
= \frac{p}{R^2 T \infty} U_j \frac{\delta^2}{R_0} \left[ \frac{U_j}{U_j \delta U_j} \right] \left[ \int_0^\eta \left[ \frac{(\lambda + f_\sigma)}{D} - \frac{(\lambda + f_\sigma)}{D} \right] \eta d \eta + \lambda' \int_0^\eta \left[ \frac{1}{D} - \frac{(\lambda + f_\sigma)}{D} \right] \eta d \eta + \delta' \int_0^\eta \left[ \frac{\partial f_\sigma}{\partial \eta} + \frac{(\lambda + f_\sigma)}{D} \eta \frac{\partial D}{\partial \eta} \right] \eta d \eta + \frac{p'}{p} \int_0^\eta \left[ \frac{(\lambda + f_\sigma)}{D} \right] \eta d \eta + \int_{U'} \left[ \frac{(\lambda + f_\sigma)}{D} \right] \eta d \eta \right]
\]
in which

\[ V_1 = \int_0^\eta \frac{(\lambda + f_o)}{D} \eta \, d\eta \]
\[ V_5 = 0 \]

\[ V_2 = \int_0^\eta - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial \eta} \frac{U_j}{U_j} \eta \, d\eta \]
\[ V_6 = \int_0^\eta - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial \eta} \frac{1}{U_j} \eta \, d\eta \]

\[ V_3 = \int_0^\eta \frac{1}{D} \eta \, d\eta \]
\[ V_{10} = \int_0^\eta \frac{\partial f_o}{\partial \eta} \eta^2 \, d\eta \]

\[ V_4 = \int_0^\eta - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial \lambda} \eta \, d\eta \]
\[ V_{11} = \int_0^\eta - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial \eta} \eta^2 \, d\eta \]

With these definitions, the integral

\[ \int_0^\delta \rho v_y \frac{\partial u}{\partial y} \, dy \]

may be evaluated as follows:

\[ \int_0^\delta \rho v_y \frac{\partial u}{\partial y} \, dy = \int_0^\delta \frac{\rho v_y}{\left( \frac{pU_j}{R \frac{T}{g \infty}} \right)} \left( \frac{\delta^2}{R_o} \right) \frac{1}{U_j} \frac{U_j}{\delta} \frac{\partial f_o}{\partial \eta} \, dy \]

\[ = \int_0^1 \left[ \frac{\rho v_y}{\left( \frac{pU_j}{R \frac{T}{g \infty}} \right)} \frac{\delta^2}{R_o} \right] \frac{\partial f_o}{\partial \eta} \eta \, d\eta \]
\[ \frac{U_j}{U_j} \left[ - \int_0^1 V_1 \frac{\partial f_o}{\partial n} \eta \, d\eta - \frac{U_j}{U_j} \int_0^1 V_2 \frac{\partial f_o}{\partial n} \eta \, d\eta \right] + \lambda \left[ \frac{1}{V_3} \frac{\partial f_o}{\partial n} \eta \, d\eta - \int_0^1 \frac{\partial f_o}{\partial n} \eta \, d\eta \right] \\
+ \frac{\partial}{\partial \eta} \left[ \int_0^1 (V_{10} + V_{11}) \frac{\partial f_o}{\partial n} \eta \, d\eta \right] + \frac{p}{p} \left[ - \int_0^1 \frac{\partial f_o}{\partial n} \eta \, d\eta \right] + \frac{1}{\eta} \left[ \int_0^1 \frac{\partial f_o}{\partial n} \eta \, d\eta \right] \\
= \frac{U_j}{U_j} \left[ R_1 + \frac{U_j}{U_j} R_2 \right] + \lambda \left[ R_{34} + \frac{\partial}{\partial \eta} \left[ R_{10} + R_{11} \right] + \frac{p}{p} R_1 + \frac{1}{\eta} R_{56} \right] \\
\text{in which } R_1 = - \int_0^1 V_1 \frac{\partial f_o}{\partial n} \eta \, d\eta \hspace{1cm} R_{10} = \int_0^1 V_{10} \frac{\partial f_o}{\partial n} \eta \, d\eta \\
R_2 = - \int_0^1 V_2 \frac{\partial f_o}{\partial n} \eta \, d\eta \hspace{1cm} R_{11} = \int_0^1 V_{11} \frac{\partial f_o}{\partial n} \eta \, d\eta \\
R_{34} = - \int_0^1 (V_3 + V_4) \frac{\partial f_o}{\partial n} \eta \, d\eta \hspace{1cm} R_{56} = - \int_0^1 (V_5 + V_6) \frac{\partial f_o}{\partial n} \eta \, d\eta \\
\text{The pressure gradient term may be rewritten as follows:} \\
\int_0^1 \frac{\partial p}{\partial x} y^2 dy = \frac{p}{p} \frac{U_j}{R_T} \left[ \int_0^1 3 \frac{BP}{R_T} \int_0^1 \frac{U_j}{U_j^2} \right] \\
\text{The shear stress term may be evaluated as follows:} \\
\int_0^1 \frac{\partial (\tau y)}{\partial y} y dy = \tau y^2 \int_0^1 \delta \int_0^1 \tau y dy \]
\[ \tau = \rho \varepsilon \frac{\partial u}{\partial y} = \rho \varepsilon \frac{U_1}{\delta} \frac{\partial f_o}{\partial \eta} \quad \text{where} \ \varepsilon \ \text{is the eddy kinematic viscosity} \]

\[
\delta \int_0^\delta \frac{\partial (\tau y)}{\partial y} \, dy = \frac{1}{\rho U_j^2 \delta^3 \left( \frac{R_g T}{g' \infty R_0} \right)} \left[ -\int_0^1 \frac{1}{\rho U_j \delta} \frac{1}{2} \frac{\partial f_o}{\partial \eta} \eta d\eta - \frac{R_g T}{g' \infty R_0} \right] \]

\[
- \int_0^1 \frac{\varepsilon}{U_j \delta} \frac{R_0}{\delta} \frac{1}{D} \frac{\partial f_o}{\partial \eta} \eta d\eta
\]

\[
= - \left( \frac{\varepsilon}{U_j \delta} \right)_{\text{avg}} \left( \frac{R_0}{\delta} \right) \int_0^1 \frac{\varepsilon}{\text{avg}} \frac{1}{D} \frac{\partial f_o}{\partial \eta} \eta d\eta
\]

\[
= \frac{E A_T}{(\delta/R_o)}
\]

where \( A_T = - \int_0^1 \frac{\varepsilon}{\text{avg}} \frac{1}{D} \frac{\partial f_o}{\partial \eta} \eta d\eta \approx 0.377 \) as in incompressible jet mixing

\[ E = \left( \frac{\varepsilon}{U_j \delta} \right)_{\text{avg}} \] is the inverse of the local turbulent Reynolds number.

Assembling all the terms, the final moment-of-momentum integral equation is as follows:

\[
\frac{U_j'}{U_j} \left( Q_1 + R_1 + R_2 \frac{U_j}{U_j'} \right) + \lambda' (Q_2 + R_{34}) + \frac{\delta'}{\delta} (Q_3 + R_{10} + R_{11}) + \frac{p'}{p} \left( R_1 + \frac{BP}{U_j^2} \right) + \overline{\tau}_{56} = \frac{E A_T}{(\delta/R_o)}
\]
The values of the coefficients follow:

\[ W(5, 1) = Q_1 + R_1 + R_2 \frac{U_j}{U_{jo}} \]
\[ W(5, 2) = Q_2 + R_{34} \]
\[ W(5, 3) = Q_3 + R_{10} + R_{11} \]
\[ W(5, 4) = R_1 + \frac{BP}{U_j^2} \left( \frac{3}{U_{jo}^2} \right) \]
\[ W(5, 5) = R_{56} \]
\[ W(5, 6) = 0 \]
\[ W(5, 7) = 0 \]

**A1-6 Equation for J = 6; Boundary Layer Momentum Equation**

The boundary layer momentum equation, equation (40), is discussed in Section 3.4 of this report.

**A1-7 Equation for J = 7; Boundary Layer Moment-of-Momentum Equation**

The boundary layer moment-of-momentum equation was used to derive the shape factor equation (41). This equation is discussed in Section 3.4 of this report.

**A1-8 Initial Conditions for the Part 1 Analysis**

The initial conditions for the Part 1 analysis are established from the transition zone analysis described in Section 3.3. The initial values were set as follows:
\[ \frac{U_J}{U_{Jo}} = 1 \]
\[ \frac{T_{oJ}}{T_{oo}} = \frac{T_{o1} - T_{oo}}{T_{oo}} \quad \text{From transition zone Analysis} \]
\[ \lambda = \lambda \quad \text{From transition zone Analysis} \]
\[ \frac{\Theta}{R_o} = \frac{\Theta}{R_o} \quad \text{Calculated from Equation (14)} \]
\[ \frac{\delta}{R_o} = \frac{R}{R_o} \left( \frac{\delta}{R} \right) \quad \text{From transition zone analysis} \]
\[ H = 1.4 \quad \text{Input value to program} \]
\[ \frac{P}{P_{oo}} = \left[ 1 - S_o \frac{k}{k-1} \lambda^2 \right] \left[ 1 - \frac{\Delta P_{soD}}{P_{oo}} \right] \]

A2 - Part 2 - Downstream of Jet Attachment

The general form of the flow equations, as described in Section 3.4, is as follows:

\[ \sum_{K=1}^{6} W(J, K) \times Y(K) = V(J) \]

The 6 variables employed in Part 2 are tabulated below. The superscript (') represents the derivative of the variable with respect to the streamwise coordinate.

\[ K = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]
\[ Y(K) = \left( \frac{U_c}{U_{Jo}} \right)^' \left( \frac{U_o}{U_c} \right)^' \left( \frac{\gamma'}{R_o} \right)^' \left( \frac{P}{P_{oo}} \right)^' \left( \frac{\gamma'}{T_{oo}} \right)^' \]

where \( P_{oo} \) and \( T_{oo} \) are the stagnation pressure and stagnation temperature for the wall streamline at the end of Part 1 just as the jet reaches the duct wall.
The variable \( Y(2) \) remains zero throughout the Part 2 analysis; this variable is a redundant parameter which remains from an earlier version of the computer program.

The \( W(J,K) \) coefficients and \( V(J) \) terms are determined in this section of the appendix.

A2-1 Equation for \( J = 1 \); Continuity Equation

\[
W_o + W_1 = \int_0^R \rho u \cdot 2\pi y dy
\]

\[
\frac{W_o + W_1}{\pi g_o} = \frac{p}{R g T_\infty} \frac{U_c R^2}{U_o} \int_0^1 \frac{T_\infty}{T} \frac{u}{U_c} 2\pi d\eta
\]

Now \( \frac{u}{U_c} = f_2(\eta) + \gamma g_2(\eta) \)

\[
\frac{T}{T_\infty} = 1 + \sqrt{f_0(\eta)} - S_o \left( \frac{U_c}{U_o} \right)^2 \left[ f_2(\eta) + \gamma g_2(\eta) \right]^2
\]

where \( \frac{T_o - T_\infty}{T_o j_o} = \sqrt{f_0(\eta)} \) (Free jet temperature profile) is assumed to hold in Part 2 as a simplification of the analysis.

The value of \( T_\infty \) used in the definitions of \( BP \) and \( S_o \), and throughout the Part 2 analysis, is the stagnation temperature for the wall streamline at the axial position selected. \( T_\infty \) varies with \( x \) in Part 2.
Let
\[ D = \frac{T}{T_{oo}} \]

\[ BP = \frac{R \cdot \frac{T_{oo}}{2}}{U_{jo}} \]

The continuity equation may be rewritten as follows:

\[ \frac{T_{oo}}{P_{ooi}} \frac{R_g}{R_o^2} \frac{W_o + W}{\pi g_o U_{jo}} = \frac{\rho}{P_{ooi}} \left( \frac{T_{oo}}{T_{ooi}} \right) \frac{U_c}{U_{jo}} \frac{R^2}{R_o^2} \cdot Z_{11} \]

where
\[ Z_{11} = \int_0^1 \frac{(f_2 + \gamma g_2) 2\eta d\eta}{D} \]

In the computer analysis, the integration is approximated by a summation across the flow:

\[ Z_{1J} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{N_i J}{D_i J} \eta_i \]

In this equation, \( N_i \) and \( D_i \) are average values of the numerator and denominator across the \( i^{th} \) equal-radius annular segment of the flow.

The following additional definitions will be used:

\[ Z_{2J} = \frac{\partial Z_{1J}}{\partial \left( \frac{U_c}{U_{jo}} \right)} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{N_i J}{D_i J} \left[ \frac{\partial}{\partial \left( \frac{U_c}{U_{jo}} \right)} \right] \eta_i \]
\[ Z_{5J} + Z_{6J} = \frac{\partial Z_{1J}}{\partial \Gamma} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{1}{D_{ij}} \frac{\partial N_{iJ}}{\partial \Gamma} \eta_i + \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{N_{iJ}}{D_{ij}} \frac{\partial D_{ij}}{\partial \Gamma} \eta_i \]

\[ Z_{7J} + Z_{9J} = \frac{\partial Z_{1J}}{\partial \gamma} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{1}{D_{ij}} \frac{\partial N_{iJ}}{\partial \gamma} \eta_i + \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{N_{iJ}}{D_{ij}} \frac{\partial D_{ij}}{\partial \gamma} \eta_i \]

\[ Z_{8J} = \frac{\partial Z_{1J}}{\partial \frac{\Gamma}{\Gamma_{oo1}}} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{N_{iJ}}{D_{ij}} \frac{\partial D_{ij}}{\partial \frac{\Gamma}{\Gamma_{oo1}}} \eta_i \]

Employing these definitions in the continuity equation, the following equation is obtained after differentiating:

\[ 0 = -\left(\frac{p}{P_{oo1}}\right)' \frac{U_c}{U_{jo}} \frac{R^2}{R_o} \frac{\partial}{\partial \Gamma} Z_{11} + \left(\frac{U_c}{U_{jo}}\right)' \frac{\partial}{\partial \frac{\Gamma}{\Gamma_{oo1}}} \frac{R^2}{R_o} \frac{Z_{11}}{\Gamma_{oo1}} + \frac{\partial}{\partial \frac{\Gamma}{\Gamma_{oo1}}} \frac{U_c \frac{R^2}{R_o}}{U_{jo} \frac{R^2}{R_o}} \left[ Z_{21} \frac{U_c}{U_{jo}} + (Z_{51} + Z_{61}) \gamma' \right] \]

\[ + \frac{\partial}{\partial \frac{\Gamma}{\Gamma_{oo1}}} \frac{U_c \frac{R^2}{R_o}}{U_{jo} \frac{R^2}{R_o}} \left[ (Z_{71} + Z_{91}) \gamma' + Z_{81} \left(\frac{\Gamma}{\Gamma_{oo1}}\right)' \right] \]

Collecting terms and dividing each by \( \frac{\partial}{\partial \frac{\Gamma}{\Gamma_{oo1}}} \frac{U_c \frac{R^2}{R_o}}{U_{jo} \frac{R^2}{R_o}} \)
\[ W(1,1) = \frac{Z_{11}}{\left(\frac{U}{U_{j0}}\right)} + Z_{21} \quad W(1,5) = Z_{51} + Z_{61} \]

\[ W(1,2) = 0 \quad W(1,6) = \frac{Z_{11}}{\left(\frac{T_{oo}}{T_{ooi}}\right)} \]

\[ W(1,3) = Z_{71} + Z_{91} \quad V(1) = -2 \frac{\left(\frac{R}{R_{o}}\right)}{\left(\frac{R}{R_{o}}\right)} Z_{11} \]

\[ W(1,4) = \frac{Z_{11}}{\left(\frac{P}{P_{ooi}}\right)} \]

Table A2 lists the values of \(N_i\), \(D_i\), and their derivatives which are required to evaluate the \(Z\) parameters in the previous equations.

A2.2 Equation for \(J = 2\); Energy Equation

\[
\text{constant} = \int_{0}^{R} \rho u T_o \cdot 2\pi y dy \quad \text{assuming constant specific heat throughout the flow}
\]

Using the substitutions for \(\rho\), the velocity profile functions, and \(\eta\) as in Section A2.1, the energy equation may be rewritten as follows:

\[
\text{constant} = \frac{p}{RgT_{oo}} U_c R^2 \int_{0}^{1} \left[ f_2(\eta) + \gamma g_2(\eta) \right] \frac{1}{D} T_o 2\eta d\eta
\]

As in Section A2.1, the free-jet temperature profile is assumed to hold:

\[
\frac{T_o}{T_{oo}} = 1 + \gamma f_0(\eta)
\]
With this, the energy equation becomes:

\[
\text{constant} = \frac{p}{R} \frac{U_c}{U_{jo}} R^2 \int_0^1 \frac{(f_2 + \gamma g_2)(1 + \frac{f_1^{1/2}}{D})}{2\eta d\eta}
\]

\[
\text{constant} = \frac{p}{P_{pool}} \frac{U_c}{U_{jo}} \frac{R^2}{R_0} Z_{12}
\]

where

\[
Z_{12} = \int_0^1 \frac{(f_2 + \gamma g_2)(1 + \frac{f_1^{1/2}}{D})}{2\eta d\eta}
\]

After differentiating with respect to \(\frac{X}{R_0}\), the energy equation takes the following form:

\[
0 = \left(\frac{p}{P_{pool}}\right)' \frac{U_c}{U_{jo}} \frac{R^2}{R_0} Z_{12} + \left(\frac{U_c}{U_{jo}}\right)' \frac{p}{P_{pool}} \frac{R^2}{R_0} Z_{12} + \left(\frac{R}{R_0}\right)' \frac{p}{P_{pool}} \frac{U_c}{U_{jo}} \frac{R}{R_0} Z_{12}
\]

\[
+ \frac{p}{P_{pool}} \frac{U_c}{U_{jo}} \frac{R^2}{R_0} \left[ Z_{22} \left(\frac{U_c}{U_{jo}}\right)' + (Z_{52} + Z_{62})\frac{f_2^{1/2}}{D} + (Z_{72} + Z_{92}) \gamma' + Z_{82} \frac{f_{oo}}{D_{oo}} \right]
\]

Collecting terms and dividing each by \(\frac{p}{P_{pool}} \frac{U_c}{U_{jo}} \frac{R^2}{R_0}\):

\[
W(2,1) = \frac{Z_{12}}{U_c} + Z_{22}
\]

\[
W(2,5) = Z_{52} + Z_{62}
\]

\[
W(2,2) = 0
\]

\[
W(2,6) = Z_{82}
\]

\[
W(2,3) = Z_{72} + Z_{92}
\]

\[
W(2,4) = \frac{Z_{12}}{P_{pool}}
\]

\[
V(2) = -2Z_{12} \frac{(R/R_0)}{(R/R_0)}
\]

\[
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\]
Table A2 lists the values of $N_i$, $D_i$, and their derivatives which are required to evaluate the $Z$ parameters in the equations above.

A2.3 Equation for $J = 3$; Momentum Equation

\[- \pi R^2 \frac{dp}{dx} - 2\pi R \tau_w = \frac{d}{dx} \int_o^R \rho u^2 \, 2\pi y \, dy\]

Using the previously-developed substitution for $\rho$, the velocity profile functions, and $\eta$:

\[ - R^2 \frac{dp}{dx} - 2 R \tau_w = \frac{d}{dx} \left\{ \frac{p}{R \gamma g_0} U_c^2 \int_o^1 \frac{1}{D} \left[ f_2(\eta) + \gamma g_2(\eta) \right]^2 \eta d\eta \right\} \]

Let \[ \tau_w = C_{fd} \frac{1}{2} \frac{\rho U_c^2}{g_0} \] where \[ \frac{\rho}{g_0} = \frac{p}{R \gamma g_0} \]

The momentum equation may be rewritten as follows:

\[- R^2 \frac{dp}{dx} - R C_{fd} \frac{U_c^2}{g_0} \frac{p}{R g_0} = \frac{d}{dx} \left\{ \frac{p}{R g_0} \frac{U_c^2}{g_0} \int_o^1 \frac{1}{D} \left[ (f_2 + \gamma g_2)^2 \right] 2\eta d\eta \right\} \]

where

\[ Z_{13} = \int_o^1 \frac{(f_2 + \gamma g_2)^2}{D} 2\eta d\eta \]

Normalizing:

\[- \frac{R^2}{R_o} \left( \frac{p}{P_{ooi}} \right)' - \frac{R}{R_o} C_{fd} \left( \frac{U_c}{U_{jo}} \right)^2 \frac{p}{P_{ooi}} \frac{1}{BP} = \frac{d}{dx} \left\{ \frac{p}{P_{ooi}} \frac{U_c}{R g_0} \frac{U_c^2}{g_0} \int_o^1 \frac{1}{D} \left[ (f_2 + \gamma g_2)^2 \right] 2\eta d\eta \right\} \]

Differentiating:

\[ \left[ - \left( \frac{R}{R_o} \right)^2 \left( \frac{p}{P_{ooi}} \right)' - \frac{R}{R_o} C_{fd} \left( \frac{U_c}{U_{jo}} \right)^2 \frac{p}{P_{ooi}} \frac{1}{BP} \right] = \frac{Z_{13}}{BP} \left[ \left( \frac{p}{P_{ooi}} \right)' \left( \frac{U_c}{U_{jo}} \right)^2 \frac{R^2}{R_o} \right. \]

\[ + \left( \frac{U_c}{U_{jo}} \right) \frac{2}{P_{ooi}} \frac{U_c}{U_{jo}} \frac{R^2}{R_o} \left] \right. + \text{(see next page)} \]
\[
\begin{align*}
\frac{Z_{13}}{BP} \left( \frac{R}{R_o} \right) & \quad 2 \frac{p}{P_{ooi}} \frac{U_c^2}{U_{jo}} \frac{R}{R_o} \\
- \frac{T_{oo}}{T_{ooi}} & \quad \frac{p}{P_{ooi}} \left( \frac{1}{T_{oo}} \right) \frac{U_c^2}{U_{jo}} \frac{R^2}{R_o^2} \frac{Z_{13}}{BP} \\
+ \frac{p}{P_{ooi}} \frac{U_c^2}{U_{jo}} \frac{R^2}{R_o^2} & \left[ Z_{23} \left( \frac{U_c}{U_{jo}} \right) ' + (Z_{53} + Z_{63}) \gamma ' + (Z_{73} + Z_{93}) \gamma ' + Z_{83} \frac{T_{oo}}{T_{ooi}} \right]
\end{align*}
\]

Collecting terms and dividing each by \( \frac{p}{P_{ooi}} \frac{U_c^2}{U_{jo}} \frac{R^2}{R_o} \frac{1}{BP} \):

\[
\begin{align*}
W(3, 1) & = \frac{2Z_{13}}{U_c} + Z_{23} & W(3, 5) & = Z_{53} + Z_{63} \\
W(3, 2) & = 0 & W(3, 6) & = \frac{T_{oo}}{T_{ooi}} + Z_{83} \\
W(3, 3) & = Z_{73} + Z_{93} & V(3) & = -\frac{C_{fd}}{R} - 2Z_{13} \left( \frac{R'}{R_o} \right)
\end{align*}
\]

Table A2 lists the values of \( N_i, D_i \), and their derivatives which are required to evaluate the \( Z \) parameters in the equations above.
A2.4 Equation for $J = 4$; Moment-of-Momentum Integral Equation

The moment-of-momentum integral equation is taken from Section A1.5 of this appendix:

$$\int_0^R \rho u_y \frac{\partial u}{\partial x} y dy + \int_0^R \rho v_y \frac{\partial u}{\partial y} y dy = \int_0^R \frac{\partial (\tau y)}{\partial y} y dy - \int_0^R \frac{dp}{dx} y^2 dy$$

Noting that, in Part 2, $u = U_c (f_2' + \gamma g_2)$

$$\frac{\partial u}{\partial x} = \frac{1}{R_o} U_c \left( f_2' + \gamma g_2 \right) - \frac{U_c}{R_o} \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \left( \frac{R}{R_o} \right) + \frac{U_c g_2}{R_o} \gamma'$$

$$\frac{\partial u}{\partial y} = \frac{U_c}{R} \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right)$$

then

$$\int_0^R \rho u_y \frac{\partial u}{\partial x} y dy = \int_0^1 \frac{p}{R g T_\infty} \frac{U_c (f_2' + \gamma g_2) \frac{\partial u}{\partial x} \eta^2 d\eta}{D} R^3$$

$$= \frac{p U^2_c R^3}{R g T_\infty} \frac{1}{\frac{U_c}{R} \left( f_2' + \gamma g_2 \right) \left( \frac{R}{R_o} \right) \eta^2 d\eta}$$

$$= \frac{p U^2_c R^3}{R g T_\infty} \left[ \frac{U_c}{U_c} Q_1 + \left( \frac{R}{R_o} \right) Q_3 + \gamma' Q_4 \right]$$
In which

\[ Q_1 = \int_0^1 \frac{(f_2 + \gamma g_2)^2 \eta^2 \, d\eta}{D} \]

\[ Q_3 = -\int_0^1 \frac{1}{D} \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) (f_2 + \gamma g_2)^3 \, d\eta \]

\[ Q_4 = \int_0^1 \frac{(f_2 + \gamma g_2)}{D} g_2^2 \eta^2 \, d\eta \]

\[ D = 1 + U \sqrt{\frac{f_2}{o(\eta)}} - S_o \left( \frac{U_c}{U_jo} \right)^2 (f_2 + \gamma g_2)^2 \]

Following the analysis in Section A1.5, the second integral is evaluated as follows:

\[ \rho vy = -\int_0^1 \frac{\eta}{R} \left[ \frac{p}{R_T o} + \frac{U_c (f_2 + \gamma g_2)}{D} \right] \, d\eta \]

\[ \rho vy = -\int_0^1 \left( \frac{R}{R_o R_T o} \right)^2 \left\{ \frac{p' U_c (f_2 + \gamma g_2)}{D} + \frac{pU_c (f_2 + \gamma g_2)}{D} \right\} \, d\eta \]

\[ + \frac{pU_c}{D} \left[ - \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta \left( \frac{R'}{R_o} \right) + g_2 \gamma' \right] \]

\[ - \frac{pU_c (f_2 + \gamma g_2)}{D^2} \left\{ \frac{\partial D}{\partial \eta} + U_c \frac{\partial D}{\partial U_c} + \gamma' \frac{\partial D}{\partial \gamma} - \eta \left( \frac{R'}{R_o} \right) \frac{\partial D}{\partial \eta} \right\} \, d\eta \]

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\[-\frac{\rho v y}{R T \infty U_c \frac{R^2}{R_0}}\]
\[= \frac{U_c}{U_c} \int_0^\eta \left[ \frac{(f_2 + \gamma g_2)}{D} \right. + \left. \frac{(f_2 + \gamma g_2)}{D^2} \frac{\partial D}{\partial \gamma} \right] \eta d\eta \]
\[+ \gamma' \int_0^\eta \left[ \frac{g_2}{D} \right. - \left. \frac{(f_2 + \gamma g_2)}{D^2} \frac{\partial D}{\partial \gamma} \right] \eta d\eta \]
\[+ \frac{p'}{p} \int_0^\eta \frac{(f_2 + \gamma g_2)}{D} \eta d\eta \]
\[+ \frac{U'}{U} \int_0^\eta \left[ \frac{(f_2 + \gamma g_2)}{D^2} \right. \frac{\partial \gamma}{\partial \gamma} \left. + \gamma \left( \frac{f_2 + \gamma g_2}{D} \right) \right] \eta d\eta \]
\[+ \frac{R'}{R} \int_0^\eta \left[ \frac{(f_2 + \gamma g_2)}{D} \right. \frac{\partial \gamma}{\partial \gamma} \left. + \gamma \left( \frac{f_2 + \gamma g_2}{D} \right) \right] \eta d\eta \]

in which
\[V_1 = \int_0^\eta \frac{(f_2 + \gamma g_2)}{D} \eta d\eta \]
\[V_2 = \int_0^\eta \frac{(f_2 + \gamma g_2)}{D^2} \frac{\partial D}{\partial \gamma} \eta d\eta \]
\[V_5 = 0 \]
\[V_7 = \int_0^\eta \frac{g_2}{D} \eta d\eta \]
\[V_9 = \int_0^\eta \frac{(f_2 + \gamma g_2)}{D^2} \frac{\partial D}{\partial \gamma} \eta d\eta \]
\[V_{10} = \int_0^\eta \frac{(f_2 + \gamma g_2)}{D} \frac{\partial \gamma}{\partial \gamma} \eta^2 d\eta \]
With these definitions, the integral

\[
\int_0^R \rho v y \frac{\partial u}{\partial y} \, y \, dy
\]

may be evaluated as follows:

\[
\frac{\rho v y}{\frac{\partial u}{\partial y}} \, y \, dy = \int_0^R \left[ \frac{\rho v y}{\frac{\partial u}{\partial y}} \right] \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \, \eta \, d\eta
\]

\[
= \frac{U' c}{U c} \left[ - \int_0^1 V_1 \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta \, d\eta - \frac{U c}{U_0} \int_0^1 V_2 \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta \, d\eta \right]
\]

\[
+ \gamma' \left[ - \int_0^1 (V_7 + V_9) \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta \, d\eta \right] + \frac{p'}{p} \left[ - \int_0^1 V_1 \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta \, d\eta \right]
\]

\[
+ \mathcal{T}' \left[ - \int_0^1 (V_5 + V_6) \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta \, d\eta \right] - \int_0^1 \left( V_{10} + V_{11} \right) \left( \frac{R_1'}{R_0} + \mathcal{T}' \frac{R_{56}}{R_0} \right) \eta \, d\eta
\]

\[
= \frac{U' c}{U c} \left[ R_1 + \frac{U c}{U_0} R_2 \right] + \gamma' \left[ R_{79} \right] + \frac{p'}{p} R_1 + \mathcal{T}' \frac{R_{56}}{R_0} + \left( \frac{R_1'}{R_0} \right) (R_{10} + R_{11})
\]
in which

\[ R_1 = - \int_0^1 V_1 \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta \quad \text{and} \quad R_{79} = - \int_0^1 (V_7 + V_9) \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta \]

\[ R_2 = - \int_0^1 V_2 \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta \quad \text{and} \quad R_{10} = \int_0^1 V_{10} \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta \]

\[ R_{56} = - \int_0^1 (V_5 + V_6) \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta \quad \text{and} \quad R_{11} = \int_0^1 V_{11} \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta \]

The pressure gradient term may be written as follows:

\[ \int_0^R \frac{dp}{dx} y^2 dy = \frac{p}{R g T} \frac{U_c^2 R^3}{R_o} \left[ \frac{p'}{p} \frac{1}{3} \frac{BP}{U_c^2} \frac{R}{U_j} \right] \]

The shear stress term may be evaluated as follows:

\[ \int_0^R \frac{\partial (\tau y)}{\partial y} dy = \tau y^2 \]

\[ \tau = \rho \varepsilon \frac{\partial u}{\partial y} = \rho \varepsilon \frac{U_c}{R} \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \text{ where } \varepsilon \text{ is the eddy kinematic viscosity} \]

\[ \int_0^R \frac{\partial (\tau y)}{\partial y} dy = - \frac{1}{2} C_f d \frac{R_o}{R} \frac{U_c^2 R^3}{R_o} \left[ \frac{1}{U_c^2} \right] \frac{R g T}{R_o} \frac{R_o}{p} \eta d\eta \]
\[ = -(1/2)C_{fd} \frac{R_o}{R} - \frac{R_o}{R} \int_0^1 \frac{\epsilon}{U_c R} \frac{1}{D} \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta \]

\[ = \frac{R_o}{R} A_\tau - \frac{R_o}{R} \frac{C_{fd}}{2} \]

where \( A_\tau = - \int_0^1 E_2 \left( \frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta \)

\[ E_2 = \frac{\epsilon}{U_c R} \text{ in Part 2} \]

Assembling all the terms, the final moment-of-momentum integral equation is as follows:

\[
\frac{U_c c'}{U_c} \left[ Q_1 + R_1 + \frac{U_c}{U_{Jo}} R_2 \right] + \gamma' \left[ Q_4 + R_{79} \right] + \frac{p'}{p} \left[ R_1 + \frac{1}{3} \left( \frac{Bp}{U_c^2} \right) \left( \frac{U_c}{U_{Jo}} \right) \right] + \mathcal{T}' R_{56}
\]

\[ = - \left( \frac{R'}{R} \right) \left[ Q_3 + R_{10} + R_{11} \right] + \frac{R_o}{R} A_\tau - \frac{C_{fd}}{2} \frac{R_o}{R} \]

The values of the coefficients follow:

\[ W(4, 1) = Q_1 + R_1 + \frac{U_c}{U_{Jo}} R_2 \quad W(4, 5) = R_{56} \]

\[ W(4, 2) = 0 \quad W(4, 6) = 0 \]
\[ W(4,3) = Q_4 + R_79 \]
\[ V(4) = -\frac{R'}{R_0} \left[ Q_3 + R_{10} + R_{11} \right] + \frac{R_0}{R} A_T - \frac{C_{fd}}{2} \frac{R_0}{R} \]

\[ W(4,4) = R_1 + \frac{1}{3} \frac{BP}{(\frac{U_c}{U_{jo}})^2} \]

A2.5 Equation for \( J = 5 \); Centerline Velocity - Temperature Relationship

The experimental measurements made during this investigation have shown, as in figure 10, that for any value of \( \frac{x}{R_0} \) in part 2,

\[ \frac{T_j}{T_{jo}} \approx \frac{U_c}{U_{jo}} \times \text{const} \quad \text{Note } U_c = U_j \text{ in Part 2 Because } U_0 = 0 \text{ is assumed} \]

\[ \frac{1}{T_j} \frac{\partial (T_j)}{T_{jo}} = \frac{1}{U_c} \frac{\partial (U_c)}{U_{jo}} \]

now

\[ \frac{T_j}{T_{jo}} = \mathbf{U} \frac{T_{ooi}}{T_{jo}} \frac{T_{oo}}{T_{ooi}} \]

so

\[ \left( \frac{T_j'}{T_{jo}} \right) = \frac{T_{ooi}}{T_{jo}} \left[ \mathbf{U}' \frac{T_{oo}}{T_{ooi}} + \mathbf{U} \left( \frac{T_{oo}}{T_{ooi}} \right)' \right] \]
\[
\left( \frac{T_i'}{T_{ij}} \right) = \frac{T_{oo}}{T_{ooi}} + \frac{T_{oo}}{T_{ooi}} \left( \frac{T_{oo}}{T_{ooi}} \right)'
\]

finally,
\[
\frac{U'}{U} + \frac{T_{oo}}{T_{ooi}} = \left( \frac{U_c}{U_{jo}} \right)'
\]

The values of the coefficients follow:

\[
W(5, 1) = \frac{1}{\left( \frac{U_c}{U_{jo}} \right)} \quad W(5, 5) = -\frac{1}{U}
\]

\[
W(5, 2) = 0 \quad W(5, 6) = -\frac{1}{T_{oo}} \left( \frac{T_{oo}}{T_{ooi}} \right)
\]

\[
W(5, 3) = 0 \quad V(5) = 0
\]

\[
W(5, 4) = 0
\]

A2.6 Equation for \( J = 6; Wall\ Velocity = 0 \)

The value of \( U_o \) is assumed to be zero throughout Part 2. Therefore,
\[
\frac{U_o'}{U_c} = 0
\]

and \( W(6, 1) = W(6, 3) = W(6, 4) = W(6, 5) = W(6, 6) = V(6) = 0 \)

\[
W(6, 2) = 1
\]
### Table A1

Values Required to Determine Z Parameters in Part 1

<table>
<thead>
<tr>
<th></th>
<th>( J = 2 )</th>
<th>( J = 3 )</th>
<th>( J = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Momentum</td>
<td>Continuity</td>
<td>Energy</td>
</tr>
<tr>
<td>( N_{ij} )</td>
<td>((\lambda + f_{oi})^2)</td>
<td>(\lambda + f_{oi})</td>
<td>((\lambda + f_{oi})(1 + \mathcal{T} f_{oi})^{1/2})</td>
</tr>
<tr>
<td>( \frac{\partial N_{ij}}{\partial \lambda} )</td>
<td>(2 (\lambda + f_{oi}))</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{\partial N_{ij}}{\partial \mathcal{T}} )</td>
<td>0</td>
<td>0</td>
<td>((\lambda + f_{oi}) f_{oi})^{1/2}</td>
</tr>
<tr>
<td>( D_{ij} )</td>
<td>(1 + \mathcal{T} f_{oi}^{1/2} - S_o \frac{U_j^2}{U_{jo}} (\lambda + f_{oi})^{2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{\partial D_{ij}}{U_j} )</td>
<td>(-2 S_o \frac{U_j}{U_{jo}} (\lambda + f_{oi})^{2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{\partial D_{ij}}{U_{jo}} )</td>
<td></td>
<td>(-2 S_o \frac{U_j^2}{U_{jo}} (\lambda + f_{oi}))</td>
<td></td>
</tr>
<tr>
<td>( \frac{\partial D_{ij}}{\partial \lambda} )</td>
<td>(-2 S_o \frac{U_j^2}{U_{jo}} (\lambda + f_{oi}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{\partial D_{ij}}{\partial \mathcal{T}} )</td>
<td>(\frac{1}{2} f_{oi})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Table A2
Values Required to Determine Z Parameters in Part 2

<table>
<thead>
<tr>
<th></th>
<th>J = 1</th>
<th>J = 2</th>
<th>J = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Continuity</td>
<td>Energy</td>
<td>Momentum</td>
</tr>
<tr>
<td>(N_{ij})</td>
<td>((f_{2i} + \gamma g_{2i}))</td>
<td>((f_{2i} + \gamma g_{2i})(1 \cdot T f_{oi}^{1/2}))</td>
<td>((f_{2i} + \gamma g_{2i})^2)</td>
</tr>
<tr>
<td>(\frac{\partial N_{ij}}{\partial \gamma})</td>
<td>(g_{2i})</td>
<td>(g_{2i}(1 + T f_{oi}^{1/2}))</td>
<td>(2g_{2i}(f_{2i} + \gamma g_{2i}))</td>
</tr>
<tr>
<td>(D_{ij})</td>
<td>(1 \cdot T \sqrt{f_{oi}(\gamma)} - S_o \left[ \frac{U_c}{U_{jo}} \right]^2 \left[ f_{2i}(\gamma) + \gamma g_{2i}(\gamma) \right]^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{\partial D_{ij}}{\partial \frac{U_c}{U_{jo}}})</td>
<td>(-2 \frac{U_c}{U_{jo}} S_o \left[ f_{2i} + \gamma g_{2i} \right]^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{\partial D_{ij}}{\partial f_{oi}})</td>
<td>(f_{oi}^{1/2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{\partial D_{ij}}{\partial \gamma})</td>
<td>(-2S_o \left[ \frac{U_c}{U_{jo}} \right]^2 \left( f_{2i} + \gamma g_{2i} \right) g_{2i} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{\partial D_{ij}}{\partial \left( \frac{T_{oo}}{T_{ooi}} \right)})</td>
<td>(- \left( \frac{U_c}{U_{jo}} \right)^2 \left( f_{2i} + \gamma g_{2i} \right)^2 \frac{S_o}{T_{oo}^{3/2}} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix B

THE COMPUTER PROGRAM

B.1 General Description

The computer program has 10 sections. The general functions of each section are described below.

MAIN: The program begins and ends in MAIN. Input data concerning the jet pump geometry, inlet gas flow properties, free jet velocity profile, and stations along the mixing tube where output values are desired, are all read in by MAIN and by two subroutines called by MAIN--DIFFEQ and SUB. MAIN converts the units of the input parameters into other units which are more convenient for subsequent analysis.

After conversion of units, MAIN computes the primary and secondary flow conditions at the top-hat section as described in Section 3.3 of this report. Then MAIN sets up the initial trial values for the velocity profile after transition and calls VBO4A to perform the iterations required to obtain an accurate solution for the profile.

After the transition zone analysis has been completed, MAIN sets up the initial conditions for the flow analysis upstream of the point of jet attachment to the wall. It also defines the stations along the mixing tube for which data will be printed out. MAIN then calls RUNGE to carry out the solution for the remainder of the flow analysis.

SUB: The first section of SUB, called when \( J = 3 \), reads in data on the mixing tube geometry--inner diameter vs. length. The diameters are converted to radii and all radii and length values are made non-dimensional by dividing by \( R_0 \). The second section of SUB, called when \( J = 1 \) or \( 2 \), finds the duct radius and slope at any axial position \( x \) specified as an input value to the subroutine.
The procedure used is linear interpolation between the nearest upstream and downstream radii which were read as input data by the first section of SUB.

**CALXFG:** The purpose of CALXFG is to perform the computations required to set up the three transition zone equations (27), (33), and (35) for solution by VBO4A. The three equations and derivatives of each of the three equations with respect to the three variables \( U_r, \lambda, \) and \( \delta/R_{eff} \) are computed in CALXFG.

**VBO4A, VDO2A, and SPNIST:** These subroutines are library routines employed to solve the three simultaneous non-linear algebraic equations (27), (33), and (35). A two-page discussion of these subroutines is included at the end of section B.3.

**DIFFEQ:** The DIFFEQ subroutine is divided into two parts. Part I establishes the 7 simultaneous equations (39) which must be solved to determine the flow conditions upstream of jet attachment. The equations used are outlined in section 3.4 and detailed in appendix A.1. When the simultaneous equations are set up, DIFFEQ calls subroutine SIMQ to solve the equations for the values of the 7 derivatives in equation (38). Then subroutine RUNGE is called to integrate the derivatives using Runge-Kutta-Merson techniques. This integration yields the values of \( U_j, U_o, \delta, p, T_{oj}, \Theta, \) and \( H \) at stations closely spaced along the duct.

Part 2 of DIFFEQ establishes the 6 simultaneous equations (48) which must be solved to determine the flow conditions in the mixing tube downstream of jet attachment to the wall. The equations used are outlined in section 3.5 and detailed in appendix A.2. Subroutine SIMQ is called to solve for the 6 derivatives in equation (47). Then subroutine RUNGE integrates the derivatives to find the values of \( U_c, \gamma, p, T_{oj}, \) and \( T_{oo} \) at stations closely spaced along the duct.

**SIMQ:** This is a library subroutine which is called by DIFFEQ to solve simultaneous linear equations to find the values of the \( Y(K) \) derivatives in equations (38) and (47).
**RUNGE:** The RUNGE subroutine performs a Runge-Kutta-Merson integration procedure to integrate the derivatives of the \( Y(K) \) quantities which are developed by DIFFEQ and SIMQ as described above. RUNGE also calls the subroutine PRINT to print the desired output values of jet pump flow parameters at each mixing tube station \( (XOUT) \) which has been specified by input data and equations in MAIN.

**PRINT:** This subroutine contains instructions for printing the computer jet pump flow parameters at selected stations along the mixing tube downstream of the transition zone.

### B.2 Input Data Format

The input data to the program must be prepared according to the following sequence:

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Parameters</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NS</td>
<td>3I2</td>
</tr>
<tr>
<td>2</td>
<td>( \text{GG}(I), I = 1, NS )</td>
<td>10F5.4</td>
</tr>
<tr>
<td>3</td>
<td>SDLOSS, ASD</td>
<td>2F10.3</td>
</tr>
<tr>
<td>4</td>
<td>( \text{THETA}, SHAPE, VISC, RZERO )</td>
<td>4F10.6</td>
</tr>
<tr>
<td>5</td>
<td>1-CARD MESSAGE identifying solution desired</td>
<td>80H</td>
</tr>
<tr>
<td>6</td>
<td>DELTAX, XTUBE, TURBNO, NSUB, NGAM, XCORE, ANOZ</td>
<td>3F10.4, 2I5, 2F10.5</td>
</tr>
<tr>
<td>7</td>
<td>POO, TOO, PO1, T01, AMASS1, AMASSO, AG, RG</td>
<td>8F7.3</td>
</tr>
<tr>
<td>8 to ( 8 + I )</td>
<td>( X(I), A(I) ) } omit if ( \text{NSUB} = 2 )</td>
<td>2F15.4</td>
</tr>
<tr>
<td>( 8 + I + 1 )</td>
<td>0.0</td>
<td>2F15.4</td>
</tr>
</tbody>
</table>

Cards 1 through 7 are required for each solution desired. The cards from 8 on are required to define a new mixing tube geometry for analysis. If the same mixing tube geometry is to be used for additional solutions with altered flow conditions, the cards from 8 on do not have to be included for these additional solutions. The input parameter \( \text{NSUB} \) tells the computer whether the cards from 8 on are included with a data set, i.e., whether the same mixing tube geometry is to be used for additional solutions.
The input parameters are described below.

NS  number (= 10) of equal-radial-increment strips used to approximate the jet mass flow, momentum, and energy integrals across the jet
GG(I) average values of \( \frac{U}{U_0}(\eta) \) for \( I = NS \) equal-radial-increment strips for the turbulent pipe flow velocity profile
SDLOSS suction duct loss coefficient; \( K_L \) in equation (55)
ASD suction duct area, \( ft^2 \); \( A_{SD} \) in equation (55)
THETA boundary layer momentum thickness at \( x = 0 \), ft
SHAPE boundary layer shape factor at \( x = 0 \)
VISC gas kinematic viscosity for secondary flow at inlet, \( ft^2/sec \)
RZERO mixing tube radius at nozzle exit section; \( x = 0 \), ft
DELTAX steps of \( x/R_o \) at which data printouts are desired in the mixing tube
XTUBE mixing tube length, ft
TURBNO turbulent Reynolds number value = 147
NSUB control instruction: if 1, a new mixing tube geometry is read in
if 2, the tube geometry from the previous solution will be used.
NGAM control instruction: if 0, incompressible flow solution (not operable)
if 1, compressible flow solution
XCORE length of the transition zone divided by \( R_o \)
ANOZ primary nozzle exit flow area, \( ft^2 \)
POO stagnation pressure upstream of the suction duct losses, psia
TOO stagnation temperature of the secondary flow, \( °R \)
PO1 stagnation pressure of the primary flow, psia
TO1 stagnation temperature of the primary flow, \( °R \)
AMASS1  mass flow rate of the primary flow, lbm/sec
AMASSO  mass flow rate of the secondary flow, lbm/sec
AG      specific heat ratio of the gas
RG      gas constant, ft-lbf/lbm-°R
X(I)    x stations along the mixing tube at which A(I) values are defined, ft
A(I)    diameter of the mixing tube at the corresponding x station, ft

B.3 Output Data

A complete sample of output data from the computer program is given in section B.5 of this appendix. The first section of the output repeats the input data and thus requires no comment. The remainder of the data is summarized below.

F (I)’s: values of \( f_0 (\eta)\) at \( \eta = 0.05, 0.15, 0.25, \ldots, 0.95 \)

CONDITIONS AT BEGINNING OF THE TRANSITION SECTION

Lists values of \( U_{oo}, \rho_{oo}, U_{joo}, T_{oo}, p_{oo} \) (psfa and in H₂O), \( \lambda_{oo} = \frac{U_{oo}}{U_{joo}} \), and primary jet momentum = \( W_1 U_1 \) where \( U_1 \) is the velocity achieved by isentropic expansion of the primary flow to the static pressure at the end of the accommodation process, \( p_{oo} \).
The next portion of the printout monitors the solution by VB04A of equations (27), (33), and (35) for the transition zone. Each iteration employing CALXFG is recorded. The VARIABLES are the values of $U_r$, $\lambda$, and $\delta/R_{eff}$ determined during the particular iteration reported. The FUNCTIONS are the values of the functions:

\[
\frac{C_{\text{mass\_new}} - C_{\text{mass\_old}}}{C_{\text{mass\_old}}}, \quad \frac{C_{\text{mom\_new}} - C_{\text{mom\_old}}}{C_{\text{mom\_old}}}, \quad \frac{P_{\text{const\_new}} - P_{\text{const\_old}}}{P_{\text{const\_old}}}
\]

where $C_{\text{mass}}$ is defined by equation (27),
$C_{\text{mom}}$ is defined by equation (32),
$P_{\text{const}} = \left( \frac{P_{o1}}{P_{oo}} \right)^{\frac{k-1}{k}}$,

($)$\text{new} is the value for the current iteration.
($)$\text{old} is the value for the previous iteration.

If these functions are computed to within ERR times the "old" value of $C_{\text{mass}}$, $C_{\text{mom}}$, or $P_{\text{const}}$ VB04A is judged to have converged satisfactorily. In the present program, ERR is set at $10^{-6}$, an excessively tight tolerance. As a result, the message "VB04A ACCURACY CANNOT BE ACHIEVED" is often printed out. Following this message, the values of the VARIABLES and FUNCTIONS for the current iteration are printed. These values are used as the first values for subsequent calculations.

Four lines of print follow the end of the VB04A material. The first line restates the values of $XX(1) = U_r$, $XX(2) = \lambda$, and $XX(3) = \delta/R_{eff}$ in numerical form. The second line compares the values of EN, a dimensionless jet pump parameter developed in reference 2, before and after transition (EN vs. EN2).

\[
EN = \frac{W_1 + W_o}{\sqrt{2\pi R_o^2 \rho_{oo} \left[ g_o(p - P_{oo})^{\pi R_{eff}} + W_1 U_1 + W_o U_o \right]}}
\]
The two values should be identical; differences which exist provide a measure of the accuracy of the transition analysis. Following the EN values, the values of $S_O$ and $B_p$ (see appendix A, section A1.1) are printed. Then the final values of $U_j$ and $U_c$ are given.

The printing continues with a tabulation of values along the mixing tube given by Part 1 of the analysis. The parameters listed are as follows:

- **$X/RZERO$**: values of $\frac{x}{R_0}$ beginning with $\frac{x_{core}}{R_0}$
- **AREA**: local value of $\frac{\pi R^2_{tube}}{\pi R_0^2}$
- **PH20**: wall static pressure, in $H_2O$ relative to $P_{oo}$
- **$U_0$**: value of $U_{o}$, secondary flow velocity
- **UCENT**: value of $U_{c}$, velocity of flow at the centerline
- **UR**: value of $U_c/U_{co}$
- **LAMBDA**: value of $\lambda = \frac{U_o}{U_j}$
- **DELTA/R**: value of $\frac{\delta}{R_0}$
- **TOCENT**: value of stagnation temperature at the duct centerline
- **TOWALL**: value of stagnation temperature in the secondary flow outside the mixing region
- **THETA/RO**: value of $\frac{\theta}{R_0}$
- **SHAPE**: value of $H$

When the Part 1 analysis indicates that the jet reaches the wall, the message "DELTA/R = 1 -- DIFFERENTIAL EQUATIONS CHANGE" is printed. The local value of $U_j$, called CL, is also printed. Next, two lines are printed as follows:
F2(I)'s: values of \( f_2(\eta) \) at \( \eta = 0.05, 0.15, 0.25, \ldots, 0.95 \)

G2(I)'s: values of \( g_2(\eta) \) at \( \eta = 0.05, 0.15, 0.25, \ldots, 0.95 \)

The printing concludes with the tabulated results obtained from the Part 2 analysis. The parameters listed are as follows:

X/RZERO, AREA, PH20, UCENT, UR, TOCENT, TOWALL; same as in Part 1

TOWALL/TOO; stagnation temperature of wall streamline divided by secondary flow inlet stagnation temperature, \( T_{oo} \)

AUGMENT: the value of the local momentum flux, \( \int_0^R \frac{\rho u^2}{g_o} \ 2\pi y \ dy \), divided by the primary jet momentum, \( W_1U_1 \), which is printed out earlier.

GAMMA; local value of \( \gamma \)
B. 3 Listing

C*******************************************************************************
C DODE: ANALYSIS OF FLOW BEHAVIOR IN AXISYMMETRIC COMPRSIBLE FLOW
C EJECTORS WITH VARIABLE AREA MIXING TUBES
C*******************************************************************************

COMMON TURBO, CF, EN, NPRT, C, JRUNGE, NGAM
1, POO, TOO, T01, AMACH, AG, RG
2, AMASS1, AMASS0, T
3, S00, CMAS, CMAS, CFNR
4, S0, RP, CM, VISC
COMMON ZA, ZR, UU, DDU, DDIR
5, MASAT, RZERO, AFAC, CFD, HD, F, A1, AZERO, TF, FP, GP
7, SDDLSS, ASD
8, AUGL, ULLCEN

DIMENSION Y(10), YL(10), YMIN(10), XOUT(201), MARK(5)

1, D(501), F(10)

DIMENSION FF(3), XX(3), ERR(3), AA(3, 10), WORK(10)
1, A1(10), F(10), TF(10)
5, F(101), G(101)

REAL HASPAT

C JET MIXING WITH PRIMARY- AND SECONDARY-STREAMS OF THE SAME PERFECT GAS

C AND CORRECT NOZZLE EXPANSION

C READ DATA

10 CONTINUE

CALL DIFFEQ (I, X, Y, DY)

READ5, 18, (EM=200) THETA, SHAPE, VISC, RZERO

THETA = INLET ALL MOMENTUM THICKNESS, FT

SHAPE = INLET ALL SHAPE FACTOR

VISC = KINEMATIC VISCOSITY, FT**2/SEC

RZERO = INLET DUCT RADIUS, FT

18 FORMAT (4F10.6)

READ5, 19

ONE CARD TO IDENTIFY THE SOLUTION

19 FORMAT (80H)

C DUCT GEOMETRY AT INLET

A = 3, 1416*RZERO*RZERO

APPROXIMATE NOZZLE AREA FT**2

READ5, (15) DELTAX, XTUBE, TURBO, NSUB, NGAM, XCORE, ANG

NGAM = 0 SUPRESS GAMMA = 1 DO NOT

C NSUB = 1 READ NEW MIXING TUBE PROFILE DATA = 2 DO NOT

15 FORMAT (3F10.4, 2F10.4)

READ5, 16, (POO, TO0, PO1, TO1, AMASS1, AMASS0, AG, RG)

C PO = PSIA

C TO = DEGR

C AMASS = LBM/SEC

C RG = FT-LBF/LBM-DEGR

16 FORMAT (8F7.3)

WRITE(6, 19)

WRITE6, 17, DELTAX, SHAPE, RZERO, XTUBE, NSUB, THETA, TURNO, NGAM, VISC,

AAG, RG, PO0, TO0, PO1, TO1, AMASS0, AMASS1, ANG, XCORE

17 FORMAT (/, 5X, THDDELTA=, F10.4, 3X, 6SHAPF=, F10.4, 6X, 6HRZER=, F10.4, 6,

12HTF, /, 5X, 6HTF=, F10.4, 4X, 5HNSUB=, 15X, 6HTHETA=, F10.4, 6X, 4HTF, /,

25X, 7THBNO=, F10.4, 4X, 5HMAS=, 15X, 5HVIS=, F10.4, 4HF*2/SEC, /,

39X, 5HAM=, F10.4, 26X, 5HMG=, F10.4, 15HFT-LBF/LBM-DEGR, /,

48X, 4HP00=, F10.4, 4HPSIA, /, 4X, 4HT00=, F10.4, 4HDEGR, /,

5AX, 4HP01=, F10.4, 4HPSIA, /, 4X, 4HT01=, F10.4, 4HDEGR, /,

65X, 7HAMASS0=, F10.4, 7HLSB/SEC, /, 5X, 7HAMASS1=, F10.4, 7HLSB/SEC, /

85
C
C       NS = 10
C
C       JF0LY = 5
C
C       CALCULATION OF F(I) FROM POLYNOMIAL
C
C       AAZERO = 1.04
C       AI(1) = -0.0175
C       AI(2) = -8.321
C       AI(3) = 16.596
C       AI(4) = -12.7877
C       AI(5) = 3.6058
C
C       CTR = .05
C
C       DC B37 I=1, NS
C       CTR = CTR + 0.1
C
C       F(I) = AI(JPOLY)
C       FP(I) = FLOAT(JPOLY)*F(I)
C
DO 808 J = 2, JF0LY
C       JJ = JF0LY - J + 1
C       F(I) = F(I)*CTR + AI(JJ)
C       FP(I) = FP(I)*CTR + FLOAT(JJ)*AI(JJ)
C
808 CONTINUE
C
C       F(I) = F(I)*CTR + AAZERO
C       F(I) = SORT(F(I))
C
807 CONTINUE
C
C       WRITE (6, 403) (F(I), I =1, NS)
C
403  FORMAT (5X,6HF(I) ','S, 5X, 10F10.4)
C
A = 3.1416*RZERO*RZERO
C
C       PO1 = P01 * 144.
C       PO0 = POO*144.
C       RG = RG*32.2
C       AMASS1=AMASS1/32.2
C       AMASS0=AMASS0/32.2
C
C       XYZ = 0.0
C
C       GC TO (10, 11), NSUB
C
C       CALL SUR(XYZ, R, DR, 3, RZERO)
C
C       JRUNGE = 4
C       NPART = 1
C
C
C       TO DETERMINE TOP HAT PARAMETERS
C
C       TO DETERMINE EFFECTIVE DUCT AREA AT END OF CORE
C
C       CALL SUR(XCORE, R, DR, 2, RZERO)
C       THETA = THETA + XCORE*RZERO
C       RCORE = R*RZERO - THETA*SHAPE
C       ACORE = 3.1416*PCORE*RCORE
C       AFFF = ACORE - ANOZ
C       AM = ACORE
C       APAC = 1.
C       AG = AC/(AC-1.)
C       RH00 = PO0/(RG*TOO)
C       CALL SUR(XCORE, R, DR, 2, RZERO)
C       PO0 = PO0 - SDLOSS*AMASS0*2.*12.*RHC0*ASC**2.1
\[ RHO_0 = \frac{P_0}{(RG*TO_0)} \]

**FIRST APPROXIMATION FOR OUTER VELOCITY**
\[ U_0 = \frac{AMASSO}{(RHO*AEFF)} \]
\[ TO_0 = 100 - (AG-1)*5*U_0 + U_0/(AG*RG) \]
\[ P = P_0/(100/TO_0) \]

**SECOND APPROXIMATION FOR OUTER VELOCITY**
\[ U_0 = \frac{AMASSO}{(RHO*AEFF)} \]
\[ TO = 100 - (AG-1)*5*U_0 + U_0/(AG*RG) \]
\[ P = P_0/(100/TO_0) \]

**ASSUME PRIMARY STREAM EXPANDS TO OUTER STREAM PRESSURE**
\[ T_1 = T_01*(P/PC1)**(1/(AG1)) \]
\[ RHOC = P/(RG*T_01) \]
\[ U_1 = \text{SORT}(2, \text{AGG}*RG*(T_01-T_1)) \]

**FIRST GUESS OF PARAMETERS AFTER CONSTANT AREA TRANSITION**
\[ U_{J00} = U_1 - U_0 \]
\[ T = (T_01 - T_00)/T_00 \]
\[ DOO = 0.5*(AG1-1)*U_{J00} + U_0/(AG*RG) \]
\[ CMOM = (P-P_0) + AMASS1*U_1 + AMASSO*U_0 \]
\[ CM = CMOM/A^2 \]
\[ CENT = AMASS1*T01 + AMASSO*T00 \]
\[ EN = CMAS/SORT(2, CMOM*RHOC) \]
\[ W_2 = AMASSO*U_0 + U_0 \]
\[ CMAS = CMAS/W_2 \]
\[ CMOD = CMOD/W_2*U_0 \]
\[ CMOD = CMOD/W_2*U_0 \]

**FIRST GUESS OF PARAMETERS AFTER CONSTANT AREA TRANSITION**
\[ \text{UR} = U_1/U_{J00} \]
\[ Y2 = \text{LAMBDA} \]
\[ DCR = \text{DELTA}_R \]
\[ \text{UR} = 1 \]
\[ Y2 = 110/U_{J00} \]
\[ PH20 = -(P_0-P)*1.93 \]
\[ THOM = AMASS1*U_1 \]
\[ RHOC = RHOC*32.2 \]

**FORMATS**
\[ XX = \text{SORT}(3, \text{AGG}*RG*(100/TO_1)) \]

**DO 121 J = 1, 3**
\[ \text{ERR}(J) = 1, E-6*XX(J) \]
\[ IP = 1 \]
\[ MAX = 120 \]
\[ \text{CALL}_V804A(3, 3, XX, ERR, AK, 300000, 1, P, MAX, 3, WORK) \]

**CHECK CALCULATION OF EN AFTER TRANSITION**
\[ CMA_2 = (F(1) + CMA_1)W_2 \]

\[ CMO_2 = (F(2) + CMO_1)W_2 + U_0 \]

\[ EN_2 = CMA_2/\text{SORT}(2, \cdot \cdot \cdot )CMO_2 \cdot A \cdot RHOM \cdot \cdot \cdot \cdot \cdot \]

\[ \text{WRITE}(6, 69)XX(1), XX(2), XX(3), EN, FN_2 \]

\[ \text{WRITE}(6, 69)FR, 10X, 3HXX(2), F15.10, 3X, 6HXX(3), = F15.10, 10X, 3HEN, = F10.4, 10X, 4HEN2, = F10.4 \]

\[ C = CM \times XX(1) \]

\[ SO = 500 \times XX(1) \times XX(1) \]

\[ BP = (AG - 1) / (2 \cdot AG \times SO) \]

\[ UJO = XX(1) \times UJO_0 \]

\[ UCENT = UJO_0 \times 1 + XX(2) \]

\[ \text{WRITE}(6, 69)SC, BP, UJO, UCENT \]

\[ \text{WRITE}(6, 69)SC, F10.4, \cdot \cdot \cdot \]

\[ C = \text{SOFTWARE} \cdot XX(1) \times XX(1) \]

\[ Y(1) = 1.0 \]

\[ Y(2) = XX(2) \]

\[ Y(3) = XX(3) \times R \]

\[ Y(4) = Y(2) \times Y(2) \times SO \times (AG - 1) \times P02 / P00 \]

\[ Y(5) = (TO1 - TO0) / TO0 \]

\[ Y(6) = 1.0 \]

\[ Y(7) = \text{THETA} / RZERO \]

\[ Y(8) = \text{SHAPE} \]

\[ D = 4.08 \times Y(3) \times (1.0 + Y(2)) \]

\[ D = \text{SCORE} \]

\[ XOUT(1) = D \]

\[ XTR = XOUT / RZERO \]

\[ M = 2 \times XTR \]

\[ DC \cdot 5 - 2 \cdot M \]

\[ XOUT(1) = XOUT(1) + \text{DELTAX} \]

\[ IF(XOUT(1) = GX \times XTR) \text{GO TO 300} \]

\[ \text{CONTINUE} \]

\[ M = I - 1 \]

\[ DC \cdot 8 \cdot K = 1.8 \]

\[ YMIN(K) = 0.01 \]

\[ TOL(K) = 0.00001 \]

\[ TOL(K) = 0.0001 \]

\[ \text{CONTINUE} \]

\[ YMIN(5) = 1.0 \]

\[ YMIN(6) = 1.0 \]

\[ YMIN(7) = 1.0 \]

\[ YMIN(8) = 10.0 \]

\[ \text{MARK}(1) = 1 \]

\[ \text{MARK}(2) = M \]

\[ \text{MARK}(4) = 0 \]

\[ H = \text{CALL RUNGE}(8, D, Y, TOL, YMIN, H, XOUT, \text{MARK}) \]

\[ \text{GO TO 100} \]

\[ \text{STOP} \]

\[ \text{END} \]

\[ \text{SUBROUTINE CIFFO}(N, X, Y, DY) \]

\[ \text{COMMON TURNO, CF, EN, NPAR, T, JRUNGE, NSAM} \]

\[ 1, P00, P01, TO0, TO1, \text{MACCH, AG, RG} \]
C
DIVNS = 2./FLOAT(NS)
DIVDEL = FLOAT(NS)
RETURN
C
Y1 = UJ/UJZFC
ALIT 55)
C
Y2 = LAMDA = UD/UJ
ALIT 57)
C
Y4 = P/PQINITIAL
ALIT 59)
C
Y5 = (TOCENTER--TOO)/100
ALIT 60)
C
Y6 = TOO/TOCINITIAL
ALIT 61)
C
Y7 = B. L. MOMENTUM THICKNESS/ZERO
C
Y8 = B. L. SHAPE FACTOR
C
CONTINUE
ALIT 620)
C
IF (LM) 913,913,914
ALIT 64)
913
CONTINUE
ALIT 64)
914
L = M = 1
ALIT 64)
Y1 = Y(1)
Y2 = Y(2)
Y3 = Y(3)
Y4 = Y(4)
Y5 = Y(5)
Y6 = Y(6)
Y7 = Y(7)
Y8 = Y(8)

Y11 = Y(1) * Y(1)
Y22 = Y(2) * Y(2)
Y33 = Y(3) * Y(3)

DIVY1 = 1. / Y1
DIVY11 = DIVY1 * DIVY1
DIVY4 = 1. / Y4
DIVY6 = 1. / Y(6)
S2 = Y2 / (1. - SO * Y(1) + Y22) + 2. * SO * Y11 + S2 * S2

CALL SUB(X, R, DR, 2, RZERO)

R = R - Y7 * Y8

NOW BRANCH TO PART 1 OR PART 2

GO TO (11, 201), NPART

CONTINUE

PART 1

IF (Y2) 409, 410, 411

WRITE(6, 1000)

1000 FORMAT(10X, 46HRECALCULATION PRESENT. CALCULATION NOT CORRECT)

DERU = 0.
DYY(7) = 0.
DYY(8) = 0.

GO TO 412

410 CONTINUE

Y7 = Y7 + .0001
RTH = Y2 * Y1 * Y7 * VISC
CF = 0.246 / (RTH)**.768*10.**1.678*Y8)
CFD = CF

412 CONTINUE

DIVY3 = 1. / Y3
E = E

1 * (1. + 1.5 * (1. - 2.718**(-1.1 * Y2)))

HD = E

J = 1.
PDD = CONSTANT

J = 2.
MOMENTUM INTEGRAL

J = 3.
CONTINUITY INTEGRAL

J = 4.
ENERGY INTEGRAL

J = 5.
MOMENTUM HALF INTEGRAL

J = 7.
B.L. MOMENT EQUATION FOR

J = 8.
B.L. MOMENT-OF-MOMENTUM EQUATION

13 CONTINUE

DCNRY = Y33 / (R + R)

C CALCULATION OF EN PART 1

2A = 0.
2B = 0.

CTR = -0.05
DC 369 I = 1, NS
CTR = CTR + 1.
Y2FI = Y2 + FI(1)
FS = TF(1)
ANA = Y2FI
ANB = Y2FI*Y2FI
D = I. + Y(5)*FS - SO*Y11*Y2FI*Y2FI
DIVD = 1. / D
ZA = ZA + DIVNS*ANA*DIVD
1 = CTR
ZB = ZB + DIVNS*ANB*DIVD
1 = CTR
369 CONTINUE
S2 = Y2/(1.-SO*Y11*Y2*Y2)
AMAS = ZA*DONR + (1.-DONR)*S2
AMOH = ZB*DONR
EN = AMAS/SQRT((AMAS + BP*(1.-1./Y(4)))*DIVV11)
RLOCAL = R*RZERO
RLOCAL = RLOCAL*RLOCAL
MASRAT = AMAS*Y4*POO*Y1*UJO*RLOCAL *3.1416/IRG*T00*AMASS1 - 1.
C
W(1,1) = Y22*Y1  
W(1,2) = Y11*Y2  
W(1,3) = 0.  
W(1,4) = BP*(1.-SO*Y11*Y22)*DIVV4 
W(1,5) = 0.  
W(1,6) = 0.  
W(1,7) = 0.  
W(1,8) = 0.  
V(1,1) = 0.  
J = 1
71 J = J+1  
71 = D  
E2 = 0.  
E3 = 0.  
E4 = 0.  
E5 = 0.  
E6 = 0.  
CTR = -.05  
DO 66 I=1,NS 
CTR = CTR + .1 
FS = TF(11)
F1 = F(I)
Y2FI*Y2+F(1) 
JJ = J-1  
GO TO (270,70,67), JJ  
270 CONTINUE 
AN = Y2FI*Y2FI 
D2N = Y2FI*2.  
DSN = 0.  
GO TO 68 
70 AN = Y2FI 
D2N = 1.  
DSN = 0.  
GO TO 68 
67 AN = Y2FI*(1.+Y5*FS) 
D2N = 1.+Y5*FS 
DSN = Y2FI*FS 
68 D = 1.+Y5*FS - SO*Y11*Y2FI*Y2FI 
DIVD = 1. / D 
DIVDD = DIVD*DIVD 
D2D = -2.*SO*Y11*Y2FI
\( D10 = -2 \cdot Y1 \cdot Y2 ) \cdot Y2 \cdot F1 \cdot Y2 \cdot F1 \)

\( DS0 = FS \)

\( ZI = 71 \cdot DVNS \cdot AN \cdot [DIVD \cdot \text{CTR}] \)

\( Z2 = 72 \cdot DVNS \cdot AN \cdot D10 \cdot [DIVD \cdot \text{CTR}] \)

\( Z3 = 73 \cdot DVNS \cdot D2N \cdot [DIVD \cdot \text{CTR}] \)

\( Z4 = 74 \cdot DVNS \cdot AN \cdot D20 \cdot [DIVD \cdot \text{CTR}] \)

\( Z5 = 75 \cdot DVNS \cdot D5N \cdot [DIVD \cdot \text{CTR}] \)

\( Z6 = 76 \cdot DVNS \cdot D50 \cdot [DIVD \cdot \text{CTR}] \)

\[ W(J,1) = DONR \cdot ((2 + Y1 + Y2) \cdot DIVV1 + Z2) + (1 - DONR) \cdot ((2 + Y2 + DS2Y2)) \]

\[ W(J,2) = DONR \cdot (Z3 + Z4) + (1 - DONR) \cdot (S2 + Y2 + DS2Y2) \]

\[ W(J,3) = 2 \cdot DONR \cdot (Z1 - Y2 + S2) \cdot DIVY3 \]

\[ W(J,4) = DONR \cdot (Z1 - Y2 + S2) \cdot DIVY4 \]

\[ 1 + BP \cdot DIVY4 \cdot DIVY1 \]

\[ W(J,5) = DONR \cdot (Z5 + Z6) \]

\[ W(J,7) = -2 \cdot S2 \cdot Y8 \cdot R \cdot Y2 \]

\[ W(J,9) = -2 \cdot S2 \cdot Y7 \cdot R \cdot Y2 \]

\[ V(J,1) = -2 \cdot SP \cdot R \div R \cdot Y2 \cdot S2 \]

\[ \text{GO TO } 212 \]

\[ W(J,1) = DONR \cdot (Z1 - Y2 + Z21 + (1 - DONR) \cdot (S2 + DIVY1 + DS2Y1)) \]

\[ W(J,2) = DONR \cdot (Z3 + Z4) + (1 - DONR) \cdot (S2 + DIVV1 + DS2Y1) \]

\[ W(J,3) = 2 \cdot DONR \cdot (Z1 - Y2 + S2) \cdot DIVY3 \]

\[ W(J,4) = DONR \cdot (Z1 - Y2 + S2) \cdot DIVY4 \]

\[ W(J,5) = DONR \cdot (Z5 + Z6) \]

\[ W(J,7) = -2 \cdot S2 \cdot Y8 \cdot R \]

\[ W(J,9) = -2 \cdot S2 \cdot Y7 \cdot R \]

\[ V(J,1) = -2 \cdot SP \cdot R \div R \cdot S2 \]

\[ \text{GO TO } 212 \]

\[ W(J,1) = DONR \cdot (Z1 - Y2 + Z21 + (1 - DONR) \cdot (S2 + DIVY1 + DS2Y1)) \]

\[ W(J,2) = DONR \cdot (Z3 + Z4) + (1 - DONR) \cdot (S2 + DIVV1 + DS2Y1) \]

\[ W(J,3) = 2 \cdot DONR \cdot (Z1 - Y2 + S2) \cdot DIVY3 \]

\[ W(J,4) = DONR \cdot (Z1 - Y2 + S2) \cdot DIVY4 \]

\[ W(J,5) = DONR \cdot (Z5 + Z6) \]

\[ W(J,7) = -2 \cdot S2 \cdot Y8 \cdot R \]

\[ W(J,9) = -2 \cdot S2 \cdot Y7 \cdot R \]

\[ V(J,1) = -2 \cdot SP \cdot R \div R \cdot S2 \]

\[ \text{GO TO } 212 \]

\[ W(J,1) = DONR \cdot (Z1 - Y2 + Z21 + (1 - DONR) \cdot (S2 + DIVY1 + DS2Y1)) \]

\[ W(J,2) = DONR \cdot (Z3 + Z4) + (1 - DONR) \cdot (S2 + DIVV1 + DS2Y1) \]

\[ W(J,3) = 2 \cdot DONR \cdot (Z1 - Y2 + S2) \cdot DIVY3 \]

\[ W(J,4) = DONR \cdot (Z1 - Y2 + S2) \cdot DIVY4 \]

\[ W(J,5) = DONR \cdot (Z5 + Z6) \]

\[ W(J,7) = -2 \cdot S2 \cdot Y8 \cdot R \]

\[ W(J,9) = -2 \cdot S2 \cdot Y7 \cdot R \]

\[ V(J,1) = -2 \cdot SP \cdot R \div R \cdot S2 \]

\[ \text{GO TO } 212 \]

\[ W(J,1) = DONR \cdot (Z1 - Y2 + Z21 + (1 - DONR) \cdot (S2 + DIVY1 + DS2Y1)) \]

\[ W(J,2) = DONR \cdot (Z3 + Z4) + (1 - DONR) \cdot (S2 + DIVV1 + DS2Y1) \]

\[ W(J,3) = 2 \cdot DONR \cdot (Z1 - Y2 + S2) \cdot DIVY3 \]

\[ W(J,4) = DONR \cdot (Z1 - Y2 + S2) \cdot DIVY4 \]

\[ W(J,5) = DONR \cdot (Z5 + Z6) \]

\[ W(J,7) = -2 \cdot S2 \cdot Y8 \cdot R \]

\[ W(J,9) = -2 \cdot S2 \cdot Y7 \cdot R \]

\[ V(J,1) = -2 \cdot SP \cdot R \div R \cdot S2 \]

\[ \text{GO TO } 212 \]
\[
R1 = 0.0
V1 = 0.0
V2 = 0.0
V3 = 0.0
V4 = 0.0
V5 = 0.0
V6 = 0.0
V7 = 0.0
V8 = 0.0
V9 = 0.0
V10 = 0.0
V11 = 0.0
\\
CTR = -0.25*DIVNS
\\
1 \rightarrow I+1
CTR = CTR + 0.5*DIVNS
FS = YF(I)
Y2FI = Y2*F(I)
\\
\text{CCC} = \text{DIVNS}*CTR*5
D71 = \text{CCC}*AN*D1V
D72 = \text{CCC}*AN*D1+D1VDD
D73 = \text{CCC}*D2N+DIVD
D74 = \text{CCC}*AN*D2+DIVD
D75 = \text{CCC}*D5N+DIVD
D76 = \text{CCC}*AN*D5+DIVD
D710 = \text{FDER}*CTR*\text{CCC}*DIVD
D711 = \text{CCC}*CTR*Y2FI*DIVD*2.5*SD*Y11*Y2FI*FDER
V1 = Z1 + 0.5*D71
V2 = Z2 + 0.5*D72
V3FI = Y2 + 0.5*D73
V4 = Z4 + 0.5*D74
V5 = Z5 + 0.5*D75
V6 = Z6 + 0.5*D76
V10 = Z10 + 0.5*D710
V11 = Z11 + 0.5*D711
Z1 = Z1 + D71
Z2 = Z2 + D72
Z3 = Z3 + D73
Z4 = Z4 + D74
Z5 = Z5 + D75
Z6 = Z6 + D76
Z10 = Z10 + D710
Z11 = Z11 + D711
\\
492 \text{ CCC} = \text{DIVNS}*CTR*CTR
\\
1 \rightarrow 5
\text{ATAU} = .377
\text{ATAU} = .0333
Q1 = Q1 + \text{CCC}*Y2FI*DIVD
\text{Y2FI} = Y2 + Q1
Q2 = Q2 + \text{CCC}*Y2FI*DIVD
Q3 = Q3 + \text{CCC}*CTR*Y2FI*FDER*DIVD
\]
R1 = R1 + CCC*V1*FDER
R2 = R2 + CCC*V2*FDER
R34 = R34 + CCC*(V3 + V4)*FDER
R56 = R56 + CCC*(V5 + V6)*FDER
R10 = R10 - CCC*V10*FDER
R11 = R11 - CCC*V11*FDER

IF (1-NS) 224, 225, 225

CONTINUE

W(J,1) = (Q1 + R1)*DIVY1 + R2
W(J,2) = Q2 + R3
W(J,3) = (Q3 + R10 + R11)*DIVY3
W(J,4) = R1*DIVY4 + B*DIVY4*DIVY11*333
W(J,5) = R56
W(J,7) = 0.
W(J,8) = 0.
V(J,1) = E*DIVY3*ATAU
MM = MM + 1

CONTINUE

CONTINUE

B.L. MOMENTUM EQUATION

W(7,1) = (2. + Y8)*Y7*DIVY1*Y2
W(7,2) = (2. + Y8)*Y7
W(7,3) = 0.
W(7,4) = 0.
W(7,5) = 0.
W(7,6) = 0.
W(7,7) = Y2
W(7,8) = 0.

V(7,1) = -5*CF*Y2

B.L. MOMENT-OF-MOMENTUM EQUATION

W(8,1) = W(8,2)*DIVY1*Y2
W(8,3) = 0.
W(8,4) = 0.
W(8,5) = 0.
W(8,6) = 0.
W(8,7) = 0.
W(8,8) = Y2
V(8,1) = (Y8*Y8 - 1.)*Y2/Y7*5*Y8*CF*.06*(Y8 - 1.)/(Y8 + 3.1)
1 *RTH**.1)

IF (Y(2) 200, 201, 201
200 W(1,1) = 0.
W(1,2) = 0.
W(1,3) = 0.
W(7,2) = 0.
V(7,1) = 0.
W(8,1) = 0.
W(8,2) = 0.
V(8,1) = 0.
201 CONTINUE

CONTINUE

IF (Y8 - 2.1 482, 481, 481
481 W(7,1) = 0.
W(7,2) = 0.
W(7,7) = 1.
V(7,1) = 0.
W(8,1) = 0.
W(8,2) = 0.
W(8,8) = 1.

94
SOLVING SIMULTANEOUS EQUATIONS BY SIMQ SUBROUTINE

C

NN = 7
C

DO 101 J = 1, NN
C

BU = Y(J, 1)
C

DO 101 J = 1, NN
C

IJ = I + NN*(J - 1)
C

AIJ = W(I, J)
C

101 CONTINUE
C

CALL SIMQ (A, R, NN, KS)
C

18

DY(1) = DY(7)
C

DY(7) = DY(6)
C

DY(6) = 0.
C

249 CONTINUE
C

JRUNGE = JRUNGE + 1
C

IF (JRUNGE - 5) 14, 15, 15
C

15

JRUNGE = 0
C

16

CALL PRINT (N, XOUT, Y, DY, 100)
C

Y(3) = 0.
C

Y3 = Y(3)
C

NPART = 2
C

BPPT1 = BP
C

SOPT1 = SO
C

DLSTAR = Y8*Y7
C

DELTA = DLSTAR/(Y8 + 1)/(Y8 - 1)
C

POW = 0.5*(Y8 - 1)
C

R = DLSTAR
C

EVALUATE NEW VELOCITY PROFILES
C

CTR = -0.5
C

DO 876 I = 1, NS
C

CTR = CTR + 0.1
C

IF (1.0 - CTR - DLSTAR/R) 875, 875, 874
C

875 F(I) = 0.
C

TF(I) = 0.
C

876 CONTINUE

C

COLLAPSE FROM 8X8 TO 7X7 MATRIX
C

DO 687 I = 1, 8
C

W(I, 1) = W(I, 7)
C

W(I, 7) = W(I, 8)
C

687 CONTINUE
C

V(6, 1) = V(7, 1)
C

V(7, 1) = V(8, 1)
C

667 CONTINUE
C

W[1, 1! = O.
C

W(l, 2! = O.
C

CONY INUF
C

COLLAPSE FROM BX8- TO 7X-'TMATRIX ....
C

V(6, 1| = V(7, 1)
C

V(7, 1) = V(8, 1)
C

CONTINUE
GO TO 840
C1TR1 = CTR1/(1.-POLSTAR/P)
        F(I) = A(I*POLY)
DO 873 J=2,JPOLY
        JJ = JPOLY - J + 1
        F(I) = F(I)*C1TR1 + A(I*JJ)
        CONTINUE
        F(I) = F(I)*C1TR1 + AZERO
        TF(I) = SQRT(F(I))
840 IF (BL-1) < 0.860, 0.861
        BL = (P/Delta**1.-C1TR1)**Pow
        IF (BL-1) < 0.860, 0.861
        CONTINUE
     F(I) = (F(I) + Y2*BL)/(1. + Y2)
        G(I) = G(I) - F(I)
876 CONTINUE
        WRITE(6,403) (F(I),I=1,NS)
        WRITE(6,406) (G(I),I=1,NS)
407 WRITE(6,407) //X,7HX/RAZERG,5X,4HAREA,5X,4HPH2C,5X,354HTOWALL
            3NTIDEGR) TCWALL(DEG)
        L = NS - 1
        DO 877 I=2,L
            FP(I) = DIVDEL*F(I+1)-F(I-1)
            GP(I) = DIVDEL*G(I+1)-G(I-1)
        CONTINUE
FP(I) = DIVDEL*(F(2) - F(I))
GP(I) = DIVDEL*(G(2) - G(I))
FP(NS) = DIVDEL*(F(NS) - F(NS-1))
GP(NS) = DIVDEL*(G(NS) - G(NS-1))
Y1 = Y1*Y1 + Y2
Y11 = Y(I)*Y(I+1) + Y2
DIVY11 = 1./Y11
E21 = E/(I* + Y2)
E21 = E21**2
CFD1 = CFD(I, + Y2)*I* + Y2)1
Y2 = O.
Y2 = O.
Y(7) = 0.
Y8 = O.
Y(8) = O.
C C PART 2
C TURBULENT PRANDTL NUMBER = PR
PR = 1.
C Y2=GAMMA
-20 CONTINUE
Y23 = Y(I) + Y(3)
SO = SO +Y(1) + Y(6)1
DSQ = SQRT(Y(6))
BP = BCF+Y(6)
C C PHASE-OUT THE BUEISH DISPLACEMENT-THICKNESS INHERITED FROM PART-1
    DY(7) = 0.
DY(I) = 0

C CALCULATION OF EN

ZA = 0

ZB = 0

CTR = -.05

DO 370 I = 1,NS

CTR = CTR + .1

YFGG = Y2 + F(I) + Y3*G(I)

FS = TF(I)

ANA = YFGG

ANB = YFGG*YFGG

D = 1. + Y5*FS + Y6*Y11*YFGG*YFGG

DIVD = 1./D

ZA = ZA + DIVNS*ANA*DIVD

I = *CTR

ZB = ZB + DIVNS*ANB*DIVD

END

370 CONTINUE

AMAS = ZA

AMOM = ZB

EN = AMAS/SQRT(AMOM + BP*I1.-1./Y(4)*DIVY(1))

RLOCAL = R*ZERO

RLOCAL = RLOCAL*RLOCAL

MASRAT = AMAS*Y4*POO*Y1*UJO*RLOCAL *3.1416/(RG*TOD*Y6*AMASS1) - 1.

UM = AMAS*Y1

RM = UM*2.*P/VISC

CFDF = AMAS*AMAS*04A*RM**(-20)

CFD = CFDF

C C J=1  CONTINUITY

C C J=2  ENERGY

C C J=3  MOMENTUM

C C J=4  MOMENT OF MOMENTUM EQUATION

C C J=5  T' / T = UJ' / UJ

C C J=6  D(LAMBDA)/DX = 0.

J=0

86 J=J+1

87 Z1=0

Z2=0

Z3=0

Z4=0

Z5=0

Z6=0

Z7=0

Z8=0

Z9 = 0.

CTR = -.05

I=0

82 I=I+1

CTR = CTR + .1

FS = TF(I)

YFGG = Y2 + F(I) + Y3*G(I)

GO TO (81,83,41),J

81 AN=YFGG

D2N=1.

D5N=0

D3N=G(I)

GO TO 85

ALIT?57
| \( Z2 = 0 \) | ALIT 50 |
| \( Z3 = 0 \) | ALIT 60 |
| \( Z4 = 0 \) | ALIT 70 |
| \( Z5 = 0 \) | ALIT 80 |
| \( Z6 = 0 \) | ALIT 90 |
| \( Z7 = 0 \) | ALIT 100 |
| \( Z8 = 0 \) | ALIT 110 |
| \( Z9 = 0 \) | ALIT 120 |
| \( Z10 = 0 \) | ALIT 130 |
| \( Z11 = 0 \) | ALIT 140 |
| \( Q1 = 0 \) | ALIT 150 |
| \( Q2 = 0 \) | ALIT 160 |
| \( Q3 = 0 \) | ALIT 170 |
| \( Q4 = 0 \) | ALIT 180 |
| \( R1 = 0 \) | ALIT 190 |
| \( R2 = 0 \) | ALIT 200 |
| \( R34 = 0 \) | ALIT 210 |
| \( R56 = 0 \) | ALIT 220 |
| \( R79 = 0 \) | ALIT 230 |
| R11 = 0 | ALIT 240 |
| RTAU = 0 | ALIT 250 |
| \( V1 = 0 \) | ALIT 260 |
| \( V2 = 0 \) | ALIT 270 |
| \( V3 = 0 \) | ALIT 280 |
| \( V4 = 0 \) | ALIT 290 |
| \( V5 = 0 \) | ALIT 300 |
| \( V6 = 0 \) | ALIT 310 |
| \( V7 = 0 \) | ALIT 320 |
| \( V8 = 0 \) | ALIT 330 |
| \( V9 = 0 \) | ALIT 340 |
| \( V10 = 0 \) | ALIT 350 |
| \( V11 = 0 \) | ALIT 360 |

\[
2FMAX = -5*CFD*.45/(FP\!(5) + GP\!(5))
\]

\[
HD = (1.+Y33)*E2I + Y33*E2FMAX
\]

\[
CTR = \cdot .25*DIVNS
\]

924

\[
I = I + 1
\]

\[
CTR = CTR + 0.5*DIVNS
\]

\[
FS = TF(I)
\]

\[
Y2FI = Y2 + F(I)
\]

\[
1 + Y3*G(I)
\]

\[
FDER = FP(I) + Y3*GP(I)
\]

\[
GDER = FP(I) + GP(I)
\]

\[
2F = -5*CFD*CTR/GGDER
\]

\[
E = (1.+Y33)*E2I + Y33*E2F
\]

\[
AN = Y2FI
\]

\[
D2N=1.
\]

\[
D5N=0.
\]

\[
E=1.+Y5*FS-SD*Y11*Y2FI*Y2FI
\]

\[
DIVO=1./D
\]

\[
DIVDD=DIVO*DIVD
\]

\[
D2D=2.*SD*Y11*Y2FI
\]

\[
D1D=2.*SD*Y11*Y2FI
\]

\[
D5D=FS
\]

\[
D6D=-50*Y11*Y2FI*G(I)
\]

\[
CCC = DIVNS*CTR*5
\]

\[
D2I = CCC*AN*DIVD
\]

\[
D22 = -CCC*AN*D1D*DIVDD
\]

\[
D23 = CCC*D2N*DIVD
\]

\[
D24 = -CCC*AN*D2D*DIVDD
\]
D75 = CCC+DSN+DIVD
D76 = -CCC+AM+DSD+DIVDO
D77 = CCC+DSN+DIVD
D78 = -CCC+DIVDO+AM+D60
D79 = -CCC+D6C+AM+DIVD
D80 = FDER*CTR*CCC+DIVD
D81 = CCC*CTR+Y2FI*DIVDD+Z2*50*Y11*Y2FI*FDER
RTAI = RTAU-E*FDER*CCC+DIVD

V1 = Z1 + 0.5*Z11
V2 = Z2 + 0.5*Z22
V3 = Z3 + 0.5*Z33
V4 = Z4 + 0.5*Z44
V5 = Z5 + 0.5*Z55
V6 = Z6 + 0.5*Z66
V7 = Z7 + 0.5*Z77
V8 = Z8 + 0.5*Z88
V9 = Z9 + 0.5*Z99
V10 = Z10 + 0.5*Z110
V11 = Z11 + 0.5*Z111
V12 = Z12 + 0.5*Z122
V13 = Z13 + 0.5*Z133
V14 = Z14 + 0.5*Z144
V15 = Z15 + 0.5*Z155
V16 = Z16 + 0.5*Z166
V17 = Z17 + 0.5*Z177
V18 = Z18 + 0.5*Z188
V19 = Z19 + 0.5*Z199
V20 = Z20 + 0.5*Z200
V21 = Z21 + 0.5*Z211
V22 = Z22 + 0.5*Z222
V23 = Z23 + 0.5*Z233
V24 = Z24 + 0.5*Z244
V25 = Z25 + 0.5*Z255
V26 = Z26 + 0.5*Z266
V27 = Z27 + 0.5*Z277
V28 = Z28 + 0.5*Z288
V29 = Z29 + 0.5*Z299
V30 = Z30 + 0.5*Z300
V31 = Z31 + 0.5*Z311
V32 = Z32 + 0.5*Z322
V33 = Z33 + 0.5*Z333
V34 = Z34 + 0.5*Z344
V35 = Z35 + 0.5*Z355
V36 = Z36 + 0.5*Z366
V37 = Z37 + 0.5*Z377
V38 = Z38 + 0.5*Z388
V39 = Z39 + 0.5*Z399
V40 = Z40 + 0.5*Z400
V41 = Z41 + 0.5*Z411
V42 = Z42 + 0.5*Z422
V43 = Z43 + 0.5*Z433
V44 = Z44 + 0.5*Z444
V45 = Z45 + 0.5*Z455
V46 = Z46 + 0.5*Z466
V47 = Z47 + 0.5*Z477
V48 = Z48 + 0.5*Z488
V49 = Z49 + 0.5*Z499
V50 = Z50 + 0.5*Z500
V51 = Z51 + 0.5*Z511
V52 = Z52 + 0.5*Z522
V53 = Z53 + 0.5*Z533
V54 = Z54 + 0.5*Z544
V55 = Z55 + 0.5*Z555
V56 = Z56 + 0.5*Z566
V57 = Z57 + 0.5*Z577
V58 = Z58 + 0.5*Z588
V59 = Z59 + 0.5*Z599
V60 = Z60 + 0.5*Z600
V61 = Z61 + 0.5*Z611
V62 = Z62 + 0.5*Z622
V63 = Z63 + 0.5*Z633
V64 = Z64 + 0.5*Z644
V65 = Z65 + 0.5*Z655
V66 = Z66 + 0.5*Z666
V67 = Z67 + 0.5*Z677
V68 = Z68 + 0.5*Z688
V69 = Z69 + 0.5*Z699
V70 = Z70 + 0.5*Z700
V71 = Z71 + 0.5*Z711
V72 = Z72 + 0.5*Z722
V73 = Z73 + 0.5*Z733
V74 = Z74 + 0.5*Z744
V75 = Z75 + 0.5*Z755
V76 = Z76 + 0.5*Z766
V77 = Z77 + 0.5*Z777
V78 = Z78 + 0.5*Z788
V79 = Z79 + 0.5*Z799
V80 = Z80 + 0.5*Z800
V81 = Z81 + 0.5*Z811
V82 = Z82 + 0.5*Z822
V83 = Z83 + 0.5*Z833
V84 = Z84 + 0.5*Z844
V85 = Z85 + 0.5*Z855
V86 = Z86 + 0.5*Z866
V87 = Z87 + 0.5*Z877
V88 = Z88 + 0.5*Z888
V89 = Z89 + 0.5*Z899
V90 = Z90 + 0.5*Z900
V91 = Z91 + 0.5*Z911
V92 = Z92 + 0.5*Z922
V93 = Z93 + 0.5*Z933
V94 = Z94 + 0.5*Z944
V95 = Z95 + 0.5*Z955
V96 = Z96 + 0.5*Z966
V97 = Z97 + 0.5*Z977
V98 = Z98 + 0.5*Z988
V99 = Z99 + 0.5*Z999
V100 = Z100 + 0.5*Z100

925 CONTINUE
\[
\begin{align*}
W(5,6) &= -\frac{1}{\gamma_6} \\
V(5,1) &= 0 \\
W(6,1) &= 0 \\
W(6,2) &= 1 \\
V(6,3) &= 0 \\
W(6,4) &= 0 \\
W(6,5) &= 0 \\
W(6,6) &= 0 \\
V(6,1) &= 0 \\
\end{align*}
\]

CONTINUE

CONTINUE

SOLVING SIMULTANEOUS EQUATIONS BY SIMQ SUBROUTINE

\[
\begin{align*}
\text{NN} &= 6 \\
\text{DO} 102 \text{ J}=1,\text{NN} \\
R(J) &= V(J,1) \\
\text{DC} 102 \text{ I}=1,\text{NN} \\
IJ &= I + \text{NN} \times (J-1) \\
102 \text{ CONTINUE} \\
\text{CALL SIMQ} (A, R, \text{NN, KS}) \\
\text{DO 103} \text{ I}=1,\text{NN} \\
-103 \text{ DO} Y(I) *=A(I) \\
165 \text{ CONTINUE} \\
-166 \text{ CONTINUE} \\
\text{RETURN} \\
\end{align*}
\]

SUBROUTINE PRINT (N, XOUT, YOUT, DY, J)

DIMENSION YOUT(10), XOUT(200), DY(50), Y(10), YSAVE(10)

COMMON TUR, EN, NPART, C, JRUNGE, NGAM

\[
\begin{align*}
\text{COMMON ZA, ZB, UJO, DORIN} \\
6 \text{, MASRT, RZERO, AFAC, CFD, HO, F, A1, AZERO, TF, PP, GP} \\
7 \text{, SLOSS, ASC} \\
8 \text{, AUG1, U1, UCENT} \\
\text{REAL MASRT} \\
\end{align*}
\]

CALL SUA (X, R, OR, 2, RZERO)

CHECK FOR INITIAL PRINT

WRITE (6, 50)

PRINT FOR ALL PARTS

UJ = YOUT(1) * UJO

PHZ0 = YOUT(4) * POO * 1.93

UO = YOUT(1) * YOUT(2) * UJO

SUM = UO + UJ

UR = SIM/UCENT

TOWALL = YOUT(6) * 100

TOCENT = TOWALL + YOUT(5) + TOWALL

101
AREA = **2.
GO TO (29,30), NPART
29 WRITE(6,55) X, AREA, PH20, UD, SUM, UR, YOUT(2), YOUT(3), TOCENT, TOWALL,
1 YOUT(7), YOUT(8)
GO TO 31
30 AUG2 = 3.1416* (YOUT(4) * POO) / BP + (YOUT(1) ** 2) * AUG1
1* (X/RZERO) ** 2.*
AUG = (AUG2 / (AMASS1 * U1))
WRITE(6, 56) X, AREA, PH20, YOUT(6), SUM, UR, AUG, YOUT(3), TOCENT, TOWALL
CONTINUE
TF = (J - 100) / 40; 41, 40
WRITE (6, 60) UJ
RETURN
! FORMAT(1/4X,7X/RZERO, 5X,4XAREA, 5X,4PH20, 5X,24HUSD/SEC)UCE
1INT(FT/SEC), 4X,2HUR, 5X,6HFLAMDA, 6X,7HDELTA/R, 2X,34HTOCENT(DEGR) TOW
2ALL(DEGR) THETA/R, 4X,5XSHAPE)
1, F7.4, 5X, F7.1, 5X, F7.4, 4X, F7.3, 5X
1, F7.4, 5X, F7.1, 5X, F7.4, 4X, F7.3, 5X
-60 FORMAT(/52H DELTA/R = 1 -- DIFFERENTIAL EQUATIONS CHANGE,
115X, SHKL = ,FT0.5//)
END
SUBROUTINE CALXFG (J, X, F, GG)
COMMON TURBNO, CF, FN, NPART, C, JRUNGE, NGAM
1, POO, P01, TOD, TO1, AMACH, AG, RG
2 , AMASS1, AMASS0, T
3, SO2, CMAS, CMOM, CEMR
4, SO, BP, CM, VISC1
COMMON ZA, ZR, UJO, DORIN
6, MASRAT, RZERO, AFAC, CFD, HD, F, A1, AZERO, TF, FP, GP
7, SDLoss, ASD
6, AUG1, U1, UCENT
REAL MASRAT
DIMENSION X(3), FF(3), GG(3), F(10)
1, AL1(10), DI1(10), D2V(10), D1D(10), D2D(10)
1, AL1(10), TF(10)
5, *FP(10), CP1(10)
UR = X(11)
Y2 = X(2)
DOR = X(3)
Z1 = 0.
Z2 = 0.
Z3 = 0.
Z4 = 0.
Z5 = 0.
CTR = -0.05
S10 = 1./(1 - SOO * UR * UR * Y2 * Y2)
S20 = Y2 * S10
DS20Y2 = S10 * S20 * S20 * SOO * UR * UR
DS20UR = S20 * S20 * Y2 * SOO * UR * 2.
DO 89 I = 1, 10
CTR = CTR + 1.
F5 = TF(I)
GO TO (60, 61, 62), J
60 AN11 = Y2 + F(I)
D2N11 = 1.
GO TO 63
61 AN11 = (Y2 + F(I)) * (Y2 + F(I))
PCONST = (PO1/PO2)**((AG-1)/AG)
DIVON = 1./PCONST
88 CONTINUE
GO TO (25, 26, 27), J
25 FF(1) = FF(1) - CMAS
GO TO 28
26 FF(2) = FF(2) - CMAS
GO TO 29
27 CONTINUE
FF(3) = FF(3) - PCONST
28 CONTINUE
C TO NORMALIZE THE FUNCTIONS FF() AND ITS DERIVATIVES GG()
FF(J) = FF(J)*DIVON
DO 111 I = 1, 3
GG(I) = GG(I)/DIVON
111 CONTINUE
C CALCULATION OF INITIAL VALUE OF DELTA/R (APPROXIMATE)
DORINV = SORT ((CMON + 0.5*Y2*Y2 - Y2*S20)/(Z1 - S20*Y2))
RETURN
SUBROUTINE SUB(xx, r, dr, j, rzero)
DIMENSION A(50), X(50), D(50), D0(50)
GO TO (22, 21, 1), J
1 I = 0
C READ X AND DIAM.
C XX = XIRZ, R = RIRZERD
C
5 FORMAT(15X, 12HP, X(II), A(I))
-11 I = I + 1
READ (5, 10) X(II), A(I)
-10 FORMAT (2F15.4)
IF (A(I)) 11, 12, 11
-12 I END = I - 1
SCALE = 1.*RZERD
DO 50 J = I, END
WRITE (6, 10) X(I), A(I)
50 X(I) = X(I)*5.*SCALE
WRITE (6, 15)
15 FORMAT(///)
I = 1
C FIND R AND DR
C
-2 IF (XX - X(I)) 20, 23, 22
20 T = I - 1
IF (XX - X(I)) 20, 23, 23
23 IA = I + 1
GO TO 24
-22 I = I + 1
IF (XX - X(I)) 25, 23, 22
25 IA = I
IB = I - 1
-24 OR = (A(IA) - A(IB))/(X(IA) - X(IB))
R = A(IB) + (XX - X(IB))*OR
RETURN
END
SUBROUTINE RINCG (N, X, Y, TOL, YMIN, H, XOUT, MARK)
C FIRST ORDER DIFF. EQ. ROUTINE--ADJUSTS STEP SIZE
DIMENSION Y(I), YMIN(I), TOL(I), SU(I), XOUT(I), KLT(I), KUT(I), MARK(I)

DIMENSION DY(I), YA(I), FA(I), FR(I), FC(I), YKEEP(I)

KBTW = 1
KRTI = 1
 NCOUNT = 15
J = MARK(2)

230 DO 250 I = 1, N
250 SUB(I) = TOL(I)/32.0

10 IF (MAX = J) 20, 30, 30
20 RETURN

30 A = XOUT(I) - X
B = ABS (2.0*6*E)

35 IF (A + B) 40, 35, 35
40 J = J + 1
GO TO 10

50 CONTINUE

CALL PRINT (N, XOUT, Y, DY, J)
J = J + 1
GO TO 10

60 IF (A = 1.5*H) 70, 70, 80
70 H = A
GO TO 1000

80 IF (A = 3.0*H) 90, 1000, 1000
90 H = 3.0*H

C
DO RUNGE-KUTTE-MERSON INTEGRATION

-1060 XA = X + H/3
XB = X + 5*H
CALL DIFSEQ (N, X, Y, DY)
X = X + H
DO 1030 I = 1, N
YKEEP(I) = Y(I)

1030 YAI(I) = Y(I) + FA(I)/3.
CALL DIFSEQ (N, X, YA, DY)
DO 1040 I = 1, N

1040 YAI(I) = Y(I) + FA(I)/6 + H*DY(I)/6
CALL DIFSEQ (N, X, YA, DY)
DO 1050 I = 1, N

1050 YAI(I) = Y(I) + 1.25*FA(I) + 375*FB(I)
CALL DIFSEQ (N, XB, YA, DY)
DO 1060 I = 1, N

1060 FC(I) = H*DY(I)

-1060 YAI(I) = Y(I) + 5*FA(I) - 1.5*FB(I) + 2.0*FC(I)
CALL DIFSEQ (N, X, YA, DY)
DO 1100 I = 1, N

1100 Y = Y(I) + FA(I)/6 + 6666666667*FC(I) + H*DY(I)/6.

-1661 U = Y(I)
IF (ABS (U) - YMIN(I)) 1110, 1110, 1100

1100 KLOW = 2
GO TO 1130

1110 IF ( F = ABS (SUB(I)*U) ) 1110, 1120, 1120

105
1120 KBTWN = 2
1130 CONTINUE
1135 GO TO (100, 1135), KLOW
1140 NCOUNT = NCOUNT - 1
1150 IF (NCOUNT) 1150, 1150, 1170
1155 PRINT 1160, X, H
1156 PRINT 1169, (I, Y(I), DY(I), I = 1, N)
1160 FORMAT (5BH4STEP SIZE HALVED 15 TIMES CONSECUTIVELY SINCE LAST PPI
1165 FORMAT (41H4STEP SIZE BECAME TOO SMALL FOR COMPUTER, 29H IT HAS BEEN HALVED, 29H TIMES CONSECUTIVELY, 29H PROGRAM TERMINATED AT
1170 FORMAT (13,7X, EI6.8,4X, EI6.8) ....................
1174 Y(I) = YKEEP(I)
1175 KBTWN = 1
1176 KLOW = 1
1177 M = 15 - NCOUNT
1180 NCOUNT = 15
1190 H = 2.*H
1200 KBTWN = 1
1210 CONTINUE
1220 PRINT (HOUT (50) = X
1230 CALL PRINT (N, XOUT, Y, DY, JK)
1240 IF (JK) 10, 10, 20
1250 FORMAT (15H X = , E16.8, 4X, 4HY(I), 15X, 13X, 4HY(I), 16X, 8HY(I), 16X)
1260 FORMAT (15X, I3, 7X, 2(EI6.8, 4X))
1270 ENDSUBROUTINE SIMOA(A,B,N,KS)
1280 SUBROUTINE SIMQ(A,B,N)
1290 PURPOSE
1300 OBTAIN SOLUTION OF A SET OF SIMULTANEOUS LINEAR EQUATIONS, AX=B
1310 USAGE CALL SIMQ(A,B,N,KS)
1320 X(50) = X
1330 CALL PRINT (N, XOUT, Y, DY, JK)
1340 IF (JK) 10, 10, 20
1350 FORMAT (15H X = , E16.8, 4X, 4HY(I), 15X, 13X, 4HY(I), 16X, 8HY(I), 16X)
1360 FORMAT (15X, I3, 7X, 2(EI6.8, 4X))
1370 ENDSUBROUTINE SIMQ(A,B,N)
DESCRIPTION OF PARAMETERS

A - MATRIX OF COEFFICIENTS STORED COLUMNWISE. THESE ARE DESTROYED IN THE COMPUTATION. THE SIZE OF MATRIX A IS N BY N.

B - VECTOR OF ORIGINAL CONSTANTS (LENGTH N). THESE ARE REPLACED BY FINAL SOLUTION VALUES, VECTOR X.

M - NUMBER OF EQUATIONS AND VARIABLES. N MUST BE GT. ONE.

KS - OUTPUT DIGIT

& FOR A SINGULAR SET OF EQUATIONS

N - NUMBER OF EQUATIONS AND VARIABLES. N MUST BE GT. ONE.

8 - FOR A NORMAL SOLUTION

REMITS _ LITIS10

MATRIX A MUST BE GENERAL. IF MATRIX IS SINGULAR, SOLUTION VALUES ARE MEANINGLESS. AN ALTERNATIVE SOLUTION MAY BE OBTAINED BY USING MATRIX INVERSION (MINV) AND MATRIX PRODUCT (GMPRD).

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED

NONE

METHOD

METHOD OF SOLUTION IS BY ELIMINATION USING LARGEST PIVOTAL ELEMENT. EACH STAGE OF ELIMINATION CONSISTS OF INTERCHANGING ROWS WHEN NECESSARY TO AVOID DIVISION BY ZERO OR SMALL ELEMENTS.

THE FORWARD SOLUTION TO OBTAIN VARIABLE N IS DONE IN N STAGES. THE BACK SOLUTION FOR THE OTHER VARIABLES IS CALCULATED BY SUCCESSIVE SUBSTITUTIONS. FINAL SOLUTION VALUES ARE DEVELOPED IN VECTOR B, WITH VARIABLE 1 IN B(1), VARIABLE 2 IN B(2), ..., VARIABLE N IN B(N).

IF NO PIVOT CAN BE FOUND EXCEEDING A TOLERANCE OF 0.0, THE MATRIX IS CONSIDERED SINGULAR AND KS IS SET TO 1. THIS TOLERANCE CAN BE MODIFIED BY REPLACING THE FIRST STATEMENT.

************

FORWARD SOLUTION

TOL=0.0
KS=0
JL=-N
DO 65 J=1,N
  JL=J+1
  B!A=Ol
  IT=J+J
  DO 30 I=J,N
C
SEACH FOR MAXIMUM COEFFICIENT IN COLUMN

  IJ=IT+I
  IF(ABS(B!A)-ABS(A(IJ))) .GE. 20,30,30
  20 B!A=A(IJ)
  IMAX=1
  30 CONTINUE

TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)

IF(ABS(B!A)-TOL) .GE. 35,35,40

107
**SUBROUTINE VBO4A**

```
DIMENSION A(3,101, W(I01, F(31, E(3), X(31

IC=0
IKFG=0
IC=IC+1
RETURN
END
```
1  KK=N
     DO 3 I=1,N
     KK=KK+1
2 3  DO 2 J=1,I
     A(I,J)=0.
     W(KK)=0.
     FF=0.
     IFG=IFG+1
    4  DO 5 K=1,M
      CALL CALXFG (K,X,F,W)
     KK=N
2  5  DO 6 J=1,I
     A(I,J)=0.
     W(KK)=0.
      KK=KK+1
    6  DO 6 J=1,I
     A(I,J)=A(I,J)+W(I)*W(J)
     W(KK)=W(KK)+A(I,J)*W(J)
     4  FF=FF+F(K)*F(K)
      GO TO (7,8),IS
2  7  DC 45 I=2,N
     IK = I - 1
2  8  DC 45 J=1,N
     A(IK,J) = A(J,IK)
     CALL SPNISTA,N,N,IK)
     KK=KK+1
    9  DO 10 J=1,N
     W(KK)=W(KK)+EM
    10  DO 10 I=1,N
     W(KK)=W(KK)+EM
     KK=KK+1
    11  IF (EM-ESCALE) .GE. 12,12,13
     EM = MAX(EM,ABS(W(KK)/E(I)))
2  12  GO TO 15
    13  IF (EM-ESCALE) .GE. 12,12,13
     EM = EM / E(M)
     KK=KK+1
    14  DO 11 J=1,N
     W(KK)=W(KK)+EM
    11  DO 11 J=1,N
     15  IF (EM-ESCALE) .GE. 12,12,13
     EM = EM / E(M)
     KK=KK+1
    16  DO 16 J=1,N
     W(KK)=W(KK)+EM
    17  IF (EM-ESCALE) .GE. 12,12,13
     EM = EM / E(M)
     KK=KK+1
    18  IF (PRINT) .GE. 19,19,20
      IF (PRINT) .GE. 19,19,20
     WRITE(6,26)IKFG
     FORMAT (///SX,18HV904A FINAL VALUES)
     IF = 2
     IF = 2
     WRITE(6,21)
     FORMAT (///SX,18HV904A FINAL VALUES)
     IF = 2
     IF = 2
     WRITE(6,26)IKFG
     26  FORMAT (///SX,18HV904A FINAL VALUES)
     WRITE(6,21)
     IF = 2
     IF = 2
     WRITE(6,26)IKFG
     26  FORMAT (///SX,18HV904A FINAL VALUES)
80  RETURN
WRITE(6,24) (X(I),I=1,N)
24 FORMAT (5X,9HVARIABLES/,5E24,14)
WRITE(6,25) (F(I),I=1,M)
25 FORMAT (5X,9VFCONCTIONS/,5E24,14)
GO TO (38,191),IFS
38 IF=IPSET
-3 TC=IC+1
-4 ITEST=3
FFX=FF
XP=0.
XC=0.
1S=1
-7 GG=0.
KK=NNP
-DO 34 I=1,N
X(I)=X(I)+XP*W(KK)
34 KK=KK+1
XP=XC
GO TO 1
39 IF (FF-FFX) 39,40,40
40 FORMAT (15X,G14)
ICON=1
26 IF (XP-XC) 35,8,35
25 IS=2
GO TO 32
40 WRITE(6,41)
41 FORMAT (/)VRO4A-ACCURACY CANNOT BE ACHIEVED)
END
SUBROUTINE VDO2A (ITEST,XC,FF,GG,6,0,0,0,3,1+)
GO TO (32,33,33,33,33),ITEST
-32 XP=XC-XP
KK=NNP
DO 34 I=1,N
X(I)=X(I)+XP*W(KK)
34 KK=KK+1
XP=XC
GO TO 1
39 IF (FF-FFX) 39,40,40
40 FORMAT (15X,G14)
ICON=1
26 IF (XP-XC) 35,8,35
25 IS=2
GO TO 32
GO TO (B,9I,IS-
9 XA_X
...
FA=F
GA=G

12 X=XA-SIGNF(XINC,GA)
XINC=XINC+XINC
GO TO 3
8 IF (F-FA) 13,14
13 DUM=FA
FA=F
DUM=FA
DUM=GA
GA=G

14 IF (GA*(X-XA)) 15,16
15 XINC=2
XINC=X
16 Z=3*(FA-F)/(X-XA)+G+GA
W=Z-Z-G+GA
IF (W) 20,20,17
17 W=SIGNF(SORTF(W),X-XA)
XP=X-(X-XA)*(W+2)/(G-GA+W+W)
18 GO TO (21,22),1INC
21 IF (ABSFXP-XA)-ABSFXINC(XINC)) 23,12
22 IF (ABSFXP-XA)-ABSFXINC(XINC)) 23,24
24 X=0.5*(XINC+XA)
IF ((X-XA)*(XINC-X)) 25,25,3
25 ITEST=3
GO TO 11

23 X=XP
IF (ARSF(XP-XA)-ARSF(XINC)) 19,26
26 IF (ARSF(XP-XA)-ARSF(XINC)) 19,19
19 ITEST=2
GO TO 11
20 GO TO (12,24),1INC
END
SUBROUTINE SPNIST(U,I,J,K)
DIMENSION U(3,3),V(3,3)

2 I=J+K
WRITE(6,4)

4 FORMAT(10HO-MODIFY-VRO4A )
STOP
1 V(1,1) = U(2,2)*U(3,3) - U(3,2)*U(2,1)
2 V(2,2) = U(1,1)*U(3,3) - U(3,1)*U(2,1)
3 V(3,3) = U(1,1)*U(2,2) - U(2,1)*U(1,1)
4 V(1,2) = V(2,1)
5 V(2,3) = V(3,2)
6 V(3,1) = U(1,1)*U(3,2) - U(2,1)*U(1,2)
7 V(1,3) = U(1,2)*V(1,1) + V(1,1)*U(2,1) + U(1,1)*U(3,2)
8 DET = U(1,1)*V(1,1) + U(2,1)*V(2,1) + U(3,1)*V(3,1)
9 DO 5 L=1,3
10 M=1,3
5 U(L,M) = V(L,M)/DET
RETURN
END
## B.4 Typical Sets of Input and Output Data

### Input Data:

<table>
<thead>
<tr>
<th>Value</th>
<th>10</th>
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<tbody>
<tr>
<td>0.9950</td>
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<tr>
<td>0.9750</td>
<td>9650</td>
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<tr>
<td>0.9550</td>
<td>9450</td>
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<tr>
<td>0.9350</td>
<td>9250</td>
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M = 21.0

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<td>1</td>
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<tr>
<td>2.50</td>
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</table>

<table>
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<tr>
<th>Value</th>
<th>10</th>
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<tr>
<td>0.9950</td>
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<tr>
<td>0.9750</td>
<td>9650</td>
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M = 17.0

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<td>1.40</td>
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Output Data

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<td>M</td>
<td>21.0 - .02</td>
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| DELTA | 0.5000 |
| R | 0.0230 |
| PTUBE | 5.2981 |
| TURBNO | 147.0000 |
| AG | 1.4000 |
| POD | 14.7000PSIA |
| TOD | 552.0000DEGK |
| P01 | 348.0000PSIA |
| T01 | +2431.4000DEGK |
| AMSS | 2.32301BM/SEC |
| AMASS | 0.11301BM/SEC |
| RMISS | 0.0040FT**2 |
| NMISS | 2.6000 |

| FILTS | 0.9806 |
|       | 0.8589 |
|       | 0.6848 |
|       | 0.5054 |
|       | 0.3482 |
|       | 0.2251 |
|       | 0.1367 |
|       | 0.0768 |
|       | 0.0367 |
|       | 0.0092 |

CONDITIONS AT BEGINNING OF THE TRANSITION SECTION

| U0 | 273.7FT/SEC |
| V0 | 309.7FT/SEC |
| PH0 | 1340.501KPA |
| T0 | 90.4DEGK |
| LAMBDA | 0.0949 |
| PH2 | 17.491KPA |
| NMISS | 2.6000 |
| MMISS | 0.0040FT**2 |

ITERATION 0

VARIALBLES

FUNCTIONS

ITERATION 1

VARIALBLES

IM CALLS OF CALXFG
<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>FUNCTIONS</th>
<th>ITERATION</th>
<th>3M CALLS OF CALXRG</th>
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**VBA3A Accuracy Cannot Be Achieved**

**VBA3A Final Values**

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<th>ITERATION</th>
<th>8M CALLS OF CALXRG</th>
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<td>0.3572011464017E-01</td>
<td>-0.161928835806E-01</td>
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<th>1.0972810496</th>
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<td>BF</td>
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**WZERO** | **AREA** | **PHSO** | **U78/100** | **V10** | **VR** | **LAMBDA** | **DELTAR** | **FRET** | **THERMAL** | **THRESH** | **SHARE** |
<table>
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REFERENCES


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<tr>
<th>Primary Flow</th>
<th>Flow Parameter</th>
<th>Instrumentation Used to Measure Parameter</th>
<th>How Recorded</th>
<th>Required for Determining</th>
<th>Data Reduction Procedure</th>
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<tr>
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<td>$P_{o1}$</td>
<td>Bourdon Tube Gage</td>
<td>Manually</td>
<td>Jet Pump Input Conditions</td>
<td>None Needed</td>
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<tr>
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<td>$T_{o1}$</td>
<td>Thermocouple and Bridge</td>
<td>Manually</td>
<td>Jet Pump Input Conditions</td>
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<td>$W_{L}$</td>
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<td>Standard calibration curves provided by flowmeter manufacturer</td>
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<td>Secondary Flow Temperature</td>
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<td>Secondary Flow Rate</td>
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<td>Manometer Board</td>
<td>Photographically</td>
<td>Mixing Tube and Diffuser Static Pressures</td>
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<td>Velocity and Temperature Profiles</td>
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</tbody>
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### Table 2

**Pressure Tap Locations and Final Mixing Tube Dimensions**

<table>
<thead>
<tr>
<th>Static Pressure Tap No.</th>
<th>Stagnation Pressure Traverse No.</th>
<th>Pressure Tap Location figure 4 x-inches</th>
<th>Dimensionless Location $x/R_o$ ($R_o = 2.670''$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.46</td>
<td>0.172</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2.21</td>
<td>0.828</td>
</tr>
<tr>
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<td>4.71</td>
<td>1.76</td>
</tr>
<tr>
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<td>1</td>
<td>6.71</td>
<td>2.51</td>
</tr>
<tr>
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<td></td>
<td>9.71</td>
<td>3.63</td>
</tr>
<tr>
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<td>2</td>
<td>12.21</td>
<td>4.57</td>
</tr>
<tr>
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<td></td>
<td>14.71</td>
<td>5.51</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>17.21</td>
<td>6.45</td>
</tr>
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<td>9</td>
<td></td>
<td>20.46</td>
<td>7.66</td>
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<td>22.21</td>
<td>8.32</td>
</tr>
<tr>
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<td>4, 5</td>
<td>24.71</td>
<td>9.25</td>
</tr>
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<td></td>
<td>27.21</td>
<td>10.19</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>29.71</td>
<td>11.13</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>32.21</td>
<td>12.06</td>
</tr>
<tr>
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<td></td>
<td>34.71</td>
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<td></td>
<td>37.21</td>
<td>13.94</td>
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<td>39.71</td>
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<td>42.21</td>
<td>15.81</td>
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<tr>
<td>19</td>
<td></td>
<td>44.46</td>
<td>16.65</td>
</tr>
<tr>
<td>20</td>
<td>6</td>
<td>47.21</td>
<td>17.68</td>
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<tr>
<td>21</td>
<td></td>
<td>49.71</td>
<td>18.62</td>
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<td>22</td>
<td></td>
<td>51.69</td>
<td>19.36</td>
</tr>
<tr>
<td>23</td>
<td></td>
<td>57.69</td>
<td>21.60</td>
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</tr>
<tr>
<td>25</td>
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<td>69.69</td>
<td>26.09</td>
</tr>
</tbody>
</table>

#### Measured Mixing Tube Dimensions

<table>
<thead>
<tr>
<th>$x$ (in)</th>
<th>Dia. (in)</th>
<th>$x/R_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.341</td>
<td>0</td>
</tr>
<tr>
<td>19.578</td>
<td>3.643</td>
<td>7.34</td>
</tr>
<tr>
<td>28.578</td>
<td>3.645</td>
<td>10.7</td>
</tr>
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<td>45.224</td>
<td>5.355</td>
<td>16.9</td>
</tr>
<tr>
<td>50.578</td>
<td>5.356</td>
<td>18.95</td>
</tr>
<tr>
<td>63.578</td>
<td>6.956</td>
<td>23.8</td>
</tr>
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</table>
# TABLE 3
STATIC PRESSURE VALUES MEASURED ALONG THE MIXING TUBE

<table>
<thead>
<tr>
<th>Entrainment Ratio</th>
<th>17.0</th>
<th>19.4</th>
<th>21.0</th>
<th>23.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Flow Rate, lbm/min</td>
<td>6.76</td>
<td>6.76</td>
<td>6.76</td>
<td>6.76</td>
</tr>
<tr>
<td>Secondary Flow Rate, lbm/min</td>
<td>115.1</td>
<td>131.4</td>
<td>142.0</td>
<td>160.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Static Pressure Tap No.</th>
<th>x/Ro station</th>
<th>all values in inches of water gage with respect to $P_{oo}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.172</td>
<td>- 7.66</td>
</tr>
<tr>
<td>2</td>
<td>0.828</td>
<td>- 8.25</td>
</tr>
<tr>
<td>3</td>
<td>1.76</td>
<td>- 8.85</td>
</tr>
<tr>
<td>4</td>
<td>2.51</td>
<td>- 9.15</td>
</tr>
<tr>
<td>5</td>
<td>3.63</td>
<td>-10.9</td>
</tr>
<tr>
<td>6</td>
<td>4.57</td>
<td>-12.1</td>
</tr>
<tr>
<td>7</td>
<td>5.51</td>
<td>-14.45</td>
</tr>
<tr>
<td>8</td>
<td>6.45</td>
<td>-17.7</td>
</tr>
<tr>
<td>9</td>
<td>7.66</td>
<td>-23.0</td>
</tr>
<tr>
<td>10</td>
<td>8.32</td>
<td>-22.7</td>
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<tr>
<td>11</td>
<td>9.25</td>
<td>-22.1</td>
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<td>15</td>
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<td>-13.55</td>
</tr>
<tr>
<td>16</td>
<td>13.94</td>
<td>- 8.85</td>
</tr>
<tr>
<td>17</td>
<td>14.87</td>
<td>1.4</td>
</tr>
<tr>
<td>18</td>
<td>15.81</td>
<td>3.5</td>
</tr>
<tr>
<td>19</td>
<td>16.65</td>
<td>5.0</td>
</tr>
<tr>
<td>20</td>
<td>17.68</td>
<td>5.7</td>
</tr>
<tr>
<td>21</td>
<td>18.62</td>
<td>6.0</td>
</tr>
<tr>
<td>22</td>
<td>19.36</td>
<td>6.6</td>
</tr>
<tr>
<td>23</td>
<td>21.60</td>
<td>9.0</td>
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<tr>
<td>24</td>
<td>23.85</td>
<td>10.3</td>
</tr>
<tr>
<td>25</td>
<td>26.09</td>
<td>11.1</td>
</tr>
</tbody>
</table>
### TABLE 4

**VELOCITY AND TEMPERATURE PROFILES AT TRAVERSE STATION 1**

<table>
<thead>
<tr>
<th>Traverse Probe Position</th>
<th>From Traverse Probe Data</th>
<th>Analytical Predictions for $X_{core} = 2.5R_o$</th>
<th>Analytical Predictions for $X_{core} = 2.0R_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y/R$</td>
<td>$p/P$</td>
<td>Mach number</td>
</tr>
<tr>
<td>0.949</td>
<td>0.967</td>
<td>0.218</td>
<td>0.991</td>
</tr>
<tr>
<td>0.837</td>
<td>0.966</td>
<td>0.223</td>
<td>0.990</td>
</tr>
<tr>
<td>0.707</td>
<td>0.965</td>
<td>0.226</td>
<td>0.990</td>
</tr>
<tr>
<td>0.548</td>
<td>0.964</td>
<td>0.229</td>
<td>0.990</td>
</tr>
<tr>
<td>0.447</td>
<td>0.964</td>
<td>0.230</td>
<td>0.990</td>
</tr>
<tr>
<td>0.316</td>
<td>0.949</td>
<td>0.273</td>
<td>0.985</td>
</tr>
<tr>
<td>0.224</td>
<td>0.830</td>
<td>0.522</td>
<td>0.948</td>
</tr>
<tr>
<td>0.100</td>
<td>0.544</td>
<td>0.975</td>
<td>0.840</td>
</tr>
<tr>
<td>0</td>
<td>0.311</td>
<td>1.45</td>
<td>0.705</td>
</tr>
<tr>
<td>0.100</td>
<td>0.524</td>
<td>1.01</td>
<td>0.831</td>
</tr>
<tr>
<td>0.224</td>
<td>0.897</td>
<td>0.398</td>
<td>0.969</td>
</tr>
<tr>
<td>0.316</td>
<td>0.962</td>
<td>0.237</td>
<td>0.989</td>
</tr>
<tr>
<td>0.447</td>
<td>0.962</td>
<td>0.236</td>
<td>0.989</td>
</tr>
<tr>
<td>0.548</td>
<td>0.963</td>
<td>0.232</td>
<td>0.989</td>
</tr>
<tr>
<td>0.707</td>
<td>0.964</td>
<td>0.229</td>
<td>0.990</td>
</tr>
<tr>
<td>0.837</td>
<td>0.964</td>
<td>0.229</td>
<td>0.990</td>
</tr>
<tr>
<td>0.949</td>
<td>0.966</td>
<td>0.222</td>
<td>0.990</td>
</tr>
</tbody>
</table>
Figure 1  Jet Mixing in a Converging-Diverging Duct
Figure 2. Velocity Profiles in the Transition Zone

Jet State \( \infty \)

Top-Hat Profile

Note: Value of \( U_0 \) varies with \( x \).

Nozzle Exit Plane

Isentropic Accomodation

\[ \delta'' = \theta \cdot H_0 \]

\[ x_{core} \]
Figure 4
Mixing Tube Geometry
Figure 6  Jet Pump Test Instrumentation
Figure 8 Measured Velocity Profiles in Mixing Tube
<table>
<thead>
<tr>
<th>Traverse Location</th>
<th>$x/r_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ 2</td>
<td>4.57</td>
</tr>
<tr>
<td>□ 3</td>
<td>6.45</td>
</tr>
<tr>
<td>○ 4</td>
<td>9.25 $90^\circ$</td>
</tr>
<tr>
<td>⊙ 5</td>
<td>9.25 $\text{A}p\text{a}r\text{t}$</td>
</tr>
<tr>
<td>▽ 6</td>
<td>17.68</td>
</tr>
</tbody>
</table>

![Graph showing measured temperature profiles in mixing tube](image)

Figure 9  Measured Temperature Profiles in Mixing Tube
Figure 10. Variation of Centerline Temperature and Velocity along the Mixing Tube (Test Results)
Figure 11  Comparison of Analytical and Experimental Mixing Tube Static Pressure Variations ($\frac{x_{core}}{R_o} = 2.5$, entrainment reduced by 2%)
Figure 12  Comparison of Analytical and Experimental Mixing Tube Static Pressure Variations ($x_{\text{core}} = 2.5$, secondary flow from bellmouth calibration)
Figure 13  Comparison of Analytical and Experimental Mixing Tube Static Pressure Variations ($x_{core} = 2.0$, entrainment reduced by 2%)
Figure 14  Comparison of Analytical and Experimental Values of Centerline Temperature and Velocity Along the Mixing Tube
Figure 15  Comparison of Experimental and Analytical
Velocity Profiles
Figure 16  Comparison of Experimental and Analytical Temperature Profiles
Figure 17. Comparison of "Measured" and Predicted Mach Number Profiles at Traverse Station 1, $x = 2.51R_o$